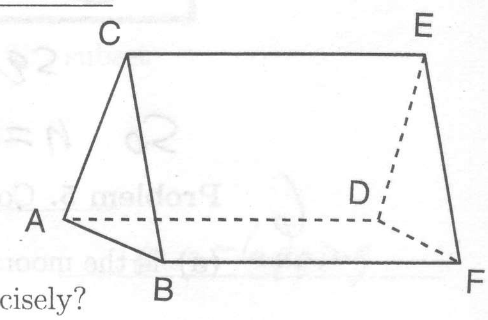
MTH 218–6816 Exam 1

February 23, 2011 Professor Ilya Kofman

NAME: Key

7 **Problem 1.**



Suppose $CE \cong AD \cong BF$ and $CE \,||\, AD \,||\, BF$.

2 (a) If $BCEF$ is a rectangle, what is this figure called precisely?

Right triangular prism

2 (b) If $\angle CBF$ is obtuse, what is this figure called precisely?

Oblique triangular prism

3 (c) Verify Euler's formula for this figure.

$v = 6$
$e = 9$
$f = 5$

$v - e + f = 2$
$6 - 9 + 5 = 2$

6 **Problem 2.** (a) If a prism has a 50–gon for its base, how many vertices, edges and faces does it have? Verify Euler's formula for this prism.

$V = 100$
$e = 150$
$f = 52$

$v - e + f = 2$
$100 - 150 + 52 = 2$

6 (b) If a prism has 120 edges, how many vertices and faces does it have? Verify Euler's formula for this prism.

$V = 80$
$e = 120$
$f = 42$

$v - e + f = 2$
$80 - 120 + 42 = 2$

6 **Problem 3.** (a) What is the measure of an interior angle of a regular decagon?

$$\frac{180(n-2)}{n} = \frac{180 \cdot 8}{10} = 144^\circ$$

(b) Explain why it cannot be the face of a regular polyhedron.

3 decagons/vertex $\rightarrow$ $3 \times 144 = 432 > 360$

Angle sum too big, must be less than 360.

25

125

6 **Problem 4.** One semiregular tiling of the plane consists of these <u>three</u> regular polygons at every vertex: a dodecagon (12–gon), a square, and what other polygon? Justify.

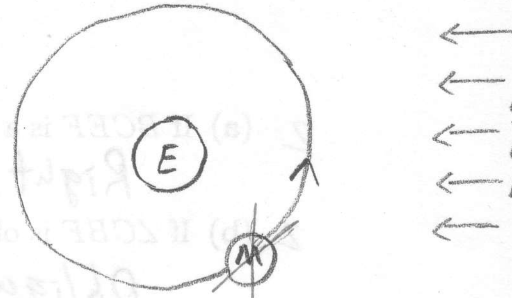
12-gon : $\frac{180 \cdot 10}{12} = 150^\circ$

square : 90°

$360 - (150 + 90) = 120^\circ$

So $n = 6$, $\frac{180 \cdot 4}{6} = 120$ ie hexagon.

6 **Problem 5.** Consider the earth and moon as shown.



(a) Is the moon new, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, or full?

(b) Is it waxing or waning?

12 **Problem 6.** Find the missing angle measures indicated by letters in the diagram below. Two parallel lines are indicated by arrows.

