ME 323 – Mechanics of Materials
Examination #2
November 5th, 2020

Purdue University | Mechanical Engineering

Name (Print) ______________________________
(Last) (First)

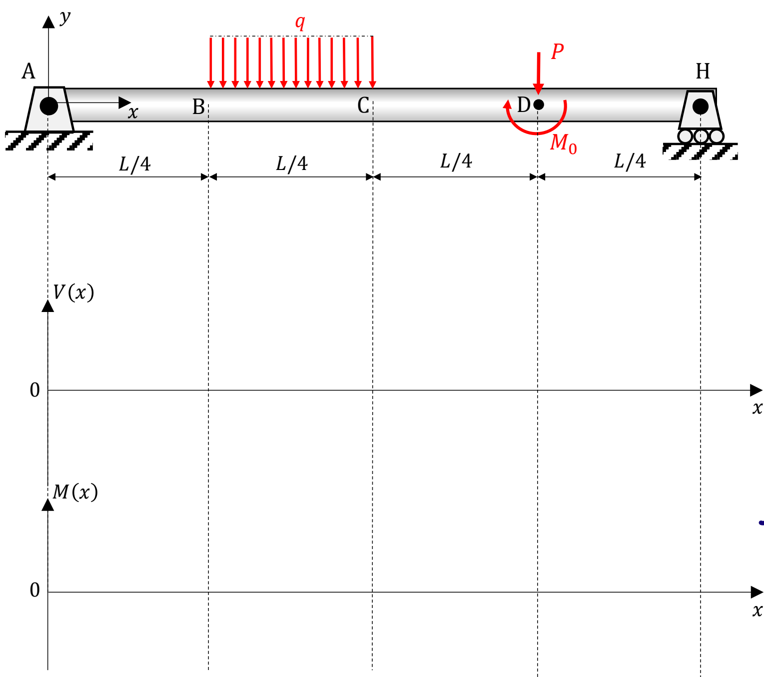
PROBLEM # 1 (25 points)

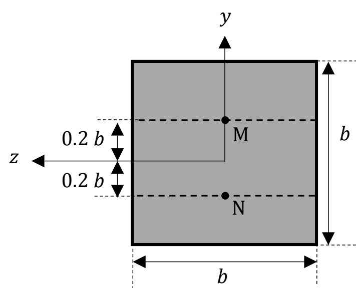
A simply supported beam AH is subject to a constant distributed load q over the section BC, a moment M_0 and a concentrated force P at D. The cross section of the beam is shown below.

The parameters are following: L=8 ft., q=10 lb/ft, M_0=40 lb×ft, P=10 lb, b= 2 in.

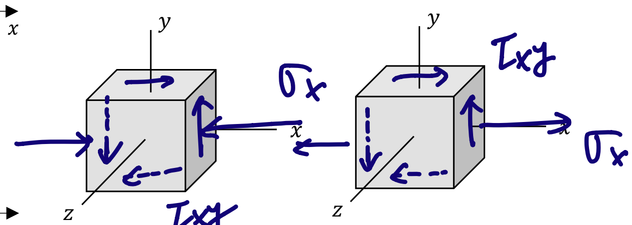
a) Draw the shear force and bending moment diagrams. Mark the values at the cross sections A, B, C, D, and H, and the maximum and minimum values along the beam.

b) Determine the stress state at the points M and N which are located at the cross section C. Sketch their stress state on the given stress elements.





Cross section



Stress element M

Stress element N

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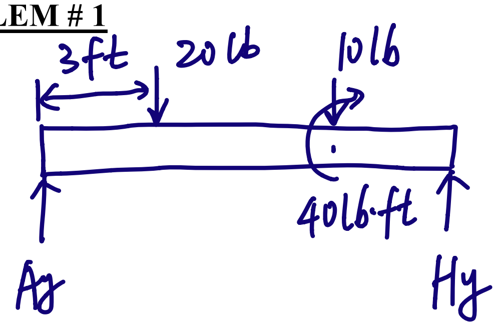
PROBLEM # 1

FBD:



