BINOMIAL DISTRIBUTION

INTRODUCTORY CALCULATIONS

Question 1

The random variable X has binomial distribution $B(20, 0.4)$.

Determine each of the following.

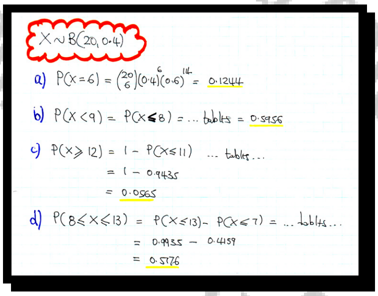
a) $P(X = 6)$.

b) $P(X < 9)$.

c) $P(X \geq 12)$.

d) $P(8 \leq X \leq 13)$.

$\boxed{0.1244}$, $\boxed{0.5956}$, $\boxed{0.0565}$, $\boxed{0.5776}$



$X \sim B(20, 0.4)$

a) $P(X=6) = \binom{20}{6}(0.4)^6(0.6)^{14} = \underline{0.1244}$

b) $P(X<9) = P(X \leqslant 8) = \ldots$ tables $= \underline{0.5956}$

c) $P(X \geqslant 12) = 1 - P(X \leqslant 11)$ ... tables ...

$= 1 - 0.9435$

$= \underline{0.0565}$

d) $P(8 \leqslant X \leqslant 13) = P(X \leqslant 13) - P(X \leqslant 7) = \ldots$ tables ...

$= 0.9935 - 0.4159$

$= \underline{0.5776}$

Question 2

The random variable X has binomial distribution $B(15, 0.35)$.

Determine each of the following.

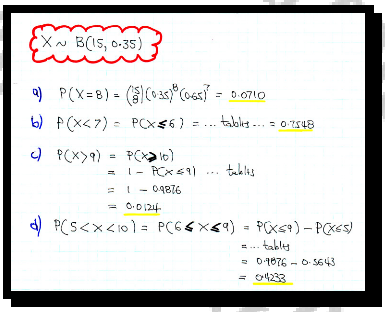
a) $P(X = 8)$.

b) $P(X < 7)$.

c) $P(X > 9)$.

d) $P(5 < X < 10)$.

$\boxed{0.0710}$, $\boxed{0.7548}$, $\boxed{0.0124}$, $\boxed{0.4233}$



$X \sim B(15, 0.35)$

a) $P(X=8) = \binom{15}{8}(0.35)^8(0.65)^7 = 0.0710$

b) $P(X<7) = P(X \leq 6) = \ldots$ tables $\ldots = 0.7548$

c) $P(X>9) = P(X \geq 10)$
$= 1 - P(X \leq 9)$ ... tables
$= 1 - 0.9876$
$= 0.0124$

d) $P(5<X<10) = P(6 \leq X \leq 9) = P(X \leq 9) - P(X \leq 5)$
$= \ldots$ tables
$= 0.9876 - 0.5643$
$= 0.4233$

Question 3

The random variable X has binomial distribution $\mathrm{B}(30, 0.3)$.

Determine each of the following.

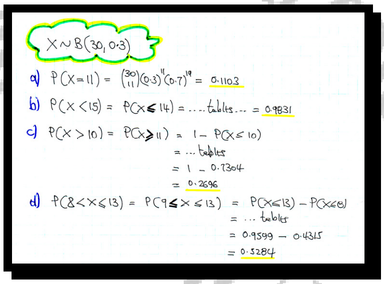
a) $\mathrm{P}(X = 11)$.

b) $\mathrm{P}(X < 15)$.

c) $\mathrm{P}(X > 10)$.

d) $\mathrm{P}(8 < X \le 13)$.

$\boxed{0.1103}$, $\boxed{0.9831}$, $\boxed{0.2696}$, $\boxed{0.5284}$



$X \sim B(30, 0.3)$

a) $P(X=11) = \binom{30}{11}(0.3)^{11}(0.7)^{19} = 0.1103$

b) $P(X<15) = P(X \le 14) = \ldots$ tables $\ldots = 0.9831$

c) $P(X>10) = P(X \ge 11) = 1 - P(X \le 10)$
$= \ldots$ tables
$= 1 - 0.7304$
$= 0.2696$

d) $P(8 < X \le 13) = P(9 \le X \le 13) = P(X \le 13) - P(X \le 8)$
$= \ldots$ tables
$= 0.9599 - 0.4315$
$= 0.5284$

Question 4

The random variable X has binomial distribution $\mathrm{B}(40,0.2)$.

Determine each of the following.

a) $\mathrm{P}(X=5)$.

b) $\mathrm{P}(X<13)$.

c) $\mathrm{P}(X>10)$.

d) $\mathrm{P}(8<X<14)$.

$\boxed{0.0854}$, $\boxed{0.9568}$, $\boxed{0.1608}$, $\boxed{0.3875}$

$X \sim B(40,0.2)$

a) $P(X=5) = \binom{40}{5}(0.2)^5(0.8)^{35} = 0.0854$

b) $P(X<13) = P(X \leqslant 12) = \ldots \text{tables} \ldots = 0.9568$

c) $P(X>10) = P(X \geqslant 11) = 1 - P(X \leqslant 10)$
$= \ldots \text{tables} \ldots$
$= 1 - 0.8392$
$= 0.1608$

d) $P(8<X<14) = P(9 \leqslant X \leqslant 13) = P(X \leqslant 13) - P(X \leqslant 8)$
$= \ldots \text{tables} \ldots$
$= 0.9806 - 0.5931$
$= 0.3875$

Question 5

The probability of a customer ordering the colour of a particular model of new car in silver is 0.2.

Find the probability that in next 30 random orders there will be …

a) … exactly 10 orders in silver.

b) … at most 8 orders in silver.

c) … no more than 11 orders in silver.

$\boxed{0.0355}$, $\boxed{0.8713}$, $\boxed{0.9905}$

X = orders in silver
$X \sim B(30, 0.2)$

(a) $P(X=10) = \binom{30}{10}(0.2)^{10}(0.8)^{20} = 0.0355$

(b) $P(X \leq 8) = \ldots$ from tables $= 0.8713$

(c) $P(X \leq 11) = \ldots$ from tables $= 0.9905$

Question 6

The probability of Mathew being late to school is 0.15.

Find the probability that in next 25 mornings he will arrive late to school …

a) … on exactly 6 occasions.

b) … on less than 6 occasions.

c) … at least twice.

$\boxed{0.0920}$, $\boxed{0.8385}$, $\boxed{0.9069}$

X = lates to school
$X \sim B(25, 0.15)$

(a) $P(X=6) = \binom{25}{6}(0.15)^{6}(0.85)^{19} = 0.0920$

(b) $P(X<6) = P(X \leq 5) = \ldots$ tables $\ldots\ 0.8385$

(c) $P(X \geq 2) = 1 - P(X \leq 1) = \ldots$ tables $\ldots = 1 - 0.0931$
$= 0.9069$

Question 7

A certain brand of sweets are sold in bags of 15.

The typical proportion of orange sweets in these bags is 1 in 5.

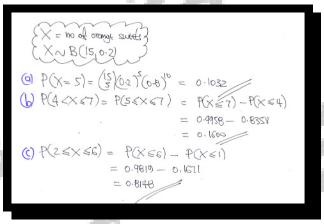
Find the probability that in a random bag of these sweets there will be …

a) … exactly 5 orange sweets.

b) … more than 4 but at most 7 orange sweets.

c) … at least 2 but no more than 6 orange sweets.

$\boxed{0.1032}$, $\boxed{0.1600}$, $\boxed{0.8148}$



X = no of orange sweets
$X \sim B(15, 0.2)$

(a) $P(X=5) = \binom{15}{5}(0.2)^5(0.8)^{10} = 0.1032$

(b) $P(4 < X \leqslant 7) = P(5 \leqslant X \leqslant 7) = P(X \leqslant 7) - P(X \leqslant 4)$
$= 0.9958 - 0.8358$
$= 0.1600$

(c) $P(2 \leqslant X \leqslant 6) = P(X \leqslant 6) - P(X \leqslant 1)$
$= 0.9819 - 0.1671$
$= 0.8148$

Question 8

Chris sells a charity magazine to raise money for the homeless, by approaching people coming out of a train station. The probability that he will have a successful sale is 0.09.

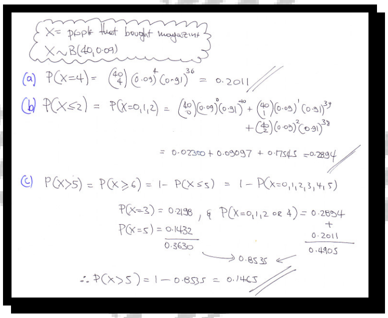
Determine the probability that if Chris approaches 40 people at random, he will sell the magazine to …

a) … exactly 4 people.

b) … at most 2 people.

c) … more than 5 people.

$\boxed{0.2011}$, $\boxed{0.2894}$, $\boxed{0.1465}$



X = people that bought magazine

$X \sim B(40, 0.09)$

(a) $P(X=4) = \binom{40}{4}(0.09)^4(0.91)^{36} = 0.2011$

(b) $P(X \le 2) = P(X=0,1,2) = \binom{40}{0}(0.09)^0(0.91)^{40} + \binom{40}{1}(0.09)^1(0.91)^{39} + \binom{40}{2}(0.09)^2(0.91)^{38}$

$= 0.02300 + 0.09097 + 0.17545 = 0.2894$

(c) $P(X>5) = P(X \ge 6) = 1 - P(X \le 5) = 1 - P(X=0,1,2,3,4,5)$

$P(X=3) = 0.2198$, & $P(X=0,1,2 \text{ or } 4) = 0.2894 + 0.2011 = 0.4905$

$P(X=5) = 0.1432$

$0.2198 + 0.1432 = 0.3630$

$0.3630 + 0.4905 = 0.8535$

$\therefore P(X>5) = 1 - 0.8535 = 0.1465$

Question 9

The probability that Anna wakes up before her alarm rings is 0.4.

a) Find the mean and variance of the number of times that Anna wakes up before her alarm rings, in the next 7 mornings.

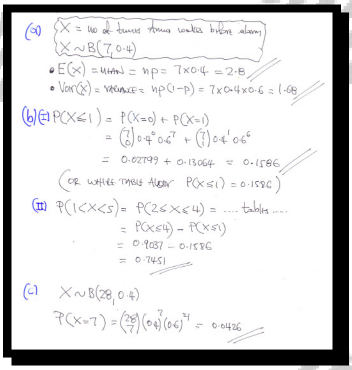
b) Determine the probability that in the next 7 mornings, Anna will wake up before her alarm rings …

i. … at most once.

ii. … in more than 1 but less than 5 mornings.

c) Calculate the probability that in the next 4 <u>weeks</u> Anna will wake up before her alarm rings on exactly 7 mornings.

