Lecture contents

- P-n junction
  - Electrostatics

Junctions: general approaches, conventions

- metal-semiconductor contact
- p-n homojunctions
- heterojunctions

=>

- Formation of potential barriers
- Different from bulk material

Formalism includes the following phenomena:

- Electrostatics (Gauss law)

$$\nabla \cdot \mathcal{E} = \frac{\rho}{\varepsilon_0 \varepsilon}$$

$$\varepsilon_0 \varepsilon \xrightarrow{replace} \varepsilon$$

$$\varepsilon\varepsilon_0 = \varepsilon \cdot 8.85 \cdot 10^{-14} \frac{F}{cm}$$

Poisson equation

$$-\nabla^2 \phi = \nabla \cdot \mathcal{E} = \frac{\rho}{\varepsilon} \quad \text{SI}$$

- Statistics (Boltzman for analytical description)

$$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

$$-\nabla^2 \phi = \nabla \cdot \mathcal{E} = \frac{4\pi\rho}{\varepsilon} \quad \text{CGS}$$

- Continuity equations:

$$\frac{\partial n}{\partial t} = G_n - R_n + \frac{1}{q} \nabla \cdot J_n \qquad \frac{\partial p}{\partial t} = G_p - R_p - \frac{1}{q} \nabla \cdot J_p$$

- Current equations
  - Drift and diffusion currents

$$J_n = nq\mu_n \mathcal{E} + qD_n \nabla n$$
$$J_p = nq\mu_p \mathcal{E} - qD_p \nabla p$$

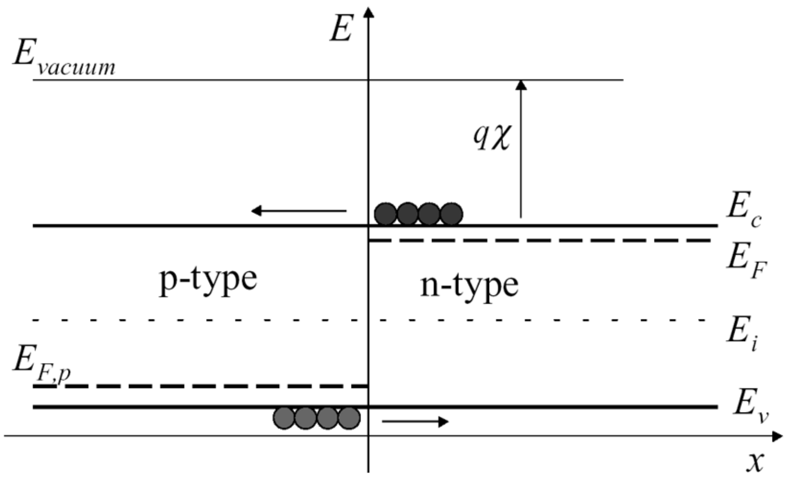
    - Einstein relation (in non-degenerate semiconductor)

$$D = \frac{k_B T}{q} \mu \longrightarrow J_n = \mu_n \left(nq\mathcal{E} + k_B T \nabla n \right)$$

  - Thermionic current
  - Tunneling current

Formation of p-n junction

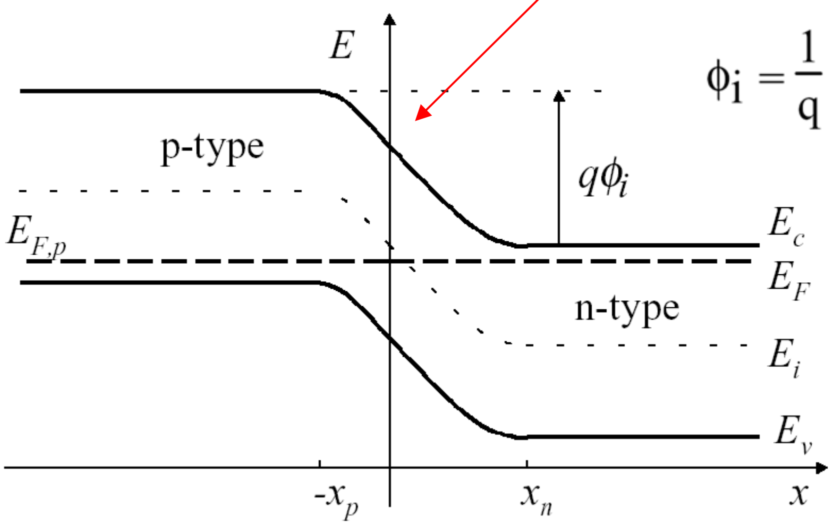
Flat band diagram before charge redistribution



