ECE 301: Signals and Systems
Homework Solution #1

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Problem 1

Determine the values of P_∞ and E_∞ for each of the following signals:

(a) $x_1(t) = e^{-2t}u(t)$

(b) $x_2(t) = e^{j(2t+\pi/4)}$

(c) $x_3(t) = cos(t)$

(d) $x_1[n] = (\frac{1}{2})^n u[n]$

(e) $x_2[n] = e^{j(\pi/2n+\pi/8)}$

(f) $x_3[n] = cos(\frac{\pi}{4}n)$

Solution

(a) $E_\infty = \int_0^\infty e^{-2t}dt = \frac{1}{4}$. $P_\infty = 0$, because $E_\infty < \infty$.

(b) $x_2(t) = e^{j(2t+\pi/4)}$, $|x_2(t)| = 1$. Therefore,

$$E_\infty = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{-\infty}^{\infty} dt = \infty.$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x_2(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} dt = \lim_{T\to\infty} 1 = 1.$$

(c) $x_3(t) = cos(t)$. Therefore,

$$E_\infty = \int_{-\infty}^{\infty} |x_3(t)|^2 dt = \int_{-\infty}^{\infty} cos^2(t)dt = \infty.$$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x_3(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} cos^2(t)dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} (\frac{1+cos(2t)}{2})dt = \frac{1}{2}.$$

(d) $x_1[n] = (\frac{1}{2})^n u[n]$, $|x_1[n]|^2 = (\frac{1}{4})^n u[n]$. Therefore,

$$E_\infty = \sum_{n=-\infty}^{\infty} |x_1[n]|^2 = \sum_{n=0}^{\infty} (\frac{1}{4})^n = \frac{4}{3}.$$

$P_\infty = 0$, because $E_\infty < \infty$.

(e) $x_2[n] = e^{j(\pi/2n+\pi/8)}$, $|x_2[n]|^2 = 1$. Therefore,

$$E_\infty = \sum_{n=-\infty}^{\infty} |x_2[n]|^2 = \sum_{n=-\infty}^{\infty} 1 = \infty.$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x_2[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} 1 = 1.$$

(f) $x_3[n] = cos(\frac{\pi}{4}n)$. Therefore,

$$E_\infty = \sum_{n=-\infty}^{\infty} |x_3[n]|^2 = \sum_{n=-\infty}^{\infty} cos^2(\frac{\pi}{4}n) = \infty.$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x_3[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} cos^2(\frac{\pi}{4}n) = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} (\frac{1+cos(\frac{\pi}{2}n)}{2}) = \frac{1}{2}.$$

Problem 2

A continuous-time signal $x(t)$ is shown in Figure 6. Sketch and label carefully each of the following signals:

(a) $x(4 - \frac{t}{2})$

(b) $[x(t) + x(-t)]u(t)$

(c) $x(t)[\delta(t + \frac{3}{2}) - \delta(t - \frac{3}{2}))]$

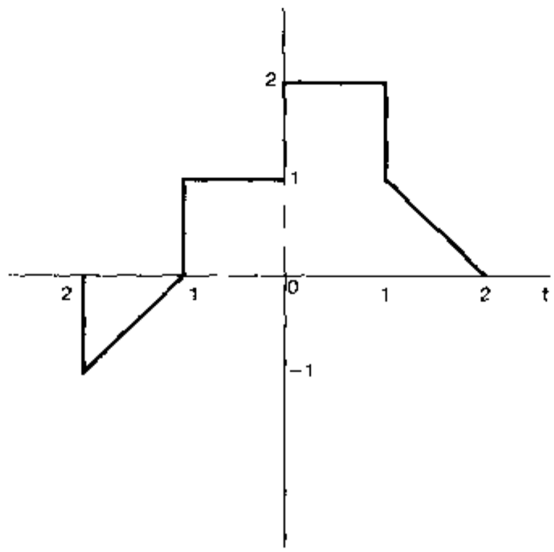

Figure 1: The continuous-time signal $x(t)$.