$\boxed{\mathrm{E}(\mathrm{X}) = 2.8}$, $\boxed{\mathrm{Var}(\mathrm{X}) = 1.68}$, $\boxed{0.1586}$, $\boxed{0.7451}$, $\boxed{0.0426}$



(a) X = no of times Anna wakes before alarm
$X \sim B(7, 0.4)$

- $E(X) = \text{MEAN} = np = 7 \times 0.4 = 2.8$
- $\mathrm{Var}(X) = \text{VARIANCE} = np(1-p) = 7 \times 0.4 \times 0.6 = 1.68$

(b)(I) $P(X \leq 1) = P(X=0) + P(X=1)$
$= \binom{7}{0} 0.4^0 0.6^7 + \binom{7}{1} 0.4^1 0.6^6$
$= 0.02799 + 0.13064 = 0.1586$
(OR WHILE TABLE ALLOW $P(X \leq 1) = 0.1586$)

(II) $P(1 < X < 5) = P(2 \leq X \leq 4) = \ldots$ tables $\ldots$
$= P(X \leq 4) - P(X \leq 1)$
$= 0.9037 - 0.1586$
$= 0.7451$

(c) $X \sim B(28, 0.4)$
$P(X=7) = \binom{28}{7} (0.4)^7 (0.6)^{21} = 0.0426$

Question 10

The probability that Phil will walk to work is 0.1.

The last 20 days that Phil went to work are examined.

a) Determine the mean and variance of the number of days that Phil walked to work in the last 20 working days.

b) Find the probability that in the last 20 working days Phil walked to work …

i. … on exactly 2 days.

ii. … on at most 3 days.

iii. … on more than 2 days.

$\boxed{E(X)=2}$, $\boxed{\text{Var}(X)=1.8}$, $\boxed{0.2852}$, $\boxed{0.8670}$, $\boxed{0.3231}$

X = day that Phil walks
$X \sim B(20, 0.1)$

(a) $E(X) = np = 20 \times 0.1 = 2$
$\text{Var}(X) = np(1-p)$ $20 \times 0.1 \times 0.9 = 1.8$

(b) (I) $P(X=2) = \binom{20}{2}(0.1)^2(0.9)^{18} = 0.2852$

(II) $P(X \leq 3) = \ldots 0.8670$

(III) $P(X>2) = P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.6769 = 0.3231$

Question 11

The probability of a random customer ordering a vegetarian meal in the Ampala restaurant is 0.2.

a) Find the mean and variance of the vegetarian meals in a total of 40 orders.

b) Determine the probability that in 50 orders there will be …

i. … exactly 11 vegetarian orders.

ii. … more than 11 vegetarian orders.

iii. … at least 6 but no more than 11 vegetarian orders.

$\boxed{E(X)=8}$, $\boxed{\text{Var}(X)=6.4}$, $\boxed{0.1271}$, $\boxed{0.2893}$, $\boxed{0.6627}$

(a) X = vegetarian order

$X \sim B(40, 0.2)$

$E(X) = np = 40 \times 0.2 = 8$

$\text{Var}(X) = np(1-p) = 8 \times 0.8 = 6.4$

(b) (I) $X \sim B(50, 0.2)$

$P(X=11) = \binom{50}{11}(0.2)^{11}(0.8)^{39} = 0.1271$

(II) $P(X>11) = P(X \geqslant 12) = 1 - P(X \leqslant 11) = 1 - 0.7107 = 0.2893$

(III) $P(6 \leqslant X \leqslant 11) = P(X \leqslant 11) - P(X \leqslant 5) = 0.7107 - 0.0480$

$= 0.6627$

Question 12

The probability of a biased dice landing on 6 is 0.4.

a) Find the mean and variance of the number of "sixes" in 50 throws.

b) Determine the probability that in 50 throws there will be …

i. … exactly 22 "sixes".

ii. … less than 17 "sixes".

iii. … more than 25 sixes.

$\boxed{\mathrm{E}(X)=20}$, $\boxed{\mathrm{Var}(X)=12}$, $\boxed{0.0959}$, $\boxed{0.1561}$, $\boxed{0.0573}$

(a) X = "six" on a dice throw

$X \sim B(50, 0.4)$

$\mathrm{E}(X) = np = 50 \times 0.4 = 20$

$\mathrm{Var}(X) = np(1-p) = 50 \times 0.4 \times 0.6 = 12$

(b) (I) $P(X=22) = \binom{50}{22}(0.4)^{22}(0.6)^{28} = 0.0959$

(II) $P(X<17) = P(X \leqslant 16) = \dots$ tables $= 0.1561$

(III) $P(X>25) = P(X \geqslant 26) = 1 - P(X \leqslant 25)$

$= \dots$ tables $\dots = 1 - 0.9427 = 0.0573$

Question 13

The probability of a letter posted first class, out of a certain company, is 0.35.

a) Find the mean and variance of the number of letters posted first class in a batch of 14 letters.

b) Find the probability that in a batch of 14 letters there will be …

i. … exactly 5 first class letters.

ii. … at least 6 first class letters.

iii. … more than 3 but no more than 9, first class letters.

$\boxed{\mathrm{E}(X)=4.9}$, $\boxed{\mathrm{Var}(X)=3.185}$, $\boxed{0.2178}$, $\boxed{0.3595}$, $\boxed{0.7735}$

(a) X = FIRST CLASS LETTER
$X \sim B(14, 0.35)$

$E(X) = np = 14 \times 0.35 = 4.9$

$\mathrm{Var}(X) = np(1-p) = 14 \times 0.35 \times 0.65 = 3.185$

(b) (I) $P(X=5) = \binom{14}{5}(0.35)^5(0.65)^9 = 0.2178$

(II) $P(X \geqslant 6) = 1 - P(X \leqslant 5) = \ldots$ AQA, MEI, OCR tables…
$= 1 - 0.6405 = 0.3595$

(III) $P(3 < X \leqslant 9) = P(4 \leqslant X \leqslant 9) = P(X \leqslant 9) - P(X \leqslant 3)$
$= \ldots$ AQA, MEI, OCR tables…
$= 0.9940 - 0.2205$
$= 0.7735$

Question 14

The probability of a carnation producing a pink flower is 0.35.

a) Find the mean and variance of the number of pink flowers produced in 20 carnation blooms.

b) Find the probability that in 20 carnations that are about to produce flowers, we will obtain …

i. … at least 2 pink flowers.

ii. … no more than 5 pink flowers.

iii. … between 3 and 7 pink flowers, inclusive on both ends.

$\boxed{E(X)=7}$, $\boxed{\text{Var}(X)=4.55}$, $\boxed{0.9969}$, $\boxed{0.2454}$, $\boxed{0.5889}$

(a) $X = $ "pink flower"
$X \sim B(20, 0.35)$

$E(X) = np = 20 \times 0.35 = 7$

$\text{Var}(X) = np(1-p) = 7 \times 0.65 = 4.55$

(b) (I) $P(X \geqslant 2) = 1 - P(X \leqslant 1) = 1 - 0.0031 = 0.9969$

(II) $P(X \leqslant 5) = 0.2454$

(III) $P(3 \leqslant X \leqslant 7) = P(X \leqslant 7) - P(X \leqslant 2)$
$= 0.6010 - 0.0121 = 0.5889$

Question 15

A box contains 50 coloured drawing. The proportion of red drawing pins in these boxes is 3 out of 20.

a) Find the mean and variance of the number of red drawing pins in these boxes.

b) Find the probability that in a box of 50 drawing pins there will be …

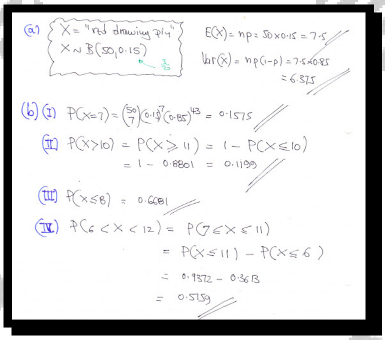
i. … exactly 7 red drawing pins.

ii. … more than 10 red drawing pins.

iii. … no more than 8 red drawing pins.

iv. … between 6 and 12, not inclusive, red drawing pins.

$\boxed{\mathrm{E}(X)=7.5}$, $\boxed{\mathrm{Var}(X)=6.375}$, $\boxed{0.1575}$, $\boxed{0.1199}$, $\boxed{0.6681}$, $\boxed{0.5759}$


(a) X = "red drawing pin"
$X \sim B(50, 0.15)$ $\frac{3}{20}$

$E(X) = np = 50 \times 0.15 = 7.5$

$Var(X) = np(1-p) = 7.5 \times 0.85 = 6.375$

(b) (I) $P(X=7) = \binom{50}{7}(0.15)^7(0.85)^{43} = 0.1575$

(II) $P(X>10) = P(X \geqslant 11) = 1 - P(X \leqslant 10) = 1 - 0.8801 = 0.1199$

(III) $P(X \leqslant 8) = 0.6681$

(IV) $P(6 < X < 12) = P(7 \leqslant X \leqslant 11) = P(X \leqslant 11) - P(X \leqslant 6) = 0.9372 - 0.3613 = 0.5759$

Question 16

The probability that Mr Smith will have coffee with his breakfast is 0.35.

a) Find the mean and variance of the number of mornings that Mr Smith has coffee with his breakfast, in the next 25 mornings.

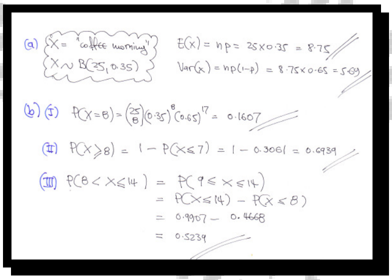
b) Find the probability that in the next 25 mornings, Mr Smith will have coffee …

i. … on exactly 8 mornings.

ii. … on at least 8 mornings.

iii. … on more than 8 but at most 14 mornings.

$\boxed{\text{E}(X)=8.75}$, $\boxed{\text{Var}(X)=5.69}$, $\boxed{0.1607}$, $\boxed{0.6939}$, $\boxed{0.5239}$



(a) $X =$ "coffee morning"
$X \sim B(25, 0.35)$

$E(X) = np = 25 \times 0.35 = 8.75$

$\text{Var}(X) = np(1-p) = 8.75 \times 0.65 = 5.69$

(b) (I) $P(X=8) = \binom{25}{8}(0.35)^8(0.65)^{17} = 0.1607$

(II) $P(X \geqslant 8) = 1 - P(X \leqslant 7) = 1 - 0.3061 = 0.6939$

(III) $P(8 < X \leqslant 14) = P(9 \leqslant X \leqslant 14)$
$= P(X \leqslant 14) - P(X \leqslant 8)$
$= 0.9907 - 0.4668$
$= 0.5239$

Question 17

The probability that Elaine will watch the evening news is 0.25.

a) Find the mean and variance of the evenings that Elaine will watch the news in the 30 days of April.

b) Find the probability that in the month of April, Elaine will watch the evening news …

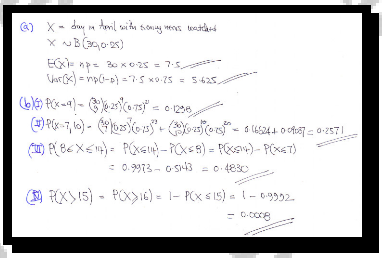
i. … exactly 9 times.

ii. … either 7 or 10 times.

iii. … between 8 and 14 times, inclusive.

iv. … for more than half the evenings of the month.