Equilibrium band diagram

Built-in potential

If non-degenerate



$$\phi_i = \frac{1}{q}\left[E_i - E_{fp} - (E_i - E_{fn})\right] = V_t \ln \left[\frac{N_d N_a}{n_i^2}\right]$$

From Van Zeghbroeck, 1998

Electrostatics: Full depletion approximation

Full depletion approximation : impurities are completely ionized within the depletion width

Neutrality requires: $N_a x_p = N_d x_n$

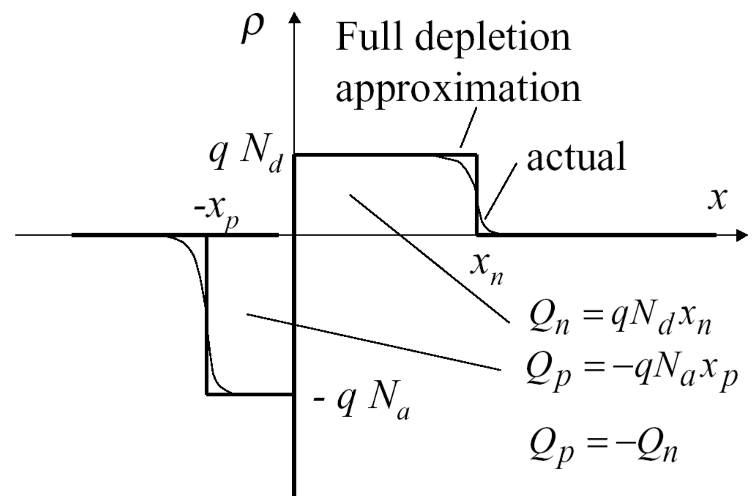
Integrating Poisson equation with boundary conditions (zero field far from the junction):

$$-\frac{d^2\phi}{dx^2} = \frac{d\mathcal{E}}{dx} = \frac{\rho}{\varepsilon}$$

for $x > 0$ $\quad \frac{d\mathcal{E}}{dx} = \frac{q}{\varepsilon} N_d \qquad \mathcal{E} = \frac{qN_d}{\varepsilon}(x - x_n)$

for x < 0 $\quad \frac{d\mathcal{E}}{dx} = -\frac{q}{\varepsilon} N_a \qquad \mathcal{E} = -\frac{qN_a}{\varepsilon}(x + x_p)$

