Summary Page week 1

charges + - opposite attract like repel

Cancelation/screening

$$F = \frac{k\, Q_1\, Q_2}{r^2}$$ **Coulombs Law**

vector sum of forces

Superposition Principle – solution (A+B)= solution (A)+ solution (B)

$$\vec{E} = \frac{\vec{F}}{q_T}$$ **Electric field** **q_T (+ test charge)**

E Field Lines

E Field behavior in/at-surface-of metals

$$\sum_{surf} E_{\perp}\, \Delta A = \frac{q_{inside}}{\varepsilon_o}$$ **Gausses' Law** $$k = \frac{1}{4\pi\, \varepsilon_o}$$

Electrostatics (stationary charges)

Back drop Newton's famous work on gravity

- attractive force at a distance - $F = G(m_1 m_2)/R^2$

electrostatic forces

- sometimes attractive ! sometimes repulsive !

du Fay 1733 ___ 2 kinds of electricity

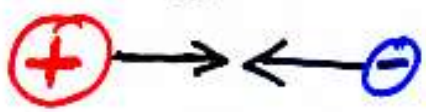
*rubbed amber −

*rubbed glass +

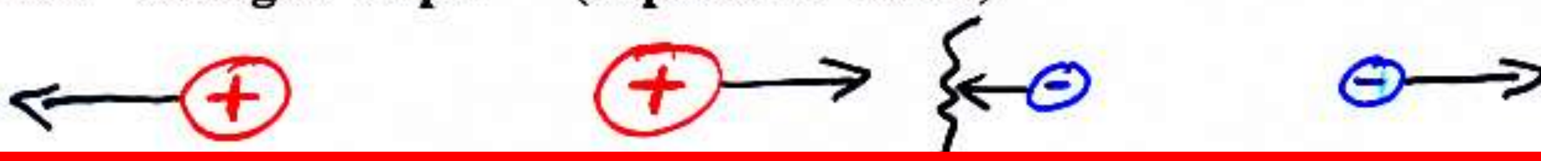
Ben. Franklin assigned opposite signs to 2 types of electricity !!

- + & - charges leads to attractive & repulsive forces

unlike charges attract (attractive force)



like charges repel (repulsive force)



- + & - charges leads to notion of "cancellation"
  {!!! charge does not disappear !! }
  + − charge close together "screen" or hide each others presence

- quantity of charge Q [measured in Coulomb = C]

Millikan Oil drop experiment (1906)
added small charges to oil drop & found smallest units of Q

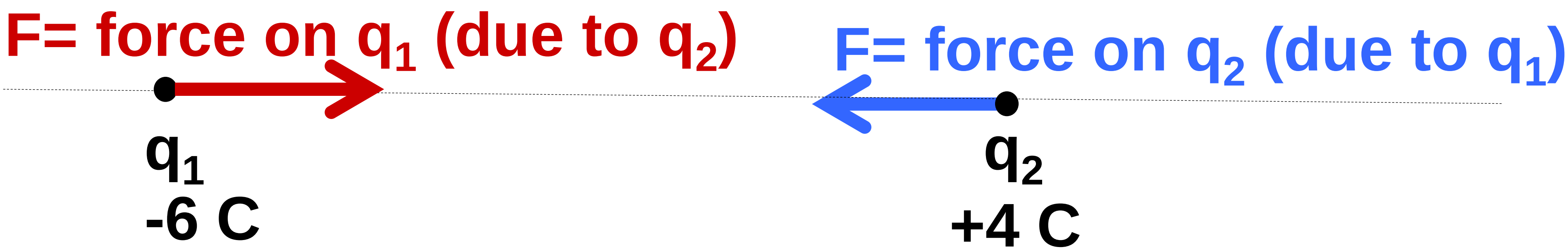
$$Q(\text{electron}, \quad e^-) = -1.6\ (10)^{-19}\ C$$

$$Q(\text{proton}, \quad p^+) = +1.6\ (10)^{-19}\ C$$

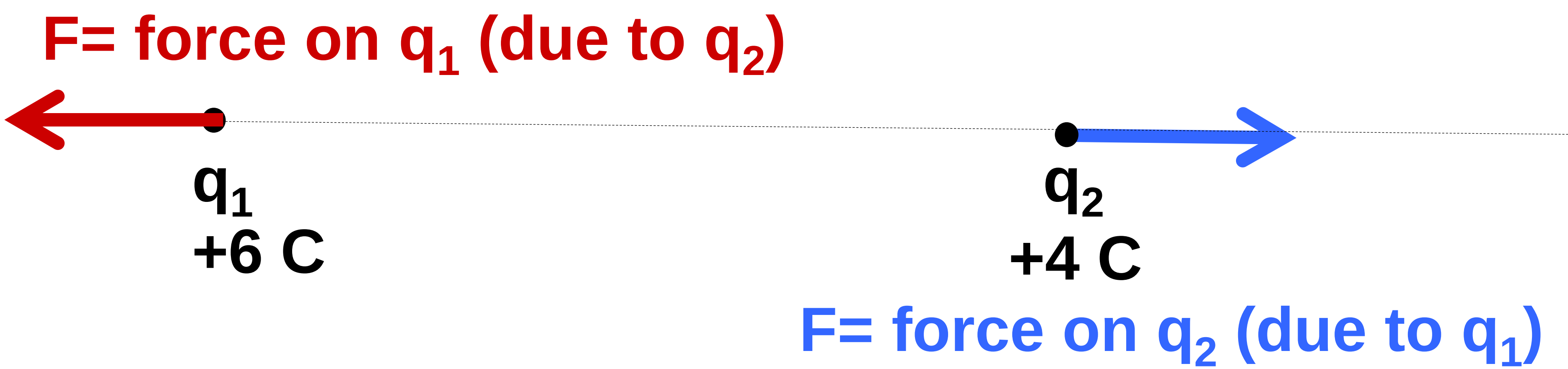
Actually 1C is huge

usually use $\underbrace{(10)^{-6} C = 1 \mu C}_{\text{micro coulomb}}$

Examples



F= F by Newton's 3'rd Law (and later, Coulomb's Law)



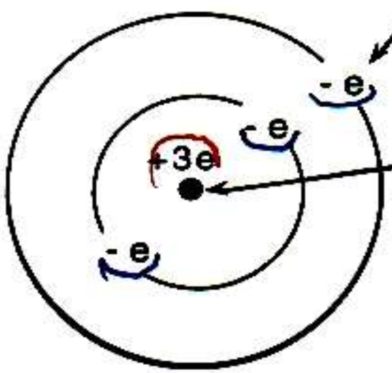
Note: forces along straight line between charges

- **All matter contains + & - charges : organized in Atoms**

Atoms

Example Li

electrons (light) e-
- charge
in orbits



Nucleus (very small)
3 protons (heavy) p+
+ charge

e = charge on electron & proton
= smallest unit of charge
= 1.6 (10)^-19 Coulombs (C)

Where does charge come from?

- **Net charge on an object ("cancellation")**

$\boxed{Q_{net} = Q_{e-} + Q_{p+}}$

neutral object	+ charged object	- charged object
Q net = 0	more protons	more electrons

- **when + & - charges become separated & charge on object "created"**

e- stripped away leaving excess +

- **Conservation of charge**
  The total charge of an isolated system can not change.

  charge can be moved around but sum is constant.

Electrostatic force

2 point charges

q_1 q_2

Coulomb's Law

$$\boxed{F = k\,\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\,\varepsilon_0}\,\frac{q_1 q_2}{r^2}}$$

$$k = 8.99\,(10)^9\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}$$

$$\varepsilon_0 = 8.85\,(10)^{-12}\,\frac{\mathrm{C}^2}{\mathrm{Nm}^2}$$

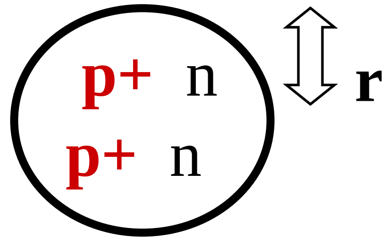
- Force depends on magnitude of both charges
  ($q_1\,q_2$ like $m_1\,m_2$ in Newton's Gravity)
- Force decreases with distance like $\boxed{1/r^2}$
  (again like Newton's Gravity)
- Force acts along straight line between point charges
  (again like Newton's Gravity)

- Force very big 10^{39} bigger than Gravity

Note: 1e- $\Rightarrow$ $-1.6\,(10)^{-19}$ C

or 1C $\Rightarrow$ $6.3\,(10)^{18}$ e^-

<u>Consider the He nucleus:</u>



About 10^{-15} m

<u>What is the repulsion force between protons?</u>

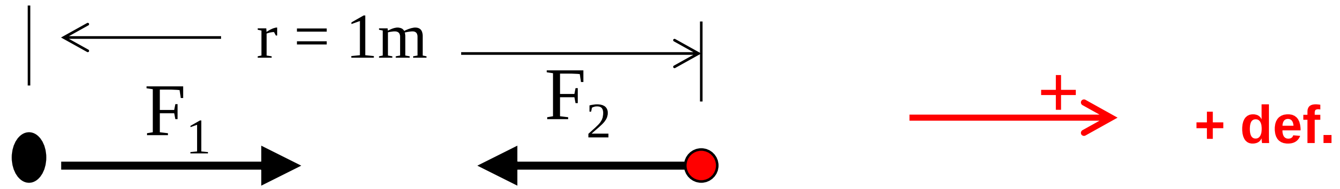
$$F = \frac{k\,q^2}{r^2} = \frac{8.99\,(10)^9\,[1.6\,(10)^{-19}]^2}{(10^{-15})^2}\left[\frac{N\cancel{m^2}}{\cancel{C^2}}\right]\left[\frac{\cancel{C^2}}{\cancel{m^2}}\right]$$

F ~ 230 N repulsive

Strong nuclear force (see end of sem.) balances this repulsion.

Example

Forces and directions on Q_1 & Q_2



$Q_1 = -6(10)^{-6}C$ $Q_2 = +4(10)^{-6}C$

-6 μC +4 μC

$$F = \frac{kQ_1Q_2}{r^2} = \frac{9(10)^9\left[6(10)^{-6}\right]\left[4(10)^{-6}\right]}{1^2} \quad \frac{N\cancel{m^2}}{\cancel{C^2}}\,\frac{\cancel{C}C}{\cancel{m^2}}$$

$= 216(10)^{-3}N$ in magnitude **signs come from + definition**

$F_1 = F_{\text{on1 due to 2}} = +.216N$ (attractive = toward other charge)

$F_2 = F_{\text{on2 due to 1}} = -.216N$ (attractive = toward other [1] charge)

Force reaction force pair NEWTON's 3rd LAW

If two one-second collections of 1 Coulomb each were concentrated at points one meter apart, the force between them could be calculated from Coulomb's Law. For this particular case, that calculation becomes

$$F = \frac{(9x10^9 N \cdot m^2 / C^2)(1C)(1C)}{1m^2} = 9x10^9 N$$

$$F = (9x10^9 N)(1lb / 4.45N)(1\,ton / 2000\,lb) = 1.01 \quad Million \quad tons!$$

If two such charges could indeed be concentrated at two points a meter apart, they would move away from each other under the influence of this enormous force, even if they had to rip themselves out of solid steel to do so!

