Basis independent vector operations: $-\vec{b}$ $5\,\vec{b}$ $\vec{a}+\vec{b}$ $\angle(\vec{a},\vec{b})$ $\vec{a}\cdot\vec{b}$ $\vec{a}\times\vec{b}$

Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

1.1 ♣ Solving problems – what physicists and engineers do.

Understanding dynamics results from **doing** problems. Many problems herein guide you to help you synthesize processes (imitation). Please **do** these problems by yourself or with colleagues/instructors and use the textbook and other resources.

Confucius 500 B.C.
"I hear and I forget.
I see and I remember.
I ______ and I understand."

"By three methods we may learn wisdom:
1^{st} by reflection, which is noblest;
2^{nd} by imitation, which is easiest;
3^{rd} by experience, which is the bitterest."

1.2 ♣ What is a vector (as defined by Gibbs circa 1897)? (Section 2.2)

Two properties (attributes) of a vector are ______ and ______ (fill in the blanks).

1.3 ♣ What is a zero vector? (Section 2.3)

A zero vector $\vec{0}$ has a magnitude of 0 ($\lvert\vec{0}\rvert = 0$).	**True/False**	(circle true or false).
A zero vector $\vec{0}$ has a direction.	**True/False**	
anyVector $+\ \vec{0}\ =$ **anyVector**	**True/False**	
A zero vector $\vec{0}$ is ***parallel*** to any vector $\vec{v}$.	**True/False**	.
A zero vector $\vec{0}$ is ***perpendicular*** to any vector $\vec{v}$.	**True/False**	.

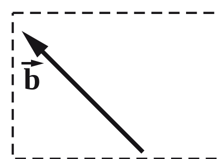
1.4 ♣ Unit vectors. (Section 2.4)

All unit vectors have a magnitude of 1 (e.g., $\lvert\hat{i}\rvert = 1$, $\lvert\hat{j}\rvert = 1$, $\lvert\hat{k}\rvert = 1$).	**True/False**
Typically, a unit vector is denoted with a hat, e.g., as $\hat{k}$ rather than $\vec{k}$.	**True/False**
All unit vectors are equal.	**True/False**
A unit vector $\hat{u}$ in the direction of the non-zero vector $\vec{v}$ is $\hat{u} = \frac{\vec{v}}{\lvert\vec{v}\rvert}$.	**True/False**
The set of unit vectors $\widehat{\text{E}}$ast, $\widehat{\text{N}}$orth, $\widehat{\text{U}}$p span 3D space (the world in which we live).	**True/False**
In general, unit vectors have units (e.g., degrees or meters or $\frac{m}{s}$ or ...).	**True/False**

1.5 ♣ Negating a vector. (Section 2.8)

Draw the vector $-\vec{b}$. Negating the vector $\vec{b}$ results in a vector with different:

magnitude   direction   orientation   sense   (circle all that apply)

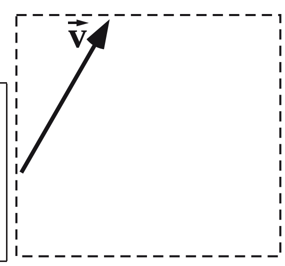


Historical note: Negative numbers (e.g., -3) were not widely accepted until 1800 A.D.

1.6 ♣ Vector magnitude and direction (orientation and sense). (Section 2.2)

The figure to the right shows a vector $\vec{v}$. **Draw** the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$, $\vec{e}$.

$\vec{a}$	Same magnitude and direction as $\vec{v}$ ($\vec{a} = \vec{v}$).
$\vec{b}$	Same magnitude as $\vec{v}$, with $\vec{b} = -\vec{v}$ (**antiparallel**, $\vec{b}$ has a different ***sense*** than $\vec{v}$).
$\vec{c}$	Same magnitude as $\vec{v}$, but different direction with $\vec{c} \neq -\vec{v}$ (different ***orientation***).
$\vec{d}$	Smaller magnitude than $\vec{v}$, but same direction as $\vec{v}$.
$\vec{e}$	Different magnitude and different direction than $\vec{v}$.



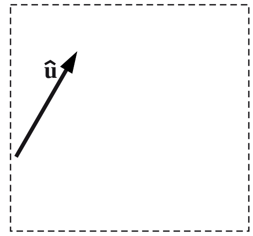
1.7 ♣ Vector magnitude and direction. (Section 2.2)

Knowing x is a real number (e.g., -3 or 0 or 7.8) and $\hat{u}$ is a horizontal unit vector →, complete ***magnitude*** with $\leq$ or $\geq$ and complete ***direction*** with $+\hat{u}$ or $-\hat{u}$.

Vector	with	Magnitude			Direction
$x\,\hat{u}$	$x \geq 0$	$\lvert x\,\hat{u}\rvert$	$\geq$	0	$+\hat{u}$
$x\,\hat{u}$	$x \leq 0$	$\lvert x\,\hat{u}\rvert$		0	
$-x\,\hat{u}$	$x \geq 0$	$\lvert -x\,\hat{u}\rvert$		0	
$-x\,\hat{u}$	$x \leq 0$	$\lvert -x\,\hat{u}\rvert$		0	

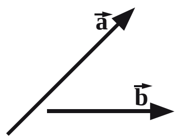
1.8 ♣ Optional: Multiplying a vector by a scalar. (Section 2.7)

The following statements involve a unit vector $\hat{\mathbf{u}}$ and a real scalar s $(s \neq 0)$.
If a statement is **true**, provide any numerical value for s that supports your answer, and if **true** also **draw** a corresponding vector, i.e., $\vec{\mathbf{a}}$ or $\vec{\mathbf{b}}$ or $\vec{\mathbf{c}}$.



$s\,\hat{\mathbf{u}}$ can have a different ***magnitude*** than $\hat{\mathbf{u}}$.	If **true** $s =$ ___,	**draw** $\vec{\mathbf{a}}$.
$s\,\hat{\mathbf{u}}$ can have a different ***direction*** than $\hat{\mathbf{u}}$.	If **true** $s =$ ___,	**draw** $\vec{\mathbf{b}}$.
$s\,\hat{\mathbf{u}}$ can have different ***magnitude and direction*** than $\hat{\mathbf{u}}$.	If **true** $s =$ ___,	**draw** $\vec{\mathbf{c}}$.

1.9 ♣ Graphical vector addition/subtraction. (Sections 2.6, 2.8)

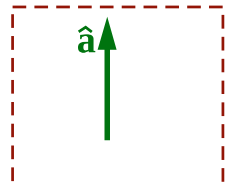


Draw $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ | **Draw** $\vec{\mathbf{a}} +$ -$\vec{\mathbf{b}}$ | **Draw** $\vec{\mathbf{b}} - \vec{\mathbf{a}}$ | **Draw** -$\vec{\mathbf{a}} - \vec{\mathbf{b}}$

1.10 ♣ Insights into orthogonal vectors via drawing. (Section 2.10)

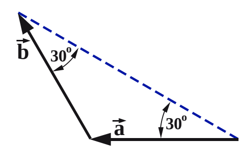
Draw unit vectors $\hat{\mathbf{b}}$ and $\hat{\mathbf{c}}$ such that:

- $\hat{\mathbf{b}}$ is perpendicular to $\hat{\mathbf{a}}$ ($\hat{\mathbf{a}}$ is vertically-upward as shown right).
- $\hat{\mathbf{c}}$ is perpendicular to $\hat{\mathbf{b}}$ but $\hat{\mathbf{c}}$ is neither parallel or perpendicular to $\hat{\mathbf{a}}$.



1.11 ♣ Angle $\angle(\vec{\mathbf{a}}, \vec{\mathbf{b}})$ between vectors. (Section 2.9)

For the figure shown right, determine the numerical value for the angle between vector $\vec{\mathbf{a}}$ and vector $\vec{\mathbf{b}}$.



Result: $\angle(\vec{\mathbf{a}},\ \vec{\mathbf{b}}) =$ ___ °

1.12 ♣ Visual representation of a vector dot-product. (Section 2.9)

Write the ***definition*** of the dot-product of a vector $\vec{\mathbf{a}}$ with a vector $\vec{\mathbf{b}}$.
Include a ***sketch*** with ***each symbol*** in your definition clearly labeled.

Result: $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \triangleq$ ___ ___ ___

Sketch should include $\vec{\mathbf{a}},\ \vec{\mathbf{b}},\ |\vec{\mathbf{a}}|,\ |\vec{\mathbf{b}}|,\ \theta$.

Knowing $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are arbitrary vectors, complete the blanks with $\leq$, $=$, or $\geq$.

When the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is	0°	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$	___	0	(**parallel**)
When the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is	90°	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$	___	0	(**perpendicular**)
When the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is	180°	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$	___	0	(**antiparallel**)
For arbitrary vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$,		$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}$	___	$\vec{\mathbf{b}} \cdot \vec{\mathbf{a}}$	

1.13 ♣ Visual representation of a vector cross-product. (Section 2.10)

Write the ***definition*** of the cross-product of a vector $\vec{\mathbf{a}}$ with a vector $\vec{\mathbf{b}}$.
Include a ***sketch*** with ***each symbol*** in your definition clearly labeled.

Result: $\vec{\mathbf{a}} \times \vec{\mathbf{b}} \triangleq$ ___ ___ ___$(\theta)\ \hat{\mathbf{u}}$

where $\hat{\mathbf{u}}$ is ___

___ and θ is ___

Sketch should include $\vec{\mathbf{a}},\ \vec{\mathbf{b}},\ |\vec{\mathbf{a}}|,\ |\vec{\mathbf{b}}|,\ \theta,\ \hat{\mathbf{u}}$.

Knowing $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are non-zero vectors, complete the blanks with $=$ or $\neq$.