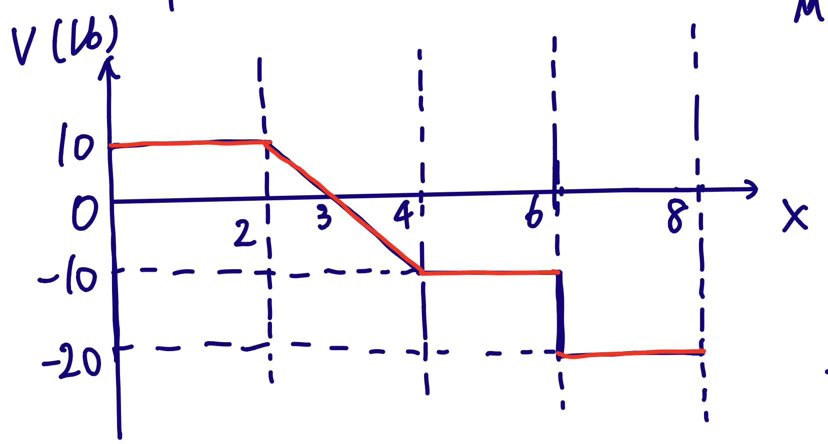
Equilibrium:

$$\Sigma F_y = A_y + H_y - 20 - 10 = 0$$

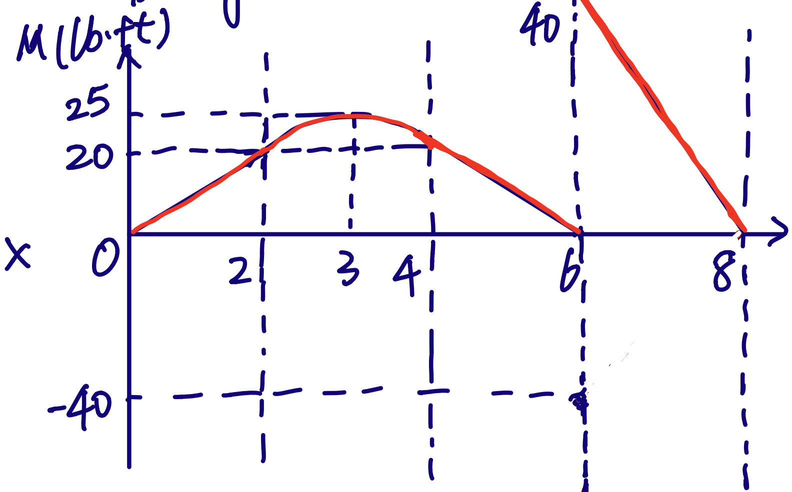
$$\Sigma M_A = -H_y \cdot 8 + 20 \times 3 + 10 \times 6 + 40 = 0$$

$H_y = 20$ lb, $A_y = 10$ lb

Shear force diagram:



Bending moment diagram:



stress states:

At M:

$$\sigma_x = -\frac{My}{I} = -\frac{20 \cdot 12\ (\text{lb} \cdot \text{in}) \cdot 0.4\ \text{in}}{\frac{1}{12} \cdot 2^4\ \text{in}^4} = -72\ \text{lb/in}^2$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{10 \times A^* \cdot \bar{y}^*}{\frac{4}{3} \times 2} = \frac{10 \times 2 \times 0.6 \times 0.7}{\frac{4}{3} \times 2} = 3.15\ \text{lb/in}^2$$

At N:

$$\sigma_x = -\frac{My}{I} = -\frac{20 \cdot 12\ (\text{lb} \cdot \text{in}) \cdot (-0.4)\ \text{in}}{\frac{4}{3}\ \text{in}^4} = 72\ \text{lb/in}^2$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{10 \times A^* \cdot \bar{y}^*}{It} = \frac{10 \times 2 \times 1.4 \times 0.3}{\frac{4}{3} \times 2} = 3.15\ \text{lb/in}^2$$

