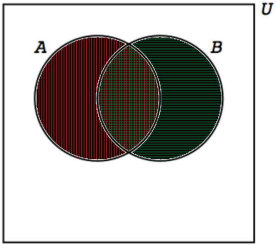
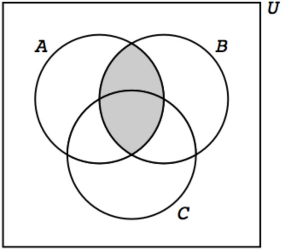
Venn Diagrams

We can visual subsets of a universal set, and how they interact/overlap, using *Venn diagrams*, as shown below.



$A \cap B$
$(A' \cup B')'$

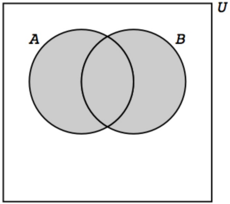


$A \cap B$
$(A' \cup B')'$

On the left, the brown shaded region is $A \cap B$. It is also $(A' \cup B')'$. On the right, the shaded area is $A \cap B$.

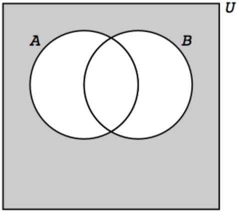
Venn Diagrams

Some more examples:



$A \bigcup B$

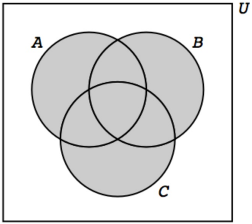
$(A' \bigcap B')'$



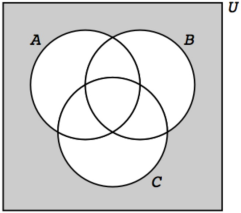
$A' \bigcap B'$

$(A \bigcup B)'$

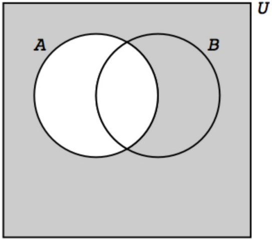
Venn Diagrams



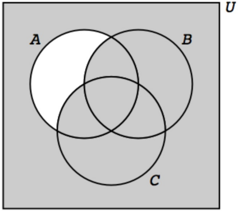
$A \cup B \cup C$
$(A' \cap B' \cap C')'$



$A' \cap B' \cap C'$
$(A \cup B \cup C)'$



A'



$A' \cup B \cup C$
$(A \cap B' \cap C')'$

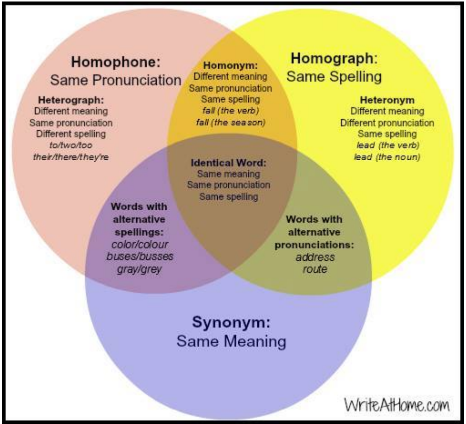
Compute the elements of various subsets

Example If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$ are subsets of the universal set $U = \{1, 2, 3, \ldots, 10\}$, list the elements of the set $A' \cup (B \cap C)$.

$A' = \{5, 6, 7, 8, 9, 10\}$, $B \cap C = \{4, 6\}$ so $A' \cup (B \cap C) = \{4, 5, 6, 7, 8, 9, 10\}$.

Venn diagrams for presentations

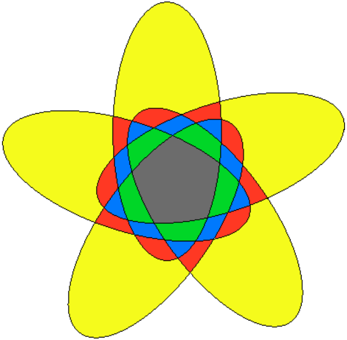
Venn diagrams using two or three sets are often used in presentations.

