Glimpses of Benoît B. Mandelbrot (1924–2010)

Edited by Michael F. Barnsley and Michael Frame

Two roads diverged in a wood, and I
— I took the one less travelled by,
And that has made all the difference.
(Robert Frost, "The Road Not Taken")

Michael F. Barnsley

Introduction

Benoît B. Mandelbrot died in Cambridge, Massachusetts, on Thursday, 14 October 2010. He was eighty-five years old and Sterling Professor Emeritus of Mathematical Sciences at Yale University. He was also IBM Fellow Emeritus (physics) at the IBM T. J. Watson Research Center. He was a great and rare mathematician and scientist. He changed the way that many of us see, describe and model, mathematically and geometrically, the world around us. He moved between disciplines and university departments, from geology to physics, to computer science, to economics and engineering, talking excitedly, sometimes obscurely, strangely vain, about all manner of things, theorizing, speculating, and often in recent years, to the annoyance of others, pointing out how he had earlier done work of a related nature to whatever it was that someone was explaining, bobbing up and down to interrupt, to explain this or that. He was an unforgettable, extraordinary person of great warmth who was also vulnerable and defensive.

Figure 1. Benoît Mandelbrot next to John Robinson's sculpture *Intuition* outside the Isaac Newton Institute, Cambridge, during the Mathematics and Applications of Fractals Program in 1999. (Photo: Findlay Kember/Isaac Newton Institute.)

Looking back, Benoît saw his life as a rough path. In [7] he recounted how his father escaped from Poland and the Nazis with a group of others and, at a certain point, went a different route through the woods, which saved his life. Benoît saw his own life in similar terms: he too took the path less travelled by, and that made him very different from most mathematicians. What he did that was different was to work in many areas, following where his geometrical intuitions led, regardless of academic boundaries. This path repeatedly risked failure and embarrassment because each discipline has

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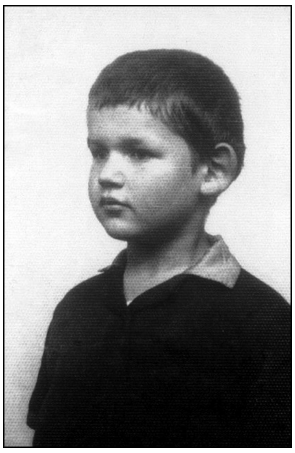

Photo courtesy of Aliette Mandelbrot.

Figure 2. Benoît, age four, in Poland.

its entrenched big guns, experts surrounded by well-constructed defenses, peer groups armed with stacks of citations.

Prior to writing both this article and [1], we emailed colleagues to ask for memories and comments on Benoît's contributions to mathematics, his influence, and personal recollections. We received replies from many—not only mathematicians, but artists, physicists, biologists, engineers, and so on. Our goal has been to put together a pair of memorial articles, something special, using the words of everyone who wrote, but, in general, editing and shortening to avoid repetitions of themes. The second article, [1], is centered on Benoît's influence and contributions to mathematics. The present article is more directed to the man himself in a personal manner. Both Michael Frame and I knew and loved Benoît: Michael Frame was his sidekick at Yale for many years, and I, Michael B., first met him in 1981 and a number of times during the following fifteen years, mainly during the early 1980s. In 1988 he came to my home in Atlanta for dinner during the Siggraph conference, together with Richard Voss, Heinz-Otto Peitgen, and others. His magical personality filled the dining room that hot summer evening, contrasted with his house in Scarsdale, where I first met him and Aliette on a February day in 1981, with snow and light gleaming off it into the windows of his book-and-Xerox-piled office.

There is a large body of written materials, available online, that are easily accessed and which recount aspects of Benoît's life, times, research, quotations, and opinions. But here we try to capture afresh the fact that he was one of us, a mathematician, and to give a glimpse and feeling, for the time that you read this, of the real and amazing man that he was.

Ian Stewart

He Began His Lecture by Shuffling His Slides

My first contact with Mandelbrot was when he phoned me to say that he'd been asked to write a popular article on fractals, didn't have time, and wondered whether I'd be interested. I accepted the invitation. From then on, he would occasionally call me when some unusually striking development in fractal geometry had happened. When *The Economist* asked me to write a feature article on applications of fractals, Benoît mailed me a stack of photocopies six inches thick, saving me weeks of work.

