Problem Sheet on K3 surfaces and IHSM

Lecture 2

(1) Find an element in $\mathrm{O}(L_{2d})$ which interchanges the two components $\mathcal{D}_{L_{2d}}$ and $\mathcal{D}'_{L_{2d}}$ of $\Omega_{L_{2d}}$.

(2) Show that an element in $g \in \mathrm{O}(L_{2d})$ can be extended to an isometry in $\mathrm{O}(L_{\mathrm{K3}})$ if and only if $g \in \tilde{\mathrm{O}}(L_{2d})$.

(3) Show that there is an exact sequence

$$0 \to \tilde{\mathrm{O}}(L_{2d}) \to \mathrm{O}(L_{2d}) \to \mathrm{O}(D(L_{2d})) \to 0.$$

(4) Find explicit (primitive) vectors in $L = 3U \oplus 2E_8(-1) \oplus \langle -2 \rangle$ with $\mathrm{div}(h) = 2$.

(5) Consider $L = 3U \oplus 2E_8(-1) \oplus \langle -2(n-1) \rangle$. How many different "types of polarization" (i.e. orbits of primitive vectors) of degree $2d$ can you find?

(6) Prove that a nef divisor h on a K3 surface S is ample if and only if it has positive degree on every (-2)-curve S (Hint: this is easy with Reider's theorem).