When the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is	0°	$\vec{\mathbf{a}} \times \vec{\mathbf{b}}$	___	$\vec{\mathbf{0}}$	(**parallel**)
When the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is	90°	$\vec{\mathbf{a}} \times \vec{\mathbf{b}}$	___	$\vec{\mathbf{0}}$	(**perpendicular**)
When the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is	180°	$\vec{\mathbf{a}} \times \vec{\mathbf{b}}$	___	$\vec{\mathbf{0}}$	(**antiparallel**)
For arbitrary vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$,		$\vec{\mathbf{a}} \times \vec{\mathbf{b}}$	___	$\vec{\mathbf{b}} \times \vec{\mathbf{a}}$	

1.14 Properties of vector dot/cross-products **Draw/show work.** $\vec{a} \neq \vec{0}$, $\vec{b} \neq \vec{0}$. **(Sections 2.9.1, 2.10)**

When $\vec{a}$ is ***parallel*** to $\vec{b}$,	$\vec{a} \cdot \vec{b} = 0$	True/False	$\vec{a} \times \vec{b} = \vec{0}$	True/False
When $\vec{a}$ is ***perpendicular*** to $\vec{b}$,	$\vec{a} \cdot \vec{b} = 0$	True/False	$\vec{a} \times \vec{b} = \vec{0}$	True/False
For arbitrary vectors $\vec{a}$ and $\vec{b}$,	$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$	True/False	$\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$	True/False

1.15 Dot-products and cross-products via definitions. **Show work.** **(Sections 2.9, 2.10)**

- **Draw** a unit vector $\hat{\mathbf{k}}$ outward-normal to the plane of the paper (perpendicular to $\vec{a}$ and $\vec{b}$).
- **Redraw** each figure to clarify $\angle(\vec{a}, \vec{b})$, the angle between $\vec{a}$ and $\vec{b}$ (useful for dot and cross-product).
- Knowing $|\vec{a}| = 2$ and $|\vec{b}| = 4$, calculate each expressions below (2^+ significant digits) using only the **definitions** of dot-product and cross-product.

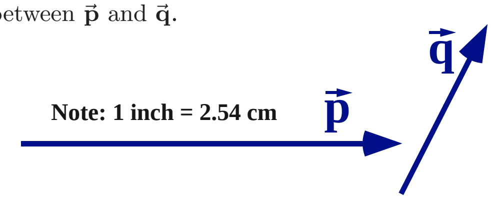
$\angle(\vec{a}, \vec{b}) = \_\_\_°$	$\angle(\vec{a}, \vec{b}) = \_\_\_°$	$\angle(\vec{a}, \vec{b}) = \_\_\_°$	$\angle(\vec{a}, \vec{b}) = \_\_\_°$
$\vec{a} \cdot \vec{b} = \_\_\_$	$\vec{a} \cdot \vec{b} = \_\_\_$	$\vec{a} \cdot \vec{b} = \_\_\_$	$\vec{a} \cdot \vec{b} = \_\_\_$
$\vec{a} \times \vec{b} = -8\,\hat{\mathbf{k}}$	$\vec{a} \times \vec{b} = \_\_\_$	$\vec{a} \times \vec{b} = \_\_\_$	$\vec{a} \times \vec{b} = \_\_\_$

1.16 Visual estimation of vector dot/cross-products. **Show work.** **(Sections 2.9, 2.10)**

Estimate the magnitude of the vector $\vec{q}$ shown below, the angle between $\vec{p}$ and $\vec{q}$, $\vec{p} \cdot \vec{q}$, and the magnitude of $\vec{p} \times \vec{q}$. **Show work** and **redraw** to clarify the angle between $\vec{p}$ and $\vec{q}$.

Result: (Provide numerical results with 1 or more significant digits).

$|\vec{p}| \approx$ 4.0 cm	$|\vec{q}| \approx \_\_\_$ cm	$\angle(\vec{p}, \vec{q}) \approx \_\_\_°$
$\vec{p} \cdot \vec{q} \approx \_\_\_\ \text{cm}^2$		$|\vec{p} \times \vec{q}| \approx \_\_\_\ \text{cm}^2$



1.17 ♣ Vector operations and units. **(Chapter 2)**

Each vector operation below involves a position vector $\vec{r}$ (with **units** of m) and/or a velocity vector $\vec{v}$ (with **units** of $\frac{m}{s}$). Determine whether the operation produces a well-defined scalar or vector or is **undefined**. If well-defined, determine the associated units.

Operation:	$-\vec{r}$	$5\,\vec{v}$	$5\,\frac{m}{s} + \vec{v}$	$\vec{r} + 2\,\vec{r}$	$\vec{r} + \vec{v}$	$5\,\frac{m}{s} \cdot \vec{v}$	$\vec{r} \cdot \vec{v}$	$\vec{r} \times \vec{v}$
Produces:	vector							
Units:	meters							

1.18 ♣ Vector exponentiation: $\vec{v}^2 = \vec{v} \cdot \vec{v}$ and $\vec{v}^3$. **(Section 2.9)**

The following is a reasonable proof that $\vec{v}^2 = \vec{v} \cdot \vec{v}$. **True/False** (if **False**, provide a proof).

$$\vec{v}^2 \triangleq |\vec{v}|^2 \qquad \vec{v} \cdot \vec{v} \underset{(2.2)}{\triangleq} |\vec{v}|\,|\vec{v}|\,\cos(0°) = |\vec{v}|^2 \qquad \vec{v}^2 = \vec{v} \cdot \vec{v}$$

Complete the proof that relates $\vec{v}^3$ to $\vec{v} \cdot \vec{v}$ raised to a real number.

Result:

$$|\vec{v}| \underset{(2.4)}{=} \sqrt{\square \cdot \square} \qquad \vec{v}^3 \triangleq |\vec{v}|^{\square} = \left(\sqrt{\square \cdot \square}\right)^{\square} = (\vec{v} \cdot \vec{v})^{\frac{3}{2}}$$

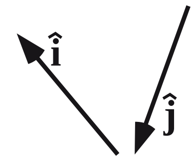
1.19 ♣ $|c\,\hat{\mathbf{a}}_x|$ Calculate vector magnitude with dot products. **(Section 2.9 and Hw 1.18)**

Show how the vector dot-product can be used to show that the magnitude of the vector $c\,\hat{\mathbf{a}}_x$ (c is a positive or **negative** number and $\hat{\mathbf{a}}_x$ is a unit vector) can be written solely in terms of c (without $\hat{\mathbf{a}}_x$).

Result:

$$|c\,\hat{\mathbf{a}}_x| = {}^{+}\sqrt{\square \cdot \square} = {}^{+}\sqrt{c^2 * \square \cdot \square} = {}^{+}\sqrt{c^2} = \text{abs}(c)$$

1.20 †(Challenge) **Magnitude of the vector $\vec{v}$.** Show work. **(Section 2.9)**



Knowing the angle between a unit vector $\hat{\mathbf{i}}$ and unit vector $\hat{\mathbf{j}}$ is 120°, calculate a numerical value for the magnitude of $\vec{v} = 3\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$.

Result: $|\vec{v}| = \sqrt{\boxed{13}}$ Note: The answer is ***not*** $\sqrt{25} = 5$.

1.21 ♣ **Property of scalar triple product.** **(Section 2.11)**

For arbitrary non-zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$: $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$ **Never/Sometimes/Always**

A property of the ***scalar triple product*** is $\vec{a} \cdot \vec{b} \times \vec{a} = 0$. **True/False**.

1.22 ♣ **Property of vector triple cross-product.** **(Sections 2.10, 2.11)**

Complete the following equation: $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\quad) - \vec{c}(\quad)$

For arbitrary vectors $\vec{a}$, $\vec{b}$, $\vec{c}$: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c} + \vec{b} \times (\vec{a} \times \vec{c})$ **True/False** (show work).

1.23 ♣ **Optional: Proof of magnitude of vector cross product property.** **(Sections 2.9, 2.10)**

Letting $\hat{\boldsymbol{\lambda}}$ be a ***unit vector*** and $\vec{v}$ be ***any vector***, prove[1] $|\vec{v} \times \hat{\boldsymbol{\lambda}}|^2 = \vec{v} \cdot \vec{v} - (\vec{v} \cdot \hat{\boldsymbol{\lambda}})^2$.

1.24 ♣ **Form the *unit* vector $\hat{u}$ having the same direction as $c\,\hat{a}_x$.** **(Section 2.4)**

Result: $\hat{u} = \dfrac{\quad}{\quad}\,\hat{a}_x$ Note: $\hat{a}_x$ is a unit vector and c is a non-zero real number, e.g., 3 or -3.

1.25 ♣ **Coefficient of $\hat{u}$ in cross products – definitions and trig functions.** **(Section 2.10)**

The ***cross product*** of vectors $\vec{a}$ and $\vec{b}$ can be written in terms of a real scalar s as $\vec{a} \times \vec{b} = s\,\hat{u}$ where $\hat{u}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ in a direction defined by the **right-hand rule**. The coefficient s of the unit vector $\hat{u}$ is inherently non-negative. **True/False**.

1.26 **Distance between a point and a line via cross-products.** Show work. **(Section 2.10.1)**

Draw a horizontally-right unit vector $\hat{a}_x$ and vertically-upward unit vector $\hat{a}_y$.

Draw a point P and a line L through P that is parallel to $\hat{u} = \frac{3}{5}\,\hat{a}_x + \frac{4}{5}\,\hat{a}_y$.

Draw a point Q whose position vector from point P is $\vec{r} = 5\,\hat{a}_x$ (also draw $\vec{r}$).

Draw the ***distance*** d between Q and L.

Calculate d with the cross-product formula in eqn (3.3).