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elefor.html#c1

- Implicit - Superposition Principle From Newton's Laws

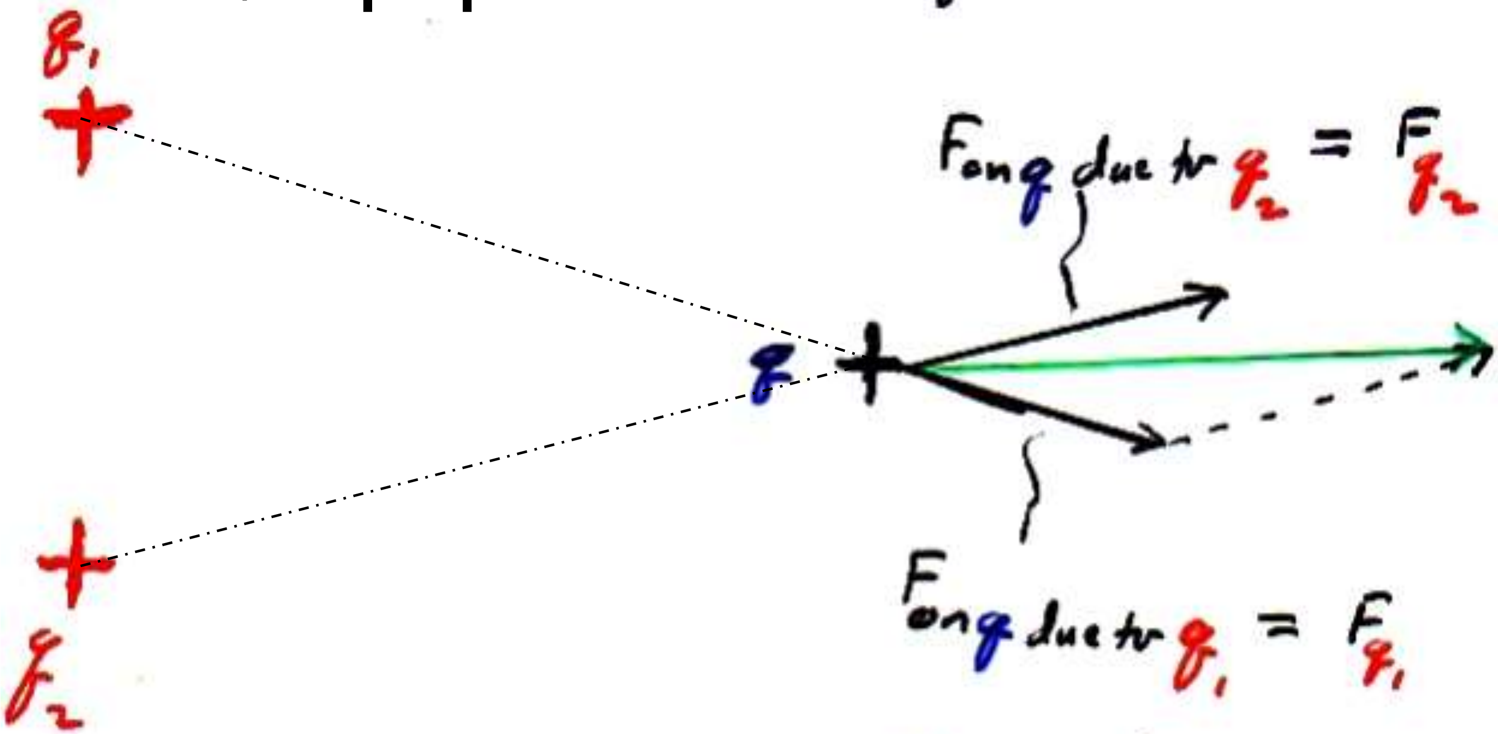
-Force between any 2 point charges - given by Coulombs Law

- To get force on any point charge, Q, due to a collection of other charges {Q1, Q2, Q2, ...}

One adds up Coulombs law forces (like vectors) for pairs Q & Q1, Q & Q2, Q & Q3, ...

typical charges in the 10^{-6} C = micro C = μ C ,range

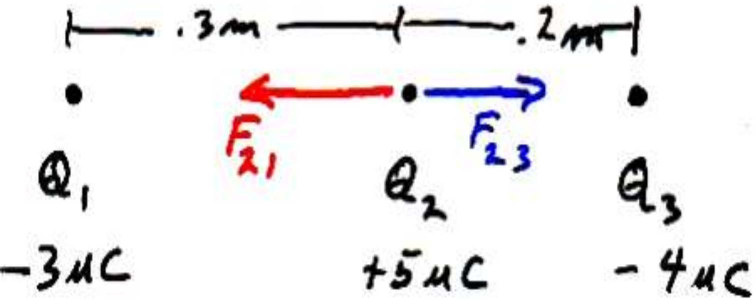
Net force → superposition of forces due to individual charges



Vector addition $\vec{F}_{net} = \vec{F}_{q_1} + \vec{F}_{q_2}$

Example: 3 co-linear charges

Consider force on Q_2



$$F_{23} = \frac{k\,Q_3 Q_2}{r_{23}^2} = \frac{9(10)^9[4(10)^{-6}][5(10)^{-6}]}{(.2)^2}\ N$$

$$= \frac{180(10)^{-3}}{.04}\ N = \boxed{4.5\ N = F_{23}}$$

attractive toward 3 to right

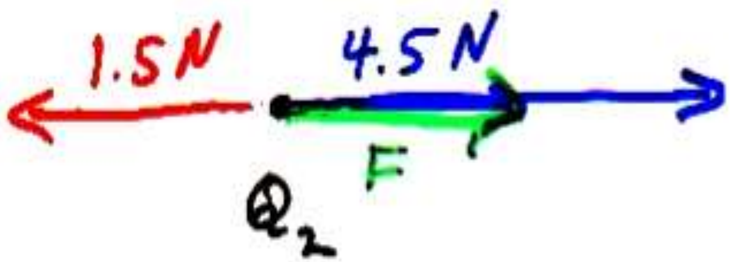
$$F_{21} = k\,\frac{Q_1 Q_2}{r_{12}^2} = \frac{9(10)^9[3(10)^{-6}][5(10)^{-6}]}{(.3)^2}\ \frac{N\,\cancel{m^2}}{\cancel{C^2}}\frac{\cancel{C}\,\cancel{C}}{\cancel{m^2}}$$

$$= -\frac{135(10)^{-3}}{.09}\ N = \boxed{1.5\ N = F_{21}}$$

attractive toward 1 to left

Consider total force F on Q_2

+ direction def.



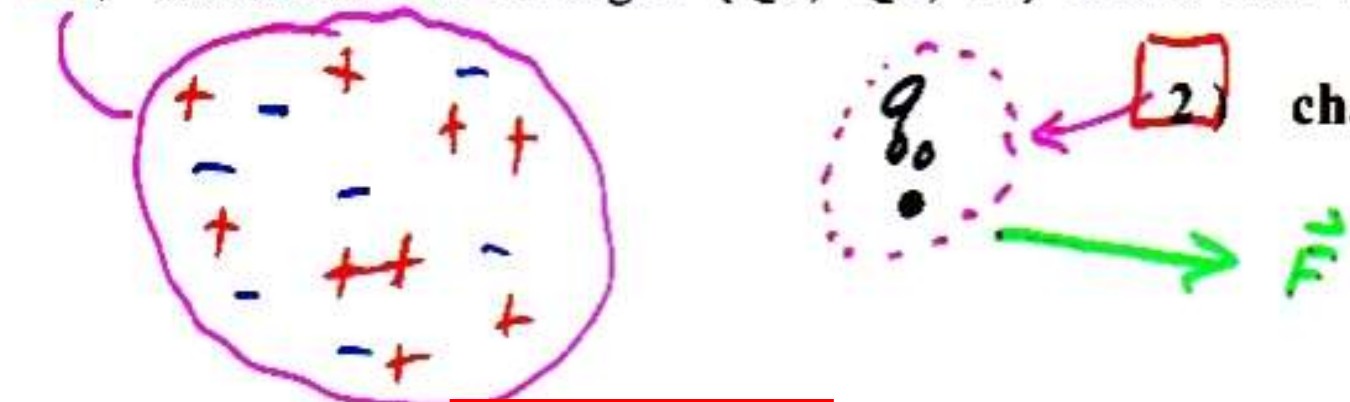
$$F = (+4.5 - 1.5)\,N = +3N$$ **to right**

1-6

Electric Field = Force per unit charge

divide universe in 2 parts [we only care about 2) q_0]

1) collection of charges {Q1, Q2, ...} exert net force F on



2) charge q_0

- Define $\vec{E}$

$$\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$$

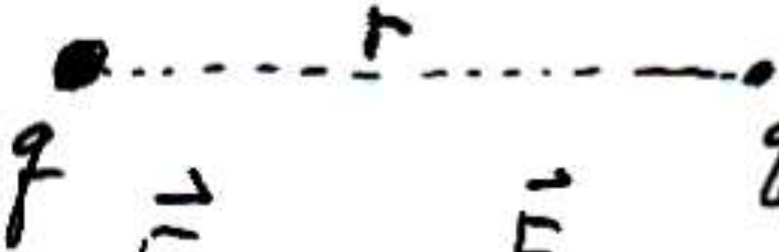
or

$$\boxed{\vec{F} = q_0 \vec{E}}$$

collection of charges create $\vec{E}$ & q_0 comes along and feels $\vec{F}$

Even if q_0 had not come along the electric field would be sitting there in space waiting for any charge that eventually happened by !!!

Electric field of one point charge



$$F_{\text{on } q_0} = \frac{k\, q\, q_0}{r^2}$$

q_0 ~ Test charge

$$\vec{E}_{\text{of } q} = \frac{\vec{F}_{\text{on } q_0}}{q_0}$$

$$E = \frac{k\, q\, \cancel{q_0}}{r^2}\, \frac{1}{\cancel{q_0}}$$

magnitude ! $E = \dfrac{k\, q}{r^2}$

Direction = direction of force on + test charge !

