2017

AP CollegeBoard

AP Calculus BC

Sample Student Responses and Scoring Commentary

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AP® CALCULUS BC
2017 SCORING GUIDELINES

Question 2

(a) $\frac{1}{2}\int_0^{\pi/2}(f(\theta))^2\,d\theta = 0.648414$

The area of R is 0.648.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\int_0^k\left((g(\theta))^2-(f(\theta))^2\right)d\theta = \frac{1}{2}\int_0^{\pi/2}\left((g(\theta))^2-(f(\theta))^2\right)d\theta$

— OR —

$\int_0^k\left((g(\theta))^2-(f(\theta))^2\right)d\theta = \int_k^{\pi/2}\left((g(\theta))^2-(f(\theta))^2\right)d\theta$

$2 : \begin{cases} 1 : \text{integral expression for one region} \\ 1 : \text{equation} \end{cases}$

(c) $w(\theta) = g(\theta) - f(\theta)$

$w_A = \dfrac{\int_0^{\pi/2} w(\theta)\,d\theta}{\frac{\pi}{2}-0} = 0.485446$

The average value of $w(\theta)$ on the interval $\left[0, \frac{\pi}{2}\right]$ is 0.485.

$3 : \begin{cases} 1 : w(\theta) \\ 1 : \text{integral} \\ 1 : \text{average value} \end{cases}$

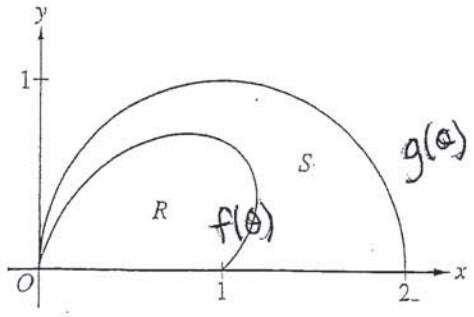
(d) $w(\theta) = w_A$ for $0 \le \theta \le \frac{\pi}{2} \Rightarrow \theta = 0.517688$

$w(\theta) = w_A$ at $\theta = 0.518$ (or 0.517).

$w'(0.518) < 0 \Rightarrow w(\theta)$ is decreasing at $\theta = 0.518$.

$2 : \begin{cases} 1 : \text{solves } w(\theta) = w_A \\ 1 : \text{answer with reason} \end{cases}$

2A1



2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $r = g(\theta) = 2\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x-axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x-axis.

(a) Find the area of R.

$$R = \frac{1}{2}\int_0^{\frac{\pi}{2}} f(\theta)^2\,d\theta \approx 0.648$$

(b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$$\frac{1}{2}\int_0^k \left[g(\theta)^2 - f(\theta)^2\right]d\theta - \frac{1}{2}\int_k^{\frac{\pi}{2}} \left[g(\theta)^2 - f(\theta)^2\right]d\theta = 0$$

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Continue question 2 on page 7.

(c) For each θ, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

$w(\theta) = 2\cos\theta - 1 - \sin\theta\cos(2\theta)$

average value $= \frac{2}{\pi}\int_0^{\frac{\pi}{2}} w(\theta)\,d\theta \approx 0.485$

(d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

$w(\theta) = 0.485446135 = B$

Calc intersect: $\searrow$ is true when $\theta = 0.517$

0.51766795

$w'(\theta)$ when $\theta = 0.517$ is negative (-0.581)

Since $w'(\theta)$ is negative

then $w(\theta)$ is decreasing at $\theta = 0.517$

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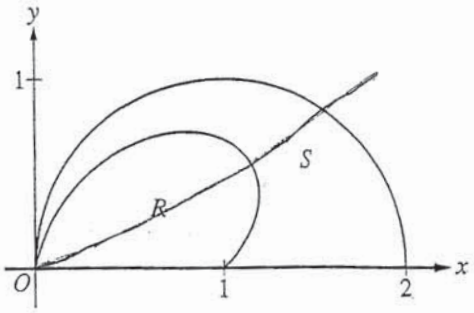
2B1



2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $r = g(\theta) = 2\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x-axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x-axis.

(a) Find the area of R.

$$\frac{1}{2}\int_0^{\pi/2}\left(1+\sin\theta\cos(2\theta)\right)^2 d\theta = 0.648$$

(b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$$\frac{1}{4}\int_0^{\pi/2}(2\cos\theta)^2 d\theta = \frac{1}{2}\int_0^{k}(2\cos\theta)^2 d\theta$$

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Continue question 2 on page 7.

