Landé g-factor Derivation.
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In the weak field limit, we assume that the magnetic dipole moment due to the electron in an atom is proportional to the total angular momentum **J**:

$$\boldsymbol{\mu}_J = -\, g_J \frac{m_B}{\hbar} \boldsymbol{J}$$

where m_B is the Bohr magneton ($\frac{e\hbar}{2m_e}$) and g_J is the Landé g factor for $\boldsymbol{J}$. To find a value of g_J, we relate $\boldsymbol{J}$ to the known values of g_L and g_S and assume the following:

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S} \qquad \boldsymbol{\mu}_J = \boldsymbol{\mu}_L + \boldsymbol{\mu}_S$$

$$\boldsymbol{\mu}_L = -\, g_L \frac{m_B}{\hbar} \boldsymbol{L} \qquad \boldsymbol{\mu}_S = -\, g_S \frac{m_B}{\hbar} \boldsymbol{S}$$

$$\boldsymbol{\mu}_J = -\, g_J \frac{m_B}{\hbar} \boldsymbol{J} = -\, g_L \frac{m_B}{\hbar} \boldsymbol{L} + -g_S \frac{m_B}{\hbar} \boldsymbol{S}$$

$$g_J \boldsymbol{J} = g_L \boldsymbol{L} + g_S \boldsymbol{S}$$

We then take the dot product with **J** and get:

$$g_J \boldsymbol{J} \bullet \boldsymbol{J} = g_L \boldsymbol{L} \bullet \boldsymbol{J} + g_S \boldsymbol{S} \bullet \boldsymbol{J}$$

$$g_J J^2 = g_L \left(L^2 + \boldsymbol{L} \bullet \boldsymbol{S}\right) + g_S \left(S^2 + \boldsymbol{L} \bullet \boldsymbol{S}\right)$$

$$g_J J^2 = g_L \left(L^2 + 0.5\left(J^2 - L^2 - S^2\right)\right) + g_S \left(S^2 + 0.5\left(J^2 - L^2 - S^2\right)\right)$$

$$J^2 = \hbar^2 J(J+1) \qquad L^2 = \hbar^2 L(L+1) \qquad S^2 = \hbar^2 S(S+1)$$

$$g_J = \frac{g_L\left(J(J+1) + L(L+1) - S(S+1)\right) + g_S\left(J(J+1) - L(L+1) + S(S+1)\right)}{2J(J+1)}$$

For the case:

$$g_L = 1$$
$$g_S = 2$$

We get:

$$g_J = \frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)}$$

And so, for the weak field limit:

$$W_z = -\boldsymbol{\mu}_J \bullet \boldsymbol{B} = g_J \omega_0 J_z$$

$$\omega_0 = -\frac{m_B}{\hbar} B_0 = -\frac{eB_0}{2m_e}$$

where ω_0 is the Larmor angular frequency.

g_J = 4/3 for 2p(j=3/2) state (L=1, S=1/2, J =3/2)

g_J = 2/3 for 2p(j=1/2) state (L=1, S=1/2, J =1/2)

Et voilà! These are eqns. 11-13 in the CTcompD12.pdf handout.