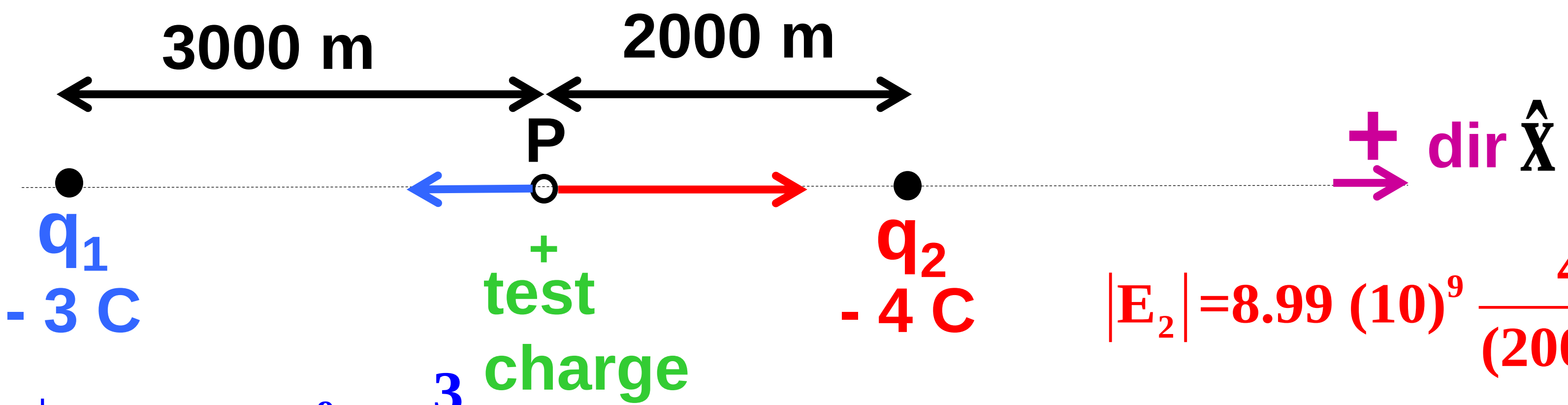
on straight line between points

Group of point charges:

add electric fields of point charges like vectors !!

Example: E at point, P, between 2 point charges

$$E = k\frac{q}{r^2}$$



$$|E_2| = 8.99\,(10)^9 \frac{4}{(2000)^2}$$

$$= 8.99 \bullet \frac{4}{2^2}(10)^3$$

$$|E_2| = 8.99(10)^3 \ \ N/C$$

$$|E_1| = 8.99\,(10)^9 \frac{3}{(3000)^2}$$

$$\frac{Nm^2}{C^2}\frac{C}{m^2} = \frac{N}{C}$$

$$= 8.99 \bullet \frac{3}{3^2}(10)^3$$

$$|E_1| = 3(10)^3 \ \ N/C$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = [-|\vec{E}_1| + |\vec{E}_2|]\ \hat{x}$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = [-3 + 8.99]\ (10)^3 \ \ \hat{x} \quad N/C$$

$$\vec{E} = 5.99\ (10)^3 \ \ \hat{x} \quad N/C$$

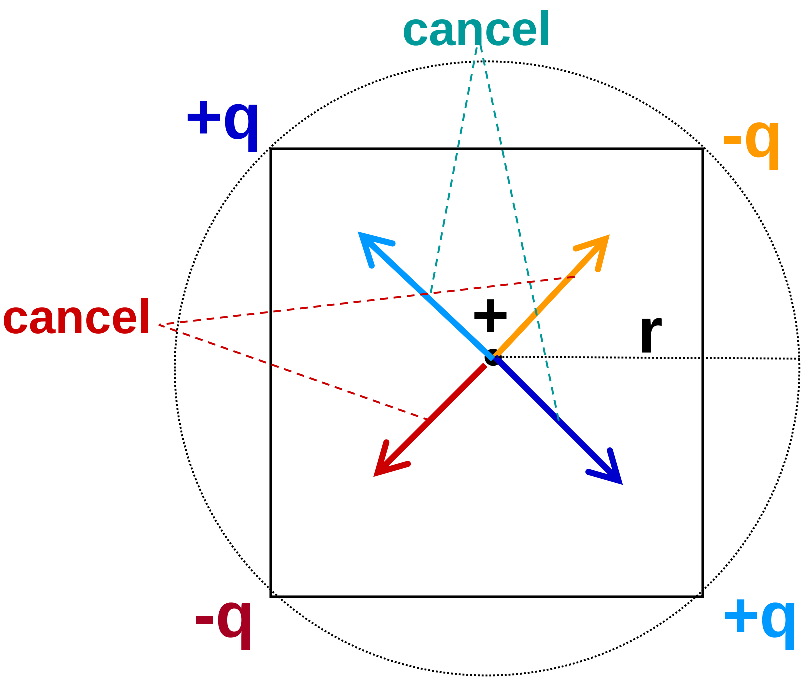
last slide 9/6/17 lec

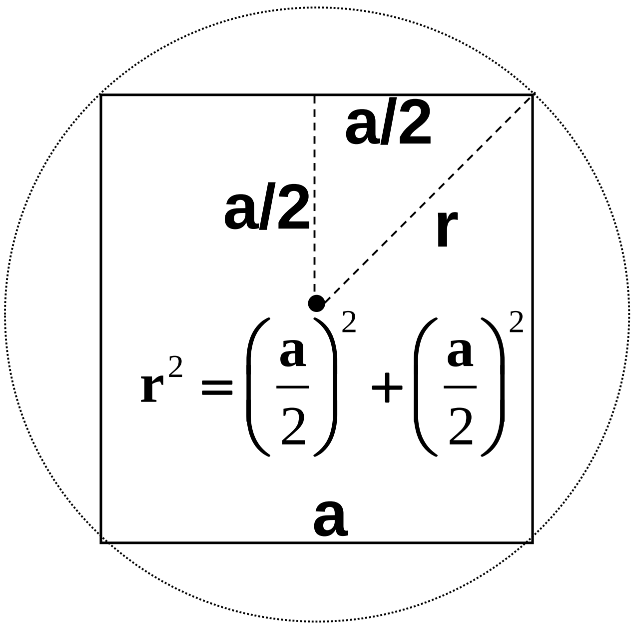
Vector addition of E fields: use symmetry to simplify when possible

Q. Electric field at center

(place + test charge at center)

homework problem help





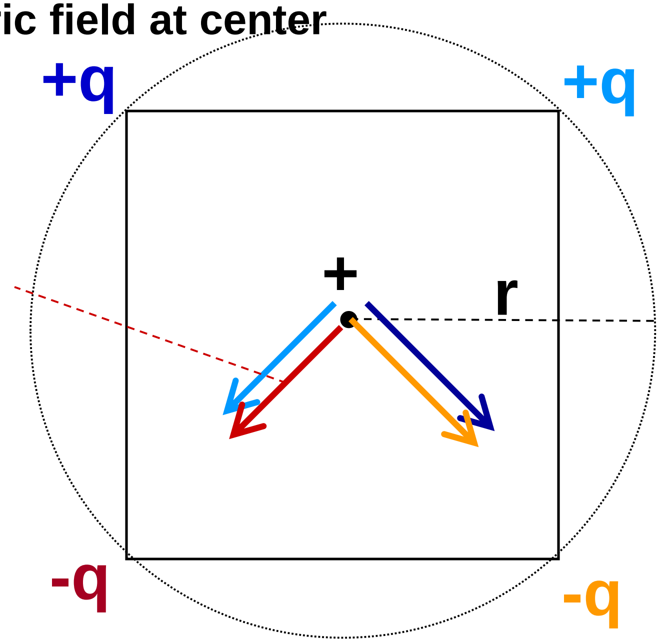
$$E = k\frac{q}{r^2}$$

q's the same r's the same ⇒ E's the same

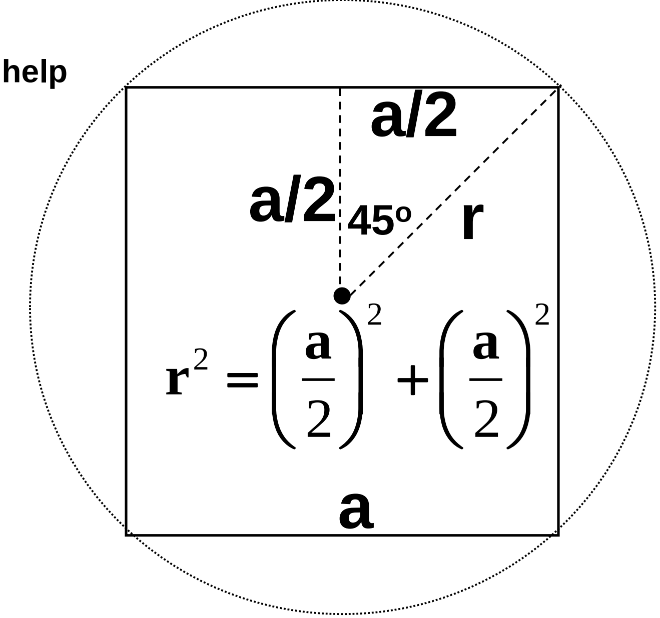
<u>E fields cancel so E_{tot}=0</u>

Vector addition of E fields: symmetry continued

Q. Electric field at center



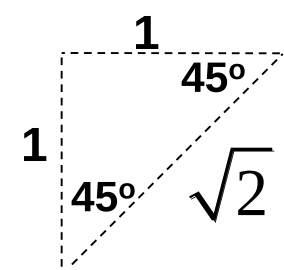
homework problem help



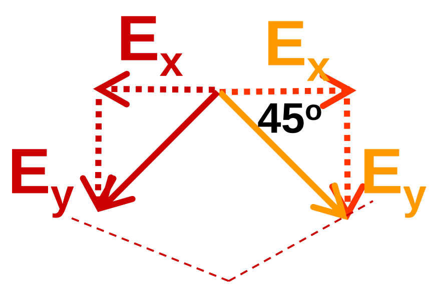
$$r^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2$$

$$E = k\frac{q}{r^2}$$

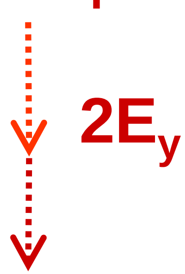
q's the same r's the same ⇒ E's the same



cancel

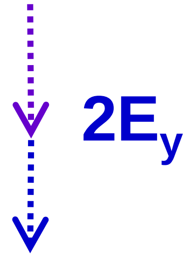


superimpose



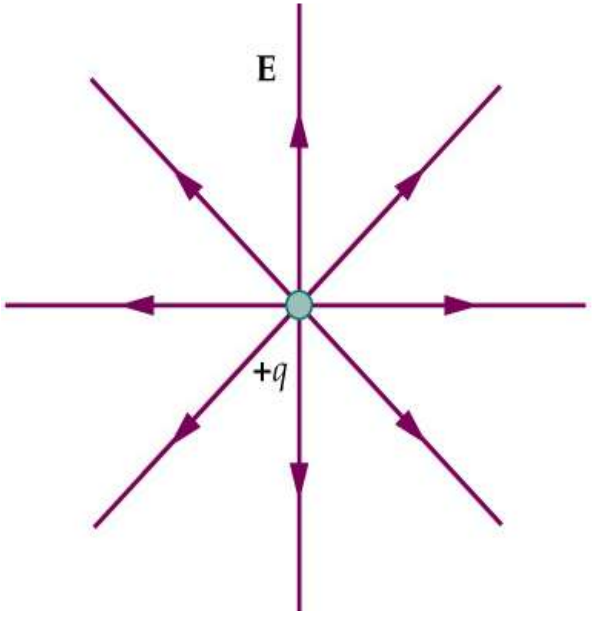
$$E_y = k\frac{q}{r^2}\sin(45^{o}) = k\frac{q}{r^2}\frac{1}{\sqrt{2}}$$