$\boxed{\text{E}(X) = 7.5}$, $\boxed{\text{Var}(X) = 5.625}$, $\boxed{0.1298}$, $\boxed{0.2571}$, $\boxed{0.4830}$, $\boxed{0.0008}$


(a) X = day in April with evening news watched

$X \sim B(30, 0.25)$

$E(X) = np = 30 \times 0.25 = 7.5$

$\text{Var}(X) = np(1-p) = 7.5 \times 0.75 = 5.625$

(b)(I) $P(X=9) = \binom{30}{9}(0.25)^9(0.75)^{21} = 0.1298$

(II) $P(X=7,10) = \binom{30}{7}(0.25)^7(0.75)^{23} + \binom{30}{10}(0.25)^{10}(0.75)^{20} = 0.16624 + 0.09087 = 0.2571$

(III) $P(8 \leq X \leq 14) = P(X \leq 14) - P(X \leq 8) = P(X \leq 14) - P(X \leq 7)$

$= 0.9973 - 0.5143 = 0.4830$

(IV) $P(X > 15) = P(X \geq 16) = 1 - P(X \leq 15) = 1 - 0.9992$

$= 0.0008$

EXAM QUESTIONS

Question 1 (**+)

During the winter months, in a certain village in Scotland, the probability of a day having severe fog is 0.06.

a) Find the probability that in a given week there will be …

i. … no day with severe fog.

ii. … three or more days with severe fog.

It is known that on a given week there were three or more days with severe fog.

b) Determine the probability that there were exactly three days with severe fog in that week.

$\boxed{0.6485}$, $\boxed{0.0063}$, $\boxed{0.9378}$

a) $\boxed{\begin{array}{l} X = \text{NUMBER OF DAYS WITH SEVERE FOG} \\ X \sim B(7, 0.06) \end{array}}$

I) $P(X=0) = \binom{7}{0}(0.06)^0(0.94)^7 = 0.6485$

II) $P(X \geqslant 3) = 1 - P(X \leqslant 2) = \dots$ CALCULATOR …
$= 1 - 0.9937\dots$
$= 0.0063$

b) CONDITIONAL PROBABILITY NOW

$P(X=3) = \binom{7}{3}(0.06)^3(0.94)^4 = 0.00590246213\dots \simeq 0.0059$

WE REQUIRE

$P(X=3 \mid X \geqslant 3) = \frac{0.0059024\dots}{0.0063} \simeq 0.9378$

Question 2 **(**+)**

Ama is a supermarket cashier. The probability that Ama will have to rescan a shopping item because the barcode reader failed to "read" it, is 0.15.

A shopping item whose bar code is read on the first attempt is called a "first time item".

Ama scans 40 shopping items.

a) Determine the probability that Ama will have more than 31 but at most 37 "first time items".

Bama is a less experienced supermarket cashier. The probability that Bama will scan a shopping item on her first attempt is 0.7.

Bama scans 50 shopping items.

b) Determine the probability that Bama will have at least 35 "first time items".

$\boxed{}$, $\boxed{0.8160}$, $\boxed{0.5692}$

a) X = NO OF FIRST TIME ITEMS

$X \sim B(40, 0.85)$

$$P(31 < X \leqslant 37) = P(32 \leqslant X \leqslant 37)$$
$$= P(X \leqslant 37) - P(X \leqslant 31)$$
... CALCULATOR ...
$$= 0.95140\ldots - 0.13540\ldots$$
$$= 0.8160$$

b) Y = NUMBER OF FIRST TIME ITEMS

$Y \sim B(50, 0.7)$

$$P(Y \geqslant 35) = 1 - P(Y \leqslant 34) = 1 - 0.4308\ldots$$
$$= 0.5692$$

Question 3 (***)

The random variable X has binomial distribution $\mathrm{B}(13, 0.16)$.

a) Determine $\mathrm{P}(X < 2)$.

Two independent observations of X are made.

b) Find the probability that exactly one of these observations is equal to 2.

$\boxed{}$, $\boxed{0.3604}$, $\boxed{0.4146}$

a) $X \sim B(13, 0.16)$

$P(X<2) = P(X \le 1) = \ldots$ CALCULATOR $\ldots = 0.3604$

b) FIRSTLY WE NEED $P(X=2)$

$P(X=2) = \binom{13}{2}(0.16)^2(0.84)^{11} = 0.29336\ldots$

HENCE WE REQUIRE

- $P(X=2) \times P(X \neq 2)$
- $P(X \neq 2) \times P(X=2)$

$\left.\right\} = (0.29336\ldots)(1 - 0.29336\ldots) \times 2$

$= 0.4146$

Question 4 (***)

The discrete random variable X has the distribution $B(15, p)$.

a) Given that $p = 0.3$, determine ...

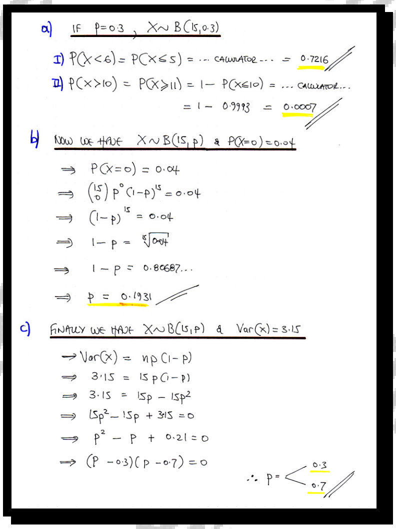
i. ... $P(X < 6)$.

ii. ... $P(X > 10)$.

b) Given instead that $P(X = 0) = 0.04$, determine the value of p, correct to three decimal places.

c) Given instead that $\text{Var}(X) = 3.15$, determine the two possible values of p.

$\boxed{}$, $\boxed{0.7216}$, $\boxed{0.0007}$, $\boxed{p \approx 0.193}$, $\boxed{p = 0.3 \text{ or } 0.7}$



a) IF $p = 0.3$, $X \sim B(15, 0.3)$

I) $P(X<6) = P(X \leqslant 5) = \ldots$ CALCULATOR $\ldots = 0.7216$

II) $P(X>10) = P(X \geqslant 11) = 1 - P(X \leqslant 10) = \ldots$ CALCULATOR$\ldots$

$= 1 - 0.9993 = 0.0007$

b) NOW WE HAVE $X \sim B(15, p)$ & $P(X=0) = 0.04$

$\Rightarrow P(X=0) = 0.04$

$\Rightarrow \binom{15}{0} p^0 (1-p)^{15} = 0.04$

$\Rightarrow (1-p)^{15} = 0.04$

$\Rightarrow 1 - p = \sqrt[15]{0.04}$

$\Rightarrow 1 - p = 0.80687\ldots$

$\Rightarrow p = 0.1931$

c) FINALLY WE HAVE $X \sim B(15, p)$ & $\text{Var}(X) = 3.15$

$\rightarrow \text{Var}(X) = np(1-p)$

$\Rightarrow 3.15 = 15p(1-p)$

$\Rightarrow 3.15 = 15p - 15p^2$

$\Rightarrow 15p^2 - 15p + 3.15 = 0$

$\Rightarrow p^2 - p + 0.21 = 0$

$\Rightarrow (p - 0.3)(p - 0.7) = 0$

$\therefore p = 0.3$ or 0.7

Question 5 (***)

The discrete random variable X has binomial distribution $\mathrm{B}(n, p)$.

Given that the mean and the standard deviation of X are both 0.95, determine the value of n.

, $\boxed{n = 19}$

$X \sim B(n,p)$

- $E(X) = \text{MEAN} = 0.95$

  $np = 0.95$

- $\text{Var}(X) = (\text{S.D})^2 = 0.95^2$

  $np(1-p) = 0.95^2$

$0.95 \times (1-p) = 0.95^2$

$1 - p = 0.95$

$p = 0.05$

& $np = 0.95$

$n \times 0.05 = 0.95$

$n = 19$ //

Question 6 (***+)

The random variable X has the binomial distribution $\mathrm{B}(n, 0.3)$.

The mean of X is three time as large as the standard deviation of X.

Determine the value of n.

, $\boxed{n = 21}$

$X \sim B(n, 0.3)$

MEAN $= np$

MEAN $= 0.3n$

STANDARD DEVIATION $= \sqrt{np(1-p)}$

STANDARD DEVIATION $= \sqrt{n \times 0.3 \times 0.7}$

STANDARD DEVIATION $= \sqrt{0.21n}$

SETTING UP AN EQUATION

$\Rightarrow$ MEAN $= 3 \times$ (STANDARD DEVIATION)

$\Rightarrow 0.3n = 3 \times \sqrt{0.21n}$

$\Rightarrow 0.09n^2 = 9(0.21n)$

$\Rightarrow 0.09n^2 = 1.89n$

$\Rightarrow n^2 = 21n$

$\Rightarrow n = 21 \quad (n \neq 0)$

Question 7 (***+)

A game is played by rolling simultaneously 15 standard fair six sided dice.

Find the probability that in a single roll of the 15 dice …

a) … no six will be obtained.

b) … exactly three dice will show a six.

c) … more than three dice will show a six.

A game is won if more than three dice show a six.

d) If ten games are played determine the probability of winning exactly 5 games.

$\boxed{}$, $\boxed{0.0649}$, $\boxed{0.2363}$, $\boxed{0.2315}$, $\boxed{0.045}$

$\boxed{\begin{array}{l} X = \text{NUMBER OF SIXES} \\ X \sim B(15, \tfrac{1}{6}) \end{array}}$

a) $P(X=0) = \binom{15}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{15} = 0.0649$

b) $P(X=3) = \binom{15}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^{12} = 0.2363$

c) $P(X>3) = P(X \geqslant 4) = 1 - P(X \leqslant 3) = \ldots \text{CALCULATOR} \ldots$

$= 1 - 0.76848\ldots = 0.2315$

d) REMODELLING WITH A NEW VARIABLE

$\boxed{\begin{array}{l} Y = \text{NUMBER OF GAMES WITH MORE THAN 3 SIXES} \\ Y \sim B(10, 0.2315) \end{array}}$

$P(Y=5) = \binom{10}{5}(0.2315)^5(1-0.2315)^5 = 0.0449$

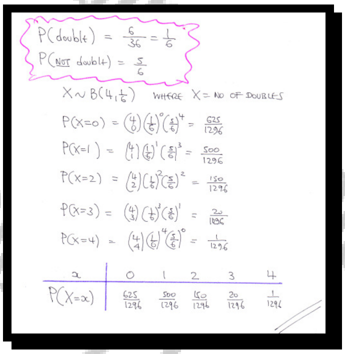
Question 8 (***+)

Two fair six sided dice are rolled and the event "the dice show the same number" is considered a favourable event.

The two dice are rolled together 4 times in a row and the random variable X represents the number of times the dice showed the same number.

Determine the probability distribution of X.

$$\boxed{\mathrm{P}(X=0)=\frac{625}{1296},\ \mathrm{P}(X=1)=\frac{500}{1296},\ \mathrm{P}(X=2)=\frac{150}{1296},\ \mathrm{P}(X=3)=\frac{20}{1296},\ \mathrm{P}(X=4)=\frac{1}{1296}}$$



$\mathrm{P}(\text{double}) = \frac{6}{36} = \frac{1}{6}$

$\mathrm{P}(\underline{\text{NOT}}\ \text{double}) = \frac{5}{6}$

$X \sim B(4, \frac{1}{6})$ WHERE X = NO OF DOUBLES

$\mathrm{P}(X=0) = \binom{4}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$

$\mathrm{P}(X=1) = \binom{4}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3 = \frac{500}{1296}$

$\mathrm{P}(X=2) = \binom{4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2 = \frac{150}{1296}$

$\mathrm{P}(X=3) = \binom{4}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 = \frac{20}{1296}$

$\mathrm{P}(X=4) = \binom{4}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^0 = \frac{1}{1296}$

x	0	1	2	3	4
$\mathrm{P}(X=x)$	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

Question 9 (***+)

The probability of a telesales representative making a sale on a customer call is 0.1.

a) Find the probability that a telesales representative achieves …

i. … no sales in 10 calls.

ii. … more than 4 sales in 20 calls.