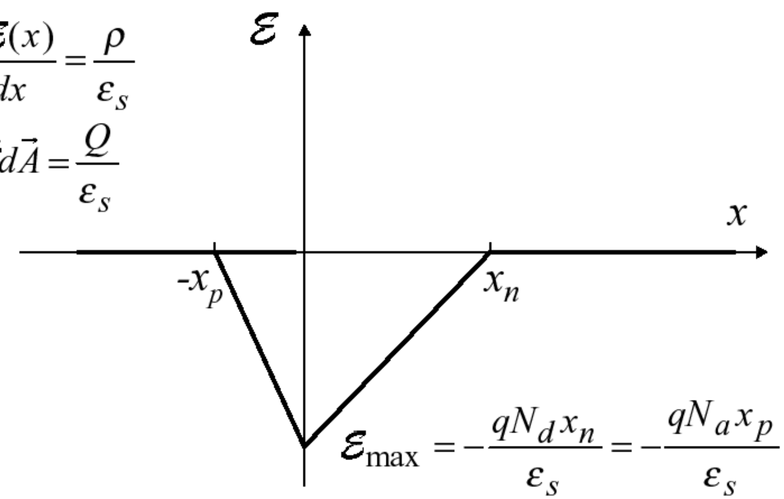
Space charge density in p-n junction



Integrating charge density => field

$$\frac{d\mathcal{E}(x)}{dx} = \frac{\rho}{\varepsilon_s}$$

$$\oint \vec{\mathcal{E}} d\vec{A} = \frac{Q}{\varepsilon_s}$$



From Van Zeghbroeck, 1998

Electrostatics: Full depletion approximation

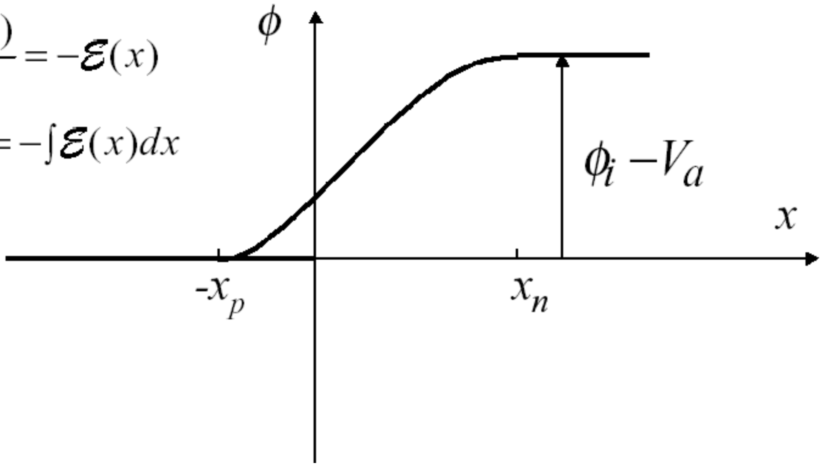
Integrating the field:

$$\phi_i - V_a = -\frac{\mathcal{E}_{max}(x_n + x_p)}{2} = \frac{qN_d x_n^2}{2\varepsilon_s} + \frac{qN_a x_p^2}{2\varepsilon_s}$$

Integrate field => potential (built-in + applied)

$$\frac{d\phi(x)}{dx} = -\mathcal{E}(x)$$

$$\phi(x) = -\int \mathcal{E}(x)dx$$



Obtain the width of depletion region

$$w = x_n + x_p = \sqrt{\frac{2\varepsilon_s(\phi_i - V_a)}{q}\left(\frac{1}{N_d} + \frac{1}{N_a}\right)}$$

$$x_n = \frac{wN_a}{N_d + N_a} \qquad x_p = \frac{wN_d}{N_d + N_a}$$

One-sided ($N_a >> N_d$)

$$W \approx x_n = \sqrt{\frac{2\varepsilon}{qN_d}(\phi_i - V_a)}$$

Electrostatics: Junction Capacitance

Capacitance per unit area [F/cm^2]

$$C = \left|\frac{dQ_n}{dV_a}\right| = \left|\frac{dQ_p}{dV_a}\right|$$

$$Q_n = qN_d x_n = -Q_p = qN_a x_p$$

$$C = \frac{\varepsilon_s}{\sqrt{\frac{2\varepsilon_s(\phi_i - V_a)}{q}\left(\frac{1}{N_d} + \frac{1}{Na}\right)}} = \frac{\varepsilon_s}{x_n + x_p} = \frac{\varepsilon_s}{w}$$

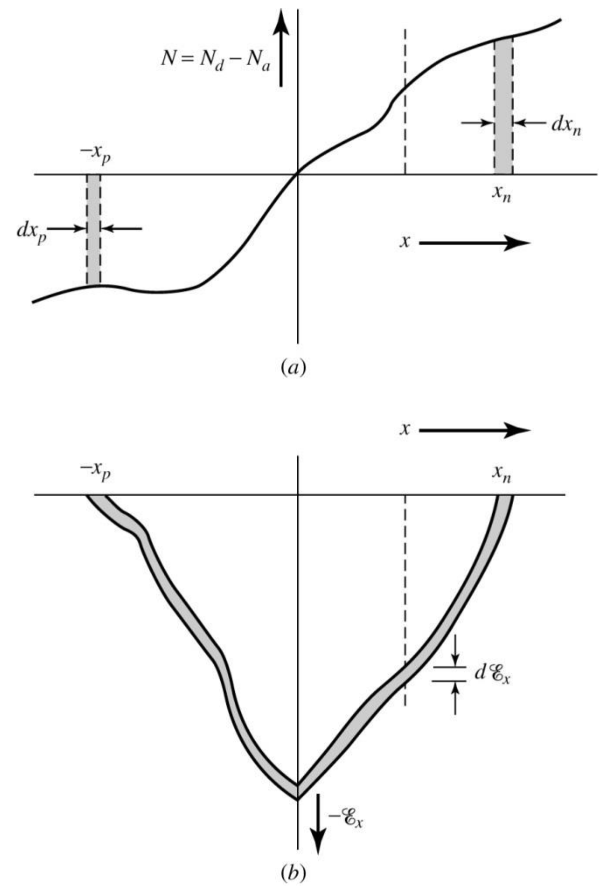


Figure 4.11 (Muller, Kamins 2003)
(*a*) Dopant concentration in an arbitrarily doped junction, showing modulation of the carrier densities at the edges of the space-charge region by an applied voltage.
(*b*) Electric-field distributions for two slightly different applied voltages.