Result: $\text{d} \underset{(3.3)}{=} \quad = \boxed{4}$ $\text{d} \underset{(3.3)}{=} \quad = \boxed{4}$

1.27 ♣ **Ranges of angles from dot-product and cross-product calculations.** **(Sections 2.9, 2.10)**

Quantity	Numerical range of values
$c = \hat{a} \cdot \hat{b}$ (assume $\hat{a}$ and $\hat{b}$ are known so a numerical value for $\hat{a} \cdot \hat{b}$ can be calculated).	$\quad \le c \le \quad$
$s = \lvert\hat{a} \times \hat{b}\rvert$ (assume $\hat{a}$ and $\hat{b}$ are known so a numerical value for $\lvert\hat{a} \times \hat{b}\rvert$ can be calculated).	$\quad \le s \le \quad$
Angle θ_c between $\hat{a}$ and $\hat{b}$ that can be uniquely determined **solely** from c.	$\quad^\circ \le \theta_c \le \quad^\circ$
Angle θ_s between $\hat{a}$ and $\hat{b}$ that can be uniquely determined **solely** from s.	$\quad^\circ \le \theta_s \le \quad^\circ$
Angle θ between $\hat{a}$ and $\hat{b}$, i.e., $\theta = \angle(\hat{a}, \hat{b})$	$\quad^\circ \le \theta \le \quad^\circ$

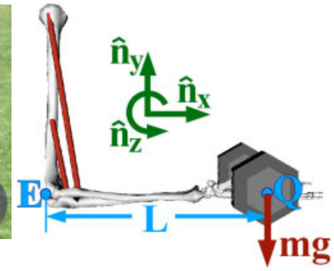
Note: The range of θ_s is smaller than the range for θ. Hence, s and θ_s are insufficient to correctly calculate θ.
What this means: Use the **dot-product** $\cdot$ to calculate an angle θ from two given/known vectors $\hat{a}$ and $\hat{b}$.

[1] One way to prove this is to write $(\vec{v} \times \hat{\boldsymbol{\lambda}})^2 = (\vec{v} \times \hat{\boldsymbol{\lambda}}) \cdot (\vec{v} \times \hat{\boldsymbol{\lambda}}) \underset{(2.13)}{=} \vec{v} \cdot [\hat{\boldsymbol{\lambda}} \times (\vec{v} \times \hat{\boldsymbol{\lambda}})]$ and then use the vector triple cross-product property $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ from Section 2.10. Alternatively, it is helpful to write $\vec{v} = \vec{v}_\perp \hat{\boldsymbol{\lambda}}_\perp + \vec{v}_{||} \hat{\boldsymbol{\lambda}}$ where $\vec{v}_\perp \hat{\boldsymbol{\lambda}}_\perp$ is the component of $\vec{v}$ that is perpendicular to $\hat{\boldsymbol{\lambda}}$ and $\vec{v}_{||} \hat{\boldsymbol{\lambda}}$ is the component of $\vec{v}$ that is parallel to $\hat{\boldsymbol{\lambda}}$.

1.28 Biomechanics: Gravity moment for curling $\vec{\mathbf{M}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$ Show work. (Section 2.10)

The figures to the right show an athlete curling a dumbbell (modeled as a particle Q of mass m). The forearm connects to the upper arm at the elbow (point E). Orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ are directed with $\hat{\mathbf{n}}_y$ vertically upward and $\hat{\mathbf{n}}_x$ from E to Q.

Description	Symbol	Type
Earth's gravitational constant	g	$g \approx 9.8 \frac{\text{m}}{\text{s}^2}$
Mass of dumbbell Q	m	Positive constant
Distance between elbow E and Q	L	Positive constant

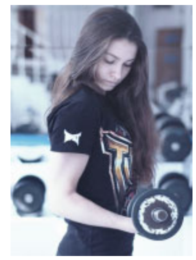


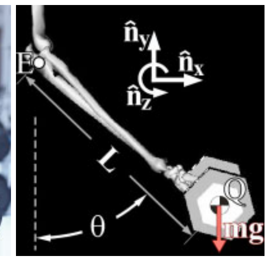
The moment of gravity forces on Q about E is $\vec{\mathbf{M}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$ where $\vec{\mathbf{F}} = \text{-m}\, g\, \hat{\mathbf{n}}_y$. Express $\vec{\mathbf{M}}$ in terms of m, g, L, $\hat{\mathbf{n}}_z$.

Result: $\vec{\mathbf{M}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} =$ ______

Now consider the forearm making an angle θ with downward vertical. Form $\vec{\mathbf{M}}$ and its magnitude $|\vec{\mathbf{M}}|$. Determine the values of θ $(0 \le \theta \le 135°)$ that produce maximum and minimum $|\vec{\mathbf{M}}|$.

To simplify $|\vec{\mathbf{M}}|$, note m, g, L are positive and for $0 \le \theta \le 135°$, $\sin(\theta) \ge 0$.





Result: (in terms of m, g, L, θ, $\hat{\mathbf{n}}_z$).

$\vec{\mathbf{M}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} =$ ______ $\quad |\vec{\mathbf{M}}| =$ ______

Max $|\vec{\mathbf{M}}| =$ ______ at $\theta =$ ______°

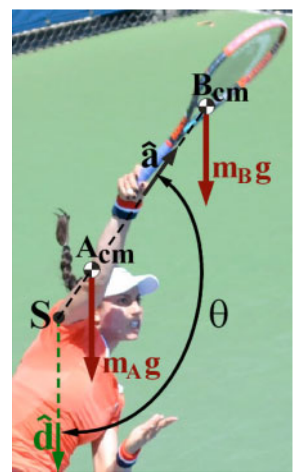
Min $|\vec{\mathbf{M}}| =$ ______ at $\theta =$ ______°

Optional: Modeling the elbow as a revolute joint, draw a ***free-body diagram (FBD)*** of the system consisting of the forearm and dumbbell.

1.29 Biomechanics: Gravity force and moment for tennis $\vec{\mathbf{M}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$ Show work. (Section 2.10)

Shown right is an athlete whose arm A swings a tennis racquet B. Point S (shoulder), A_{cm} (A's center of mass), and B_{cm} (B's center of mass) lie along a line parallel to a unit vector $\hat{\mathbf{a}}$. The unit vector $\hat{\mathbf{d}}$ is vertically-downward ↓.

Description	Symbol	Type
Earth's gravitational constant	g	$g \approx 9.8 \frac{\text{m}}{\text{s}^2}$
Mass of A, mass of B	$\text{m}_A,\ \text{m}_B$	Positive constants
Distances between S and A_{cm} and S and B_{cm}	$L_A,\ L_B$	Positive constants
Angle between $\hat{\mathbf{a}}$ and $\hat{\mathbf{d}}$	θ	$0 \le \theta \le 180°$



- Form $\vec{\mathbf{F}}_{\text{gravity}}$ (the net force on A and B due to Earth's gravity).
- Form $|\vec{\mathbf{M}}|$ (the magnitude of the moment of those gravity forces about S).

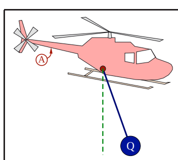
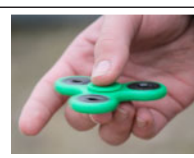
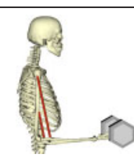
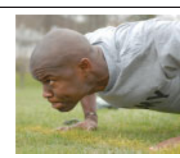
Note: $\vec{\mathbf{M}} = {}^{S}\vec{\mathbf{r}}^{A_{cm}} \times \text{m}_A\, g\, \hat{\mathbf{d}} \ + \ {}^{S}\vec{\mathbf{r}}^{B_{cm}} \times \text{m}_B\, g\, \hat{\mathbf{d}}$.

Result: $\vec{\mathbf{F}}_{\text{gravity}} = ($ ______ $)\, \hat{\mathbf{d}}$

$|\vec{\mathbf{M}}| =$ ______

Optional: Modeling the athlete grip of the racquet as a weld, draw a ***free-body diagram (FBD)*** of the racquet. Next, choose a model for the shoulder joint and draw a ***FBD*** of the system consisting of the arm and racquet.

1.30 †Optional: Draw the *free-body diagram (FBD)* for each object below.

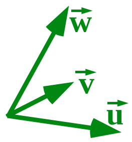
					
Helicopter A Hw 12.24	**Skidding vehicle** Hw 15.6	**Fidget spinner** B Hw 15.12, 15.13	**Forearm** B Hw 15.19	**Left balloon** Hw 16.11	**Athlete** Hw 17.20

1.31 ♣ **Using vector identities to simplify expressions** **(refer to Homework 1.14)**

One reason to treat vectors as ***basis-independent*** quantities is to simplify vector expressions ***without*** resolving the vectors into orthogonal "$\vec{\mathbf{x}}, \vec{\mathbf{y}}, \vec{\mathbf{z}}$" or "$\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$" components. Simplify the following vector expressions using mathematical properties of dot-products and cross-products.

Express results in terms of dot-products $\cdot$ and cross-products $\times$ of the arbitrary vectors $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$.

$\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$ are not necessarily orthogonal or co-planar.



Vector expression	Simplified vector expression
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \times (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	$\square\; \vec{\mathbf{u}} \times \vec{\mathbf{v}}$
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	$\square\; \vec{\mathbf{u}}^2 - \square\; \vec{\mathbf{v}}^2 + \square\; \vec{\mathbf{u}} \cdot \vec{\mathbf{v}}$
$(\vec{\mathbf{u}} - \vec{\mathbf{v}}) \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}})$	$\square - \square$
$(3\,\vec{\mathbf{u}} - 2\,\vec{\mathbf{v}}) \times (\vec{\mathbf{u}} + \vec{\mathbf{v}}) \cdot (2\,\vec{\mathbf{u}} - 7\,\vec{\mathbf{v}})$	$\square$
$(\vec{\mathbf{u}} + \vec{\mathbf{v}}) \times (\vec{\mathbf{v}} + 2\,\vec{\mathbf{w}}) \cdot (\vec{\mathbf{w}} + 2\,\vec{\mathbf{u}})$	$\square\; \vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$

1.32 ♣ **Vector concepts: Solving a vector equation?** **(Section 2.9.3)**

Shown right is a vector equation and a questionable process that solves for v_x ($\widehat{\mathbf{a}}_{\mathrm{x}}$ is a unit vector and v_x, $\dot{\theta}$, R are scalars).

$$v_x\, \widehat{\mathbf{a}}_{\mathrm{x}} = \dot{\theta}\, R\, \widehat{\mathbf{a}}_{\mathrm{x}}$$

$$v_x = \dot{\theta}\, R\, \frac{\widehat{\mathbf{a}}_{\mathrm{x}}}{\widehat{\mathbf{a}}_{\mathrm{x}}} = \dot{\theta}\, R$$

This is a valid process to solve for v_x. **True/False**.