Representatives are required to achieve a mean of at least 4 sales each day.

b) Find the least number of calls a representative should make each day, in order to achieve this requirement.

c) Calculate the least number of calls that a representative needs to make in a day for the probability of at least 1 sale, to exceed 0.98.

$\boxed{}$, $\boxed{0.3487}$, $\boxed{0.0432}$, $\boxed{n=40}$, $\boxed{n=38}$

a) I) $\boxed{\begin{array}{l}X = \text{NUMBER OF SALES IN 10}\\ X \sim B(10, 0.1)\end{array}}$

$P(X=0) = \binom{10}{0}(0.1)^0(0.9)^{10} = 0.3487$

II) $\boxed{\begin{array}{l}Y = \text{NUMBER OF SALES IN 20}\\ Y \sim B(20, 0.1)\end{array}}$

$P(Y>4) = P(Y \geq 5) = 1 - P(Y \leq 4) = \ldots$ CALCULATOR

$= 1 - 0.95682\ldots$

$= 0.0432$

b) USING "MEAN $= np$", HERE MEAN $= 4$

$\Rightarrow 4 = n \times 0.1$

$\Rightarrow n = 40$

c) HERE WE REQUIRE $W \sim B(n, 0.1)$, & $P(W \geq 1) > 0.98$

$\Rightarrow P(W \geq 1) > 0.98$

$\Rightarrow 1 - P(W=0) > 0.98$

$\Rightarrow -P(W=0) > -0.02$

$\Rightarrow P(W=0) < 0.02$

$\Rightarrow \binom{n}{0}(0.1)^0(0.9)^n < 0.02$

$\Rightarrow 1 \times 1 \times 0.9^n < 0.02$ BY LOGS OR TRIAL & IMPROVEMENT

$\Rightarrow \log(0.9^n) < \log(0.02)$

$\Rightarrow n\log(0.9) < \log(0.02)$

$\Rightarrow n > \frac{\log(0.02)}{\log(0.9)}$ $[\log(0.9) < 0]$

$\Rightarrow n > 37.12\ldots$ $\therefore n = 38$

Question 10 (***+)

A geologist is looking for fossils in rocks. In a certain area it has been established over a long period of time that 10% of the rocks contain fossils. The geologist selects twenty rocks from this area.

a) State 2 conditions that must be apply in order for a binomial model to be valid.

Find the probability that in the geologist's sample there will be …

b) … one rock containing fossils.

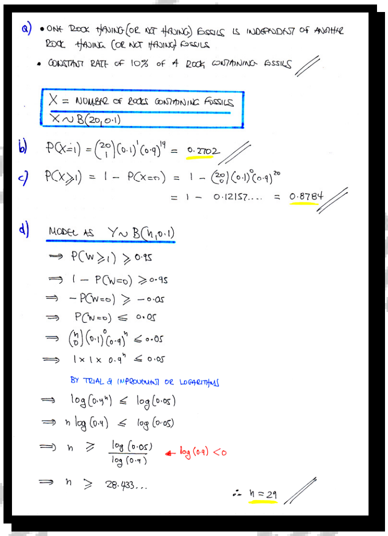
c) … at least one rock containing fossils.

The geologist selects a new sample of n rocks.

He wants to have at least a 95% chance that his new sample will contain fossils.

d) Determine the smallest value of n.

$\boxed{}$, $\boxed{\text{state as appropriate}}$, $\boxed{0.2702}$, $\boxed{0.8784}$, $\boxed{n=29}$



a) • ONE ROCK HAVING (OR NOT HAVING) FOSSILS IS INDEPENDENT OF ANOTHER ROCK HAVING (OR NOT HAVING) FOSSILS

• CONSTANT RATE OF 10% OF A ROCK CONTAINING FOSSILS

$$\boxed{\begin{array}{l} X = \text{NUMBER OF ROCKS CONTAINING FOSSILS} \\ X \sim B(20, 0.1) \end{array}}$$

b) $P(X=1) = \binom{20}{1}(0.1)^1(0.9)^{19} = 0.2702$

c) $P(X \geqslant 1) = 1 - P(X=0) = 1 - \binom{20}{0}(0.1)^0(0.9)^{20}$

$= 1 - 0.12157\ldots = 0.8784$

d) MODEL AS $Y \sim B(n, 0.1)$

$\Rightarrow P(W \geqslant 1) \geqslant 0.95$

$\Rightarrow 1 - P(W=0) \geqslant 0.95$

$\Rightarrow -P(W=0) \geqslant -0.05$

$\Rightarrow P(W=0) \leqslant 0.05$

$\Rightarrow \binom{n}{0}(0.1)^0(0.9)^n \leqslant 0.05$

$\Rightarrow 1 \times 1 \times 0.9^n \leqslant 0.05$

BY TRIAL & IMPROVEMENT OR LOGARITHMS

$\Rightarrow \log(0.9^n) \leqslant \log(0.05)$

$\Rightarrow n\log(0.9) \leqslant \log(0.05)$

$\Rightarrow n \geqslant \dfrac{\log(0.05)}{\log(0.9)}$ ← $\log(0.9) < 0$

$\Rightarrow n \geqslant 28.433\ldots$

$\therefore n = 29$

Question 11 (***+)

The table below summarizes census information about the number of children in the households of an English town.

Number of children	0	1	2	3	4 or more
Percentage of households	23	32	35	7	3

A random sample of 20 households is selected from this town.

a) Determine the probability that the sample will contain ...

i. ... 3 households with no children.

ii. ... more than half the households, with at least 2 children.

iii. ... more than 15 but at most 19 households, with at most 2 children.

A new random sample of n households is selected from this town.

The probability that this new sample contains a household with 4 or more children is more than 10%.

b) Determine the smallest value of n.

$\boxed{}$, $\boxed{0.1631}$, $\boxed{0.2493}$, $\boxed{0.8352}$, $\boxed{n=4}$

a)

NO OF CHILDREN	0	1	2	3	$\geqslant 4$
PERCENTAGE OF HOUSEHOLDS	23%	32%	35%	7%	3%

I) X = "NO KID" HOUSEHOLD
$X \sim B(20, 0.23)$

$P(X=3) = \binom{20}{3} 0.23^3\, 0.77^{17}$

$= 0.1631$

II) Y = "AT LEAST 2 KIDS" HOUSEHOLD
$Y \sim B(20, 0.45)$

$P(\text{more than half})$
$= P(Y>10) = P(Y \geqslant 11)$
$= 1 - P(Y \leqslant 10) = \dots \text{tables}$
$= 1 - 0.7507$
$= 0.2493$

III) W = "AT MOST 2 KIDS" HOUSEHOLD
$W \sim B(20, 0.9)$

MODEL WITH COMPLIMENT
$W' \sim B(20, 0.1)$

$P(15 < W \leqslant 19)$
$= P(16 \leqslant W \leqslant 19)$
$= P(1 \leqslant W' \leqslant 4)$
$= P(W' \leqslant 4) - P(W' \leqslant 0)$
$= 0.9568 - 0.1216$
$= 0.8352$

W	W'
16	4
17	3
18	2
19	1

b) V = A FOUR OR MORE KID HOUSEHOLD
$V \sim B(n, 0.03)$

$\Rightarrow P(V \geqslant 1) > 10\%$
$\Rightarrow P(V \geqslant 1) > 0.1$
$\Rightarrow 1 - P(V=0) > 0.1$
$\Rightarrow -P(V=0) > -0.9$
$\Rightarrow P(V=0) < 0.9$
$\Rightarrow \binom{n}{0}(0.03)^0(0.97)^n < 0.9$
$\Rightarrow 0.97^n < 0.9$

BY LOGARITHMS

$\Rightarrow \log 0.97^n < \log 0.9$
$\Rightarrow n \log 0.97 < \log 0.9$
$\Rightarrow n > \frac{\log 0.9}{\log 0.97}$
$\Rightarrow n > 3.459\dots$
$\Rightarrow n = 4$

OR TRIAL AND IMPROVEMENT

$n=5 \quad 0.97^5 = 0.858 < 0.9$
$n=4 \quad 0.97^4 = 0.885 < 0.9$
$n=3 \quad 0.97^3 = 0.913 > 0.9$

$\therefore n = 4$

Question 12 (***+)

The records in a doctor's surgery show that 20% of the patients that make an appointment fail to turn up.

During a given weekday there are 20 appointments to see the doctor and the doctor has enough time to see all 20 patients.

a) Find the probability that all the patients **will** turn up.

b) Find the probability that more than three patients **will not** turn up.

In order to improve efficiency in the surgery, the doctor decides to make more than 20 appointments although he still has enough time to see only 20 patients.

One day the doctor has booked 21 appointments.

c) Find the probability he will be able to see all the patients that **will** turn up.

Another day the doctor has booked 25 appointments.

d) Find the probability that the doctor **will not** be able to see one of the patients that will turn up.

$\boxed{}$, $\boxed{0.0115}$, $\boxed{0.5886}$, $\boxed{0.9908}$, $\boxed{0.1867}$

$\boxed{\begin{array}{l} X = \text{no of "appointments" which fail to turn up} \\ X \sim B(20, 0.2) \end{array}}$

a) $P(X=0) = \binom{20}{0}(0.2)^0(0.8)^{20} = 0.0115$

b) $P(X>3) = P(X \geq 4) = 1 - P(X \leq 3) = \dots$ calculator/tables
$= 1 - 0.41144\dots = 0.5886$

c) "HE WILL BE ABLE TO SEE ALL THE PATIENTS THAT TURN UP"

$X \sim B(21, 0.2)$

P(AT LEAST ONE DOES NOT TURN UP) $= P(X \geq 1)$
$= 1 - P(X=0)$
$= 1 - \binom{21}{0}(0.2)^0(0.8)^{21}$
$= 1 - 0.009223\dots$
$= 0.9908$

OR MODEL AS

$\boxed{\begin{array}{l} Y = \text{NO OF APPOINTMENTS WHICH TURN UP} \\ Y \sim B(21, 0.8) \end{array}}$

$P(Y \leq 20) = \dots$ calculator/tables
$= 0.9908$

d) $\boxed{\begin{array}{l} Y = \text{NUMBER OF PEOPLE WHICH SHOW UP} \\ Y \sim B(25, 0.8) \end{array}}$

P(NOT BEING ABLE TO SEE ONE OF THE PATIENTS WHICH TURN UP)
$= P(Y=21)$
$= \binom{25}{21}(0.8)^{21}(0.2)^4$
$= 0.1867$

Question 13 (***+)

Jane and Amber are telesales operatives for a company.

The probability of Jane making a sale on a customer call is 0.1.

a) Showing a clear method, find the probability that Jane achieves …

i. … no sales in 10 calls.

ii. … more than 4 sales in 20 calls.