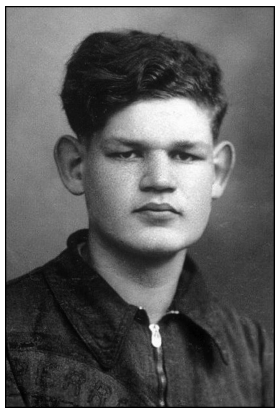

Photo courtesy of Aliette Mandelbrot.

Figure 3. Benoît, age thirteen or fourteen, in Paris.

I met him a few times. He visited the University of Warwick and began his lecture by shuffling his slides to make sure it was different from previous talks. My wife and I had breakfast with him at a conference on financial mathematics in Santa Fe. He was in great spirits because a conjecture of his had just been proved, but he knew that my wife was not a mathematician, so he took care to avoid talking shop.

David Mumford

He Opened a Door and Let in a Gale of Wind

I met Benoît when he came to Harvard as a visiting professor in 1979. At that time, the Harvard math department was an insulated place, a temple of pure math. His appearance opened a door and let in a gale of wind. He was a large man and his presence was large too. He gave lectures in a dozen departments, and every lecture dealt with a different phenomenon of nature. He seemed to have studied everything and picked up grist for his mill in every corner of the world.

I had some wonderful times socially with Benoît and his wife, Aliette. They were warm and fascinating hosts who seemed to know everyone too. I remember especially talking about Gadjusek, the discoverer of the link between cannibalism and prion diseases, who was a good friend of theirs. I last saw him at the birthday celebratory meeting in his honor at Bad Neuenahr. Surrounded by his hosts who had contributed so much to his theories, he gave a moving speech on the fact that this was his first visit to Germany since the Holocaust. Benoît was a completely unique person and scientist who cannot be pigeon-holed and his influence has been vast. I count myself very lucky to have known and worked with him.

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Kenneth Falconer

Benoît Told Me…Everyone Would Be Merry after Food and Wine

The first time that I met Benoît was at the Winter Workshop on Fractals at Les Houches in 1984. This was the first time that I had encountered the "fractal community". It was an eye-opening meeting, as I realized the wealth of ideas that was emerging from mathematicians and scientists interested in fractals. I have a vivid memory of the friendliness and encouragement shown to me by Benoît at that meeting. The highlight of the week's cuisine was the fondue, and I was scheduled to give the evening talk immediately afterwards. Benoît took me aside and told me that there was no need to be nervous, as everyone would be merry after the food and wine, so my talk was bound to be appreciated! In fact, I think that the talk did go well. This was the first occasion on which I presented my "digital sundial" theorem—that there exists a fractal such that its orthogonal projections can be essentially anything one wishes, for example, the thickened digits of the time. Along the same lines I also proposed the construction of a space station that was plainly visible to Western countries and effectively invisible to Eastern countries, to the amusement of many present. Benoît told me afterwards that he liked these examples because they gave a "visual" interpretation of an abstract mathematical theorem.

The year 1984 also saw the publication of *The Geometry of Fractal Sets* [4], which was one of the earliest books on fractals, apart from those of Benoît himself. Benoît provided very helpful comments on my manuscript, but in a review he did refer to "…the usual dry mathematics", though I don't think he meant it unkindly. In later years Benoît coauthored a number of papers employing similar formal mathematics, so I think that once his ideas had been accepted by the mathematical community, he became less concerned about the "dryness".

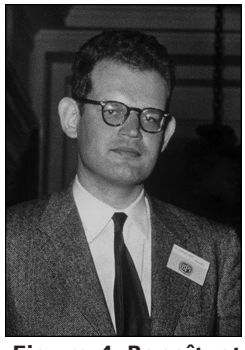

Photo courtesy of Aliette Mandelbrot.

Figure 4. Benoît, at age twenty-eight or twenty-nine, was living in France.