Explain: ______

1.33 **Change a vector equation to scalar equations.** Show work. **(Section 2.9.3)**

Draw three mutually orthogonal unit vectors $\widehat{\mathbf{p}}, \widehat{\mathbf{q}}, \widehat{\mathbf{r}}$.

Use a vector operation (e.g., $+$, $*$, $\cdot$, $\times$) to change the **vector** equation $(2x-4)\,\widehat{\mathbf{p}} = \vec{\mathbf{0}}$ into one **scalar** equation and subsequently solve the scalar equation for x.

Result: $(2x-4)\,\widehat{\mathbf{p}} = \vec{\mathbf{0}} \quad \overset{??}{\Rightarrow} \quad (2x-4) = 0 \quad \Rightarrow \quad x = 2$

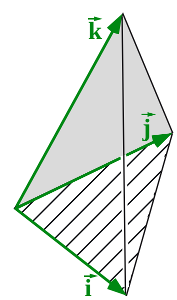
Show ***every*** vector operation (e.g., $+$, $*$, $\cdot$, $\times$) that changes the following **vector** equation into three **scalar** equations and subsequently solve the scalar equations for x, y, z.

$$(2\,x-4)\,\widehat{\mathbf{p}} + (3\,y-9)\,\widehat{\mathbf{q}} + (4\,z-16)\,\widehat{\mathbf{r}} = \vec{\mathbf{0}}$$

Result: $(2\,x-4) = 0 \qquad (3\,y-9) = 0 \qquad (\;\square\;) = 0$

$x = 2 \qquad y = 3 \qquad z = 4$

† The figure to the right shows three **non-orthogonal**, non-coplanar vectors $\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$. Show every vector operation that changes the following **vector** equation into three uncoupled **scalar** equations and subsequently solve those scalar equations for x, y, z.



$$(2\,x-4)\,\vec{\mathbf{i}} + (3\,y-9)\,\vec{\mathbf{j}} + (4\,z-16)\,\vec{\mathbf{k}} = \vec{\mathbf{0}}$$

Result: $(2\,x-4) = 0 \qquad (3\,y-9) = 0 \qquad (\;\square\;) = 0$

$x = 2 \qquad y = 3 \qquad z = 4$

Hint: think $\times$ $\cdot$, not matrix algebra.

1.34 ♣ Number of independent scalar equations from 1 vector equation. (Section 2.9.3)

The **vector** equation shown right is useful for static analyses of a system S. $\vec{\mathbf{F}}^S = \vec{\mathbf{0}}$

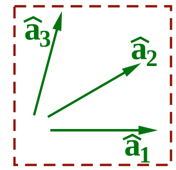
In the table to the right, box **all** integers that could be equal to the number of ***independent scalar*** equations produced by the previous vector equation. Hint: Hw 1.33. Related Hw 17.15.

System type	Integer(s)
1D (line)	0 1 2 3 4^+
2D (planar)	0 1 2 3 4^+
3D (spatial)	0 1 2 3 4^+

Note: 1D/linear means $\vec{\mathbf{F}}^S$ can be expressed in terms of one vector $\hat{\mathbf{i}}$.
2D/planar means $\vec{\mathbf{F}}^S$ can be expressed in terms of two non-parallel unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$.
3D/spatial means $\vec{\mathbf{F}}^S$ can be expressed in terms of three non-coplanar unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$.

1.35 ♣ Vector concepts: Solving a vector equation (just circle true or false and fill-in the blank).

Consider the following vector equation written in terms of the scalars x, y, z and three unique non-orthogonal **coplanar** unit vectors $\hat{\mathbf{a}}_1$, $\hat{\mathbf{a}}_2$, $\hat{\mathbf{a}}_3$.



$$(2\,x - 4)\;\hat{\mathbf{a}}_1 \;+\; (3\,y - 9)\;\hat{\mathbf{a}}_2 \;+\; (4\,z - 16)\;\hat{\mathbf{a}}_3 \;=\; \vec{\mathbf{0}}$$

The **unique** solution to this vector equation is $x = 2,\; y = 3,\; z = 4$. **True/False**.

Explain: $\hat{\mathbf{a}}_2$ can be expressed in terms of $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_3$ (i.e., $\hat{\mathbf{a}}_2$ is a linear combination of $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_3$).
Hence the vector equation produces ______ linearly independent scalar equations.

1.36 ♣ Gibbs (≈1900 AD) vectors revolutionizes Euclidean geometry (300 BC). (Sections 2.9.2, 2.10.1, 2.11.1)

For each geometrical quantity shown right, circle the vector operation(s) (dot-product, cross-product, or both) that is ***most*** useful for their calculation.

Length: $\cdot$ $\times$	Angle: $\cdot$ $\times$
Area: $\cdot$ $\times$	Volume: $\cdot$ $\times$

1.37 ♣ Order of operations with vector dot products (•) and cross products (×). (Chapter 2)

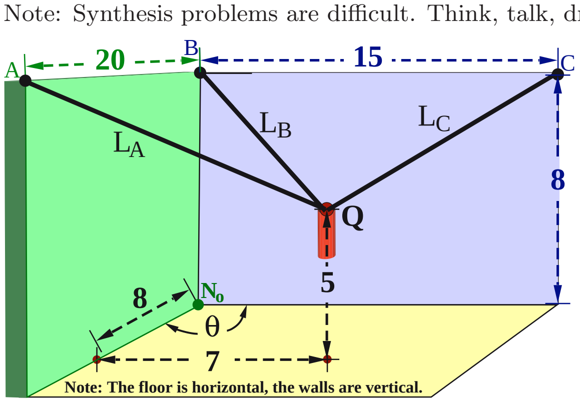
Create a valid expression by adding parentheses to each expression or ~~cross-out~~ the expression if it is inherently invalid.
Example: $3 * \vec{\mathbf{a}} + \vec{\mathbf{b}} \;\Rightarrow\; (3 * \vec{\mathbf{a}}) + \vec{\mathbf{b}}$.

$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} + \vec{\mathbf{c}}$	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{c}}$	$\vec{\mathbf{a}} + 5 \times \vec{\mathbf{c}}$
$\vec{\mathbf{a}} \times \vec{\mathbf{b}} + \vec{\mathbf{c}}$	$\vec{\mathbf{a}} \times \vec{\mathbf{b}} \cdot \vec{\mathbf{c}}$	$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \cdot \vec{\mathbf{c}}$

1.38 † Microphone cable lengths (non-orthogonal walls) "It's just geometry". Show work.

A microphone Q is attached to three pegs A, B, C by three cables. Knowing the peg locations, microphone location, and the angle θ between the vertical walls, express L_A, L_B, L_C solely in terms of numbers and θ. Next, complete the table by calculating L_B when $\theta = 120°$.

Hint: To do this **efficiently**, use one set of **non-orthogonal** unit vectors (do **not** use an **orthogonal** set of unit vectors).
Hint: Use the distributive property of the vector dot-product as shown in Section 2.9.1 and Homework 2.4.
Note: Synthesis problems are difficult. Think, talk, draw, sleep, walk, get help, ... (if necessary, read Section 3.4).



Distance between A and B	20 m
Distance between B and C	15 m
Distance between N_o and B	8 m
Distance along back wall (see picture)	7 m
Q's height above N_o	5 m
Distance along side wall (see picture)	8 m
L_A: Length of cable joining A and Q	16.9 m
L_B: Length of cable joining B and Q	8.1 m
L_C: Length of cable joining C and Q	14.2 m

Result: $L_A = \sqrt{202 - \_\_\_\_ \cos(\theta)}$ $\quad L_B = \sqrt{122 + 112\cos(\theta)}$ $\quad L_C = \sqrt{\_\_\_\_ - 128\,\_\_\_\_}$

Vocabulary: In this ***inverse kinematics*** analysis, the position of "end-effector" Q is known and you determine the cable lengths.

- Knowing β is defined as the angle between lines $\overline{B\,N_o}$ and $\overline{B\,Q}$, show $\beta \approx$ 68.33°.

Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

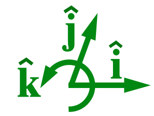
2.1 ♣ Right-handed, orthogonal, unit vectors. (Section 4.1)

Draw a set of right-handed orthogonal (mutually perpendicular) unit vectors consisting of $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$. In other words, draw $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ so that $\widehat{\mathbf{n}}_y$ is perpendicular (orthogonal) to $\widehat{\mathbf{n}}_x$ and $\widehat{\mathbf{n}}_z = \widehat{\mathbf{n}}_x \times \widehat{\mathbf{n}}_y$.

2.2 ♣ Adding and subtracting vectors. (Sections 2.6, 2.8)

Given: Vectors $\vec{\mathbf{p}}$ and $\vec{\mathbf{q}}$ expressed in terms of unit vectors $\widehat{\mathbf{i}}, \widehat{\mathbf{j}}, \widehat{\mathbf{k}}$. Form the vector sums and differences below.

$$\vec{\mathbf{p}} = a\,\widehat{\mathbf{i}} + b\,\widehat{\mathbf{j}} + c\,\widehat{\mathbf{k}} \qquad \vec{\mathbf{q}} = x\,\widehat{\mathbf{i}} + y\,\widehat{\mathbf{j}} + z\,\widehat{\mathbf{k}}$$



Result:

$$\vec{\mathbf{p}} + \vec{\mathbf{q}} = (a+x)\,\widehat{\mathbf{i}} + (\_\_\_)\,\widehat{\mathbf{j}} + (\_\_\_)\,\widehat{\mathbf{k}} \qquad \vec{\mathbf{p}} - \vec{\mathbf{q}} = (a-x)\,\widehat{\mathbf{i}} + (\_\_\_)\,\widehat{\mathbf{j}} + (\_\_\_)\,\widehat{\mathbf{k}}$$

2.3 ♣ Words: Physical vectors and column matrices. (Section 2.12, Hw 1.2)

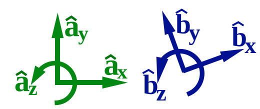
True/False As defined by Gibbs and for $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$, physical vectors have magnitude and direction.