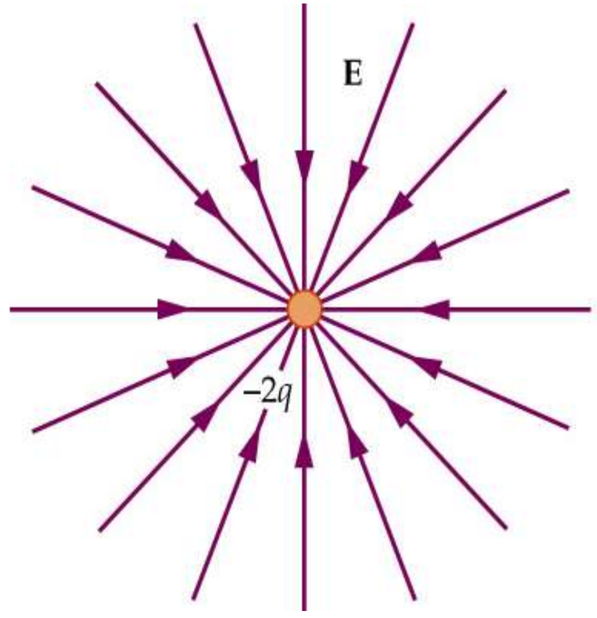
similarly



-y direction

$$E_{tot} = 4E_y \quad (-\hat{y})$$



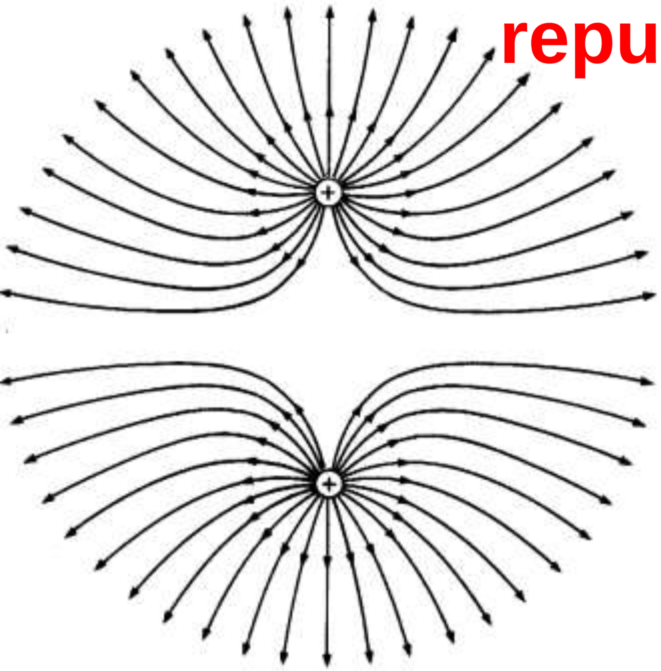


E Field Lines

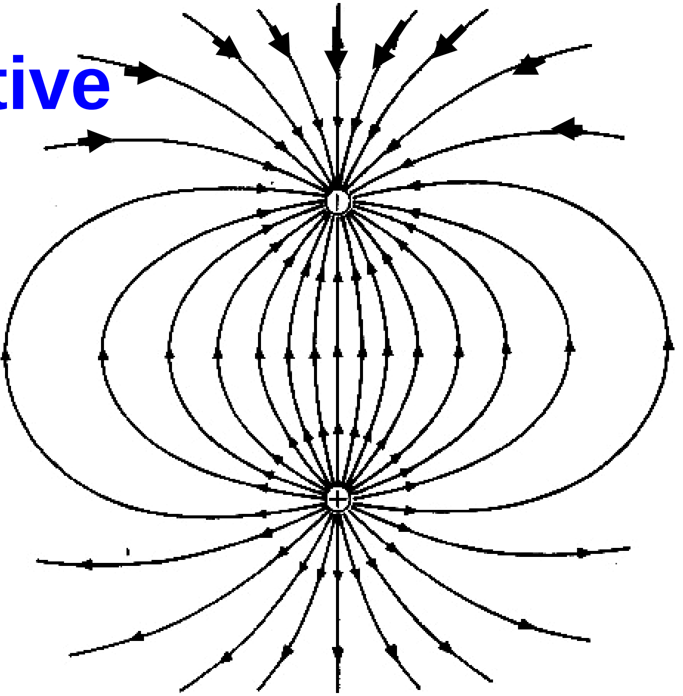
- direction = force direction on + test charge (away from +) (toward -)

- line density α field magnitude

repulsive



attractive

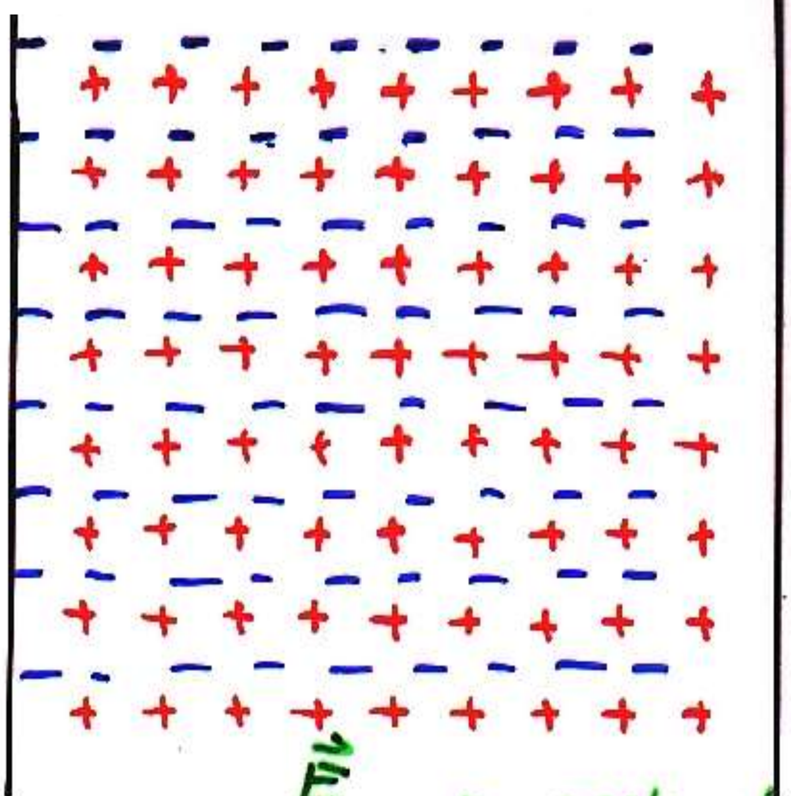


Static (no charge motion) condition

Metal

+ fixed on lattice

− electron fluid



$\vec{E}$ → apply electric field $(t=0)$

⊖ electrons flow

[+ fixed]

For Metal

① e^- (charge) free to flow

② all charge on outside surface (static conditions)

③ $\vec{E}$ inside metal $\equiv 0$ (static conditions)

④ $\vec{E}$ field at surface of metal is $\perp$ to the surface

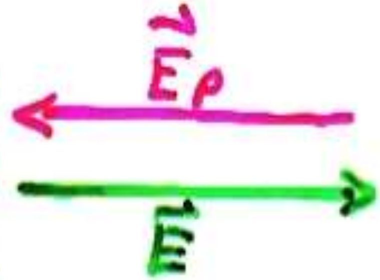
t = later

e^- no place to go

t = later

e^- no place to go

- interior charge screened
- unbalanced surface charge
  - creates polarization field



$$\vec{E}_{tot} = \vec{E} + \vec{E}_p$$

$$\boxed{\vec{E}_{tot} = 0 \quad !!}$$

total electric field in metal = 0!!

[as long as charge not flowing]

For Metal

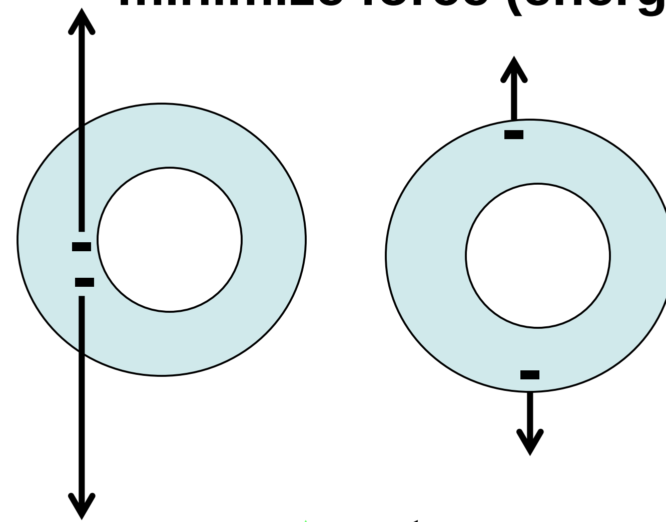
① e^- (charge) free to flow

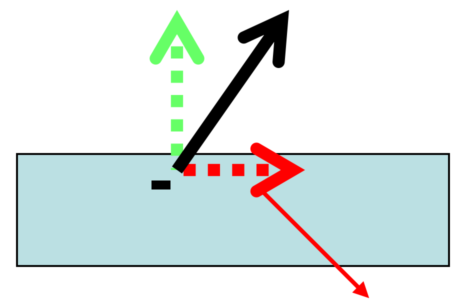
② all charge on outside surface (static conditions)

③ $\vec{E}$ inside metal $\equiv 0$ (static conditions)

④ $\vec{E}$ field at surface of metal is $\perp$ to the surface

minimize force (energy)





would cause motion (violates static)

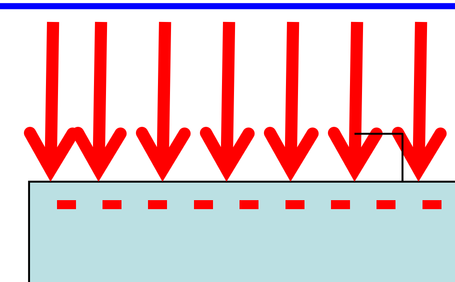
⑤ **E concentrate at sharp points (low radius of curvature)**

for later

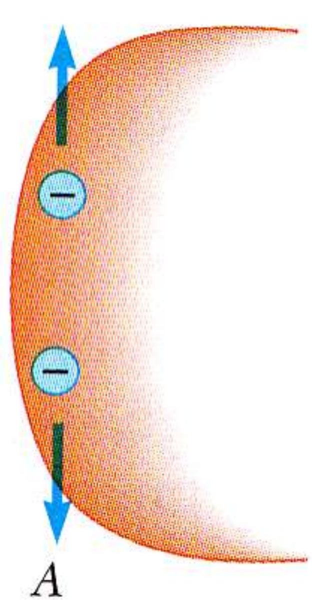
∞ flat plane

Electric field constant

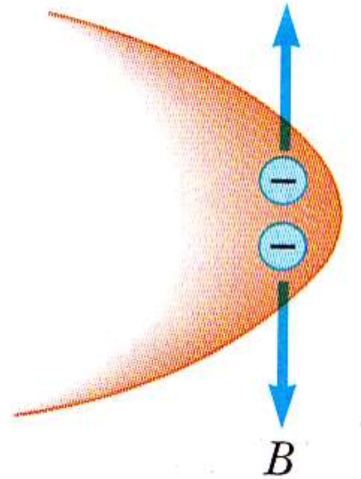
Electric field $\perp$ surface!!!!