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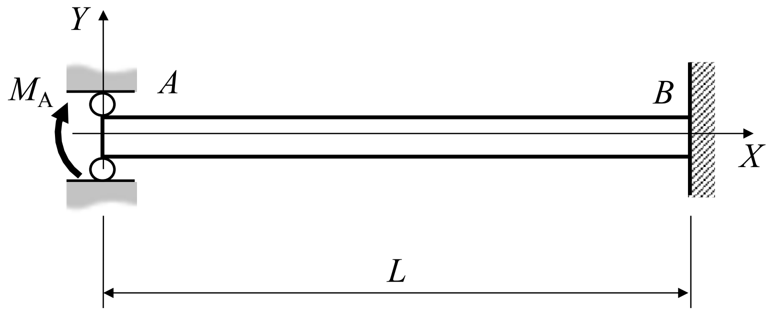
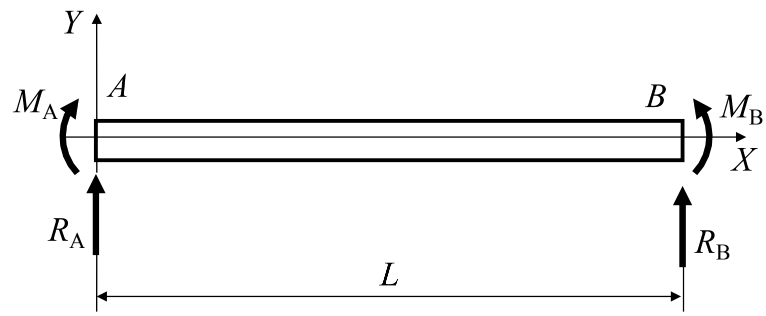
PROBLEM # 2 (25 points).



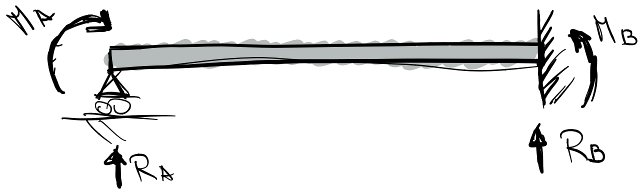
The linearly elastic beam shown in the figure supports a couple M_A at end A. The beam is homogeneous, with Young's modulus E, and has constant cross-section, with moment of inertia I.

(a) Using the following free body diagram, write the equations of equilibrium and identify whether the structure is statically determinate or indeterminate.



Using the second-order integration method:

(b) Determine the bending moment $M(x)$ of the beam (as a function of the reactions at A, the external loads and the geometric parameters).
(c) Determine the slope $v'(x)$ and deflection $v(x)$ of the beam.
(d) Indicate the boundary conditions at supports A and B.
(e) Solve for the reaction at A, i.e., R_A.

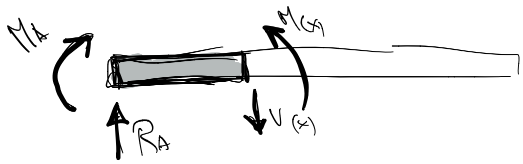


$\uparrow\oplus \quad \sum F_y = 0 = R_A + R_B$

$\circlearrowleft\oplus \quad \left(\sum M\right)_B = 0 = M_A + R_A \cdot L - M_B$

2 equations & 3 unknowns (R_A, R_B, M_B)

$\Rightarrow$ Statically Indeterminate



$\Rightarrow M(x) = M_A + R_A x$

Second order integration method

$$EI\,v'' = M_A + R_A x$$

$$EI\,v' = M_A x + \frac{1}{2} R_A x^2 + C_1$$

$$EI\,v = M_A \frac{x^2}{2} + R_A \frac{x^3}{6} + C_1 x + C_2$$

Boundary Conditions

$$\boxed{\begin{array}{l} v(0) = 0 \\ v(L) = 0 \\ v'(L) = 0 \end{array}}$$

$\Rightarrow C_2 = 0$

$\Rightarrow M_A \frac{L^2}{2} + R_A \frac{L^3}{6} + C_1 L = 0$

$\Rightarrow M_A L + \frac{1}{2} R_A L^2 + C_1 = 0$

Solve for R_A

$$\begin{cases} M_A \frac{L}{2} + R_A \frac{L^2}{6} + C_1 = 0 \\ M_A L + \frac{1}{2} R_A L^2 + C_1 = 0 \end{cases}$$

$\boxed{\text{2x2 system of equations}}$

$$M_A \frac{L}{2} + \frac{2}{6} R_A L^2 = 0 \Rightarrow \boxed{R_A = -\frac{3M_A}{2L}}$$