True/False In math (linear algebra), a column matrix is called a "vector".

True/False The physical vector $\widehat{\mathbf{a}}_x + 2\,\widehat{\mathbf{a}}_y + 3\,\widehat{\mathbf{a}}_z$ can be written $[\widehat{\mathbf{a}}_x \;\; \widehat{\mathbf{a}}_y \;\; \widehat{\mathbf{a}}_z] * \begin{bmatrix}1\\2\\3\end{bmatrix}$.
Note: $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$ are the orthogonal unit vectors shown below.

True/False The physical vector $\widehat{\mathbf{a}}_x + 2\,\widehat{\mathbf{a}}_y + 3\,\widehat{\mathbf{a}}_z$ is equal to the column matrix $\begin{bmatrix}1\\2\\3\end{bmatrix}$.

True/False $\widehat{\mathbf{a}}_x + 2\,\widehat{\mathbf{a}}_y + 3\,\widehat{\mathbf{a}}_z \;+\; 4\,\widehat{\mathbf{b}}_x + 5\,\widehat{\mathbf{b}}_y + 6\,\widehat{\mathbf{b}}_z = \begin{bmatrix}1\\2\\3\end{bmatrix} + \begin{bmatrix}4\\5\\6\end{bmatrix} = \begin{bmatrix}5\\7\\9\end{bmatrix}$
($\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$ and $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ are shown right).



- Complete the following statement with one equal sign $=$ and one not-equal sign $\neq$.

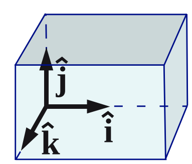
$$\widehat{\mathbf{a}}_x + 2\,\widehat{\mathbf{a}}_y + 3\,\widehat{\mathbf{a}}_z \;\_\_\_\; [\widehat{\mathbf{a}}_x \;\; \widehat{\mathbf{a}}_y \;\; \widehat{\mathbf{a}}_z] * \begin{bmatrix}1\\2\\3\end{bmatrix} \;\_\_\_\; \begin{bmatrix}1\\2\\3\end{bmatrix}$$

For more on vector spaces and physical vectors vs. column matrices, see Malcolm Shuster's "*Tutorial on Vectors and Attitude*" in IEEE Control Systems Magazine, Volume 29 issue 2, April 2009. Historical note: Physical vectors were invented by Gibbs circa 1890 and he co-opted the word vector from part of Hamilton's quaternion invented in 1844. Matrix algebra was invented circa 1850. The word **vector** has multiple related meanings, which causes ongoing confusion. As Shuster noted, the "failure to recognize the difference between physical vectors and column vectors has led sometimes to errors in spacecraft mission support software."

2.4 ♣ Dot products with orthogonal unit vectors. (Sections 2.9, 2.9.4)

Given: Vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ expressed in terms of right-handed orthogonal unit vectors $\widehat{\mathbf{i}}, \widehat{\mathbf{j}}, \widehat{\mathbf{k}}$, with:

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \underbrace{(a\,\widehat{\mathbf{i}} + b\,\widehat{\mathbf{j}} + c\,\widehat{\mathbf{k}})}_{\vec{\mathbf{v}}} \cdot \underbrace{(x\,\widehat{\mathbf{i}} + y\,\widehat{\mathbf{j}} + z\,\widehat{\mathbf{k}})}_{\vec{\mathbf{w}}}$$



- Use the ***distributive property*** for dot products to write $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$ in terms of $\widehat{\mathbf{i}} \cdot \widehat{\mathbf{i}}$, $\widehat{\mathbf{i}} \cdot \widehat{\mathbf{j}}$, etc.

Next, use the ***definition*** of the dot product to calculate $\widehat{\mathbf{i}} \cdot \widehat{\mathbf{i}}$, $\widehat{\mathbf{i}} \cdot \widehat{\mathbf{j}}$, etc. (below-right).

Result:

$$\begin{aligned}\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} &= a\,x\;\widehat{\mathbf{i}} \cdot \widehat{\mathbf{i}} + a\,y\;\widehat{\mathbf{i}} \cdot \widehat{\mathbf{j}} + \_\_\_\;\widehat{\mathbf{i}} \cdot \widehat{\mathbf{k}} \\ &+ b\,x\;\widehat{\mathbf{j}} \cdot \widehat{\mathbf{i}} + b\,y\;\_\_\_ \cdot \_\_\_ + \_\_\_\;\_\_\_ \cdot \_\_\_ \\ &+ c\,x\;\widehat{\mathbf{k}} \cdot \widehat{\mathbf{i}} + c\,y\;\_\_\_ \cdot \_\_\_ + \_\_\_\;\_\_\_ \cdot \_\_\_\end{aligned}$$

$\widehat{\mathbf{i}} \cdot \widehat{\mathbf{i}} = 1$	$\widehat{\mathbf{i}} \cdot \widehat{\mathbf{j}} = \_\_\_$	$\widehat{\mathbf{i}} \cdot \widehat{\mathbf{k}} = \_\_\_$
$\widehat{\mathbf{j}} \cdot \widehat{\mathbf{i}} = 0$	$\widehat{\mathbf{j}} \cdot \widehat{\mathbf{j}} = \_\_\_$	$\widehat{\mathbf{j}} \cdot \widehat{\mathbf{k}} = \_\_\_$
$\widehat{\mathbf{k}} \cdot \widehat{\mathbf{i}} = 0$	$\widehat{\mathbf{k}} \cdot \widehat{\mathbf{j}} = \_\_\_$	$\widehat{\mathbf{k}} \cdot \widehat{\mathbf{k}} = \_\_\_$

- Simplify $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$ and use its special dot-product formula for the calculations that follow.

Result: $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = a\,x + b\,y + \_\_\_$

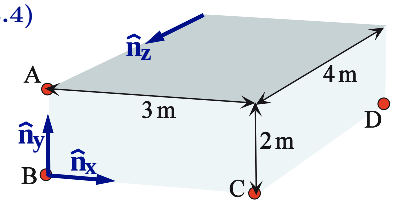
Use this special dot-product formula to calculate $\vec{\mathbf{v}} \cdot \vec{\mathbf{w}}$ when $\widehat{\mathbf{i}}, \widehat{\mathbf{j}}, \widehat{\mathbf{k}}$ are **orthogonal unit** vectors.

Given	Calculate		
$\vec{\mathbf{p}} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}$	$\vec{\mathbf{p}} \cdot \vec{\mathbf{q}} = 2\,x + 3\,y + \square\, z$	$\vec{\mathbf{p}} \cdot \vec{\mathbf{p}} = 29$	$\lvert\vec{\mathbf{p}}\rvert = \sqrt{29}$
$\vec{\mathbf{q}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$	$\vec{\mathbf{p}} \cdot \vec{\mathbf{r}} = \square$	$\vec{\mathbf{q}} \cdot \vec{\mathbf{q}} = x^2 + \square + \square$	$\lvert\vec{\mathbf{q}}\rvert = \sqrt{\square}$
$\vec{\mathbf{r}} = 5\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}} + 7\,\hat{\mathbf{k}}$	$\vec{\mathbf{q}} \cdot \vec{\mathbf{r}} = \square$	$\vec{\mathbf{r}} \cdot \vec{\mathbf{r}} = \square$	$\lvert\vec{\mathbf{r}}\rvert = \sqrt{110}$

2.5 ♣ Perpendicular vectors. **(Note: $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are orthogonal unit vectors). (Section 2.9)**

Draw two vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ that are perpendicular. .
When $\vec{\mathbf{v}} = x\,\hat{\mathbf{i}} + 2\,\hat{\mathbf{j}} + 3\,\hat{\mathbf{k}}$ is perpendicular to $\vec{\mathbf{w}} = 4\,\hat{\mathbf{i}} + 5\,\hat{\mathbf{j}} + 6\,\hat{\mathbf{k}}$, $x = \square$.

2.6 Dot products to calculate distance and angles. **(Sections 2.9, 3.4)**



The figure to the right shows a block with sides of length 2 m, 3 m, 4 m and points A, B, C located at corners. Right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ are directed with $\hat{\mathbf{n}}_x$ from B to C and $\hat{\mathbf{n}}_y$ from B to A.