E concentrate at sharp points (low radius of curvature)

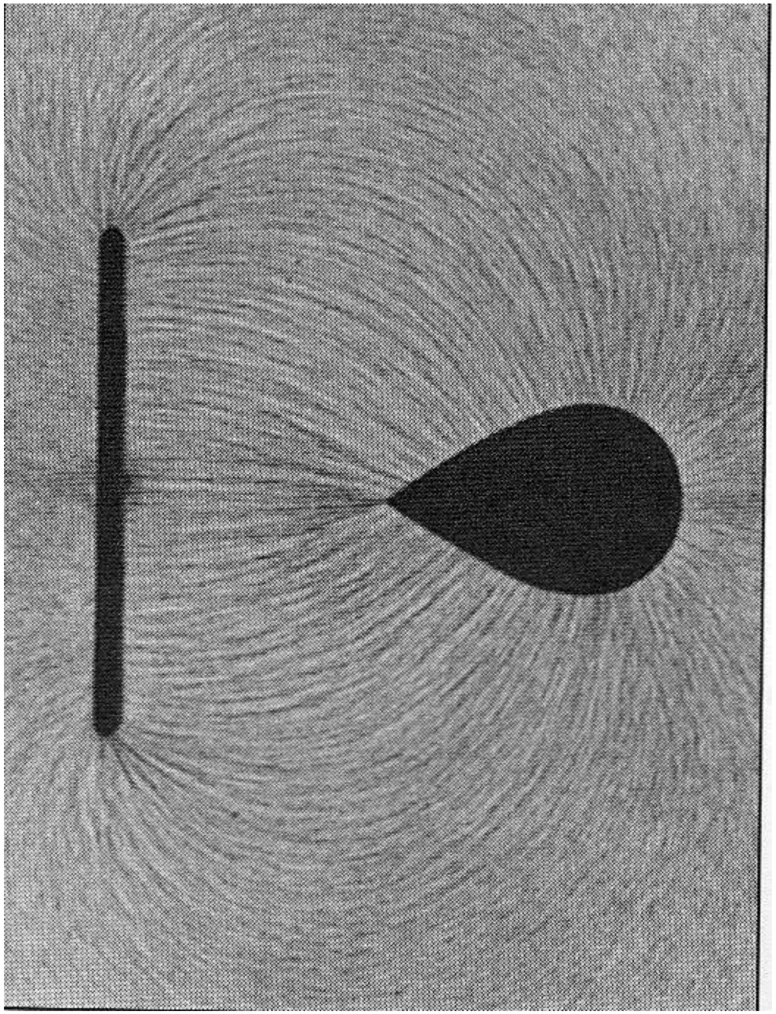


can move apart -decrease density

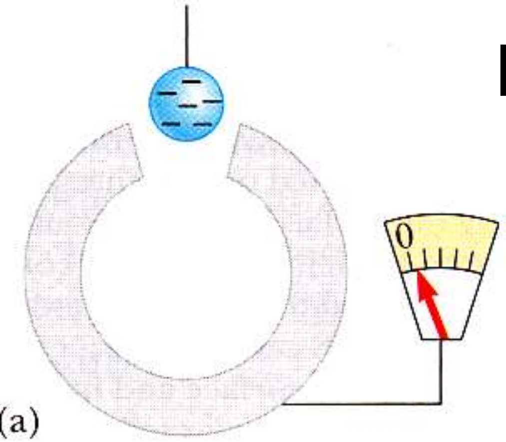


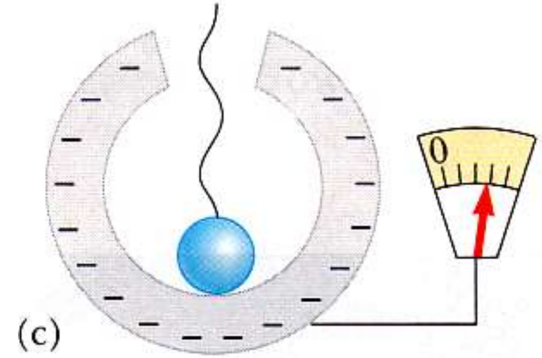
can't move apart (surf. contains)

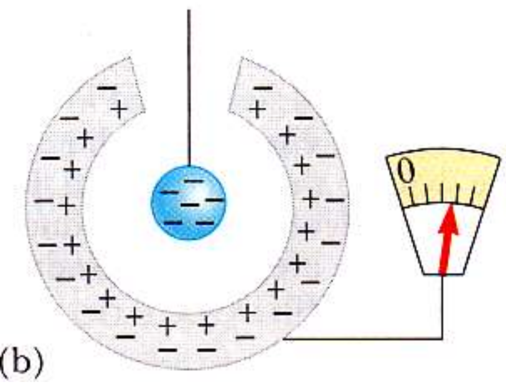
enough E can actually rip charges from surface

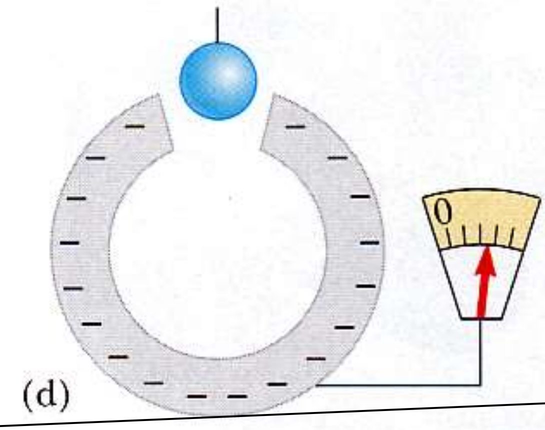


Faraday Bucket

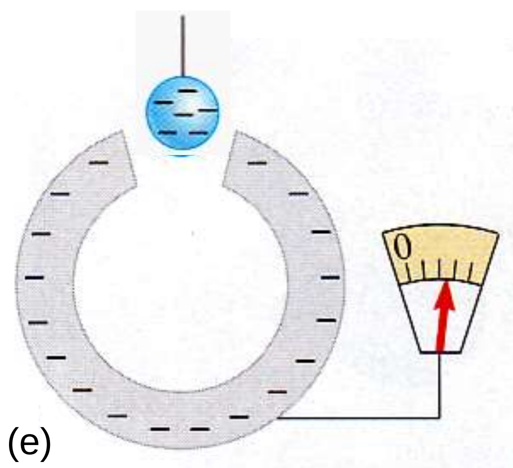


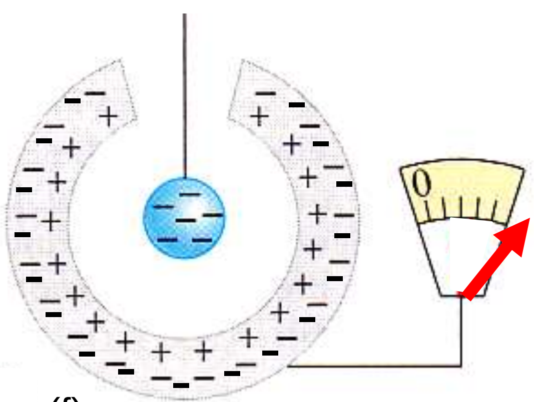


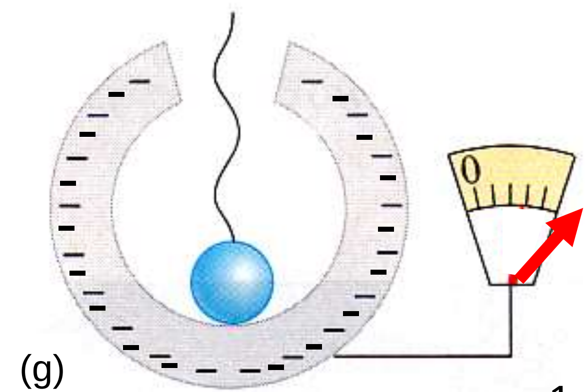




can accumulate charge !!



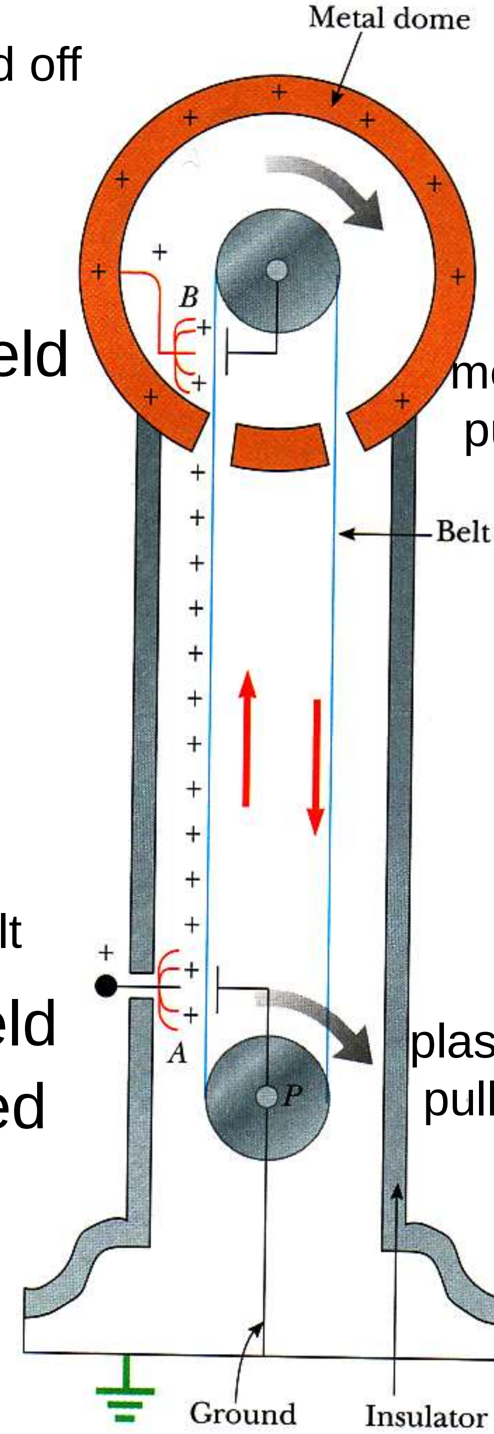




Van de Graff generator

+ pulled off belt

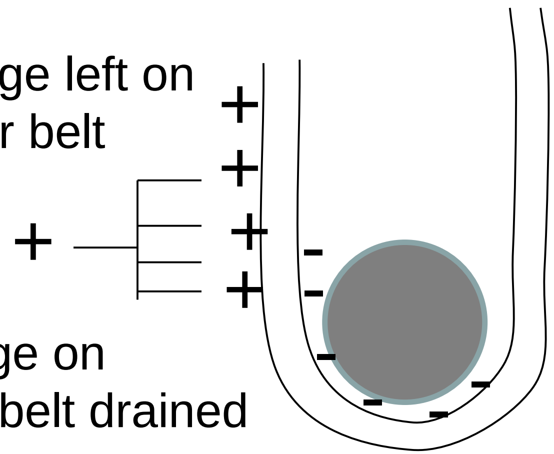
continuous charge accumulation !!