The probability of Amber making a sale on a customer call is 0.15.

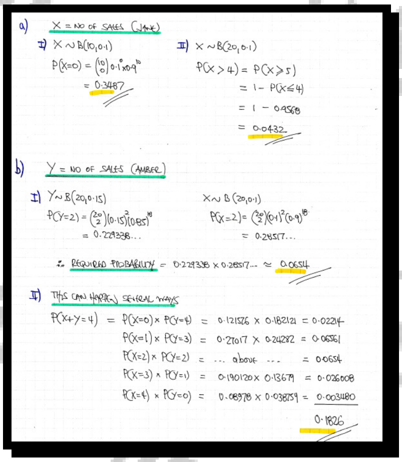
b) If Jane and Amber make 20 calls each, determine the probability that they will make

i. … 2 sales each.

ii. … a total of 4 sales between them.

c) Calculate the least number of calls that Amber needs to make for the probability of at least 1 sale, to exceed 0.99.

$\boxed{}$, $\boxed{0.3487}$, $\boxed{0.0432}$, $\boxed{0.0654}$, $\boxed{0.1826}$, $\boxed{n = 29}$



a) X = NO OF SALES (JANE)

I) $X \sim B(10, 0.1)$

$P(X=0) = \binom{10}{0} 0.1^0 \times 0.9^{10}$

$= 0.3487$

II) $X \sim B(20, 0.1)$

$P(X>4) = P(X \geqslant 5)$

$= 1 - P(X \leqslant 4)$

$= 1 - 0.9568$

$= 0.0432$

b) Y = NO OF SALES (AMBER)

I) $Y \sim B(20, 0.15)$

$P(Y=2) = \binom{20}{2}(0.15)^2(0.85)^{18}$

$= 0.229338\ldots$

$X \sim B(20, 0.1)$

$P(X=2) = \binom{20}{2}(0.1)^2(0.9)^{18}$

$= 0.28517\ldots$

$\therefore$ REQUIRED PROBABILITY $= 0.229338 \times 0.28517\ldots \approx 0.0654$

II) THIS CAN HAPPEN SEVERAL WAYS

$P(X+Y=4) = P(X=0) \times P(Y=4) = 0.121576 \times 0.182121 = 0.02214$

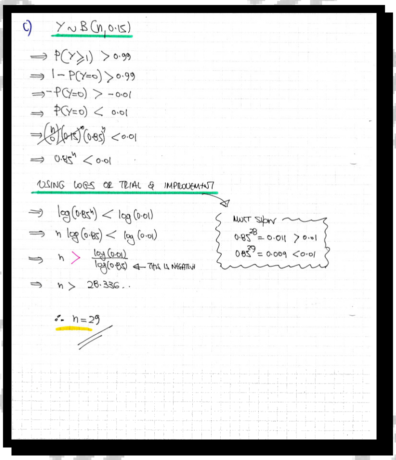
$P(X=1) \times P(Y=3) = 0.27017 \times 0.24282 = 0.06561$

$P(X=2) \times P(Y=2) = \ldots$ above $\ldots = 0.0654$

$P(X=3) \times P(Y=1) = 0.190120 \times 0.13679 = 0.026008$

$P(X=4) \times P(Y=0) = 0.08978 \times 0.038759 = 0.003480$

0.1826



c) $Y \sim B(n, 0.15)$

$\Rightarrow P(Y \geqslant 1) > 0.99$

$\Rightarrow 1 - P(Y=0) > 0.99$

$\Rightarrow -P(Y=0) > -0.01$

$\Rightarrow P(Y=0) < 0.01$

$\Rightarrow \cancel{\binom{n}{0}(0.15)^0}(0.85)^n < 0.01$

$\Rightarrow 0.85^n < 0.01$

USING LOGS OR TRIAL & IMPROVEMENT

$\Rightarrow \log(0.85^n) < \log(0.01)$

$\Rightarrow n \log(0.85) < \log(0.01)$

$\Rightarrow n > \frac{\log(0.01)}{\log(0.85)}$ ← THIS IS NEGATIVE

$\Rightarrow n > 28.336\ldots$

MUST SHOW

$0.85^{28} = 0.011 > 0.01$

$0.85^{29} = 0.009 < 0.01$

$\therefore n = 29$

Question 14 (****)

General purpose hooks are sold in boxes of 50.

The probability that a hook will be defective is 0.05.

10 boxes of hooks are examined.

Determine, to 2 significant figures, the probability that half of these boxes will contain more than 5 defective hooks each.

$\boxed{}$, $\boxed{0.000016}$

X = NO OF DEFECTIVE HOOKS IN A BOX

$X \sim B(50, 0.05)$

$P(X>5) = P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.96222\ldots$

$= 0.037776\ldots$

Y = A BOX WITH MORE THAN 5 DEFECTIVE HOOKS

$Y \sim B(10, 0.037776)$

$P(Y=5) = \binom{10}{5}(0.037776)^5(0.96222)^5 = 0.000016$

Question 13 (****)

a) State four conditions for making a binomial distribution an appropriate model for statistical work.

Roy and Ned are door to door salesmen.

The probability of Roy making a sale on a house visit is 0.05.

b) Showing a clear method, find the probability that Roy achieves …

i. … no sales in 15 visits.

ii. … more than 3 sales in 20 visits.

c) Calculate the least number of visits that Roy needs to make for the probability of at least 1 sale, to exceed 0.99.

The probability of Ned making a sale on a house visit is 0.15.

d) If Roy and Ned make 20 house visits **each**, determine the probability that they will make …

i. … 2 sales each.

ii. … a total of 5 sales between them.

Ned makes 240 house visits in a given week.

e) Use a Normal approximation, to find the probability that Ned achieves more than 30 sales in these 240 visits.

$\boxed{}$, $\boxed{0.4633}$, $\boxed{0.0159}$, $\boxed{n=90}$, $\boxed{0.0433}$, $\boxed{0.167}$, $\boxed{0.8399}$

a)
- FIXED TRIALS
- TWO OUTCOMES
- CONSTANT PROBABILITY OF SUCCESS
- INDEPENDENCE OF EVENTS

b) X = NUMBER OF SALES (ROY)

I) $X \sim B(15, 0.05)$

$P(X=0) = \binom{15}{0}(0.05)^0(0.95)^{15}$

$= 0.4633$

II) $X \sim B(20, 0.05)$

$P(X>3) = P(X \geqslant 4)$

$= 1 - P(X \leqslant 3)$

$= 1 - 0.9841$

$= 0.0159$

c) WE REQUIRE THAT

$\Rightarrow P(X \geqslant 1) > 0.99$

$\Rightarrow 1 - P(X=0) > 0.99$

$\Rightarrow -P(X=0) > -0.01$

$\Rightarrow P(X=0) < 0.01$

$\Rightarrow \binom{n}{0}(0.05)^0(0.95)^n < 0.01$

$\Rightarrow 0.95^n < 0.01$

$\Rightarrow \log(0.95^n) < \log(0.01)$

$\Rightarrow n \log(0.95) < \log(0.01)$

$\Rightarrow n > \dfrac{\log(0.01)}{\log(0.95)}$ (DIVIDED BY NEGATIVE)

$\Rightarrow n > 89.78\ldots$

$\therefore n = 90$

d) ORGANIZING THE OUTCOMES USING Y = SALES MADE BY ROY

$X \sim B(20, 0.05)$ & $Y \sim B(20, 0.15)$

I) $P(X=2) \times P(Y=2) = \binom{20}{2}(0.05)^2(0.95)^{18} \times \binom{20}{2}(0.15)^2(0.85)^{18} = 0.0433$

II) $P(X=0) = \binom{20}{0}(0.05)(0.95)^5 = 0.3585$

$P(X=1) = \binom{20}{1}(0.05)^1(0.95)^4 = 0.3774$

$P(X=2) = \binom{20}{2}(0.05)^2(0.95)^3 = 0.1887$

$P(X=3) = \binom{20}{3}(0.05)^3(0.95)^2 = 0.0596$

$P(X=4) = \binom{20}{4}(0.05)^4(0.95)^1 = 0.0133$

$P(X=5) = \binom{20}{5}(0.05)^5(0.95)^0 = 0.0022$

$P(Y=0) = \binom{20}{0}(0.15)^0(0.85)^5 = 0.0388$

$P(Y=1) = \binom{20}{1}(0.15)^1(0.85)^4 = 0.1368$

$P(Y=2) = \binom{20}{2}(0.15)^2(0.85)^3 = 0.2293$

$P(Y=3) = \binom{20}{3}(0.15)^3(0.85)^2 = 0.2428$

$P(Y=4) = \binom{20}{4}(0.15)^4(0.85)^1 = 0.1821$

$P(Y=5) = \binom{20}{5}(0.15)^5(0.85)^0 = 0.1028$

SO WE HAVE

(0,5)	(5,0)	(1,4)	(4,1)	(2,3)	(3,2)
↓	↓	↓	↓	↓	↓
0.0369	0.0001	0.0687	0.0018	0.0458	0.0137

ADDING TO OBTAIN THE DESIRED RESULT OF 0.167

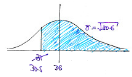
e) FINALLY APPROXIMATE BY A NORMAL DISTRIBUTION

$Y \sim B(240, 0.15)$

MEAN $= 240 \times 0.15 = 36$

VARIANCE $= 36 \times 0.85 = 30.6$

$W \sim N(30, 30.6)$



$P(Y>30) = P(Y \geqslant 31)$

$= P(W > 30.5)$

$= P\left(Z > \frac{30.5-36}{\sqrt{30.6}}\right)$

$= \Phi(-0.9943)$

$= 0.8399$

Question 15 (****)

In a certain village market, eggs are sold in boxes of 12.

The probability that a box will contain double yolked eggs is 0.15.

Determine the probability that…

a) … an egg picked at random from a box will be double yolked.

b) … in 20 boxes at least 13 boxes but less than 19 boxes will <u>not</u> contain double yolked eggs.

$\boxed{\text{S-X}}$, $\boxed{0.0135}$, $\boxed{0.8185}$

a) $\boxed{\begin{array}{l} X = \text{NUMBER OF DOUBLE YOLKED EGGS} \\ X \sim B(12, p) \end{array}}$

$\Rightarrow P(X \geqslant 1) = 0.15$

$\Rightarrow 1 - P(X=0) = 0.15$

$\Rightarrow 0.85 = P(X=0)$

$\Rightarrow \binom{12}{0} p^0 (1-p)^{12} = 0.85$

$\Rightarrow 1 \times 1 \times (1-p)^{12} = 0.85$

$\Rightarrow 1 - p = +\sqrt[12]{0.85}$

$\Rightarrow 1 - p = 0.98654\ldots$

$\Rightarrow p = 0.01345$

b) $\boxed{\begin{array}{l} Y = \text{NUMBER OF BOXES WITHOUT DOUBLE YOLKED EGGS} \\ Y \sim B(20, 0.85) \end{array}}$

$$P(13 \leqslant Y < 19) = P(13 \leqslant Y \leqslant 18)$$
$$= P(Y \leqslant 18) - P(Y \leqslant 12)$$
$$= 0.82444\ldots - 0.00592$$
$$= 0.8185$$

Question 16 (****)

Daniel takes part in a quiz where he has to answer 12 Geography questions.