Electrostatics: Full depletion vs. exact solution

For exact electrostatic solution Poisson equation should be integrated with free carriers:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{q}{\varepsilon_s}(p - n + N_d^+ - N_a^-)$$

As in the case of MS junction, the depletion layer width on each side is increased by the Debye length

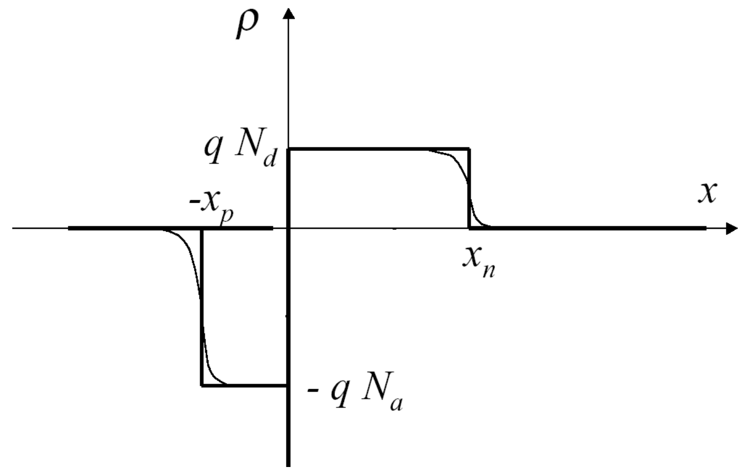
$$L_D = \left(\frac{\varepsilon}{qn}\frac{kT}{q}\right)^{1/2}$$

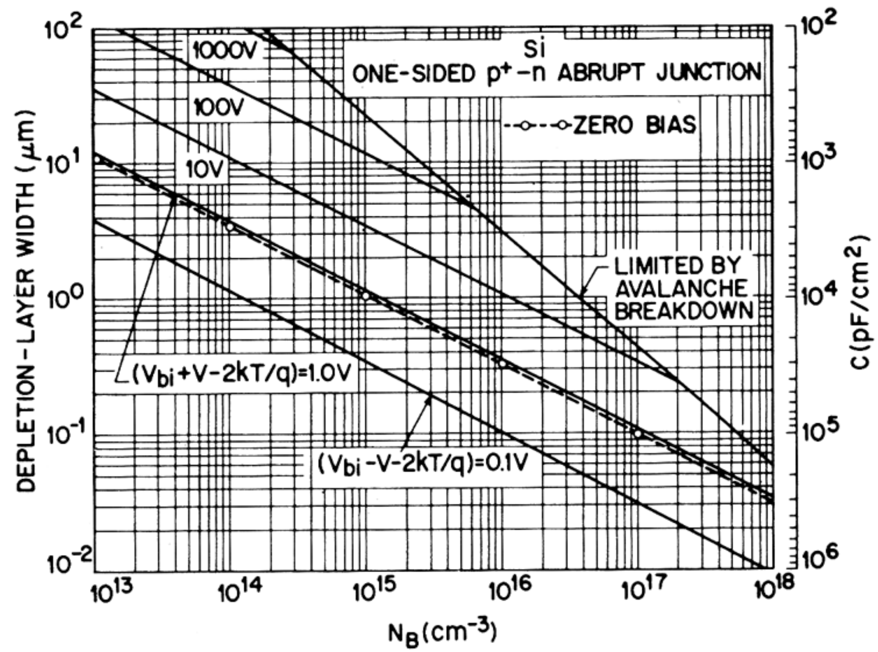
$$W = x_n + x_p = \sqrt{\frac{2\varepsilon}{q}\frac{N_d + N_a}{N_d N_a}(\phi_i - V_a - 2V_t)}$$

Or for one-sided junction:

$$W = \sqrt{\frac{2\varepsilon}{qN_d}(\phi_i - V_a - 2V_t)} = L_D\sqrt{2\left(\frac{\phi_i}{V_t} - \frac{V_a}{V_t} - 2\right)}$$

Space charge: Full depletion vs. exact solution





From Sze, 1981