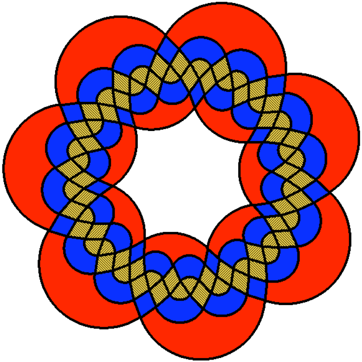


Venn Diagram of Word Relationships

Venn diagrams for presentations

Venn diagrams of more sets are possible, but tend to be confusing as a presentation tool because of the number of possible interactions. The following diagrams show Venn diagrams for five sets on the left and for 7 sets on the right.





The Inclusion-Exclusion Principle

For any finite set, S, we let $n(S)$ denote the number of objects in S.

Example If $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{5, 6, 7, 8, 9, 10\}$ then $n(A) = 7$ and $n(B) = 6$

The Inclusion Exclusion Principle

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}.$$

Example Check that this works for A and B from the example above.

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $n(A \cup B) = 10$.
$A \cap B = \{5, 6, 7\}$, $n(A \cap B) = 3$.

$$10 = 7 + 6 - 3$$

The Inclusion-Exclusion Principle

Note that if two sets A and B do not intersect, then $n(A \cap B) = 0$ and hence $n(A \cup B) = n(A) + n(B)$.

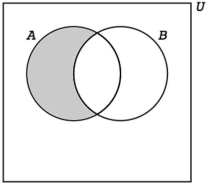
Formula 1 Now apply this to a set and its complement to get

$$\boxed{n(A') = n(A^c) = n(U) - n(A)}$$

where U is the universal set.

Formula 2 The shaded region below is $A \cap B^c$ and $(A \cap B^c) \cap (A \cap B) = \emptyset$ so

$$\boxed{n(A \cap B^c) = n(A) - n(A \cap B)}$$

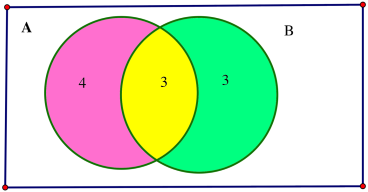


Venn diagrams and the Inclusion Exclusion Principle

We can sometimes use the inclusion-exclusion principle either as an algebraic or a geometric tool to solve a problem. We can use a Venn diagram to show the number of elements in each basic region to display how the numbers in each set are distributed among its parts.

Venn diagrams and the Inclusion Exclusion Principle

With $A = \{1,2,3,4,5,6,7\}$, $B = \{5,6,7,8,9,10\}$ as above we saw that $n(A \cap B) = 3$, hence the 3 in the region of intersection, the yellow bit.



- Since $n(A) = 7$, formula 2 says that for the magenta region, $n(A \cap B^c) = 7 - 3$.
- Similarly, since $n(B) = 6$, formula 2 says that for the green region $n(A^c \cap B) = 6 - 3$.
- Note $10 = n(A \cup B) = 4 + 3 + 3$.

More Inclusion-Exclusion

In general, the Inclusion-Exclusion Principle is an equation relating four numbers. Hence if you know three of them, you can work out the fourth.

Example Let A and B be sets, such that $n(A) = 10$ and $n(B) = 12$ and $n(A \cup B) = 15$, then how many elements are in the set $A \cap B$?

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ so
$15 = 10 + 12 - n(A \cap B)$ OR $n(A \cap B) = 7$.

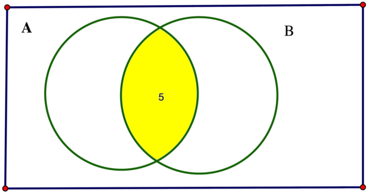
More Inclusion-Exclusion

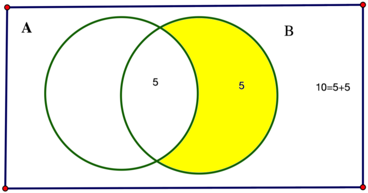
Example Let A and B be sets, such that $n(A \cup B) = 20$, $n(B) = 10$ and $n(A \cap B) = 5$, then how many elements are in the set A? (Solve this using both methods: algebra and a Venn diagram)

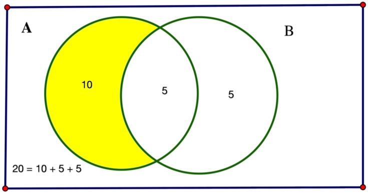
Algebra: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ so $20 = n(A) + 10 - 5$ OR $n(A) = 15$.