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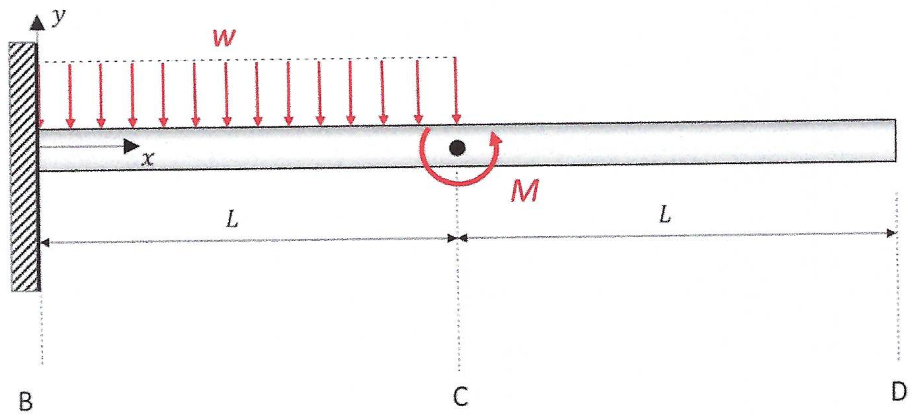
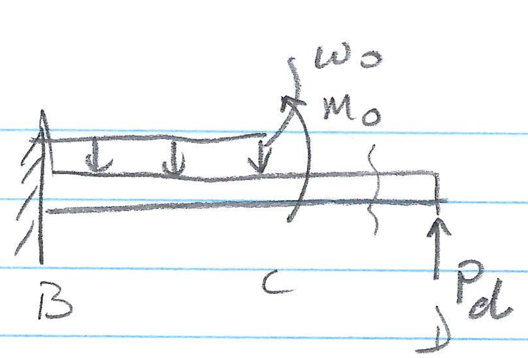
PROBLEM # 3 (25 points).

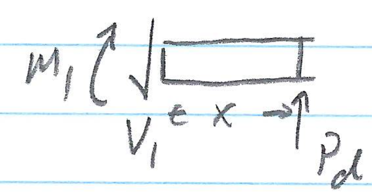
A cantilevered beam BCD is subjected to a distributed load w (in the unit of load/length) between BC and a concentrated moment M at point C. The structure is made of a material with elastic modulus E, second moment of area I and cross-sectional area A.

1) Assuming **the shear strain energy due to bending is negligible, use Castigliano's theorem** to determine the vertical (y-direction) deflection of point D.
2) Use the attached superposition tables to calculate the deflection of point D using **the superposition method.**





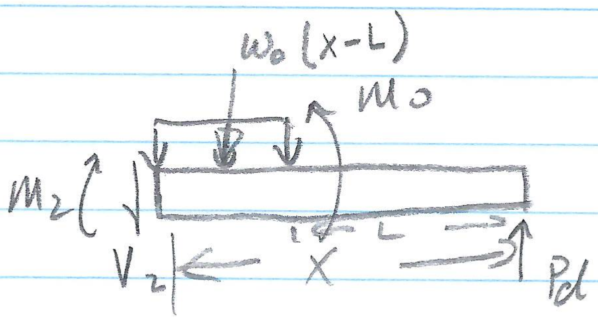
CD



$$-M_1 + P_d x = 0$$

$$\boxed{M_1 = P_d x}$$

BC



$$\Sigma M_K = -M_2 - \frac{w_0 (x-L)^2}{2} + M_0 + P_d x$$

$$\boxed{M_2 = M_0 - \frac{w_0}{2}(x-L)^2 + P_d x}$$

$$v_D = \frac{1}{EI}\int_0^L M_1 \underbrace{\frac{\partial M_1}{\partial P_d}}_{x} dx + \frac{1}{EI}\int_L^{2L} M_2 \underbrace{\frac{\partial M_2}{\partial P_d}}_{x} dx$$

$$= \frac{1}{EI}\int_0^L (\cancel{P_d}^{\,0} x) x \, dx + \frac{1}{EI}\int_L^{2L} \left(M_0 - \frac{w_0}{2}(x-L)^2 + \cancel{P_d}^{\,0} x\right) x \, dx$$

$$= \frac{1}{EI}\int_{L}^{2L} \left[M_o x - \frac{w_o}{2}\left(x^3 - 2Lx^2 + L^2 x\right)\right] dx$$

$$= \frac{1}{EI}\left\{\left[M_o \frac{x^2}{2}\right]_{L}^{2L}\right.$$

$$\left. - \frac{w_o}{2}\left[\frac{x^4}{4} - 2L\frac{x^3}{3} + L^2\frac{x^2}{2}\right]_{L}^{2L}\right\}$$

$$= \frac{1}{EI}\left\{\left[\frac{M_o}{2}\left(4L^2 - L^2\right)\right.\right.$$

$$- \frac{w_o}{2}\left[\underbrace{\frac{16L^4 - L^4}{4}}_{\frac{15L^4}{4}} - \underbrace{\frac{2L}{3}\left(8L^3 - L^3\right)}_{\frac{14}{3}L^4} + \underbrace{\frac{L^2}{2}\left(4L^2 - L^2\right)}_{\frac{3}{2}L^4}\right]$$