Solution

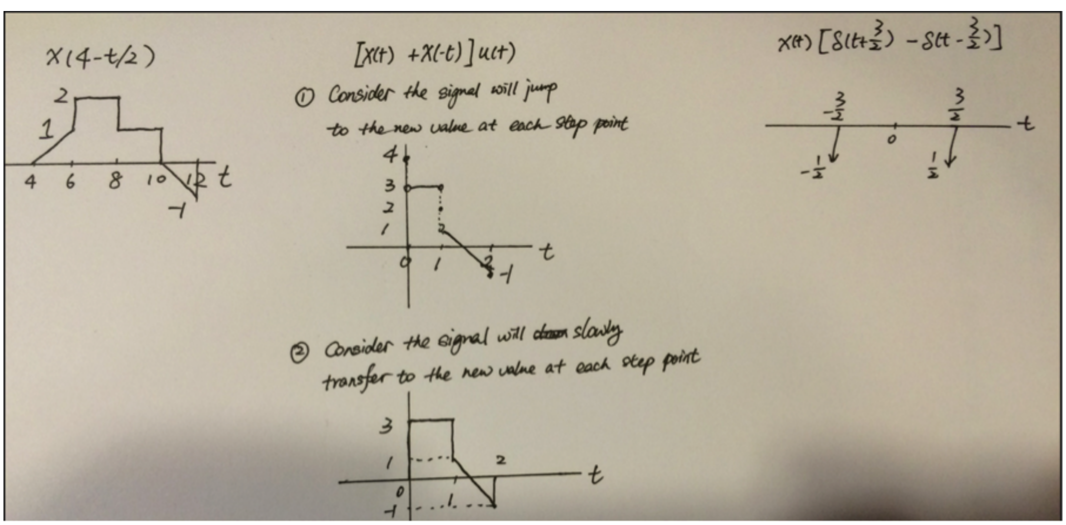

Figure 2: Sketches for the resulting signals.

Problem 3

A discrete-time signal $x[n]$ is shown in Figure 3. Sketch and label carefully each of the following signals:

(a) $x[3n]$

(b) $x[n]u[3-n]$

(c) $x[n-2]\delta[n-2]$

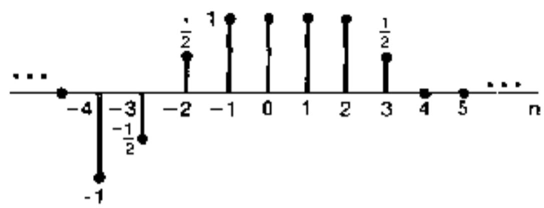

Figure 3: The discrete-time signal $x[n]$.

Solution

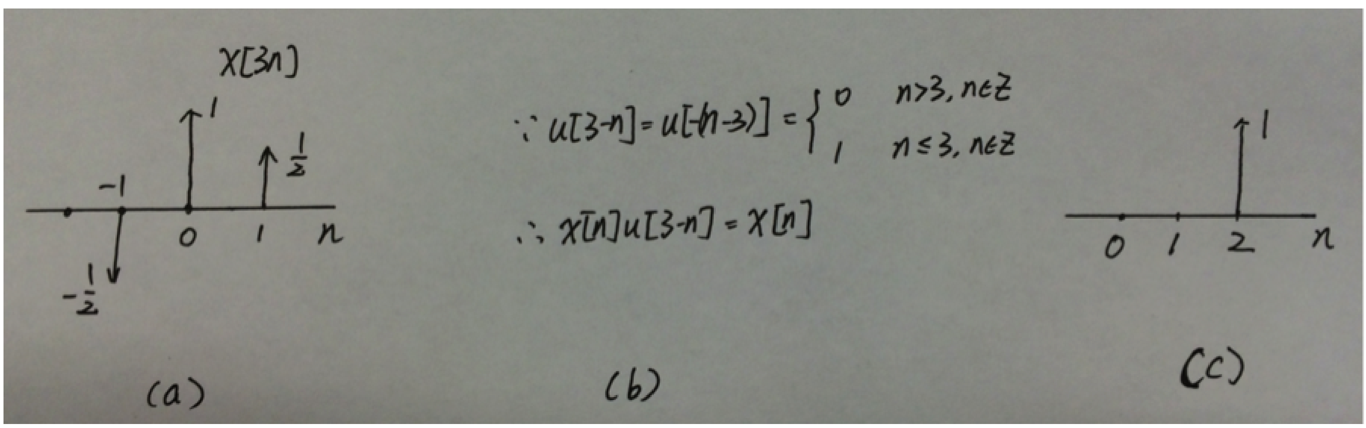

Figure 4: Sketches for the resulting signals.

Problem 4

Deternmine and sketch the even and odd parts of the signals depicted in Figure 5. Label your sketches carefully.