sharp points – big electric field

metal pulley

+ charge left on rubber belt



- charge on rubber belt drained

- static charge induced on inside of rubber belt

sharp points – big electric field

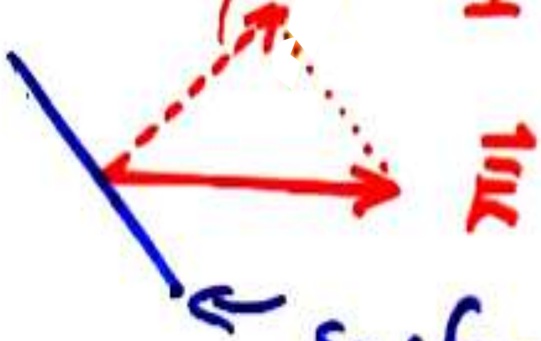
charge transferred

plastic pulley

Electric Flux

- Electric Flux, φ_E , through a surface of area, A

$E_\perp$ = component of $\vec{E}$ $\perp$ to surface

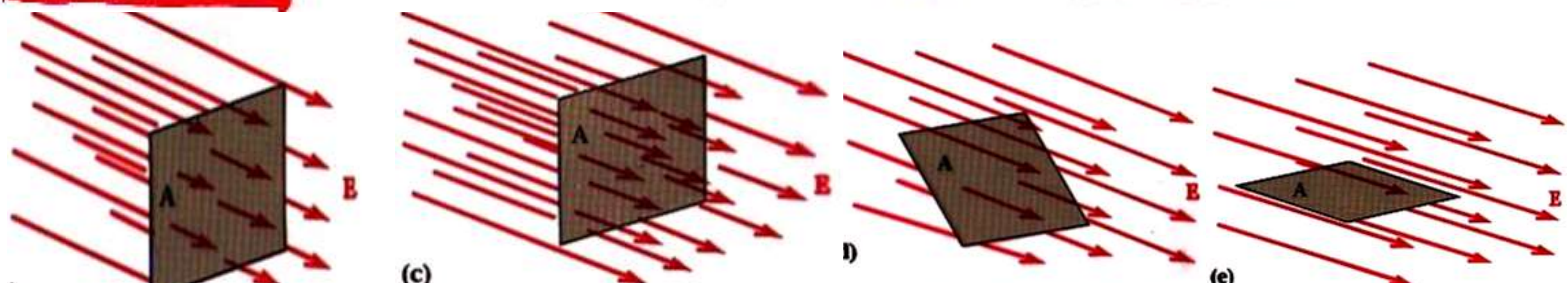


surface (side view)
(surface area A)

$$\boxed{\varphi_E = A E_\perp}$$

$\{$units $m^2 \frac{N}{C}\}$

Gauss's Law



Larger field, larger flux

Area not perpendicular to field, smaller flux

Area parallel to field, zero flux

$E_\perp = 0$!

$\Rightarrow$ $\varphi_E = 0$

Gauss's Law

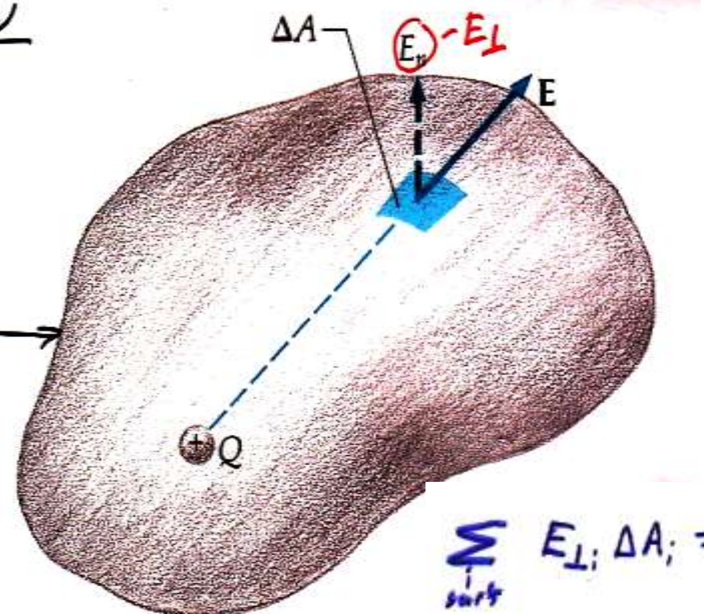
The net electric flux (φ_E) through any closed surface is directly proportional to the net electric charge (Q) enclosed by that surface.

$$\boxed{\varphi_E = \frac{Q_{inside}}{\varepsilon_o}}$$

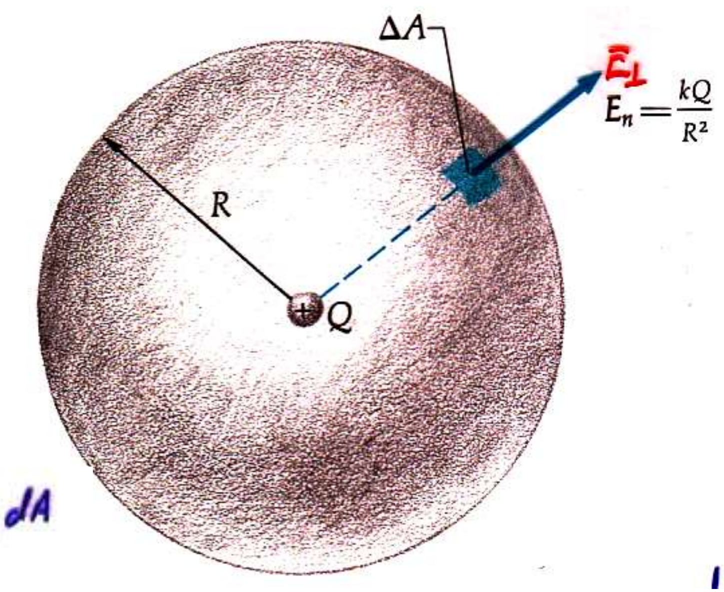
$$\boxed{\sum_{surf} E_\perp \, \Delta A = \frac{Q_{inside}}{\varepsilon_o} = \varphi_E}$$

Gauss's Law

any Gaussian Surface

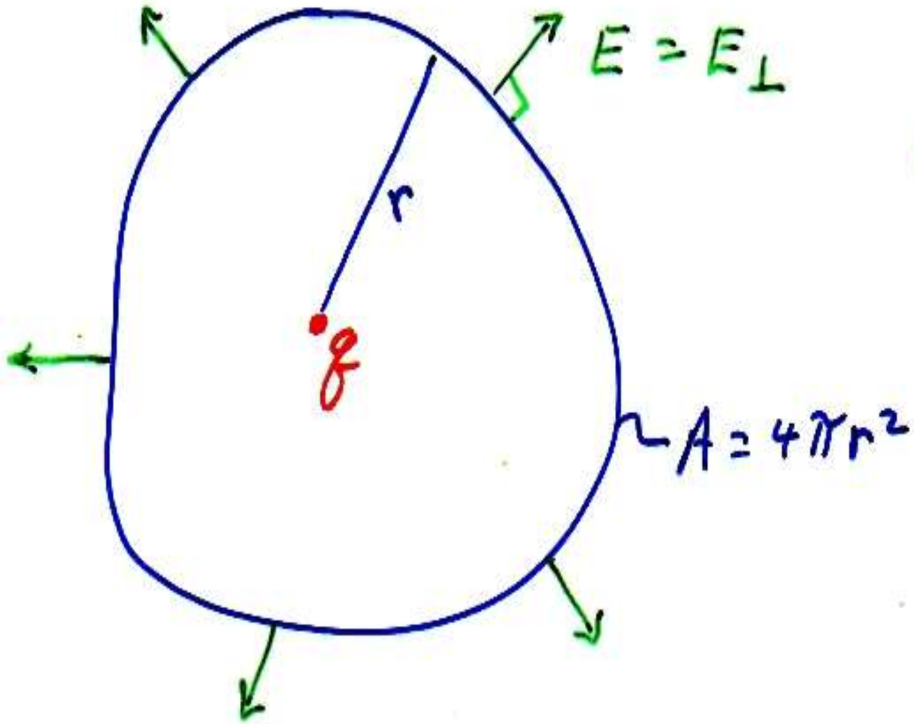


Special surface sphere



$$\sum_{i\ surf} E_{\perp i} \Delta A_i = \varphi_E = \oint_{surf} E_\perp \, dA$$

Point Charge (Gauss's Law Applications cont.)



$$E_\perp \underbrace{A}_{} = \frac{q}{\varepsilon_0}$$

$$E \, \overbrace{4\pi r^2}^{} = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}}$$

Note: add q_0 at a r and find force on q_0

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r^2}$$

Coulombs Law

$\Updownarrow$

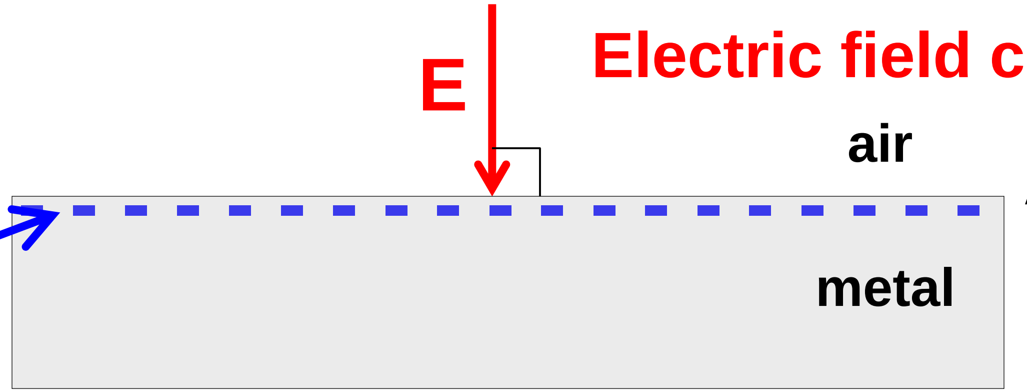
Gauss's Law

Surface charge density & E field for metal

Electric field ⊥ surface!!!!