The probability that Daniel answer any one of these questions correctly is assumed to be constant at 0.8. Daniel wins £100 for each correct answer but loses £50 for an incorrect answer.

Let X denote the number of questions Daniel answers correctly.

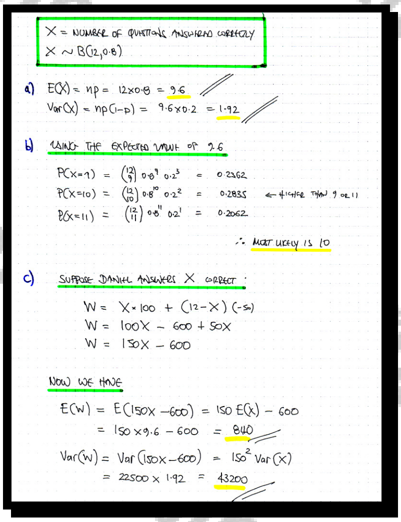
Let W denote total amount that Daniel wins at the end of the quiz.

a) Find the mean and variance of X.

b) Determine, indicating your reasoning, the most likely value of X.

c) Calculate the mean and variance of W

$\boxed{\mathrm{E}(X)=9.6}$, $\boxed{\mathrm{Var}(X)=1.92}$, $\boxed{10}$, $\boxed{\mathrm{E}(W)=840}$, $\boxed{\mathrm{Var}(W)=43200}$



X = NUMBER OF QUESTIONS ANSWERED CORRECTLY

$X \sim B(12, 0.8)$

a) $E(X) = np = 12 \times 0.8 = 9.6$

$Var(X) = np(1-p) = 9.6 \times 0.2 = 1.92$

b) USING THE EXPECTED VALUE OF 9.6

$P(X=9) = \binom{12}{9} 0.8^9\, 0.2^3 = 0.2362$

$P(X=10) = \binom{12}{10} 0.8^{10}\, 0.2^2 = 0.2835$ ← HIGHER THAN 9 OR 11

$P(X=11) = \binom{12}{11} 0.8^{11}\, 0.2^1 = 0.2062$

∴ MOST LIKELY IS 10

c) SUPPOSE DANIEL ANSWERS X CORRECT:

$W = X \times 100 + (12-X)(-50)$

$W = 100X - 600 + 50X$

$W = 150X - 600$

NOW WE HAVE

$E(W) = E(150X - 600) = 150E(X) - 600$

$= 150 \times 9.6 - 600 = 840$

$Var(W) = Var(150X - 600) = 150^2 Var(X)$

$= 22500 \times 1.92 = 43200$

Question 17 (****)

The random variable X has the distribution $\mathrm{B}(20, 0.8)$.

a) Find $\mathrm{P}(X > 14)$.

The random variable Y has the distribution $\mathrm{B}(4, 0.4)$.

b) Find $\mathrm{P}(Y = 3)$.

Three independent observations Y_1, Y_1 and Y_1 are recorded.

c) Determine $\mathrm{P}(Y_1 + Y_3 + Y_3 = 10)$.

$\boxed{}$, $\boxed{0.8042}$, $\boxed{0.1536}$, $\boxed{0.00249}$

a) $X \sim B(20, 0.8)$

$P(X > 14) = P(X \geq 15) = 1 - P(X \leq 14)$

$= 1 - 0.19579 = 0.8042$

b) $Y \sim B(4, 0.4)$

$P(Y = 3) = \binom{4}{3} 0.4^3 0.6^1 = 0.1536$

c) LISTING OUTCOMES

- 3, 3, 4 (3 WAYS)
- 4, 4, 2 (3 WAYS)

$P(Y = 2) = \binom{4}{2} 0.4^2 0.6^2 = 0.3456$

$P(Y = 3) = 0.1536$ (PART b)

$P(Y = 4) = \binom{4}{4} 0.4^4 0.6^0 = 0.0256$

$P(Y_1 + Y_2 + Y_3 = 10) = P(3,3,4) + P(3,4,3) + P(4,3,3) + P(4,4,2) + P(4,2,4) + P(2,4,4)$

$= 0.1536^2 \times 0.0256 \times 3 + 0.0256^2 \times 0.3456 \times 3$

$= 0.00249$

Question 18 (****)

Of a particular variety of rose bush, half produce red flowers and the rest produce white flowers.

A selection of 9 rose bushes of this variety, are bought.

a) Find probability that more than 6 of these bushes will produce red flowers.

Another selection of 24 rose bushes of this variety are bought.

b) Determine the probability that more bushes will produce red flowers than white flowers.

You may not use cumulative probabilities or distributional approximations in this part.

$\boxed{}$, $\boxed{0.0898}$, $\boxed{0.4194}$

a) $\boxed{\begin{array}{l} X = \text{number of "red producing flowers" bushes} \\ X \sim B(9, 0.5) \end{array}}$

$P(X>6) = P(X \geqslant 7) = 1 - P(X \leqslant 6)$

$= 1 - 0.910156... = 0.0898$

b) WORK AS FOLLOWS

- $P(X=12) = \binom{24}{12}(0.5)^{12}(0.5)^{12} = 0.16118...$
- $P(X>12) = P(X = 12, 13, 14, ..., 24)$
- $P(X<11) = P(X = 0, 1, 2, 3, ... 11)$

NOTE THAT $P(X=0) = P(X=24) = \binom{24}{0}(0.5)^{0}(0.5)^{24} = \binom{24}{24}(0.5)^{24}(0.5)^{0}$

$P(X=1) = P(X=23) = \binom{24}{1}(0.5)^{1}(0.5)^{23} = \binom{24}{23}(0.5)^{24}(0.5)^{1}$

ETC

$\therefore P(X<11) + P(X=12) + P(X>12) = 1$

$P(X>12) + 0.16118... + P(X>12) = 1$

$2P(X>12) = 0.83881...$

$P(X>12) = 0.4194$

OR IN "SIMPLER LANGUAGE" AS $p=0.5$

$P(\text{"red > white"}) = P(\text{"white > red"})$

$\therefore$ REQUIRED PROBABILITY $= \dfrac{1 - P(X=12)}{2}$

Question 19 (****)

The discrete random variables X and Y are independent from one another and are defined as

$$X \sim \mathrm{B}(16, 0.25) \quad \text{and} \quad Y \sim \mathrm{Po}(2).$$

a) Find the value of $\mathrm{Var}(XY)$.

b) Determine $\mathrm{P}(XY = 3)$

$\boxed{\mathrm{Var}(XY) = 50}$, $\boxed{\mathrm{P}(XY = 3) \approx 0.0659}$

a) WORK AS FOLLOWS

- $X \sim B(16, 0.25)$
- $E(X) = np = 16 \times 0.25 = 4$
- $\mathrm{Var}(X) = np(1-p) = 4 \times 0.75 = 3$

- $Y \sim Po(2)$
- $E(Y) = 2$
- $\mathrm{Var}(Y) = 2$

NOW WE HAVE

- $E(X^2) = \mathrm{Var}(X) + [E(X)]^2$
- $E(X^2) = 3 + 4^2 = 19$

- $E(Y^2) = \mathrm{Var}(Y) + [E(Y)]^2$
- $E(Y^2) = 2 + 2^2 = 6$

$E(XY) = E(X)E(Y) = 4 \times 2 = 8$

$E((XY)^2) = E(X^2Y^2) = E(X^2)E(Y^2) = 19 \times 6 = 114$

FINALLY WE HAVE

$\Longrightarrow \mathrm{Var}(XY) = E((XY)^2) - [E(XY)]^2$

$\Longrightarrow \mathrm{Var}(XY) = 114 - 8^2$

$\Longrightarrow \mathrm{Var}(XY) = 50$

b) $P(XY=3) = P(X=1)P(Y=3) + P(X=3)P(Y=1)$

$= \binom{16}{1}(0.25)^1(0.75^{15}) \times \dfrac{e^{-2} \times 2^3}{3!} + \binom{16}{3}(0.25)^3(0.75)^{13} \times \dfrac{e^{-2} \times 2^1}{1!}$

$= \frac{16}{3}e^{-2}(0.75)^{15} + \frac{35}{2}e^{-2}(0.75)^{13}$

$= 0.009646\ldots + 0.0562659\ldots$

$= 0.0659$

Question 20 (****)

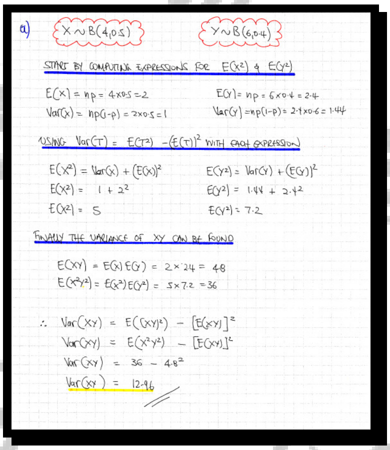
The discrete random variables X and Y are independent from one another and are defined as

$$X \sim \mathrm{B}(4, 0.5) \quad \text{and} \quad Y \sim \mathrm{B}(6, 0.4).$$

a) Find the value of $\mathrm{Var}(XY)$.

b) Determine $\mathrm{P}(XY = 0)$

$\boxed{}$, $\boxed{\mathrm{Var}(XY) = 12.96}$, $\boxed{\mathrm{P}(XY = 0) = \frac{332}{3125} = 0.10624}$



a) $X \sim B(4, 0.5)$ $\quad$ $Y \sim B(6, 0.4)$

START BY COMPUTING EXPRESSIONS FOR $E(X^2)$ & $E(Y^2)$

$E(X) = np = 4 \times 0.5 = 2$ $\qquad$ $E(Y) = np = 6 \times 0.4 = 2.4$

$\mathrm{Var}(X) = np(1-p) = 2 \times 0.5 = 1$ $\qquad$ $\mathrm{Var}(Y) = np(1-p) = 2.4 \times 0.6 = 1.44$

USING $\mathrm{Var}(T) = E(T^2) - (E(T))^2$ WITH EACH EXPRESSION

$E(X^2) = \mathrm{Var}(X) + (E(X))^2$ $\qquad$ $E(Y^2) = \mathrm{Var}(Y) + (E(Y))^2$

$E(X^2) = 1 + 2^2$ $\qquad$ $E(Y^2) = 1.44 + 2.4^2$

$E(X^2) = 5$ $\qquad$ $E(Y^2) = 7.2$

FINALLY THE VARIANCE OF XY CAN BE FOUND

$E(XY) = E(X)E(Y) = 2 \times 2.4 = 4.8$

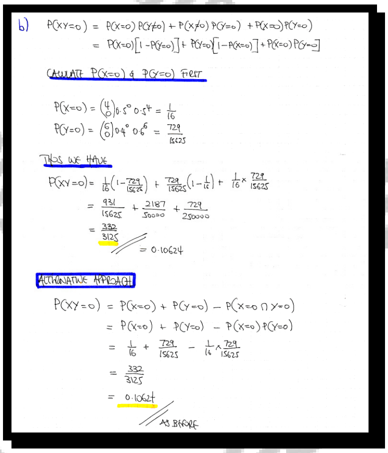
$E(X^2Y^2) = E(X^2)E(Y^2) = 5 \times 7.2 = 36$