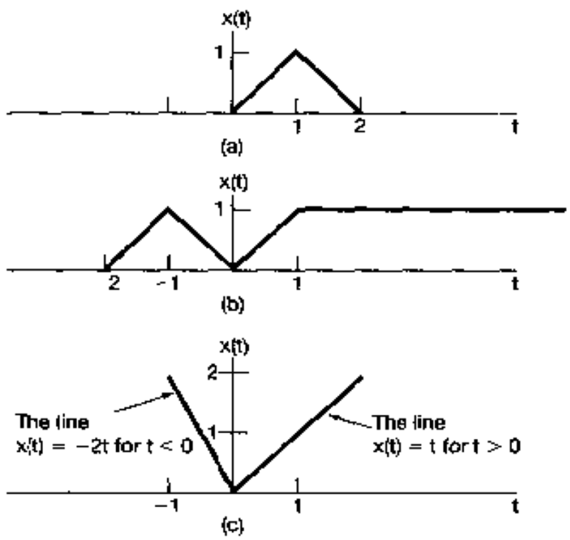

Figure 5: The continuous-time signal $x(t)$.

Solution

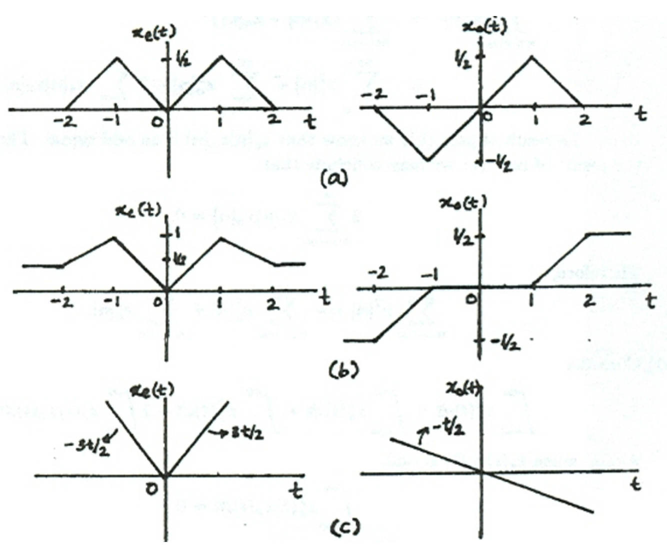

Figure 6: Sketches for the resulting signals.

Problem 5

Let $x(t)$ be the continuous-time complex exponential signal

$$x(t) = e^{jw_0 t}$$

with fundamental frequency ω_0 and fundamental period $T_0 = 2\pi/\omega_0$. Consider the discrete-time signal obtained by taking equally spaced samples of $x(t)$ - that is,

$$x[n] = x(nT) = e^{j\omega_0 nT}$$

(a) Show that $x[n]$ is periodic if and only if T/T_0 is a rational number - that is, if and only if some multiple of the sampling interval exactly equals a multiple of the period of $x(t)$.

(b) Suppose that $x[n]$ is periodic - that is, that

$$\frac{T}{T_0} = \frac{p}{q} \tag{1}$$

where p and q are integers. What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

(c) Again assuming that $\frac{T}{T_0}$ satisfies equation (1), determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.

Solution

(a) If $x[n]$ is periodic, then $e^{j\omega_0(n+N)T} = e^{j\omega_0 nT}$, where $\omega_0 = 2\pi/T_0$. This implies that

$$\frac{2\pi}{T_0}NT = 2\pi k \Rightarrow \frac{T}{T_0} = \frac{k}{N} = \text{a rational number.}$$

If $\frac{T}{T_0} = \frac{k}{N}$ = a rational number, then we have

$$\frac{T}{T_0} = \frac{k}{N} \Rightarrow \frac{2\pi}{T_0}NT = 2\pi k.$$

This implies that $e^{j\omega_0(n+N)T} = e^{j\omega_0 nT}$, where $\omega_0 = 2\pi/T_0$. $x[n]$ is periodic.

Combining the above two conditions, we can conclude that $x[n]$ is periodic if and only if T/T_0 is a rational number.

(b) If $\frac{T}{T_0} = \frac{p}{q}$ then $x[n] = e^{j2\pi n(\frac{p}{q})}$.The fundamental period is $N = q/gcd(p,q)$ (gcd refer to the greatest common divisor). The fundamental frequency is

$$\frac{2\pi}{q}gcd(p,q) = \frac{2\pi}{p}\frac{p}{q}gcd(p,q) = \frac{\omega_0 T}{p}gcd(p,q)$$

(c) We know that the fundamental period of (b) is $N = q/gcd(p,q)$, so overall $\frac{NT}{T_0} = p/gcd(p,q)$ periods of $x(t)$ is needed to obtain the samples that form a single period of $x[n]$.