2B2

(c) For each θ, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

$w(\theta) = 2\cos\theta - 1 - \sin\theta\cos(2\theta)$

$$\frac{\int_0^{\pi/2} (2\cos\theta - 1 - \sin\theta\cos(2\theta))\,d\theta}{\frac{\pi}{2} - 0} = 0.485$$

(d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

$0.485 = 2\cos\theta - 1 - \sin\theta\cos(2\theta)$

$\theta = 0.518$

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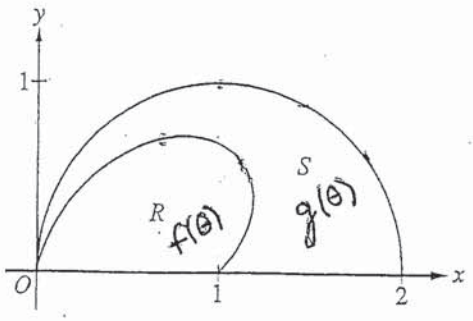
2C1



2. The figure above shows the polar curves $r = f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $r = g(\theta) = 2\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Let R be the region in the first quadrant bounded by the curve $r = f(\theta)$ and the x-axis. Let S be the region in the first quadrant bounded by the curve $r = f(\theta)$, the curve $r = g(\theta)$, and the x-axis.

(a) Find the area of R.

$$\int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\sin\theta\cos 2\theta)^2\,d\theta = .648$$

(b) The ray $\theta = k$, where $0 < k < \frac{\pi}{2}$, divides S into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$$k = \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \frac{1}{2}(2\cos\theta)^2\,d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{2}(1+\sin\theta\cos 2\theta)^2\,d\theta\right]$$

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Continue question 2 on page 7.

2C2

(c) For each θ, $0 \le \theta \le \frac{\pi}{2}$, let $w(\theta)$ be the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Write an expression for $w(\theta)$. Find w_A, the average value of $w(\theta)$ over the interval $0 \le \theta \le \frac{\pi}{2}$.

$w(\theta) = g(\theta) - f(\theta)$

$$\frac{1}{\frac{\pi}{2} - 0}\left[\int_0^{\frac{\pi}{2}} \frac{1}{2}(2\cos\theta)^2 \, d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 + \sin\theta\cos\theta)^2 \, d\theta\right]$$

$= .359$

(d) Using the information from part (c), find the value of θ for which $w(\theta) = w_A$. Is the function $w(\theta)$ increasing or decreasing at that value of θ ? Give a reason for your answer.

$.359 = (2\cos\theta - (1 + \sin\theta\cos\theta))$

$\theta =$

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AP® CALCULUS BC
2017 SCORING COMMENTARY