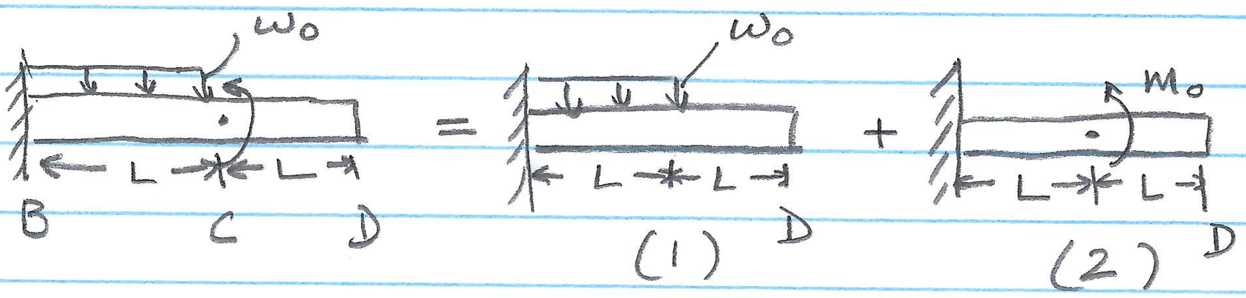
$$\frac{15L^4}{4} - \frac{14}{3}L^4 + \frac{3}{2}L^4$$

$$\left[45L^4 - 56L^4 + 18L^4\right]/12$$

$$63 - 56 = 7/12$$

$$= \frac{1}{EI}\left\{\frac{3M_oL^2}{2} - \frac{7w_oL^4}{24}\right\}$$

b) Superposition:



$$v_{D_1} = \frac{-w_0 L^3}{24EI}(8L - L) = -\frac{7w_0 L^4}{24EI}$$

$$v_{D_2} = \frac{M_0 L}{2EI}(4L - L) = \frac{3M_0 L^2}{2EI}$$

$$v_D = v_{D_1} + v_{D_2} = -\frac{7w_0 L^4}{24EI} + \frac{3M_0 L^2}{2EI}$$

PROBLEM #4 (25 Points)

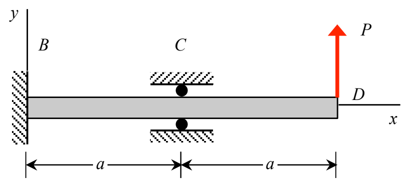
Part A – 5 points

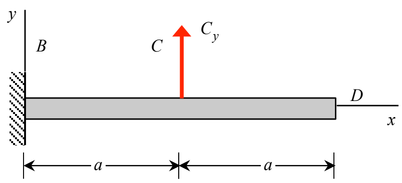
The beam shown below has a second area moment *I* for its cross-section, and is made of a material with a Young's modulus of *E*. Using the method of superposition:

a) Determine the reaction on the beam at location C. Leave your answer in terms of the load *P*.

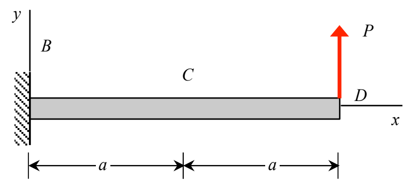
b) Determine the deflection of the beam at end D. Leave your answer in terms of *P*, *a* and *EI*.