More Inclusion-Exclusion

Geometric:







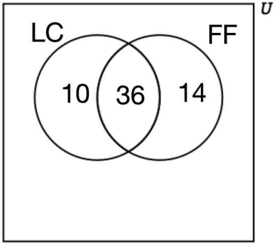
More Inclusion-Exclusion

Example A survey of a group of students, revealed that 60 of them liked at least one of the cereals, Frosted Flakes or Lucky Charms. If 50 of them liked Frosted Flakes and 46 of them liked Lucky Charms,

(a) How many of them liked both cereals?

$n(FF) = 50$; $n(LC) = 46$; $n(FF \cup LC) = 60$ so
$n(FF \cap LC) = 50 + 46 - 60 = 36.$

(b) Draw a Venn diagram showing the results of the survey.



(c) How many students liked Frosted Flakes but did not like Lucky Charms?

More Inclusion-Exclusion

Example A survey of 70 students revealed that 64 of them liked to learn visually. How many of them did not like to learn visually?

By formula 1, the answer is $70 - 64 = 6$.

Example 68 students were interviewed about their music preferences. 66 of them liked at least one of the music types, Rap, Classical and Eighties. How many didn't like any of the above music types?

$n(R \cup C \cup E) = 66$ and $n(U) = 68$, so by formula 1, then number who didn't like any of the above music types is

$$n\big((R \cup C \cup E)^c\big) = 68 - 66 = 2.$$

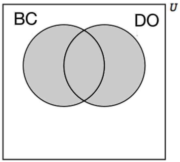
More Inclusion-Exclusion

Example In a survey of 70 students on Movie preferences, the students were asked whether they liked the movies "The Breakfast Club" and "Ferris Bueller's Day Off". (All students had seen both movies and the only options for answers were like/dislike.) 50 of the students said they liked "The Breakfast Club" and 25 of them said they didn't like "Ferris Bueller's Day Off". All students liked at least one of the movies.

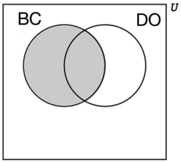
(a) How many students said they liked both movies?

(b) Display the survey results on a Venn diagram.

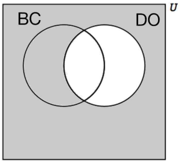
More Inclusion-Exclusion



70 Figure 1



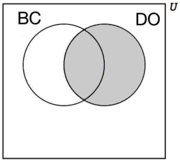
50 Figure 2



25 Figure 3

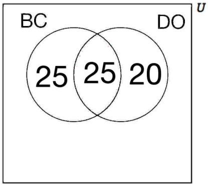
$n(U) = 70$.

From Figure 3 we see $45 = n(U) - n(DO^c)$ students did like "Ferris Bueller's Day Off".



45

Fill in the Venn diagram.

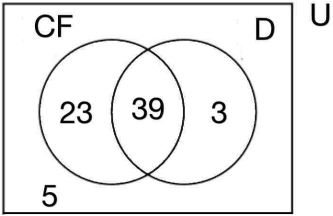


For part (a), 25.

More Inclusion-Exclusion

Example In a survey of a group of 70 movie-goers, 62 liked the movie "Catching Fire", 42 liked the movie "Divergent" and 39 liked both movies.

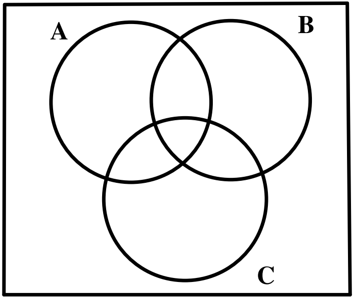
(a) Represent this information on a Venn Diagram.



(b) Use the Venn diagram to find how many of those surveyed did not like either movie. 5.

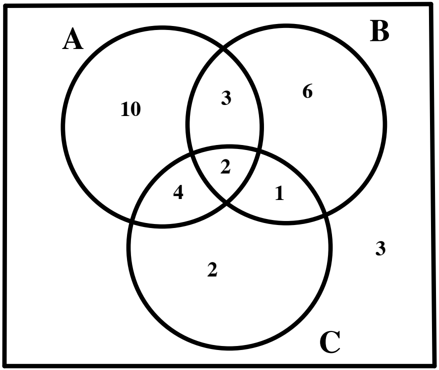
Venn Diagrams of 3 sets

A Venn diagram of 3 sets divides the universal set into 8 non-overlapping regions. We can sometimes use partial information about numbers in some of the regions to derive information about numbers in other regions or other sets.