Electric field constant

Q= tot. surface charge



A =surface area

$$\boxed{\sigma = \frac{Q}{A}}$$

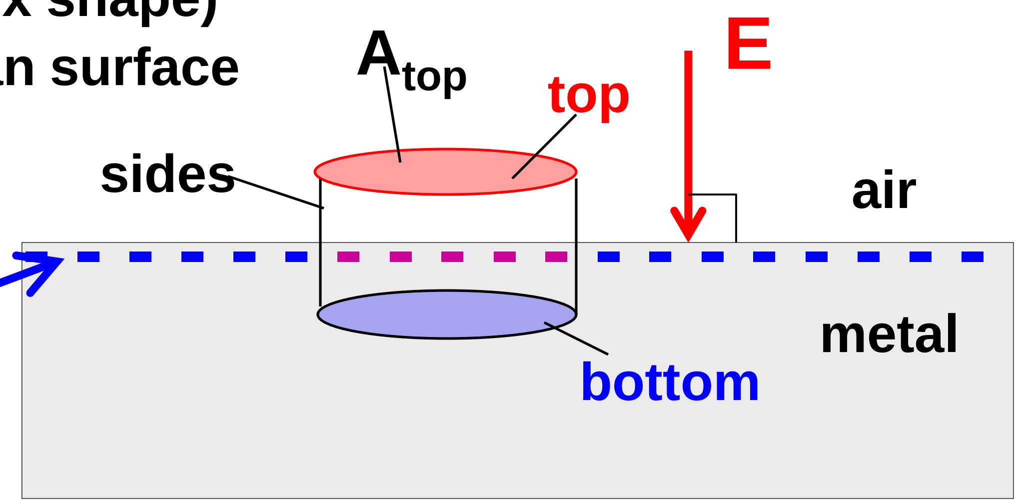
σ units C/m^2

Q. What is relation between E and σ ?

Electric field above charged metal surface

(pill box shape)

Gassian surface



Q= tot. surface charge

A_{tot} =surface area

$$\boxed{\sigma = \frac{Q}{A}} \quad \boxed{Q = \sigma A}$$

σ units C/m²

$$\varphi_{tot} = \varphi_{top} + \varphi_{bottom} + \varphi_{sides}$$

φ_{tot} through surface

Gauss

$$\varphi_{tot} = \frac{Q_{in}}{\varepsilon_0}$$

$$\varphi_{tot} = \frac{\sigma A_{top}}{\varepsilon_0}$$

E is ⊥ top

$$\varphi_{top} = E\,A_{top}$$

$\varphi_{bottom} \to 0$

E=0 in metal

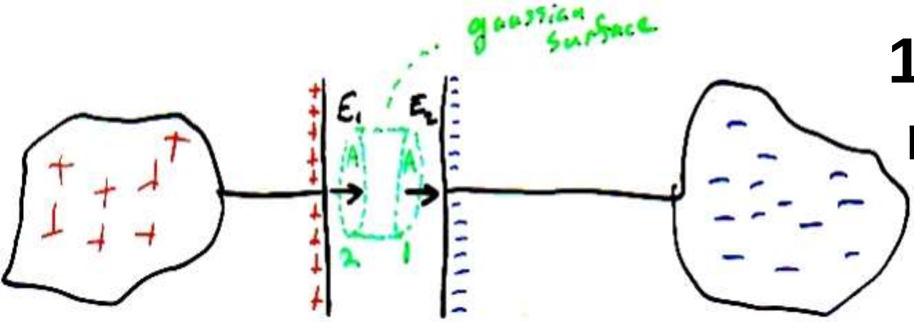
$\varphi_{sides} \to 0$

E=0 in metal

E || side in air

$$\frac{\sigma \cancel{A}_{top}}{\varepsilon_0} = E\,\cancel{A}_{top} \Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Consider 2 // plates connected to charge reservoirs.



1. The E field, by symmetry, must run straight across the gap.

2. Apply Gauss's Law

$$\Rightarrow \quad \boxed{\vec{E} = \text{constant in gap}}$$

No enclosed charge $\therefore \Phi_{net} = 0$

$$\Phi_{net} = \Phi_1 + \Phi_2 + \cancel{\Phi_{sides}} = 0$$

0 because sides // to $\vec{E}$

$$\therefore -[\Phi_2] = [\Phi_1]$$

$$-[-E_2 A] = [E_1 A]$$

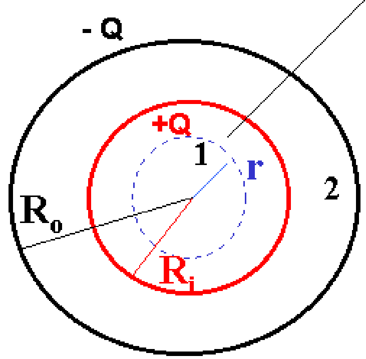
$$E_2 \cancel{A} = E_1 \cancel{A}$$

$$\boxed{E_1 = E_2} \Rightarrow \underline{E \text{ constant}} \ \underline{\text{between plates}}$$

3. Metal Plates

$$\Rightarrow \text{we know} \quad E = \frac{\sigma}{\varepsilon_0} = \frac{Q_{Tot}}{A_{Tot}} \frac{1}{\varepsilon_0}$$

hollow spherical shells: R_i (charge + Q) and R_o (charge –Q

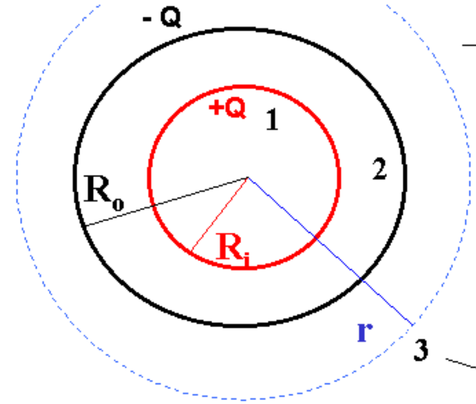


Region 1 $r < R_i$

$$Q_{in} = 0 \Rightarrow E_1 = 0$$

3 Gaussian surfaces
with r in region 1, 2 & 3

hollow spherical shells: R_i (charge + Q) and R_o (charge –Q).

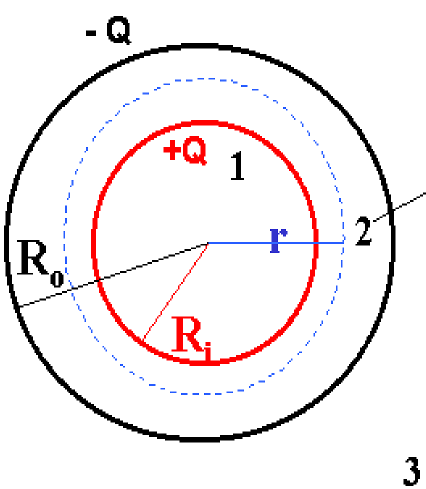


3 Gaussian surfaces
with r in region 1, 2 & 3

Region 3 $r > R_o$

$$E_3\, 4\pi r^2 = \frac{(+Q - Q)}{\varepsilon_o} = 0 \Rightarrow E_3 = 0$$

hollow spherical shells: R_i (charge + Q) and R_o (charge –Q).



Region 2 $R_i < r < R_o$

$$E_2\, 4\pi\, r^2 = \frac{+Q}{\varepsilon_o}$$

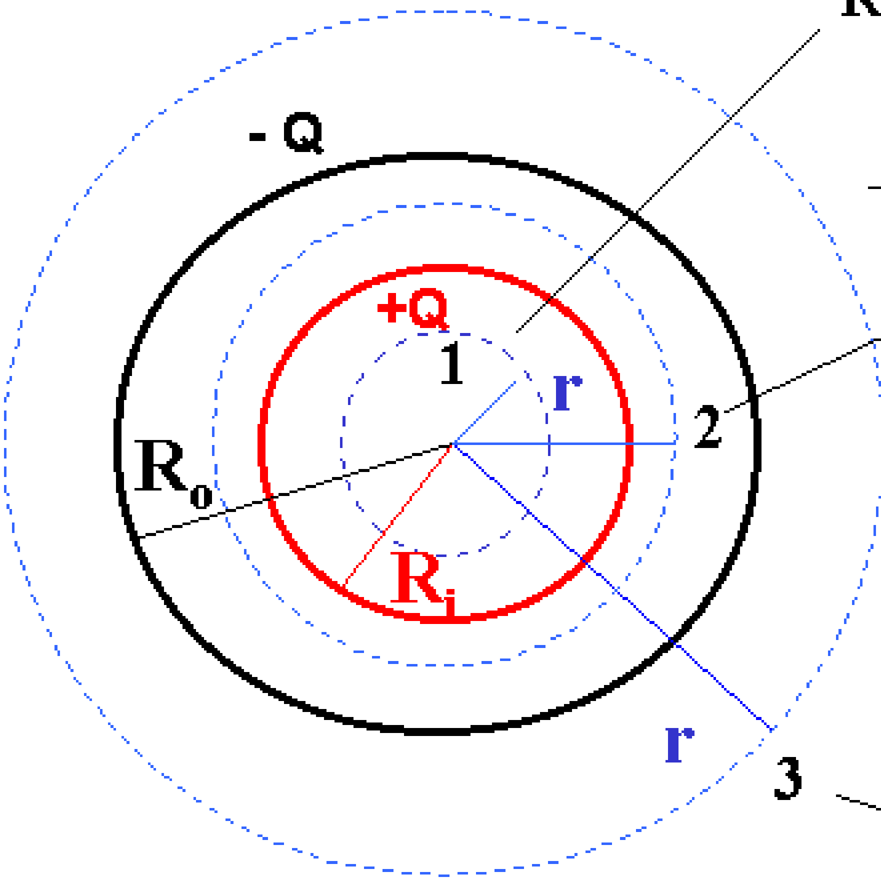
$$\Downarrow$$

$$E_2 = \frac{+Q}{4\pi\, \varepsilon_o\, r^2}$$

3 Gaussian surfaces
with r in region 1, 2 & 3

1-15a-b-c

hollow spherical shells: R_i (charge + Q) and R_o (charge –Q).