$\therefore \mathrm{Var}(XY) = E((XY)^2) - [E(XY)]^2$

$\mathrm{Var}(XY) = E(X^2Y^2) - [E(XY)]^2$

$\mathrm{Var}(XY) = 36 - 4.8^2$

$\mathrm{Var}(XY) = 12.96$



b) $P(XY=0) = P(X=0)P(Y \neq 0) + P(X \neq 0)P(Y=0) + P(X=0)P(Y=0)$

$= P(X=0)[1 - P(Y=0)] + P(Y=0)[1 - P(X=0)] + P(X=0)P(Y=0)$

CALCULATE $P(X=0)$ & $P(Y=0)$ FIRST

$P(X=0) = \binom{4}{0} 0.5^0\, 0.5^4 = \frac{1}{16}$

$P(Y=0) = \binom{6}{0} 0.4^0\, 0.6^6 = \frac{729}{15625}$

THUS WE HAVE

$P(XY=0) = \frac{1}{16}\left(1 - \frac{729}{15625}\right) + \frac{729}{15625}\left(1 - \frac{1}{16}\right) + \frac{1}{16} \times \frac{729}{15625}$

$= \frac{931}{15625} + \frac{2187}{50000} + \frac{729}{250000}$

$= \frac{332}{3125}$ $= 0.10624$

ALTERNATIVE APPROACH

$P(XY=0) = P(X=0) + P(Y=0) - P(X=0 \cap Y=0)$

$= P(X=0) + P(Y=0) - P(X=0)P(Y=0)$

$= \frac{1}{16} + \frac{729}{15625} - \frac{1}{16} \times \frac{729}{15625}$

$= \frac{332}{3125}$

$= 0.10624$

AS BEFORE

Question 21 (****)

It has been established over a long period of time that in a particular variety of rose bushes, 0.2 produce pink flowers.

A selection of 10 rose bushes of this variety, are bought.

a) Find probability that more than 4 of these bushes will produce red flowers.

b) Calculate the least number of rose bushes that need to be bought so that the probability of producing at least 1 plant with pink flowers exceeds 0.975.

Another selection of 125 rose bushes of this variety are bought.

c) **Use a Normal distribution approximation** to determine the probability that the number of bushes which will produce pink flowers will be more than 21 but no more than 30.

Finally a selection of 25 rose bushes of the same variety are considered. When these rose bushes flowered they produced 10 plants with pink flowers.

d) Stating your hypotheses clearly, test at the 1% level of significance, whether this constitutes evidence that this variety of rose bushes have a higher probability than 0.2 in producing pink flowers.

[illegible], $\boxed{0.3704}$, $\boxed{n=17}$, $\boxed{0.673}$, $\boxed{\text{not sigificant} \quad 1.73\% > 1\%}$

X = NUMBER OF BUSHES WITH PINK FLOWERS, $X \sim B(20, 0.2)$

a) $P(X>4) = P(X \geq 5) = 1 - P(X \leq 4)$
$= \ldots$ tables $\ldots$
$= 1 - 0.6296$
$= 0.3704$

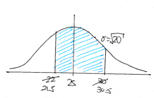
b) $P(X \geq 1) > 0.975$

$\Rightarrow 1 - P(X=0) > 0.975$
$\Rightarrow 0.025 < P(X=0)$
$\Rightarrow \binom{n}{0}(0.2)^0(0.8)^n > 0.025$
$\Rightarrow 0.8^n < 0.025$
$\Rightarrow \log(0.8^n) < \log(0.025)$
$\Rightarrow n\log(0.8) < \log(0.025)$
$\Rightarrow n > \frac{\log(0.025)}{\log(0.8)}$
[log 0.8 IS NEGATIVE]
$\Rightarrow n > 16.53\ldots$
$\therefore n = 17$

c) APPROXIMATE BY NORMAL

$X \sim B(125, 0.2)$

- MEAN $= 125 \times 0.2 = 25$
- VARIANCE $= 25 \times 0.8 = 20$



$Y \sim N(25, 20)$

$P(21 < X \leq 30)$
$= P(22 \leq X \leq 30)$
$= P(21.5 < Y < 30.5)$
$= P(Y < 30.5) - P(Y < 21.5)$
$= P(Y < 30.5) - [1 - P(Y > 21.5)]$
$= P(Y < 30.5) + P(Y > 21.5) - 1$
$= P\left(z < \frac{30.5-25}{\sqrt{20}}\right) + P\left(z > \frac{21.5-25}{\sqrt{20}}\right) - 1$
$= \Phi(1.23) + (-0.78) - 1$
$= 0.8907 + 0.7823 - 1$
$\simeq 0.673$

d) SETTING HYPOTHESES FOR $X \sim B(25, 0.2)$

$H_0: p = 0.2$
$H_1: p > 0.2$

WITH p THE PROPORTION OF PINK FLOWERING BUSHES IN GENERAL

TESTING AT 1% SIGNIFICANCE ON THE BASIS $X = 10$

$P(X \geq 10) = 1 - P(X \leq 9)$
$= 1 - 0.9827$
$= 0.0173$
$= 1.73\% > 1\%$

THERE IS NO SIGNIFICANT EVIDENCE AT 1% THAT THE PROPORTION IS GREATER THAN 0.2 — INSUFFICIENT EVIDENCE TO REJECT H_0

Question 22 (****)

*In this question you may **only** use the binomial table in the following page if you require **cumulative** probabilities.*

Elastic cat collars are available in boxes of 50 collars and in two different varieties, standard and fluorescent. The proportions of different colours of these cat collars in the two varieties are shown in the tables below.

Standard Colour	Red	Blue	White	Black
Proportion	0.175	0.4	0.325	0.1

Fluorescent Colour	Yellow	Pink	Green	Orange
Proportion	0.18	0.4	0.15	0.27

A box of 50 **standard colour** cat collars is selected at random.

a) Determine the probability that this box will contain …

i. … exactly 6 red cat collars.

ii. … at least 15 but less than 25 blue cat collars.

Next, another box of 50 **standard colour** cat collars and a box of 50 **fluorescent colour** cat collars are selected at random.

b) Determine the probability that in these 100 cat collars there will be …

i. … no black collars

ii. … fewer than 20 blue or black cat collars **and** more than 30 yellow or orange cat collars.

Finally, another box of 50 **fluorescent colour** cat collars are selected at random.

c) Determine the probability that in these cat collars there will be at least 25 but less than 33 cat collars, that are **not pink**.

$\boxed{}$, $\boxed{0.0962}$, $\boxed{0.8482}$, $\boxed{0.0052}$, $\boxed{0.0012}$, $\boxed{0.7058}$

[solutions overleaf]

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 50$, $x = 0$	0.0769	0.0052	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.2794	0.0338	0.0029	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.5405	0.1117	0.0142	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.7604	0.2503	0.0460	0.0057	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8964	0.4312	0.1121	0.0185	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000
5	0.9622	0.6161	0.2194	0.0480	0.0070	0.0007	0.0001	0.0000	0.0000	0.0000
6	0.9882	0.7702	0.3613	0.1034	0.0194	0.0025	0.0002	0.0000	0.0000	0.0000
7	0.9968	0.8779	0.5188	0.1904	0.0453	0.0073	0.0008	0.0001	0.0000	0.0000
8	0.9992	0.9421	0.6681	0.3073	0.0916	0.0183	0.0025	0.0002	0.0000	0.0000
9	0.9998	0.9755	0.7911	0.4437	0.1637	0.0402	0.0067	0.0008	0.0001	0.0000
10	1.0000	0.9906	0.8801	0.5836	0.2622	0.0789	0.0160	0.0022	0.0002	0.0000
11	1.0000	0.9968	0.9372	0.7107	0.3816	0.1390	0.0342	0.0057	0.0006	0.0000
12	1.0000	0.9990	0.9699	0.8139	0.5110	0.2229	0.0661	0.0133	0.0018	0.0002
13	1.0000	0.9997	0.9868	0.8894	0.6370	0.3279	0.1163	0.0280	0.0045	0.0005
14	1.0000	0.9999	0.9947	0.9393	0.7481	0.4468	0.1878	0.0540	0.0104	0.0013
15	1.0000	1.0000	0.9981	0.9692	0.8369	0.5692	0.2801	0.0955	0.0220	0.0033
16	1.0000	1.0000	0.9993	0.9856	0.9017	0.6839	0.3889	0.1561	0.0427	0.0077
17	1.0000	1.0000	0.9998	0.9937	0.9449	0.7822	0.5060	0.2369	0.0765	0.0164
18	1.0000	1.0000	0.9999	0.9975	0.9713	0.8594	0.6216	0.3356	0.1273	0.0325
19	1.0000	1.0000	1.0000	0.9991	0.9861	0.9152	0.7264	0.4465	0.1974	0.0595
20	1.0000	1.0000	1.0000	0.9997	0.9937	0.9522	0.8139	0.5610	0.2862	0.1013
21	1.0000	1.0000	1.0000	0.9999	0.9974	0.9749	0.8813	0.6701	0.3900	0.1611
22	1.0000	1.0000	1.0000	1.0000	0.9990	0.9877	0.9290	0.7660	0.5019	0.2399
23	1.0000	1.0000	1.0000	1.0000	0.9996	0.9944	0.9604	0.8438	0.6134	0.3359
24	1.0000	1.0000	1.0000	1.0000	0.9999	0.9976	0.9793	0.9022	0.7160	0.4439
25	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	0.9900	0.9427	0.8034	0.5561
26	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9955	0.9686	0.8721	0.6641
27	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9981	0.9840	0.9220	0.7601
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993	0.9924	0.9556	0.8389
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9966	0.9765	0.8987
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9986	0.9884	0.9405
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9947	0.9675
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9978	0.9836
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9923
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9967
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

a) I) X = NUMBER OF RED COLLARS

$X \sim B(50, 0.175)$

$P(X=6) = \binom{50}{6}(0.175)^6(0.825)^{44} = \underline{0.0962}$

II) Y = NUMBER OF BLUE CAT COLLARS

$Y \sim B(50, 0.4)$

$P(15 \leq Y < 25) = P(15 \leq Y \leq 24) = P(Y \leq 24) - P(Y \leq 14)$

$= 0.9022 - 0.0540 = \underline{0.8482}$

b) I) IRRELEVANT FOR THIS PART THE 50 FLUORESCENT COLLARS

W = NUMBER OF NON BLACK COLLARS

$W \sim B(50, 0.9)$

OR

W = NUMBER OF BLACK COLLARS

$W \sim B(50, 0.1)$

$P(W=50) = \binom{50}{50}(0.9)^{50}(0.1)^0 = \underline{0.0052}$

OR

$P(W=0) = \binom{50}{0}(0.1)^0(0.9)^{50} = \underline{0.0052}$

II) U = NUMBER OF BLUE OR BLACK COLLARS

$U \sim B(50, 0.5)$

V = NUMBER OF YELLOW OR ORANGE COLLARS

$V \sim B(50, 0.45)$

$P(U < 20) = P(U \leq 19)$

$= 0.1013$

$P(V > 30) = P(V \geq 31)$

$= 1 - P(V \leq 30)$

$= 1 - 0.9884$

$= 0.0116$

$\therefore$ REQUIRED PROBABILITY $= 0.1013 \times 0.0116 = \underline{0.0012}$

c) T = NUMBER OF NON-PINK COLLARS

$T \sim B(50, 0.6)$

$P(25 \leq T < 33) = P(25 \leq T \leq 32)$

CANNOT USE THE TABLES

REMODEL AS

S = NUMBER OF PINK COLLARS

$S \sim B(50, 0.4)$

T:	25	26	27	28	29	30	31	32
S:	25	24	23	22	21	20	19	18

$\Rightarrow P(25 \leq T < 33) = P(18 \leq S \leq 25)$

$= P(S \leq 25) - P(S \leq 17)$

$= 0.9427 - 0.2369$

$= \underline{0.7058}$

Question 23 (****)

A discrete random variable X has distribution

$$X \sim B\left(6, \tfrac{1}{3}\right).$$

a) Find $P(X = 2)$.