2pts. each

$a = $ 130

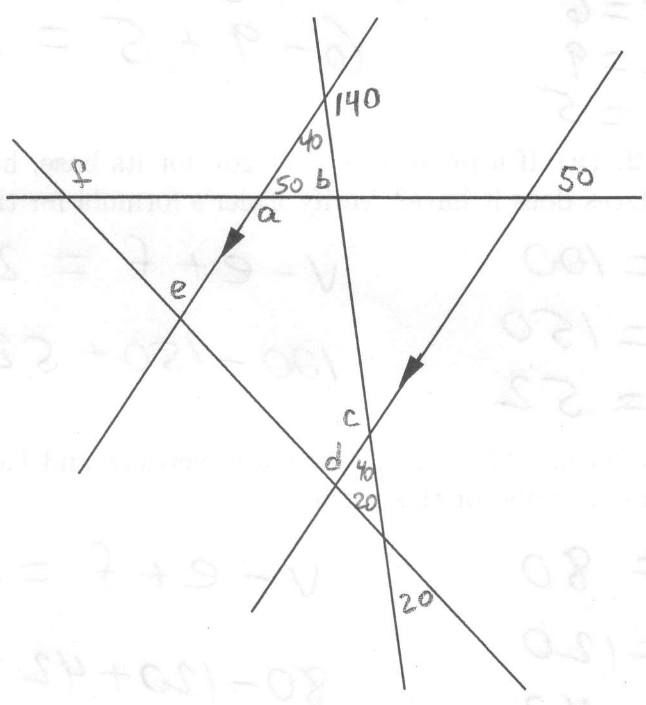
$b = $ 90

$c = $ 140

$d = $ 60

$e = $ 60

$f = $ 110



10 **Problem 7.** Convex or concave?

(a) Trapezoid Convex

(b) Obtuse triangle Convex

(c) Two regular hexagons glued along a common edge Concave

(d) Regular polyhedron Convex

(e) Oblique pyramid convex

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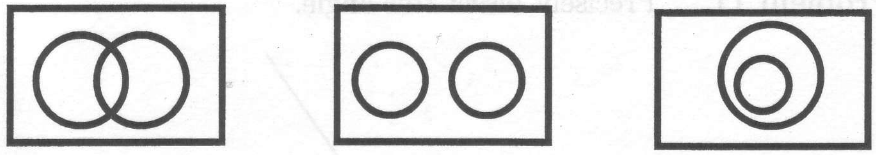
Problem 8. For each of the following pairs, identify which type of Venn diagram describes their relationship.



overlapping disjoint subset

2pts. each

(a) Rectangles and kites overlapping

(b) Rhombi and parallelograms subset

(c) Rhombi and quadrilaterals with congruent diagonals overlapping

(d) Rectangles and trapezoids subset

(e) Kites and squares subset

(f) Isosceles triangles and obtuse triangles overlapping

(g) Regular polyhedra and pyramids overlapping

(h) Prisms and pyramids disjoint

12 **Problem 9.** Among parallelograms, rectangles, rhombi, and isosceles trapezoids, list all for which the following statements always true:

2 pts. each

(a) Adjacent angles are congruent. rectangles, isosc. trapezoids

(b) Opposite angles are congruent. parallel., rectangles, rhombi

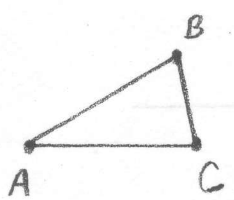
(c) Diagonals bisect angles. rhombi

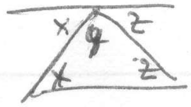
(d) Diagonals are congruent. rectangles, isosc. trapezoids

(e) Diagonals cross at right angles. rhombi

(f) Diagonals cross at midpoints. parallelograms, rectangles, rhombi

6 **Problem 10.** In $\triangle ABC$, $\angle A = 30°$ and $\angle B = 70°$. Use either Euclid's Parallel Postulate or the rotation angles method to precisely explain why $\angle C = 80°$.

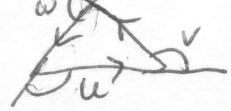




By Euclid's Parallel Postulate, there is a line parallel to AC through B. Then

$x+y+z=180 \Rightarrow 30+70+\angle C=180$

$\Leftrightarrow \angle C = 80.$



Travelling around $\triangle ABC$ means turning $360°$

$u+v+w=360 \Rightarrow 150+v+110=360$

$\Rightarrow v=100 \Rightarrow \angle C=80$

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