Region 1 $r < R_i$

$$Q_{in} = 0 \Rightarrow E_1 = 0$$

Region 2 $R_i < r < R_o$

$$E_2 \, 4\pi \, r^2 = \frac{+Q}{\varepsilon_o}$$

$$\Downarrow$$

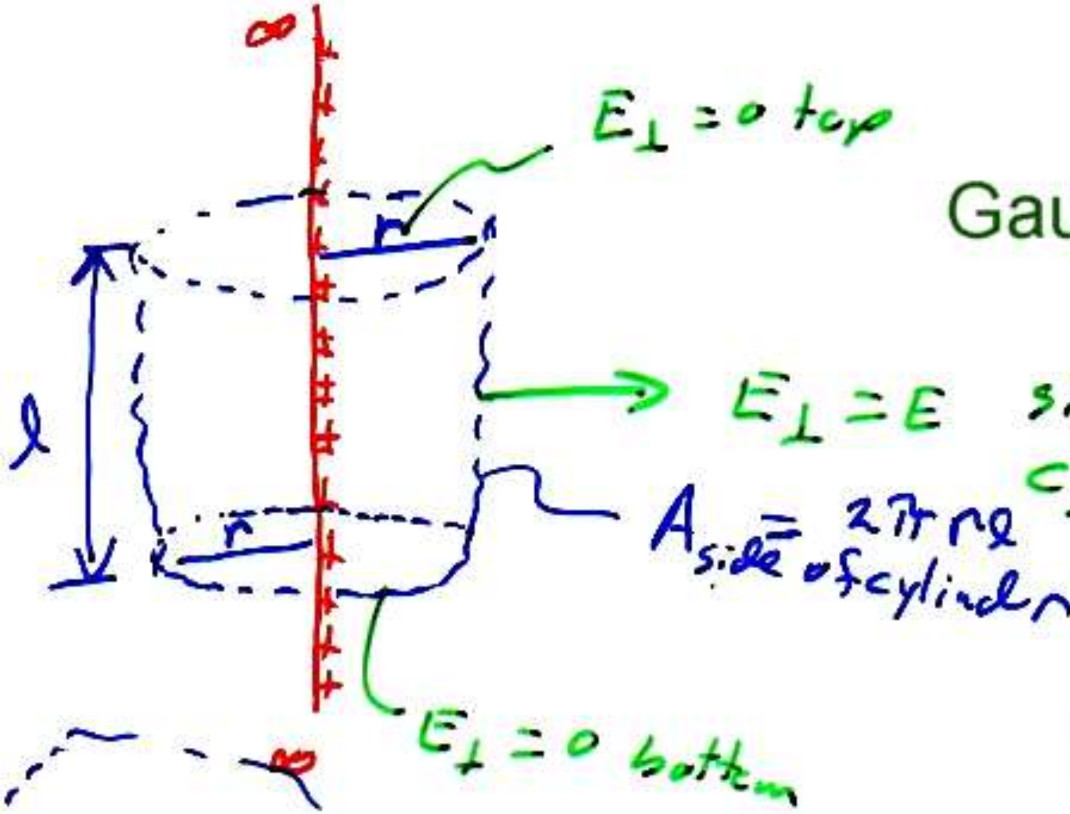
$$E_2 = \frac{+Q}{4\pi \varepsilon_o r^2}$$

3 Gaussian surfaces with r in region 1, 2 & 3

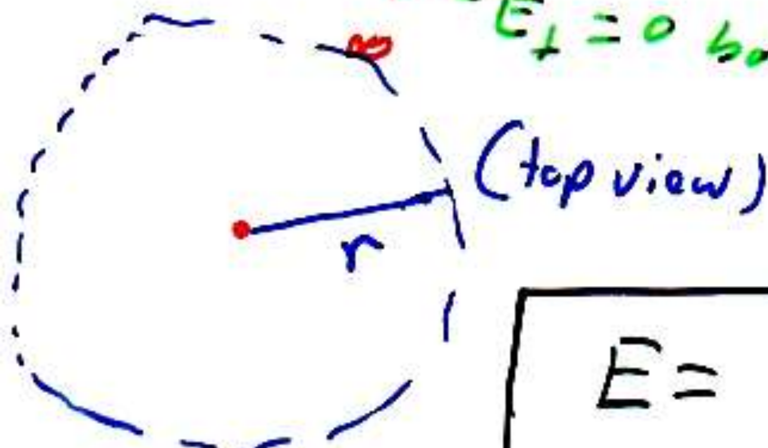
Region 3 $< R_o$

$$E_3 \, 4\pi r^2 = \frac{(+Q - Q)}{\varepsilon_o} = 0 \Rightarrow E_3 = 0$$

∞ line of charge of density λ $\left(\frac{C}{m}\right)$



Gauss's Law application



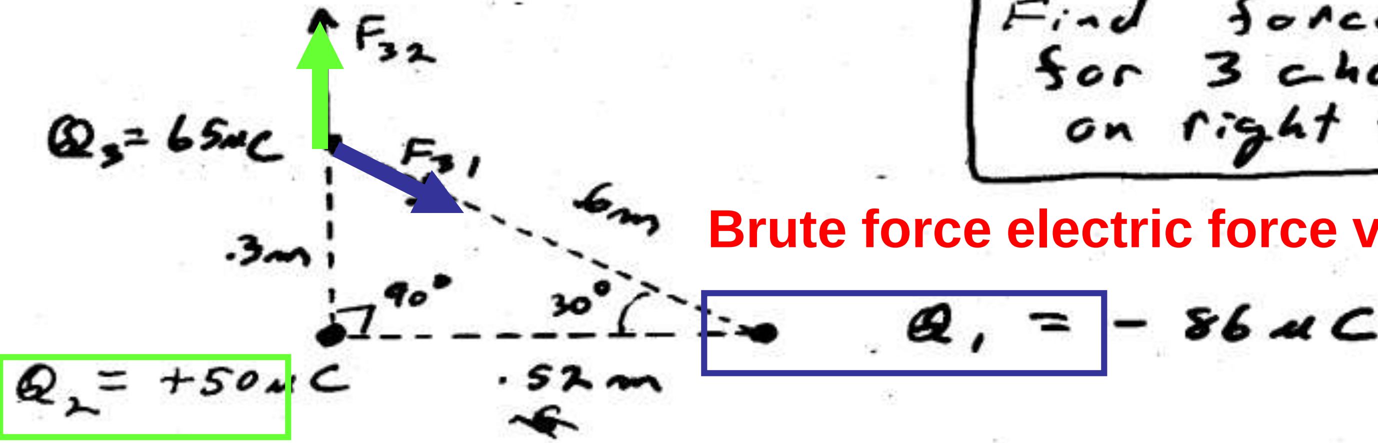
$$E\,A = \frac{Q_{encl.}}{\varepsilon_0}$$

$$E\,(2\pi r \ell) = \frac{(\lambda \ell)}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}}$$

Find force on Q_3 for 3 charges (non colinear) on right triangle

Brute force electric force vector addition



$$F_{32} = \frac{9(10)^9 [65(10)^{-6}][50(10)^{-6}]}{.3^2} = \frac{29250\ (10)^{-3}}{0.09}$$

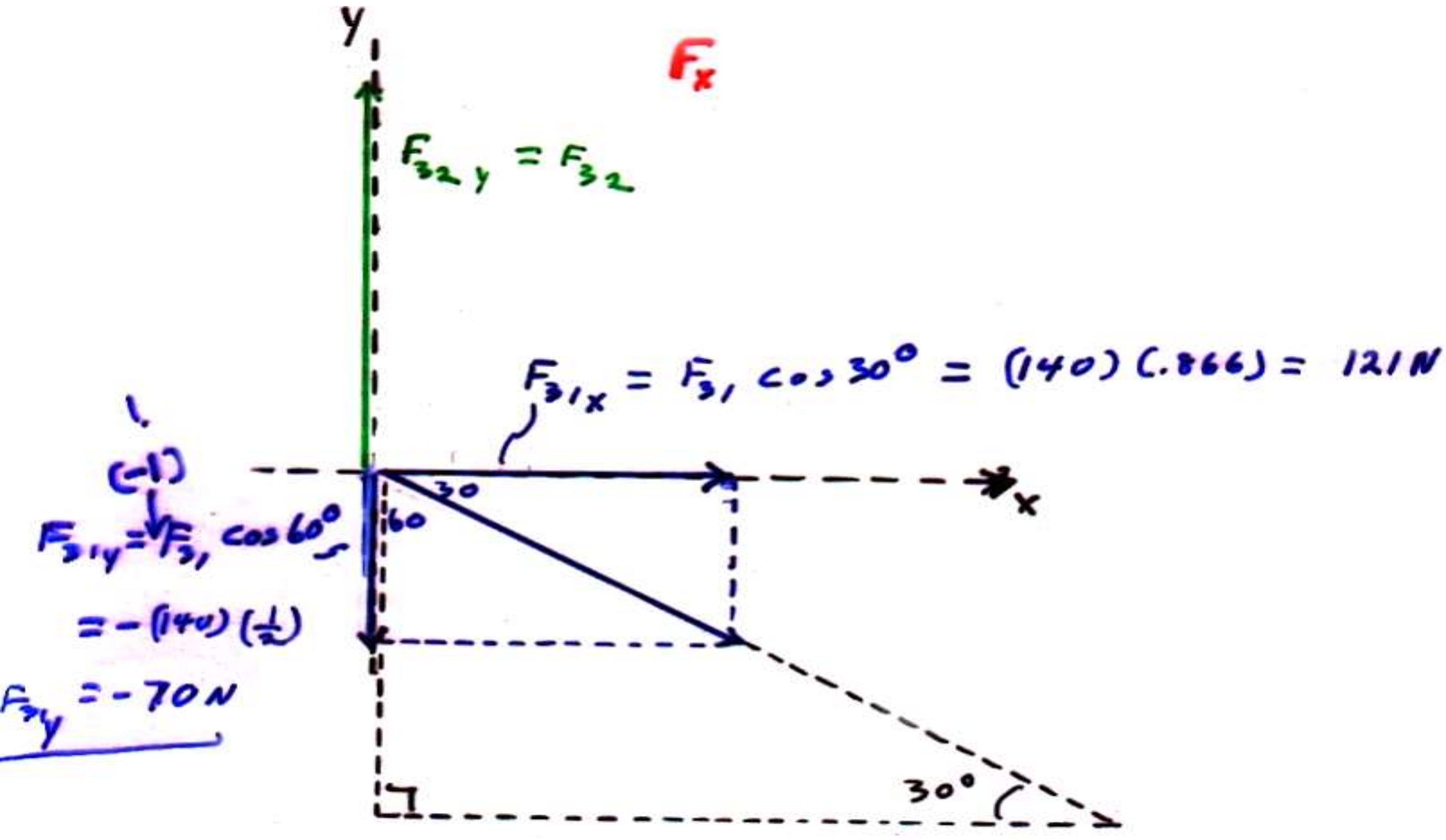
$$= \frac{29.25}{.09} = \underline{325\ N = F_{32}}$$ away from 2

$$F_{31} = \frac{9(10)^9 [65(10)^{-6}][86(10)^{-6}]}{(.6)^2} = \frac{50310\ (10)^{-3}}{0.36}$$

$$= -\frac{50.31}{.36} = \underline{140\ N = F_{31}}$$ toward 1

will assign absolute direction

1-17-extra



$F_x = F_{31x} = 121 N$

$F_y = F_{32y} + F_{31y} = 325 - 70 = 255 N$