Venn Diagrams of 3 sets

Example The following Venn diagram shows the number of elements in each region for the sets A, B and C which are subsets of the universal set U.



Find the number of elements in each of the following sets:

(a) $A \cap B \cap C$ 2

(b) B' $3+2+4+10=19$

(c) $A \cap B$ $3+2=5$

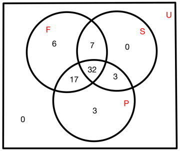
(d) C $2+4+2+1=9$

(e) $B \cup C$ $9+3+6=18$

Venn Diagrams of 3 sets

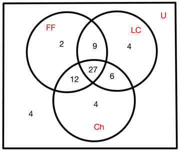
Example In a survey of a group of 68 Finite Math students, 62 liked the movie "The Fault in our Stars", 42 liked the movie "The Spectacular Now" and 55 liked the movie "The Perks of Being a Wallflower". 32 of them liked all 3 movies, 39 of them liked both "The Fault in Our Stars" and "The Spectacular Now", 35 of them liked both "The Spectacular Now" and "The Perks of Being a Wallflower" and 49 of them liked both "The Fault in Our Stars" and "The Perks of Being a Wallflower". Represent this information on a Venn Diagram. $n(U) = 68;$

$n(F) = 62; n(S) = 42; n(P) = 55.\ n(F \cap S) = 39;$
$n(P \cap S) = 35; n(F \cap P) = 49.\ n(F \cap S \cap P) = 32.$



Venn Diagrams of 3 sets

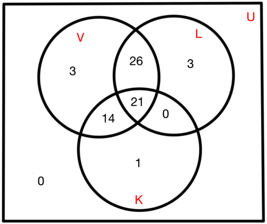
Example In a survey of a group of 68 Finite Math students (Spring 2006), 50 said they liked Frosted Flakes, 49 said they liked Cheerios and 46 said they liked Lucky Charms. 27 said they liked all three, 39 said they liked Frosted Flakes and Cheerios, 33 said they liked Cheerios and Lucky Charms and 36 said they liked Frosted Flakes and Lucky Charms. Represent this information on a Venn Diagram. How many didn't like any of the cereals mentioned? $n(U) = 68$; $n(FF) = 50$; $n(Ch) = 49$; $n(LC) = 46$. $n(FF \cap Ch) = 39$; $n(LC \cap Ch) = 33$; $n(FF \cap LC) = 36$. $n(FF \cap Ch \cap LC) = 27$.



Venn Diagrams of 3 sets

Example The results of a survey of 68 Finite Math students(Spring 2006) on learning preferences were as follows: 64 liked to learn visually, 50 liked learning through listening and 36 liked learning Kinesthetically. 21 liked using all three channels, 47 liked to learn visually and through listening, 35 liked to learn both visually and kinesthetically, 21 liked to learn through listening and kinesthetically. How many preferred only visual learning?

$n(U) = 68$; $n(V) = 64$; $n(L) = 50$; $n(K) = 36$;
$n(V \cap L) = 47$; $n(V \cap K) = 35$; $n(L \cap K) = 21$.
$n(V \cap L \cap K) = 21$.

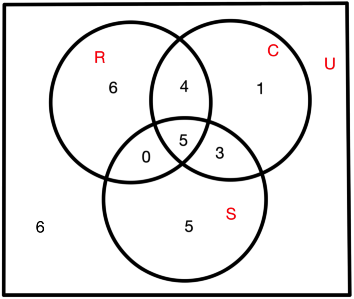


Old Exam questions for Review

1 In a group of 30 people, 15 run, 13 swim, 13 cycle, 5 run and swim, 8 cycle and swim, 9 run and cycle, and 5 do all three activities. How many of the 30 people neither run nor cycle?

(a) 8 (b) 10 (c) 9 (d) 12 (e) 11

$n(U) = 30$; $n(R) = 15$; $n(S) = 13$; $n(C) = 13$.
$n(R \cap S) = 5$; $n(C \cap S) = 8$; $n(R \cap C) = 9$.
$R \cap C \cap S = 5$.