I saw a great deal of Benoît during the program Mathematics and Applications of Fractals at the Isaac Newton Institute in Cambridge in 1999; indeed he, along with Aliette, occupied an office adjacent to mine. The four-month program was organized by Robin Ball and me, and we were delighted that Benoît stayed in Cambridge for the entire time. A number of young researchers and research students took part, and Benoît made a point of taking time to encourage them by talking to them all and discussing ideas with them individually.

I was delighted when Benoît accepted an honorary degree from the University of St. Andrews in 1999. It was a pleasure to entertain him and Aliette in my hometown, and it was clear that receiving such an honor from a Scottish university meant a great deal to him. I recall that, as we crossed the Firth of Forth on the way from the airport, he commented that the Forth Railway Bridge, constructed in 1890, displayed fine fractal features in its hierarchical structure!

For many years I met Benoît regularly at conferences; he was rarely absent from any meeting on fractals. He once paid me the biggest compliment that my lecturing has ever received: "I really liked your talk, Ken; you have such a wonderful theatrical style!"

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Ron Eglash

"That Is Not Criticism; That Is a Tribute to Your Work"

I am best known for my book *African Fractals* [3]. Needless to say, that would have been impossible without Benoît! Much of the research in ethnomathematics had been things like "how to count to 10 in Yoruba" or "African houses are shaped like a cylinder." But when I first saw aerial photos of African villages, their fractal structure was immediately obvious. That gave me a basis for a Fulbright fellowship to Africa, and once I was there I found that that recursive scaling cropped up in all sorts of artifacts and knowledge systems, from sculpture and textiles to divination and cosmology. The NSF has allowed us to develop software for teaching math and computing using fractal algorithms [13]. This work has also caught the eye of architects; for example, there are now plans for an entire university in Angola to have a fractal layout. Benoît leaves behind a legacy on many continents.

Photo courtesy of Aliette Mandelbrot.

Figure 5. Benoît at age thirty-nine or forty when he was working at IBM.

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Figure 6. Benoît Mandelbrot, Kenneth Falconer, and Keith Ball outside the Isaac Newton Institute, Cambridge, in 1999. (Photo: Findlay Kember/Isaac Newton Institute.)

Figure 7. This photo, taken by Richard Bernt, shows, from left to right, Richard Voss, Benoît B. Mandelbrot, Dietmar Saupe, and Heinz-Otto Peitgen. Richard Bernt believes it was taken in May 1987. It was the week Dietmar and Voss went sailing on his catamaran. Mandelbrot and he had talked about the fractal dimension of ocean waves and how it was similar to music. The photo was taken at UC Santa Cruz by the Applied Science Building, which had picnic tables nearby. It was near the offices of Heinz-Otto, Dietmar, and Ralph Abraham, near room 366, where they, with Richard Bernt, generated fantastic fractal images on SGI machines.

The first time I spoke to Benoît was when he visited UCSC, where I was in graduate school in the late 1980s. After his lecture, I asked him why some fractals show Euclidean shapes—the Sierpinski gasket, for example—and others show only mush or globs that show no recognizable shapes. To my surprise he said, "I have been asking myself that question for over a decade and have yet to find a satisfactory answer." The second time was a phone call; he wanted to know if I had been given tenure—he had written a recommendation for my case. We got to chatting about *African Fractals*, and he asked me if I was getting any criticism for it. So I described some of the hate mail I was receiving from critics who insisted that black people had genetically inferior brains and could not have created fractals on their own. He said, "That is not criticism; that is a tribute to your work!"

Harlan Brothers

Ways in Which Music Could Manifest Fractal Structure

Benoît's most important contribution to education was the work he did in conjunction with Michael Frame and Nial Neger in conducting the Fractal Geometry Workshops at Yale. Related collaborations included the book *Fractals, Graphics, & Mathematics Education* [5], the DVD *Mandelbrot's World of Fractals* [8], and the vast Yale website on fractal geometry [12], which contains the collection of labs called *Kitchen Science Fractals*. Thanks to Benoît's vision, countless young minds around the globe have come to appreciate mathematics through their exposure to fractal geometry.