(a) Express $\vec{\mathbf{r}}$ (position from A to C) in terms of $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and find a numerical value for $\lvert\vec{\mathbf{r}}\rvert^2$.
Next calculate the distance d between A to C (magnitude of $\vec{\mathbf{r}}$).
Result: $\vec{\mathbf{r}} = \square\,\hat{\mathbf{n}}_x - \square\,\hat{\mathbf{n}}_y$ $\quad \lvert\vec{\mathbf{r}}\rvert^2 \underset{(2.4)}{=} \vec{\mathbf{r}} \cdot \vec{\mathbf{r}} = \square\ \text{m}^2$ $\quad \text{d} = \sqrt{\square}\ \text{m}$

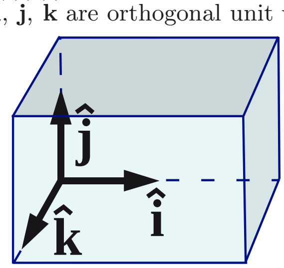
(b) Calculate the unit vector $\hat{\mathbf{u}}$ directed from A to C and the unit vector $\hat{\mathbf{v}}$ directed from A to D.
Result: $\hat{\mathbf{u}} = \dfrac{3\,\hat{\mathbf{n}}_x - \square\,\hat{\mathbf{n}}_y}{\sqrt{\square}}$ $\quad \hat{\mathbf{v}} = \dfrac{\square\,\hat{\mathbf{n}}_x - \square\,\hat{\mathbf{n}}_y - \square\,\hat{\mathbf{n}}_z}{\sqrt{\square}}$

(c) Calculate $\angle BAC$ (angle between line $\overline{AB}$ and line $\overline{AC}$) and $\angle CAD$ (angle between line $\overline{AC}$ and line $\overline{AD}$).
Result: $\angle BAC = \square^\circ$ $\quad \angle CAD = 47.97^\circ$

2.7 ♣ Construct a unit vector $\hat{\mathbf{u}}$ in the direction of each vector given below. **(Section 2.9.2)**

Vector	Unit vector $\hat{\mathbf{u}}$
$3\,\hat{\mathbf{i}}$	$\hat{\mathbf{i}}$
$-3\,\hat{\mathbf{i}}$	$\square$
$3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}}$	$\dfrac{\square - \square}{\square}$
$3\,\hat{\mathbf{i}} - 4\,\hat{\mathbf{j}} + 12\,\hat{\mathbf{k}}$	$\dfrac{\square}{\square}$
$c\,\hat{\mathbf{i}}$ c is a real non-zero number	$\dfrac{c}{\square}\,\hat{\mathbf{i}}$ or $\text{sign}(c)\,\hat{\mathbf{i}}$

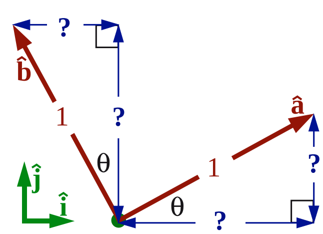
Note: $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are orthogonal unit vectors.



Ensure your last answer agrees with your first two answers, e.g., if $c = 3$ or $c = -3$.

2.8 ♣ Vector components: Sine and cosine. **(Section 1.4)**

- **Replace** each **?** in the figure to the right with $\sin(\theta)$ or $\cos(\theta)$.
- Use vector addition to express $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ in terms of $\sin(\theta)$, $\cos(\theta)$, $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$.

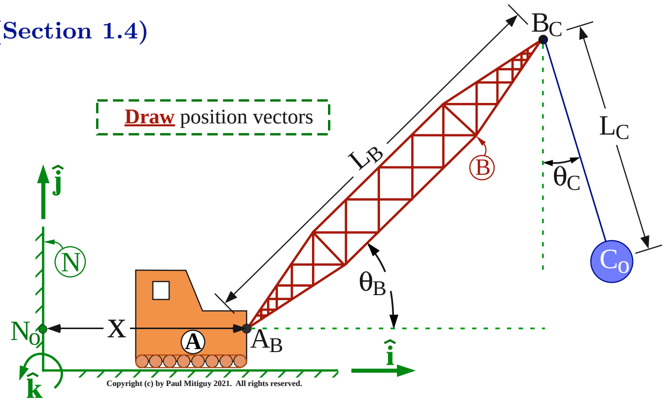


Result: $\hat{\mathbf{a}} = \square\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$
$\hat{\mathbf{b}} = \square\,\hat{\mathbf{i}} + \cos(\theta)\,\hat{\mathbf{j}}$

Reminder:
SohCahToa

2.9 ♣ Vector components for a crane-boom. (Section 1.4)

Shown right is a crane whose cab A supports a boom B that swings a wrecking ball C_o.



Right-handed orthogonal unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are directed with $\hat{\mathbf{i}}$ horizontally-right, $\hat{\mathbf{j}}$ vertically-upward, and $\hat{\mathbf{k}}$ outward-normal to the plane containing points N_o, A_B, B_C, C_o.

Draw each position vector listed below and then use your knowledge of sine/cosine to resolve these vectors into $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ components.

Position from N_o to A_B $\quad {}^{N_o}\vec{\mathbf{r}}^{A_B} = \square\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$

Position from A_B to B_C $\quad {}^{A_B}\vec{\mathbf{r}}^{B_C} = \square\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$

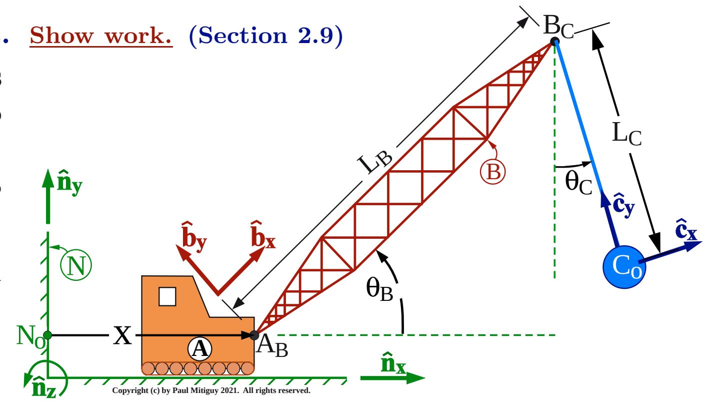
Position from B_C to C_o $\quad {}^{B_C}\vec{\mathbf{r}}^{C_o} = \square\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$

Position from N_o to B_C $\quad {}^{N_o}\vec{\mathbf{r}}^{B_C} = [\square + \square]\,\hat{\mathbf{i}} + \square\,\hat{\mathbf{j}}$

Position from N_o to C_o $\quad {}^{N_o}\vec{\mathbf{r}}^{C_o} = [\square]\,\hat{\mathbf{i}} + [L_B\,\sin(\theta_B) - L_C\,\cos(\theta_C)]\,\hat{\mathbf{j}}$

2.10 Dot products and distance calculations. Show work. (Section 2.9)

Shown right is a crane whose cab A supports a boom B that swings a wrecking ball C_o. To prevent the wrecking ball from hitting a car, the distance between N_o and point B_C (the tip of the boom) must be controlled.



To start this problem, express $\vec{\mathbf{r}}$ (the position vector from N_o to B_C) in terms of x, L_B, $\hat{\mathbf{n}}_x$, $\hat{\mathbf{b}}_x$.

Result: $\vec{\mathbf{r}} = \square\,\hat{\mathbf{n}}_x + \square\,\hat{\mathbf{b}}_x$

- ***Without*** resolving $\vec{\mathbf{r}}$ into $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$ components, use $|\vec{\mathbf{r}}| = \sqrt{\vec{\mathbf{r}} \cdot \vec{\mathbf{r}}}$ [from equation (3.1)] and the distributive property to calculate the distance between N_o and B_C in terms of x, L_B, θ_B.

Result: (if stumped, hint below).[1] **Optional:** Calculate $|\vec{\mathbf{r}}|$ when $x = 20$ m, $L_B = 10$ m, $\theta_B = 30°$.

$$\text{Distance between } N_o \text{ and } B_C: \quad |\vec{\mathbf{r}}| = \sqrt{\square^2 + \square^2 + 2\,x\,L_B\,\cos(\theta_B)} \approx 29.1 \text{ m}$$

- Homework 2.9 showed $\vec{\mathbf{r}}$ can be expressed as $\vec{\mathbf{r}} = [x + L_B\,\cos(\theta_B)]\,\hat{\mathbf{n}}_x + L_B\,\sin(\theta_B)\,\hat{\mathbf{n}}_y$.

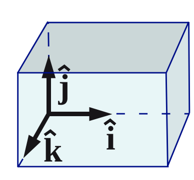
Use this expression to verify your previous result for $|\vec{\mathbf{r}}| = \sqrt{\vec{\mathbf{r}} \cdot \vec{\mathbf{r}}}$.

Result: $|\vec{\mathbf{r}}|$ simplifies to the previous result but uses an inefficient process and $\sin^2(\theta_B) + \cos^2(\theta_B) = 1$.

- **Optional:** Calculate the distance between N_o and C_o in terms of x, L_B, L_C, θ_B, and θ_C.

Result: $|{}^{N_o}\vec{\mathbf{r}}^{C_o}| = \sqrt{x^2 + L_B^2 + L_C^2 + 2\,x\,L_B\,\cos(\theta_B) + 2\,x\,L_C\,\sin(\theta_C) - 2\,L_B\,L_C\,\sin(\theta_B - \theta_C)}$

2.11 ♣ Cross products with right-handed orthogonal unit vectors. (Section 2.10)



Given: Vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ expressed in terms of right-handed orthogonal unit vectors $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, with:

$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \underbrace{(a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}})}_{\vec{\mathbf{v}}} \times \underbrace{(x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}})}_{\vec{\mathbf{w}}}$$

---

[1] Hint: The distributive property for vector dot-multiplication is $(\vec{\mathbf{a}} + \vec{\mathbf{b}}) \cdot (\vec{\mathbf{c}} + \vec{\mathbf{d}}) = \vec{\mathbf{a}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{a}} \cdot \vec{\mathbf{d}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{c}} + \vec{\mathbf{b}} \cdot \vec{\mathbf{d}}$. Use the distributive property to express $\vec{\mathbf{r}} \cdot \vec{\mathbf{r}}$ in terms of x, L_B, and $\hat{\mathbf{n}}_x \cdot \hat{\mathbf{b}}_x$. Thereafter, use the ***dot-product definition*** of $(\hat{\mathbf{n}}_x \cdot \hat{\mathbf{b}}_x)$ to form $\vec{\mathbf{r}} \cdot \vec{\mathbf{r}} = \square^2 + \square^2 + 2\,x\,L_B\,(\hat{\mathbf{n}}_x \cdot \hat{\mathbf{b}}_x) \underset{(2.2)}{=} \square^2 + \square^2 + 2\,x\,L_B\,\cos(\square)$.

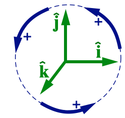
- Use the ***distributive property*** for cross products to write $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ in terms of $\hat{\mathbf{i}} \times \hat{\mathbf{i}}$, $\hat{\mathbf{i}} \times \hat{\mathbf{j}}$, etc. Next, use the ***definition*** of the cross product to calculate $\hat{\mathbf{i}} \times \hat{\mathbf{i}}$, $\hat{\mathbf{i}} \times \hat{\mathbf{j}}$, etc. (below-right).