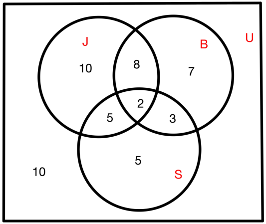
Answer is $6 + 5 = 11$ or (e)

Old Exam questions for Review

2 Out of 50 students who exercise regularly, 25 jog, 20 play basketball and 15 swim. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 2 people do all three. How many students do not do any of these activities?

(a) 10 (b) 15 (c) 4 (d) 0 (e) 2

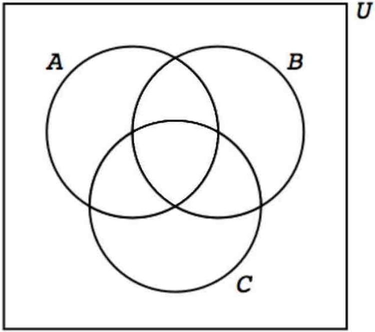
$n(U) = 50$; $n(J) = 25$; $n(B) = 20$; $n(S) = 15$.
$n(B \cap J) = 10$; $n(B \cap S) = 5$; $n(J \cap S) = 7$;
$n(B \cap J \cap S) = 2$;



Answer is 10 or (a)

Worked homework problem

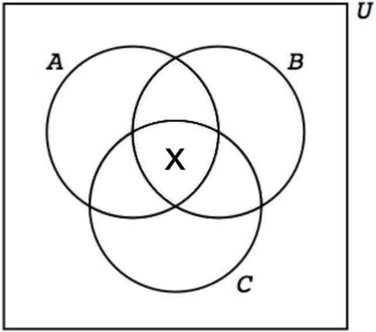
Here is an example of a type of problem from the homework. Given 3 subsets A, B and C of a universal set U, suppose $n(U) = 68$; $n(A \cup B \cup C) = 64$; $n(A) = 50$; $n(B) = 49$; $n(C) = 46$. $n(A \cap B) = 39$; $n(C \cap B) = 33$; $n(A \cap C) = 36$. Fill in the Venn diagram.



We do not know $n(A \cap B \cap C)$ or this would just be another example of earlier problems.

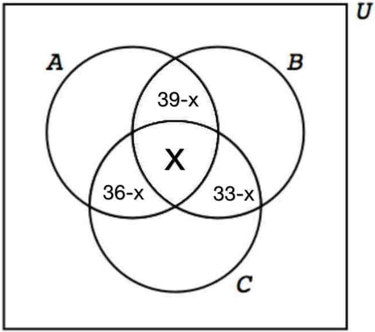
Worked homework problem

Denote $n\,(A \cap B \cap C)$ by x.



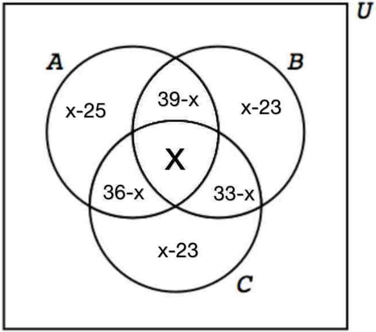
Worked homework problem

Now work out the double intersections: $n(A \cap B) = 39$; $n(C \cap B) = 33$; $n(A \cap C) = 36$.



Worked homework problem

Now work out the sets: $n(A) = 50$; $n(B) = 49$; $n(C) = 46$.



For example, if y denotes the part of A outside of $B \cup C$,

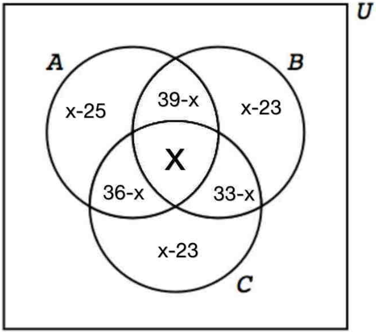
$$50 = y + (39 - x) + (36 - x) + x = y + 75 - x$$

so

$$y = x - 25$$

The others are similar.

Worked homework problem



Since

$$64 = n(A \cup B \cup C) = x + (39 - x) + (36 - x) + (33 - x) + (x - 25) + (x - 23) + (x - 23) = x + (108 - 71) = x + 37$$

Hence $n((A \cup B \cup C)^c) = 68 - 64 = 4.$

Worked homework problem