Superposition tables are provided.





loading 1



loading 2

Using the displacement at C using the loadings shown above:

$$v(a) = v_1(a) + v_2(a)$$

$$0 = \frac{1}{6}\left\{(a)^2\left[3(a) - a\right]\right\}\frac{C_y}{EI} + \frac{1}{6}\left\{(a)^2\left[3(2a) - a\right]\right\}\frac{P}{EI}$$

$$= \left[\frac{2}{6}C_y + \frac{5}{6}P\right]\frac{a^3}{EI} \;\Rightarrow\; C_y = -\frac{5}{2}P$$

The displacement at D using the loadings shown above:

$$v(a) = v_1(a) + v_2(a)$$

$$0 = \frac{1}{6}\left\{(a)^2\left[3(2a) - a\right]\right\}\frac{C_y}{EI} + \frac{1}{6}\left\{(2a)^2\left[3(2a) - (2a)\right]\right\}\frac{P}{EI}$$

$$= \frac{5}{6}\frac{a^3}{EI}C_y + \frac{16}{6}\frac{a^3}{EI}P = \left[\frac{5}{6}\left(-\frac{5}{2}\right) + \frac{16}{6}\right]\frac{Pa^3}{EI} = \frac{7}{12}\frac{Pa^3}{EI}$$

PROBLEM #4 *(continued)*

***Part B* – 8 points**

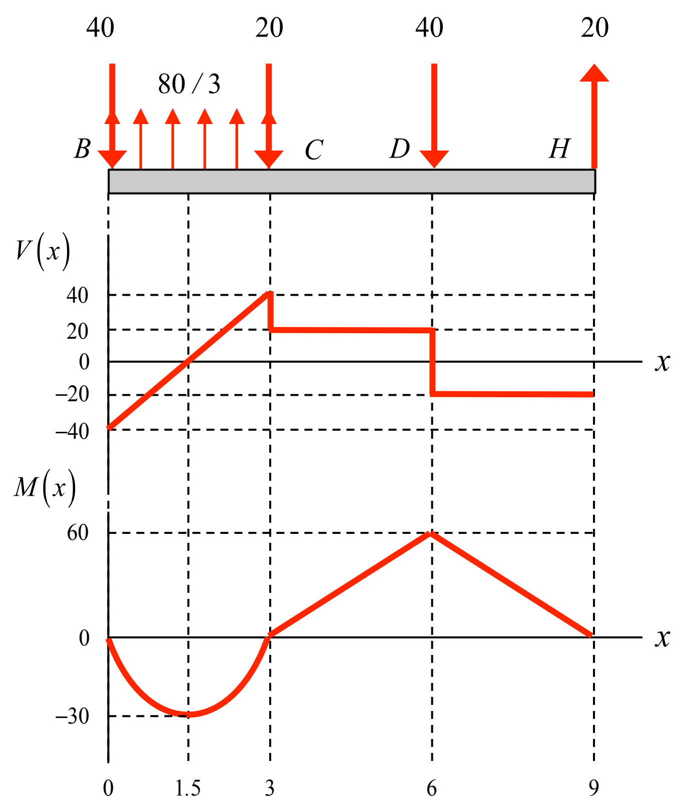
The loading on the beam shown below is not provided in the figure. The shear force $V(x)$ for the beam is given below the beam, with the shear force being provided in terms of *kips* and the position variable x in *ft*. In addition, it is known that the bending moment at the left end is $M(0) = 0$, and there are no concentrated couples applied to the beam at any locations except, possibly, at H.

For the shear force diagram provided:

a) Draw the bending diagram $M(x)$ on the axes provided.

b) Show the loading on the beam in the figure below.

No justification is needed for your answers.



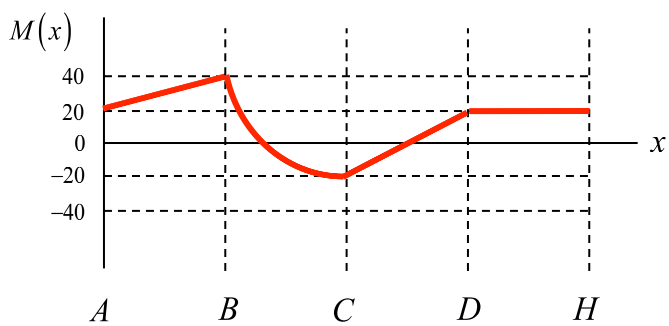
PROBLEM #4 *(continued)*

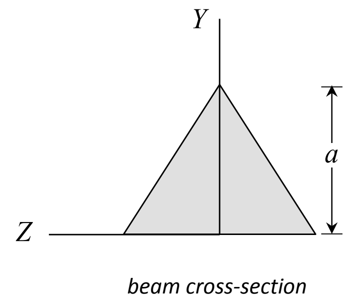
***Part C* – 6 points**

The bending moment diagram for a loaded beam is shown below. The beam is known to have the triangular cross section shown below. Provide a justification for each answer.

a) At what location(s) on the beam does the maximum *tensile* normal stress exist? Provide both x and Y components of the location of this point(s). You are *not* asked to solve for this value of stress.

b) At what location(s) on the beam does the maximum *compressive* normal stress exist? Provide both x and Y components of the location of this point(s). You are *not* asked to solve for this value of stress.