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It was Benoît who set me on the path of establishing some mathematical rigor for the term "fractal music". Prior to the summer of 2003, someone had given him and Michael Frame a CD of what purported to be fractal music. They passed it on to me. When I explained that the composer did not seem to have a solid grasp of the fundamentals, Benoît agreed, saying, "Yes, I think you are right. If you would like to look into this subject, that would be wonderful."

In collaboration with Michael Frame, the following summer we did a presentation and lab on fractal music. I have since continued to publish and present on the subject and last year appeared in the BBC documentary *Bach & Friends* [2] discussing fractal geometry and its relationship to the music of Bach. I regularly receive email from students around the world, high school through grad school, who are working on projects or have questions about fractal music.

I had been recommended to Benoît by a former student of his, Miguel Garcia, who was my professor at Gateway Community College. I will always remember our first meeting at Yale. Benoît was seated, his hands pointed in and resting on his legs. He began by saying, in his inimitable accent, "So, Miguel tells me you are not the average cookie-cutter student…" I shared some of my research,

Figure 8. Benoît Mandelbrot in an iconic pose, fingers together, appearing in the documentary *Clouds Are Not Spheres* [9]. The shot was taken in Paris in 1999. (Photo courtesy of Nigel Lesmoir-Gordon.)

and by the time I left, he had shared everything from the lesser-known work of John Venn to the sociopolitical history of Budapest dating back to the sixteenth century. Since then, over the years, through the Fractal Geometry Workshops and in numerous phone calls, Benoît continued to share his overwhelming expertise, his humor, and his wisdom in practical matters. His generosity of spirit and fundamental good nature have inspired me and helped to define who I am.

Nigel Lesmoir-Gordon

Benoît Was Superb, Inspiring and Lucid

I made the film *The Colors of Infinity* [6] in 1992. After we had finished filming Arthur C. Clarke in Sri Lanka, I interviewed Benoît at his home. He was, in essence, happy with the questions that I proposed to ask. I commented uneasily that he looked a bit formal in his suit and tie and suggested that he should dress more casually. He laughed and said that he had a blue jacket he could wear and that he would drop the tie. When he came back into the room, instead of the bright blue jacket I was expecting, he was wearing a coat of subtle grey. He took his seat, and I sat down by the camera, clutching my notes and my list of questions. He looked good. Lights, camera, action! Then everything popped. We had blown a fuse. Our electrician rerouted the lights, and we started again. After ten minutes the lights blew again. I started to get very agitated, but Benoît remained as cool as a cucumber.

We had another coffee break while the electrician did some serious rewiring. Everyone stayed calm except for me. It was essential that I made a good job of the interview for the sake of our investors, the crew, and most of all for Benoît. We managed to get started again and, save for a half-hour lunch break, worked on into the afternoon. Benoît was superb—inspiring and lucid.

It took many months to complete the postproduction; then we sent tapes off to the contributors. Benoît was generous with his praise and his expressions of gratitude. We found a distributor for the film. It went on to sell in over forty territories worldwide and has been subtitled in three foreign languages. It was shown on eighty-two PBS stations in the United States.

When Benoît was recalling his research work at IBM, he told me:

> For me the first step with any difficult mathematical problem was to program it and see what it looked like. We started programming Julia sets of all kinds. It was extraordinary great fun! And in particular, at one point, we became interested in the simplest possible transformation: $z \mapsto z^2 + c$. ... And after a few weeks we had this very strong, overwhelming impression that this was a kind of big bear we have encountered!
>
> This discovery was named after me. It is called the Mandelbrot set. I think the most important implication is that from very simple formulas you can get very complicated results.…

Benoît, Michael Frame, and I went on to make the educational DVD production [8], which was commissioned by the National Science Foundation through Yale University. This DVD concludes with Benoît addressing the camera:

> I've spent most of my life unpacking the ideas that became fractal geometry. This has been exciting and enjoyable, most times. But it also has been lonely. For years few shared my views. Yet the ghost of the idea of fractals continued to beguile me, so I kept looking through the long, dry years. So find the thing you love. It doesn't so much matter what it is. Find the thing you love and throw yourself into it. I found a new geometry; you'll find something else. Whatever you find will be yours.

