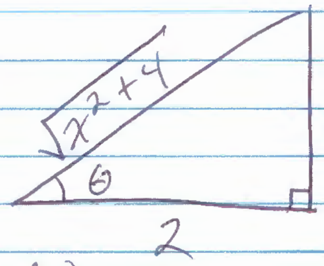
Section 7.3 Solutions    Math 31

2. $\displaystyle\int \frac{x^3}{\sqrt{x^2+4}}\,dx$

Let $x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$\sqrt{x^2+4} = 2\sec\theta$




$$= \int \frac{(2\tan\theta)^3(2\sec^2\theta\, d\theta)}{2\sec\theta}$$

$$= 8\int \tan^3\theta \sec\theta\, d\theta = 8\int \tan^2\theta\,(\sec\theta\tan\theta\, d\theta)$$

$$= 8\int (\sec^2\theta - 1)(\sec\theta\tan\theta\, d\theta),$$

Let $u = \sec\theta$

$du = \sec\theta\tan\theta\, d\theta$

$$= 8\int (u^2-1)\,du = 8\left(\tfrac{1}{3}u^3 - u\right) + C$$

$$= 8\left(\tfrac{1}{3}\sec^3\theta - \sec\theta\right) + C = 8\left(\frac{1}{3}\cdot\frac{(x^2+4)^{3/2}}{8} - \frac{\sqrt{x^2+4}}{2}\right) + C$$

$$= \boxed{\frac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C}$$

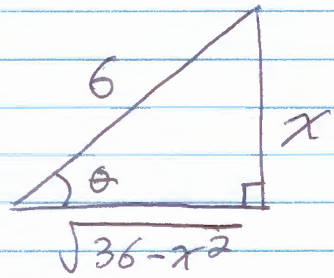
6. $\displaystyle\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$

$x = 6\sin\theta$

$dx = 6\cos\theta\, d\theta$

$\sqrt{36-x^2} = 6\cos\theta$

| $x$ | $\theta$ |
|---|---|
| 0 | 0 |
| 3 | $\pi/6$ |




$$= \int_0^{\pi/6} \frac{(6\sin\theta)(6\cos\theta\, d\theta)}{6\cos\theta}$$

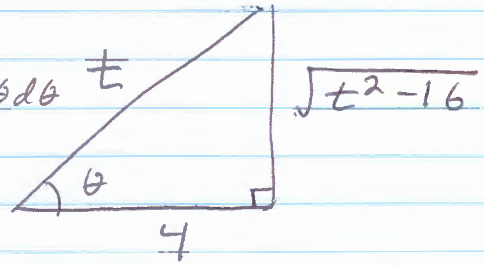
$$= 6\int_0^{\pi/6} \sin\theta\, d\theta = -6\cos\theta\Big|_0^{\pi/6} = \boxed{6 - 3\sqrt{3}}$$

8. $\displaystyle\int \frac{dt}{t^2\sqrt{t^2-16}}$

$t = 4\sec\theta$

$dt = 4\sec\theta\tan\theta\, d\theta$

$\sqrt{t^2-16} = 4\tan\theta$




$$= \int \frac{4\sec\theta\tan\theta\, d\theta}{(4\sec\theta)^2(4\tan\theta)}$$

$$= \frac{1}{16}\int \cos\theta\, d\theta = \frac{1}{16}\sin\theta + C = \boxed{\frac{1}{16}\cdot\frac{\sqrt{t^2-16}}{t} + C}$$

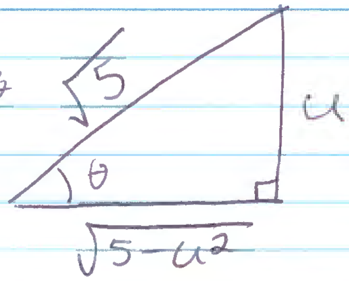
# Section 7.3 Solutions Continued

12. $\displaystyle\int \frac{du}{u\sqrt{5-u^2}}$

$u=\sqrt{5}\sin\theta$

$du=\sqrt{5}\cos\theta\,d\theta$

$\sqrt{5-u^2}=\sqrt{5}\cos\theta$




$\displaystyle = \int \frac{\sqrt{5}\cos\theta\,d\theta}{(\sqrt{5}\sin\theta)(\sqrt{5}\cos\theta)}$

$\displaystyle = \frac{1}{\sqrt{5}}\int \csc\theta\,d\theta = \frac{1}{\sqrt{5}}\ln|\csc\theta-\cot\theta|+C$

$\displaystyle = \frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5}}{u}-\frac{\sqrt{5-u^2}}{u}\right|+C = \boxed{\frac{1}{\sqrt{5}}\ln\left|\frac{5-\sqrt{5-u^2}}{u}\right|+C}$

30. $\displaystyle\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}\,dt$

Let $u=\sin t$

$du=\cos t\,dt$

| $t$ | $u$ |
|---|---|
| $0$ | $0$ |
| $\pi/2$ | $1$ |

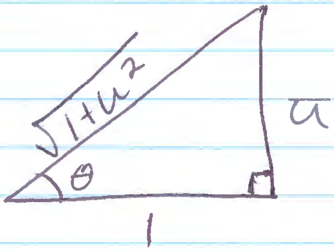
$\displaystyle = \int_0^1 \frac{du}{\sqrt{1+u^2}}$

Let $u=\tan\theta$

$du=\sec^2\theta\,d\theta$

$\sqrt{1+u^2}=\sec\theta$

| $u$ | $\theta$ |
|---|---|
| $0$ | $0$ |
| $1$ | $\pi/4$ |




$\displaystyle = \int_0^{\pi/4} \frac{\sec^2\theta\,d\theta}{\sec\theta}$

$\displaystyle = \int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta|\Big|_0^{\pi/4}$

$= \ln|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}| - \ln|\sec 0+\tan 0|$

$= \ln|\sqrt{2}+1| - \ln|1+0|$

$= \ln|\sqrt{2}+1| = \boxed{\ln(\sqrt{2}+1)}$