At location B along beam: $|M| = 40$

- At Y = 0: $|\sigma| = \left|\dfrac{M(-a/3)}{I}\right| = \dfrac{40}{3}\dfrac{a}{I}$ $(tensile)$
- At Y = a: $|\sigma| = \left|\dfrac{M(2a/3)}{I}\right| = \dfrac{80}{3}\dfrac{a}{I}$ $(compressive)$

At location C along beam: $|M| = 20$

- At Y = 0: $|\sigma| = \left|\dfrac{M(-a/3)}{I}\right| = \dfrac{20}{3}\dfrac{a}{I}$ $(compressive)$
- At Y = a: $|\sigma| = \left|\dfrac{M(2a/3)}{I}\right| = \dfrac{40}{3}\dfrac{a}{I}$ $(tensile)$

PROBLEM #4 ***(continued)***

Part D **– 6 points**

The cross-sections for Beams 1 and 2 are shown below. Let I_1 and I_2 represent the centroidal second area moment (about the z-axis) for beams 1 and 2, respectively. Each beam is experiencing the same shear force of V at the cross section. Let τ_{1B} and τ_{2B} be the shear stress at points B on Beams 1 and 2, respectively.

a) Circle the correct answer below in regard to the relative sizes of I_1 and I_2. You are *not* asked to provide numerical values for these second area moments, or justification for your answers.

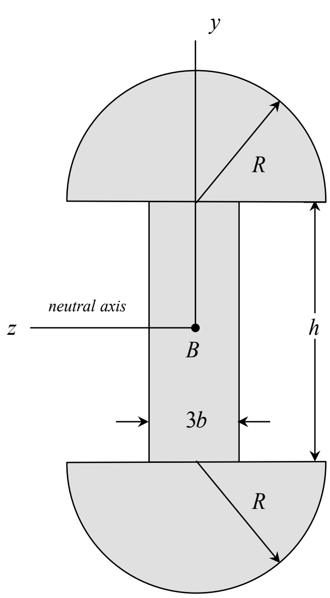
- $I_1 > I_2$
- $\boxed{I_1 = I_2}$
- $I_1 < I_2$

The vertical sections (the "webs") of the two cross-sections have the same second area moments. Likewise, the horizontal sections (the "flanges") have the same second area moments. Therefore, the second area moments are the same.

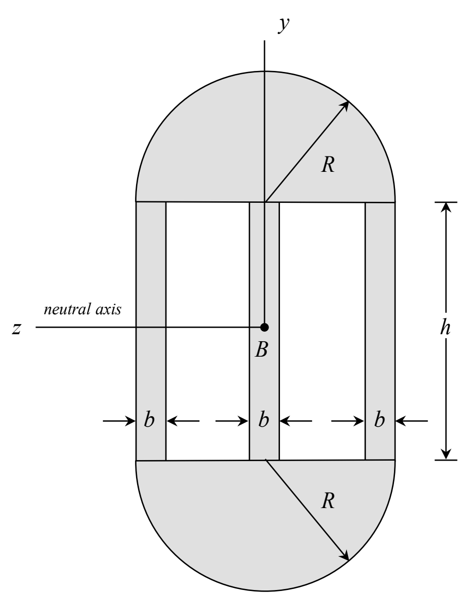
b) Circle the correct answer below in regard to the relative sizes of $|\tau_{1B}|$ and $|\tau_{2B}|$. You are *not* asked to provide numerical values for these stresses, or justification for your answers.

- $|\tau_{1B}| > |\tau_{2B}|$
- $\boxed{|\tau_{1B}| = |\tau_{2B}|}$
- $|\tau_{1B}| < |\tau_{2B}|$

The first area moments (Q) for the webs are the same, as well as for the flanges. The "thickness" of the cross sections are the same (2b). Therefore, the shear stresses are the same.



Beam 1



Beam 2