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Javier Barrallo

Not Only Should the Toy Be Built, But We Should Know How to Play with It

As Johannes Kepler used his toy, the ellipse, to explain our solar system, so did Benoît Mandelbrot use his toy, the fractal, to interpret the geometry of nature. Once Benoît explained to me: "Not only should the toy be built, but it should also be known how to play with it."

My first contact with Benoît was the invitation I sent to him to chair the international fractal art contest that bears his name. When attending the first exhibition contest at Conde Duque in Madrid, he was surprised to see a long line of people. "I am an inveterate optimist, but I never expected to see a crowd standing in a long line to admire mathematics in any of its forms," he said that night. I remember having to wait for him for over forty minutes while he signed autographs and took pictures with fans. He was more like a rock star or a Hollywood actor.

I remember while walking one beautiful autumn morning in San Sebastian, Benoît noticed a sculpture in the rocks of La Concha Bay. It was *The Comb of the Wind* by Eduardo Chillida. He immediately recognized the artist, then proceeded to tell me that born just a few miles away was Ignatius of Loyola, founder of the Jesuits; next he informed me that in the nearby port of Guetaria, the explorer who completed the first circumnavigation of the world, Juan Sebastian Elcano, was born. For a nonnative, he had remarkable cultural knowledge. He could talk about Hokusai style and immediately illustrate the Japanese character by relaying an anecdote that took place while he was dining with the empress of Japan. He told me once that Eugène Delacroix used to instruct his students that to paint a tree it was necessary to draw inside another smaller tree, and inside another, and another.

Benoît chaired three of the International Fractal Art Contests. In each case, twenty-five images were selected for exhibition. The results of the third contest, by artists of seventeen different nationalities, were exhibited in Bilbao (Spain), Buenos Aires (Argentina), and Hyderabad (India). Benoît guided our efforts to discover new ways to express fractal art. Thus, the typical filaments and spirals were reduced to an aesthetic closer to contemporary art rather than the usual fractal structures. Looking at the last exhibition contest, he said, "Many will prefer the old images, but compared with these, they look like antiques."

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Figure 9. Picture by Kerry Mitchell, a panel member image for the 2007 Benoît Mandelbrot Art Contest. Javier Barrallo noted that Benoît really enjoyed this picture.

The Benoît Mandelbrot International Fractal Art Contest also gave him the opportunity to participate for the first time in the International Congress of Mathematicians (ICM). He entered through the back door as honorary director of the fractal art contest. But when his presence became known, he raised unexpected excitement—well above any other guest speaker. Thus, Benoît Mandelbrot was invited to give the closing lecture of ICM2006, with several thousand people attending in the main auditorium. In his speech he congratulated Wendelin Werner for being a recipient of the Fields Medal as well as for being able to demonstrate one of his conjectures. In fact, he said, "This is the third time a Fields Medal was awarded for proving one of my conjectures."

For some people this may portray a smug man, but this is not true. I remember the night he turned down an invitation to a prestigious dinner with some of the best mathematicians in the world to join my group of young colleagues who had planned a beer and tapas tasting in the bustling Plaza of Santa Ana. "Could we join you?" he asked. That night we drank and laughed but mostly listened to Benoît tell fantastic stories and anecdotes from his life, science, history, and art …it was an unforgettable moment that revealed a much more approachable and intimate person than one might think.

My last conversations with Benoît dealt with the Mandelbrot set in 3D, also called Mandelbulb. Although he truly admired the gorgeous animations of the Mandelbulb and other graphics experiments, he never entered the debate on them. His era was ending and a new one was beginning.

Figure 10. Close-up and zoom on the attractor of an iterated function system comprising four projective transformations. Mandelbrot's work built on classical geometry and leads to simple mathematical models for natural objects, such as forests and leaves.

Benoît was not a conventional mathematician, but he was certainly the most brilliant mind I've had the chance to meet.

Sir Michael Berry

It Is Winter and the Trees Are Bare of Leaves

When Benoît visited the UK, he and Aliette occasionally stayed with us. My abiding memory is of his nonnegligible bulk dominating our kitchen amid a whirl of culinary activity. Fortified by a continuous supply of orange juice, he entertained and entranced us with his monologues about mathematics, his wartime experiences, his opinions of publishers and colleagues.…As I write, it is winter and the trees are bare of leaves. Frost on the branches dramatically enhances their fractility, and I remember Benoît, who taught us to see it.