Result:

$$\begin{aligned} \vec{\mathbf{v}} \times \vec{\mathbf{w}} = \; & a\,x\; \hat{\mathbf{i}} \times \hat{\mathbf{i}} \;+\; a\,y\; \hat{\mathbf{i}} \times \hat{\mathbf{j}} \;+\; \square\; \hat{\mathbf{i}} \times \hat{\mathbf{k}} \\ & +\; b\,x\; \hat{\mathbf{j}} \times \hat{\mathbf{i}} \;+\; b\,y\; \square \times \square \;+\; \square\; \square \times \square \\ & +\; c\,x\; \hat{\mathbf{k}} \times \hat{\mathbf{i}} \;+\; c\,y\; \square \times \square \;+\; \square\; \square \times \square \end{aligned}$$

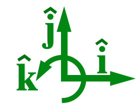
$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \vec{\mathbf{0}}$	$\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$	$\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \text{-}\hat{\mathbf{j}}$
$\hat{\mathbf{j}} \times \hat{\mathbf{i}} = \square$	$\hat{\mathbf{j}} \times \hat{\mathbf{j}} = \square$	$\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \square$
$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \square$	$\hat{\mathbf{k}} \times \hat{\mathbf{j}} = \square$	$\hat{\mathbf{k}} \times \hat{\mathbf{k}} = \square$

- Combine your previous results to calculate $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ in terms of a, b, c, x, y, z.



Result: $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \left(b\,z - \square\right)\hat{\mathbf{i}} + \left(\square - a\,z\right)\hat{\mathbf{j}} + \left(\square\right)\hat{\mathbf{k}}$

2.12 ♣ Cross products and determinants (orthogonal unit vectors). (Section 2.10.2)



Shown right are arbitrary vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ expressed in terms of right-handed orthogonal unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$. Show that calculating $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ with the ***distributive property*** of the cross product (seen in Hw 2.11) happens to be equal to the ***determinant*** of the matrix shown to the right.

$$\vec{\mathbf{v}} = a\,\hat{\mathbf{i}} + b\,\hat{\mathbf{j}} + c\,\hat{\mathbf{k}}$$
$$\vec{\mathbf{w}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$$
$$\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \det \begin{bmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a & b & c \\ x & y & z \end{bmatrix}$$

Result: $\vec{\mathbf{v}} \times \vec{\mathbf{w}} = \left(b\,z - \square\right)\hat{\mathbf{i}} + \left(\square - a\,z\right)\hat{\mathbf{j}} + \left(\square\right)\hat{\mathbf{k}}$

2.13 ♣ Cross product as skew-symmetric matrix multiplication. (Section 2.10.3)

Shown right, calculate the 3×3 ***skew symmetric matrix*** multiplied by the 3×1 column matrix. In view of Hw 2.11, the $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$ coefficients of $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ happen to be equal to the elements that result from ***skew symmetric matrix*** multiplication.

$$\begin{bmatrix} 0 & \text{-}c & b \\ c & 0 & \text{-}a \\ \text{-}b & a & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b\,z - \square \\ \square - a\,z \\ \square \end{bmatrix}$$

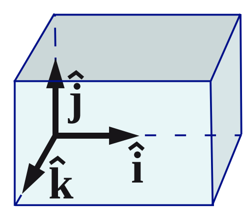
Skew symmetric matrix multiplication is an <u>inefficient</u> way to calculate a cross product **True/False**.

Note: The number of mathematical operations required to multiply the 3×3 matrix by the 3×1 matrix (including multiplication by 0) is 9 multiplications and 6 additions/subtractions whereas the number of operations to calculate the elements of the simplified answer is 6 multiplications and 3 subtractions.

Note: Using skew symmetric matrix multiplication for a cross product is useful in theory/proofs, e.g., Section 9.5.2.

2.14 ♣ Scalar triple product with bases. (Section 2.11)

The figure shows right-handed orthogonal unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, $\hat{\mathbf{k}}$.



Given	Calculate
$\vec{\mathbf{u}} = 2\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + 4\,\hat{\mathbf{k}}$	$\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{u}} = \square$
$\vec{\mathbf{v}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$	$\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \square\, z - \square\, x - 6\,y$
$\vec{\mathbf{w}} = 5\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}} + 7\,\hat{\mathbf{k}}$	$\vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}} = \square\, z - 45\,x - \square\, y$

Note: Although the order of operations in $\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{u}}$ is unambiguous, parentheses may clarify your work.

$\vec{\mathbf{u}} \times \vec{\mathbf{v}} \cdot \vec{\mathbf{w}} = \vec{\mathbf{u}} \cdot \vec{\mathbf{v}} \times \vec{\mathbf{w}}$ and it is OK to switch $\cdot$ and $\times$ in scalar triple products. **True/False**.

2.15 ♣ Optional: Scalar triple products and determinants. (Section 2.11)

Given arbitrary vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $\vec{\mathbf{c}}$ expressed in terms of **right-handed orthogonal unit** vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ as shown right, show that calculating $\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$ happens to be equal to the determinant of the matrix shown to the right.

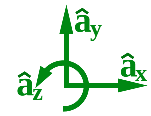
$$\begin{aligned}
\vec{\mathbf{a}} &= a_x\,\widehat{\mathbf{n}}_x + a_y\,\widehat{\mathbf{n}}_y + a_z\,\widehat{\mathbf{n}}_z \\
\vec{\mathbf{b}} &= b_x\,\widehat{\mathbf{n}}_x + b_y\,\widehat{\mathbf{n}}_y + b_z\,\widehat{\mathbf{n}}_z \\
\vec{\mathbf{c}} &= c_x\,\widehat{\mathbf{n}}_x + c_y\,\widehat{\mathbf{n}}_y + c_z\,\widehat{\mathbf{n}}_z \\
\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{c}} &= \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}
\end{aligned}$$

2.16 † Optional: Form $\vec{\mathbf{v}}$ from $\vec{\mathbf{v}} \cdot \widehat{\mathbf{u}}$ and $\vec{\mathbf{v}} \times \widehat{\mathbf{u}}$. Show work. (Table in Section 2.10)

Given: $\vec{\mathbf{v}} \cdot \widehat{\mathbf{a}}_x = 15 \qquad \vec{\mathbf{v}} \times \widehat{\mathbf{a}}_x = 5\,\widehat{\mathbf{a}}_z$

Form: Vector $\vec{\mathbf{v}}$ in terms of right-handed orthogonal unit vectors $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$.

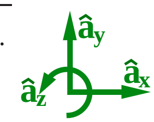
Result: $\vec{\mathbf{v}} = \boxed{}\,\widehat{\mathbf{a}}_x - \boxed{}\,\widehat{\mathbf{a}}_y$



Given: Unit vector $\widehat{\mathbf{u}} = 0.6\,\widehat{\mathbf{a}}_x + 0.8\,\widehat{\mathbf{a}}_y \qquad \vec{\mathbf{v}} \cdot \widehat{\mathbf{u}} = 15 \qquad \vec{\mathbf{v}} \times \widehat{\mathbf{u}} = 4\,\widehat{\mathbf{a}}_x - 3\,\widehat{\mathbf{a}}_y + 5\,\widehat{\mathbf{a}}_z$.

Form: Vector $\vec{\mathbf{v}}$ in terms of right-handed orthogonal unit vectors $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$.

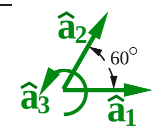
Result: $\vec{\mathbf{v}} = \boxed{}\,\widehat{\mathbf{a}}_x + \boxed{}\,\widehat{\mathbf{a}}_y - \boxed{}\,\widehat{\mathbf{a}}_z$ Note: $\vec{\mathbf{v}} \times \widehat{\mathbf{u}}$ is not arbitrary. It is perpendicular to $\widehat{\mathbf{u}}$.



Given: $\vec{\mathbf{s}} = 0.6\,\widehat{\mathbf{a}}_1 + 0.8\,\widehat{\mathbf{a}}_2 \qquad \vec{\mathbf{v}} \cdot \vec{\mathbf{s}} = 15 \qquad \vec{\mathbf{v}} \times \vec{\mathbf{s}} \approx 2.07846\,\widehat{\mathbf{a}}_1 - 3\,\widehat{\mathbf{a}}_2 + 5\,\widehat{\mathbf{a}}_3$.

Form: Vector $\vec{\mathbf{v}}$ in terms of right-handed **non-**orthogonal unit vectors $\widehat{\mathbf{a}}_1$, $\widehat{\mathbf{a}}_2$, $\widehat{\mathbf{a}}_3$.

Result: $\vec{\mathbf{v}} \approx 12.476\,\widehat{\mathbf{a}}_1 + 3.643\,\widehat{\mathbf{a}}_2 - \boxed{}\,\widehat{\mathbf{a}}_3$ (Hint: $\vec{\mathbf{s}}$ is not a unit vector).

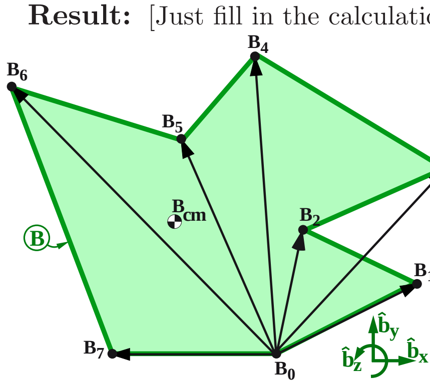


2.17 ♣ Cross products: Commercial area calculation algorithm (surveying). (Sections 2.10.1, 3.3)

Complex **planar objects** such as the polygon B below can be decomposed into triangles for important planar measurements (e.g., farming acreage, building costs, and mass and area properties of 2D objects).