Two independent observations of X, denoted by X_1 and X_2 are considered.

b) Determine the probability that the sum of these two observations will be less than 2.

Eight independent observations of X are selected at random.

c) Determine the probability that half of these observations will be a 2.

$\boxed{}$, $\boxed{\frac{80}{243} \approx 0.3292}$, $\boxed{\approx 0.0540}$, $\boxed{\approx 0.1665}$

a) $X \sim B\left(6, \frac{1}{3}\right)$

$P(X=2) = \binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4 = \frac{80}{243} \approx 0.3292$

b) WE REQUIRE $P(X_1 + X_2 < 2)$

$P(X=0) \times P(X=0) = \left[\binom{6}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6\right]^2 \approx 0.007707\ldots$

$\left.\begin{matrix} P(X=0) \times P(X=1) \\ P(X=1) \times P(X=0) \end{matrix}\right\} = \left[\binom{6}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^6\right] \times \left[\binom{6}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5\right] \times 2 \text{ WAYS}$

$\approx 0.046244\ldots$

$\therefore$ REQUIRED PROBABILITY IS $0.007707 + 0.046244 \approx 0.0540$

c) $\boxed{\begin{matrix} Y = \text{AN OBSERVATION OF 2 FROM } X \\ Y \sim B\left(8, \frac{80}{243}\right) \end{matrix}}$

$P(Y=4) = \binom{8}{4}\left(\frac{80}{243}\right)^4\left(\frac{163}{243}\right) \approx 0.1665$

Question 24 (****+)

A discrete random variable X has distribution

$$X \sim \mathrm{B}\left(8, \tfrac{1}{4}\right).$$

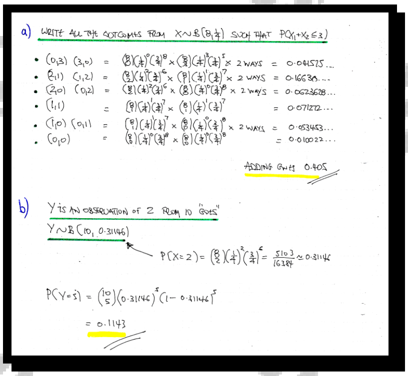
Two independent observations of X, denoted by X_1 and X_2 are considered.

a) Determine $\mathrm{P}\left(X_1 + X_2 \le 3\right)$.

Ten independent observations of X are selected at random.

b) Determine the probability that half of these observations will be a 2.

$\boxed{}$, $\boxed{\approx 0.405}$, $\boxed{\approx 0.1143}$



a) WRITE ALL THE OUTCOMES FROM $X \sim B(8, \frac{1}{4})$ SUCH THAT $P(X_1 + X_2 \le 3)$

- $(0,3)\ (3,0) = \binom{8}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^8 \times \binom{8}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^5 \times 2 \text{ WAYS} = 0.041525\ldots$
- $(2,1)\ (1,2) = \binom{8}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6 \times \binom{8}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^7 \times 2 \text{ WAYS} = 0.16638\ldots$
- $(2,0)\ (0,2) = \binom{8}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6 \times \binom{8}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^8 \times 2 \text{ WAYS} = 0.0623628\ldots$
- $(1,1) = \binom{8}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^7 \times \binom{8}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^7 = 0.071272\ldots$
- $(1,0)\ (0,1) = \binom{8}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^7 \times \binom{8}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^8 \times 2 \text{ WAYS} = 0.053453\ldots$
- $(0,0) = \binom{8}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^8 \times \binom{8}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^8 = 0.010022\ldots$

ADDING GIVES 0.405

b) Y IS AN OBSERVATION OF 2 FROM 10 "GOES"

$Y \sim B(10, 0.31146)$

$P(X=2) = \binom{8}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^6 = \frac{5103}{16384} \simeq 0.31146$

$P(Y=5) = \binom{10}{5}(0.31146)^5(1-0.31146)^5$

$= 0.1143$

Question 25 (****+)

A multiple choice test consists of 25 questions, each question having 4 responses. Each correct answer gains 3 marks but each incorrect answer loses 1 mark.

Suppose a certain candidate chooses to answer all 25 questions at random, and let X be the number of correct answers that this candidate achieves.

a) State the distribution of X, defining all relevant parameters.

Let M be the number of marks that this candidate achieves.

b) Show clearly that the number of marks that this candidate is expected to achieve is zero.

c) Determine the variance of his marks.

In order to pass in this test at least 27 marks are required.

d) Find the probability that this candidate will pass the test, briefly commenting on his approach to this test.

$\boxed{\text{Var}(M) = 75}$, $\boxed{0.0004}$

(a) X = A correct answer given at random

$X \sim B(25, \frac{1}{4})$

(b) $M = X \times 3 - (25 - X) \times 1$

$M = 3X - 25 + X$

$M = 4X - 25$

$E(M) = E(4X - 25)$

$E(M) = 4E(X) - 25$

BUT $E(X) = np = 25 \times \frac{1}{4} = \frac{25}{4}$

$\therefore E(M) = 4 \times \frac{25}{4} - 25 = 0$

(c) $\text{Var}(M) = \text{Var}(4X - 25)$

$= 16\text{Var}(X)$

BUT $\text{Var}(X) = np(1-p) = \frac{25}{4} \times \frac{3}{4} = \frac{75}{16}$

SO $\text{Var}(M) = 16 \times \frac{75}{16}$

$\text{Var}(M) = 75$

(d) $P(M \geqslant 27)$

$= P(4X - 25 \geqslant 27)$

$= P(4X \geqslant 52)$

$= P(X \geqslant 13)$

$= 1 - P(X \leqslant 12)$

$= \ldots$ looking at tables

$= 1 - 0.9996$

$= 0.0004$

NOT A GOOD APPROACH AS THE CHANCE OF PASSING IS PRACTICALLY ZERO

Question 26 (****+)

During hot days, an ice cream van sells a large number of ice cream cones containing either 1, 2 or 3 scoops of ice cream.

The respective probabilities of a customer buying a 1, 2 or 3 scoop ice cream cone are $\frac{1}{6}$, $\frac{1}{2}$ or $\frac{1}{3}$.

A random sample of n customers is examined, each customer having bought an ice cream cone from this van. The probability that more than n scoops of ice cream are ordered by these n customers is greater than 0.9999.

Determine the smallest possible value of n.

$\boxed{n=6}$

NO OF SCOOPS	1	2	3
PROBABILITY	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

PROCEED AS FOLLOWS

$\Rightarrow P(\text{MORE THAN } n \text{ SCOOPS OF ICE CREAM WERE ORDERED BY } n \text{ CUSTOMERS}) > 0.99$

$\Rightarrow P(\text{AT LEAST ONE CUSTOMER BOUGHT MORE THAN 1 SCOOP}) > 0.99$

$\Rightarrow 1 - P(\text{ALL } n \text{ CUSTOMERS BOUGHT 1 SCOOP}) > 0.99$

$\Rightarrow 1 - \left(\frac{1}{6}\right)^n > 0.9999$

$\Rightarrow -\left(\frac{1}{6}\right)^n > -0.0001$

$\Rightarrow \left(\frac{1}{6}\right)^n < 0.0001$

$\Rightarrow 6^n > 10000$

$\Rightarrow \log_{10}(6^n) > \log_{10}(10000)$

$\Rightarrow n\log_{10}6 > 4$

$\Rightarrow n > \frac{4}{\log_{10}6}$

$\Rightarrow n > 5.140388\ldots$

$\therefore n=6$

Question 27 (****+)

The probability that a student answers correctly the first question, of a multiple choice exam, is 0.2.

A random sample of 30 students which sat this exam is considered.

a) Determine the probability that more than 5 but at most 10 students answered the first question correctly.

Students earn 5 marks for answering the first question correctly but loose 2 marks for answering incorrectly.

b) Find the probability that the total number of marks scored by all 30 students, in answering the first question, is more than 17.

$\boxed{}$, $\boxed{0.5469}$, $\boxed{0.0095}$

LET X = NUMBER OF STUDENTS WHICH ANSWERED CORRECTLY THE FIRST QUESTION.

$\boxed{X \sim B(30, 0.2)}$

a) $P(5 < X \leq 10) = P(6 \leq X \leq 10)$
$= P(X \leq 10) - P(X \leq 5)$
$= 0.97438... - 0.42751...$
$= 0.5469$

b) T = TOTAL MARKS OF THE 30 STUDENTS FROM THE FIRST QUESTION

$T = X \times 5 - (30 - X) \times 2$
$T = 5X + 2X - 60$
$T = 7X - 60$

$\Rightarrow P(T > 17) = P(7X - 60 > 17)$
$= P(7X > 77)$
$= P(X > 11)$
$= P(X \geq 12)$
$= 1 - P(X \leq 11)$
$= 1 - 0.9905$
$= 0.0095$

Question 28 (*****)

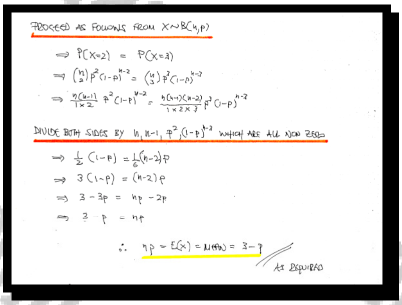
It is given that the discrete random variable X satisfies

$$X \sim \mathrm{B}(n, p).$$

Given further that $\mathrm{P}(X=2)=\mathrm{P}(X=3)$, show that

$$\mathrm{E}(X)=3-p.$$

$\boxed{}$, $\boxed{\text{proof}}$



PROCEED AS FOLLOWS FROM $X \sim B(n,p)$

$\Rightarrow P(X=2) = P(X=3)$

$\Rightarrow \binom{n}{2} p^2 (1-p)^{n-2} = \binom{n}{3} p^3 (1-p)^{n-3}$

$\Rightarrow \frac{n(n-1)}{1 \times 2} p^2 (1-p)^{n-2} = \frac{n(n-1)(n-2)}{1 \times 2 \times 3} p^3 (1-p)^{n-3}$

DIVIDE BOTH SIDES BY $n, n-1, p^2, (1-p)^{n-3}$ WHICH ARE ALL NON ZERO

$\Rightarrow \frac{1}{2}(1-p) = \frac{1}{6}(n-2)p$

$\Rightarrow 3(1-p) = (n-2)p$

$\Rightarrow 3-3p = np-2p$

$\Rightarrow 3-p = np$

$\therefore np = E(X) = \text{MEAN} = 3-p$

AS REQUIRED