Michael Frame

Epilogue: November 12, 2010

Benoît and I worked together for twenty years. We wrote papers, edited a book on fractals and education, ran summer workshops for teachers, and spent hours upon hours discussing . . . everything. These conversations were exhilarating, among the very best moments of my life. Benoît collaborated with many scientists, all much brighter than I, but our relationship was different. Deep inside, I remain an eleven-year-old kid, inhabiting a simpler world filled with mysteries, where the job of every kid is to explore. Benoît was more complicated, but with me he followed his sense of innocent wonder at the wide world. During our first freely roaming conversation, I had an image that has stayed with me through the years, that gives me some small comfort at his loss. Benoît and I were two little kids running around in a big field under a bright sky, showing each other what we found. Friends sharing the unalloyed joys of discovery.

Benoît was fascinated by complex things. His life's work revolved around finding a feature common to examples from mathematics, physics, economics, art, and music: patterns that kept recurring as he looked ever closer. Others had noticed some aspect of this before, but Benoît saw so much more: that complicated shapes can be understood dynamically as processes, not objects. Continuing to astound each new generation of students, the power of this view is remarkable.

I'll end with two more points: some of our final conversations, and what I really learned from Benoît.

When Benoît called to share the news of his diagnosis, at first he asked me to tell no one. All he wanted to discuss was how to try to finish the work that remained undone: his memoirs and projects on negative dimensions and lacunarity were much on his mind. Working from Benoît's notes, Aliette

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Figure 11. "I, Javier Barrallo, took this picture in Donostia-San Sebastián, a city in the Basque Country in northern Spain. The date was November 15, 2007. Benoît and Aliette are in Ondarreta Beach in front of a famous sculpture by Eduardo Chillida named 'The Combs of the Wind'. I remember it was a very happy day for them." (Photo courtesy of Javier Barrallo.)

Mandelbrot and Merry Morse finished the memoirs [11]. Through their considerable efforts, Benoît's story will be told.

In addition to unfinished projects, we continued to discuss some general scientific questions. Despite ample reason to think only of himself, curiosity—one of our very finest traits, the only thing that might save the species, the only thing that could make us worth saving—burned in Benoît with the brilliance it did in his youth eight decades earlier. These feelings would persist until the end.

Benoît and Aliette were very kind to Jean and me, but I cannot understand why he brought me into his world. Hundreds and hundreds of conversations, just he and I. Why? This made no sense. Surely he had better things to do with his time. But these talks have given me a detailed picture of Benoît.

What do I know for sure about Benoît? In his mind, shapes were fluid, bending, twisting, and turning without effort. He read everything, remembered everything, but dynamically, looking for connections in combinations both expected and unlikely. Familiarity with so many topics allowed Benoît to converse with anyone. With my father, a machinist, Benoît had a long discussion about annealing. Benoît loved music, especially opera, knew Charles Wuorinen's work long before Charles contacted Benoît to talk about fractal aspects of music. During the Yale memorial for Benoît, Ralph Gomory characterized Benoît as courageous, refining and extending his ideas about scaling across many disciplines, following the paths and practices of no field, ignored for years. Early in his life, Benoît wanted his own Keplerian revolution. This he achieved, but at a cost. Many years later Benoît lamented not having a large group of assistants; so much more would have been finished if the path he'd taken had not been so lonely. Still, that path got him to where he was, gave fractals to us all.

Years ago, when asked if he was a mathematician, a physicist, or an economist, Benoît replied that he was a storyteller. After Benoît died, I saw another interpretation of his answer. By emphasizing how an object grows, a fractal description of the object is a story. Twists and turns of a snowflake in a cloud, rough waves sculpting a jagged coastline, my lungs growing before I was born, the spread of galaxies throughout the deep dark of space. These share something? Benoît told us they have similar stories. Benoît told us science should tell more stories.

Did Benoît's stories change how we understand the world? Yes, indeed.

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