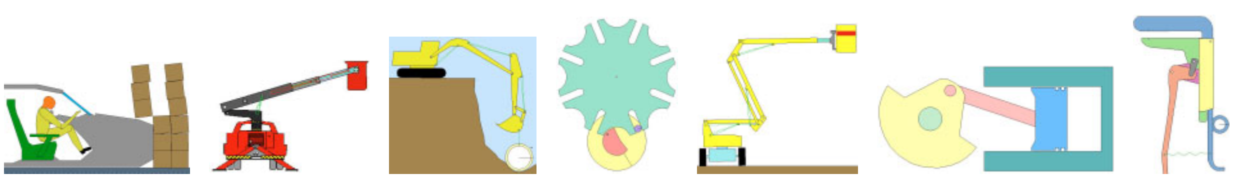
- Number the vertices sequentially in counter-clockwise fashion.
- Label a vertex B_0 and number the remaining vertices sequentially in counter
- Form ${}^{B_0}\vec{\mathbf{r}}^{B_i}$, the position from B_0 to B_i $(i = 1, 2, \ldots)$
- Calculate $\vec{\mathbf{A}}_2$ and $\vec{\mathbf{A}}_4$, the vector-areas of triangles $B_0\,B_2\,B_3$ and $B_0\,B_4\,B_5$.
- Account for overlapped areas with **positive** and **negative** vector areas.

Result: [Just fill in the calculations for $\vec{\mathbf{A}}_2$, $\vec{\mathbf{A}}_4$, and $\mathbf{A}$ using eqn (3.4)].



$$\begin{aligned}
\vec{\mathbf{r}}_1 &= {}^{B_0}\vec{\mathbf{r}}^{B_1} = 2.0\,\widehat{\mathbf{b}}_x + 2.0\,\widehat{\mathbf{b}}_y \\
\vec{\mathbf{r}}_2 &= {}^{B_0}\vec{\mathbf{r}}^{B_2} = 0.5\,\widehat{\mathbf{b}}_x + 2.5\,\widehat{\mathbf{b}}_y \\
\vec{\mathbf{r}}_3 &= {}^{B_0}\vec{\mathbf{r}}^{B_3} = 3.0\,\widehat{\mathbf{b}}_x + 4.0\,\widehat{\mathbf{b}}_y \\
\vec{\mathbf{r}}_4 &= {}^{B_0}\vec{\mathbf{r}}^{B_4} = -0.5\,\widehat{\mathbf{b}}_x + 7.0\,\widehat{\mathbf{b}}_y \\
\vec{\mathbf{r}}_5 &= {}^{B_0}\vec{\mathbf{r}}^{B_5} = -1.0\,\widehat{\mathbf{b}}_x + 5.0\,\widehat{\mathbf{b}}_y \\
\vec{\mathbf{r}}_6 &= {}^{B_0}\vec{\mathbf{r}}^{B_6} = -3.0\,\widehat{\mathbf{b}}_x + 6.0\,\widehat{\mathbf{b}}_y \\
\vec{\mathbf{r}}_8 &= {}^{B_0}\vec{\mathbf{r}}^{B_8} = -2.0\,\widehat{\mathbf{b}}_x + 0.0\,\widehat{\mathbf{b}}_y
\end{aligned}$$

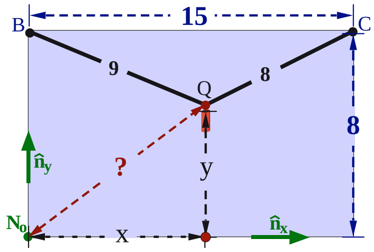
$\vec{\mathbf{A}}_1$	=	$\frac{1}{2}(\vec{\mathbf{r}}_1 \times \vec{\mathbf{r}}_2)$	=	$2\,\widehat{\mathbf{b}}_z$
$\vec{\mathbf{A}}_2$	=	$\frac{1}{2}(\vec{\mathbf{r}}_2 \times \vec{\mathbf{r}}_3)$	=	$\boxed{}.75\,\widehat{\mathbf{b}}_z$
$\vec{\mathbf{A}}_3$	=	...	=	$11.5\,\widehat{\mathbf{b}}_z$
$\vec{\mathbf{A}}_4$	=	...	=	$\boxed{}.25\,\widehat{\mathbf{b}}_z$
$\vec{\mathbf{A}}_5$	=	...	=	$4.5\,\widehat{\mathbf{b}}_z$
$\vec{\mathbf{A}}_6$	=	$\frac{1}{2}(\vec{\mathbf{r}}_6 \times \vec{\mathbf{r}}_7)$	=	$6\,\widehat{\mathbf{b}}_z$
$\vec{\mathbf{A}}$	=	$\sum_{i=1}^{6} \vec{\mathbf{A}}_i$	=	$\boxed{}$
Area	=	$\lvert\vec{\mathbf{A}}\rvert$	=	23.5



Planar objects, courtesy of Working Model and Design-Simulation Technologies.

2.18 Locating a microphone (2D). Show work. (Section 1.4)

A microphone Q is attached to two pegs B and C by two cables. Knowing the peg locations, cable lengths, and points B, C, Q, N_o all lie in the same plane, determine the distance between Q and N_o.
Do the problem with Euclidean geometry (e.g., law of cosines), then try vectors (see Hw 1.38).

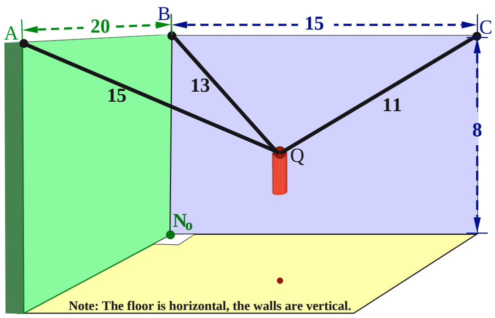


Distance between B to C		15 m
Distance between N_o to B	h	8 m
Length of cable joining B and Q	L_B	9 m
Length of cable joining C and Q	L_C	8 m
Distance between N_o and Q		9.01 m

Note: Although there are two mathematical answers to this problem, one is above the ceiling by $\approx$ 12 m and requires the cables to be in compression.

2.19 † Locating a microphone (3D).

A microphone Q is attached to three pegs A, B, and C by three cables. Knowing the peg locations and cable lengths, determine the distance between Q and point N_o. Show work.[2]



Distance between A to B		20 m
Distance between B to C		15 m
Distance between N_o to B	h	8 m
Length of cable joining A and Q	L_A	15 m
Length of cable joining B and Q	L_B	13 m
Length of cable joining C and Q	L_C	11 m
Distance between N_o and Q If Q is above ceiling, distance $\approx$ 17 m		13.3 m

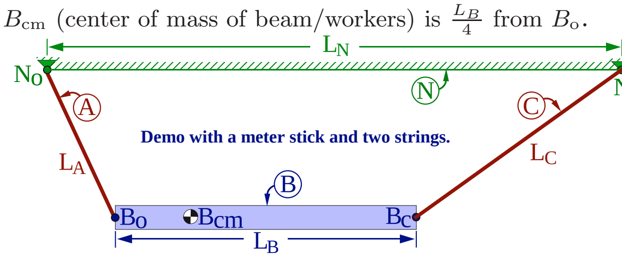
Note: This is part of the process of a camera targeting a football/baseball in a stadium or laser targeting cancer or . . .

Vocabulary: In this ***forward kinematics*** analysis, the cable lengths are known and you determine the position of "end-effector" Q.

2.20 † Cable length to keep a window-washer's beam stationary and horizontal. Show work.

A beam B is attached to the roof of a building N by two relatively light (massless) cables A and C.
Cable A attaches to the roof at point N_o of N and to the beam at point B_o of B.
Cable C attaches to the roof at point N_C of N and to the beam at point B_C of B.
N_o, B_o, B_{cm}, B_C, N_C are all in the same vertical plane. B_{cm} (center of mass of beam/workers) is $\frac{L_B}{4}$ from B_o.

Description	Symbol	Type	Value
Distance between N_o and N_C	L_N	Constant	15 m
Distance between B_o and B_C	L_B	Constant	7 m
Length of cable A	L_A	Constant	7 m
Length of cable C	L_C	Constant	? m

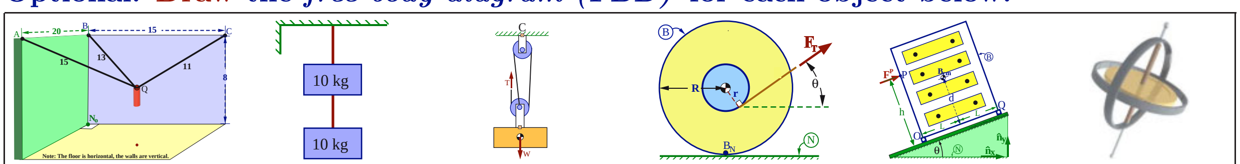


Stumped? www.MotionGenesis.com ⇒ Textbooks ⇒ Resources

Determine L_C so the beam stays horizontal.
Result: $L_C =$ 9 m

If $L_B = L_N$, intuition/analysis predicts $L_C = L_A$ (vertical cables), independent of B_{cm}'s location between B_o and B_C.

2.21 Optional: Draw the *free-body diagram (FBD)* for each object below.



Particle Q	Top block	Bottom pulley	Rolling spool B	Bureau B	Entire system
Hw 2.19	Hw 16.12	Hw 16.14	Hw 17.14	Hw 17.10	Hw 15.18

[2] Hint: See Homework 1.38 or Section 3.4. Introduce whatever **identifiers** facilitate your work.
Although nonlinear algebraic equations are usually solved with a computer, these can also be solved "by-hand".
Solution at www.MotionGenesis.com ⇒ Get Started ⇒ **2D/3D geometry**. Alternatively, go to www.WolframAlpha.com and type
Solve `x^2 + (-20+z)^2 + (-8+y)^2 = 225,  x^2 + z^2 + (-8+y)^2 = 169,  z^2 + (-15+x)^2 + (-8+y)^2 = 121`