Question 2

Overview

In this problem a polar graph is supplied for the curves $f(\theta) = 1 + \sin\theta\cos(2\theta)$ and $g(\theta) = 2\cos\theta$ for $0 \le \theta \le \frac{\pi}{2}$. Regions R, bounded by the graph of $r = f(\theta)$ and the x-axis, and region S, bounded by the graphs of $r = f(\theta)$, $r = g(\theta)$, and the x-axis, are identified on the graph. In part (a) students were asked for the area of R. Students needed to recognize that region R is traced by the polar ray segment from $r = 0$ to $r = f(\theta)$ for $0 \le \theta \le \frac{\pi}{2}$ and use the graphing calculator to evaluate the area of R as the numeric value of $\frac{1}{2}\int_0^{\pi/2}(f(\theta))^2\,d\theta$. [LO 3.4D/EK 3.4D1] In part (b) students were asked to produce an equation involving one or more integrals that can be solved for k, $0 < k < \frac{\pi}{2}$, such that the ray $\theta = k$ divides S into two regions of equal areas. Students needed to recognize that region S is traced by the polar ray segment from $r = f(\theta)$ to $r = g(\theta)$ for $0 \le \theta \le \frac{\pi}{2}$. The ray $\theta = k$ divides S into two subregions with areas $\frac{1}{2}\int_0^k\left((g(\theta))^2 - (f(\theta))^2\right)d\theta$ and $\frac{1}{2}\int_k^{\pi/2}\left((g(\theta))^2 - (f(\theta))^2\right)d\theta$. Students should have reported an equation equivalent to setting these two expressions equal to each other, or setting one of them equal to half of the area of S, which is given by $\frac{1}{2}\int_0^{\pi/2}\left((g(\theta))^2 - (f(\theta))^2\right)d\theta$. [LO 3.4D/EK 3.4D1] In part (c) $w(\theta)$ is defined as the distance between the points with polar coordinates $(f(\theta), \theta)$ and $(g(\theta), \theta)$. Students were asked to write an expression for $w(\theta)$ and to find w_A, the average value of $w(\theta)$ for $0 \le \theta \le \frac{\pi}{2}$. Students needed to recognize that $w(\theta) = g(\theta) - f(\theta)$ and use the graphing calculator to evaluate the average value $w_A = \dfrac{\int_0^{\pi/2} w(\theta)\,d\theta}{\frac{\pi}{2} - 0}$. [LO 3.4B/EK 3.4B1] In part (d) students were asked to find the value of θ for which $w(\theta) = w_A$, and to determine whether $w(\theta)$ is increasing or decreasing at that value of θ. Importing the value of w_A from part (c), students needed to use the graphing calculator to solve $w(\theta) = w_A$ to obtain $\theta = 0.517688$. Students should have reported this value rounded or truncated to three decimal places. Students should then have reported that $w(\theta)$ is decreasing at this value of θ because the calculator reports a negative value for $w'(0.517688)$.
[LO 3.4D/EK 3.4D1] This problem incorporates the following Mathematical Practices for AP Calculus (MPACs): reasoning with definitions and theorems, connecting concepts, implementing algebraic/computational processes, connecting multiple representations, building notational fluency, and communicating.

Sample: 2A
Score: 9

The response earned all 9 points: 2 points in part (a), 2 points in part (b), 3 points in part (c), and 2 points in part (d). In part (a) the first point was earned for the integral $\int_0^{\frac{\pi}{2}} f(\theta)^2\,d\theta$. The second point was earned for the answer of 0.648. In part (b) the first point was earned by either $\int_0^k\left[g(\theta)^2 - f(\theta)^2\right]d\theta$ or

Question 2 (continued)

$\int_k^{\frac{\pi}{2}} \left[g(\theta)^2 - f(\theta)^2 \right] d\theta$. The second point was earned for a correct equation using these two integral expressions. That is, the second point was earned for the student's equation $\frac{1}{2}\int_0^k \left[g(\theta)^2 - f(\theta)^2 \right] d\theta - \frac{1}{2}\int_k^{\frac{\pi}{2}} \left[g(\theta)^2 - f(\theta)^2 \right] d\theta = 0$. In part (c) the first point was earned for the student's expression $w(\theta) = 2\cos\theta - 1 - \sin\theta\cos(2\theta)$. The second point was earned for the integral $\int_0^{\frac{\pi}{2}} w(\theta)\, d\theta$. The third point was earned for the average value answer of 0.485. In part (d) the first point was earned for solving $w(\theta) = 0.4854461355$ to get $\theta = 0.517$. The second point was earned for the student's conclusion that "Since $w'(\theta)$ is negative then $w(\theta)$ is decreasing at $\theta = 0.517$."

Sample: 2B
Score: 6

The response earned 6 points: 2 points in part (a), no points in part (b), 3 points in part (c), and 1 point in part (d). In part (a) the first point was earned for the integral $\int_0^{\frac{\pi}{2}} (1 + \sin\theta\cos(2\theta))^2\, d\theta$. The second point was earned for the answer of 0.648. In part (b) the first point was not earned because the student does not present a correct integral expression with an integrand in the form $(g(\theta))^2 - (f(\theta))^2$. The student's integral expressions only involve $g(\theta)$ and do not involve $f(\theta)$. Because the first point was not earned, the student is not eligible for the second point. In part (c) the first point was earned for the student's expression $w(\theta) = 2\cos\theta - 1 - \sin\theta\cos(2\theta)$. The second point was earned for the integral $\int_0^{\frac{\pi}{2}} (2\cos\theta - 1 - \sin\theta\cos(2\theta))\, d\theta$. The third point was earned for the average value answer of 0.485. In part (d) the first point was earned for solving $w(\theta) = 0.485$ to get $\theta = 0.518$. Because the student does not provide a conclusion for whether $w(\theta)$ is increasing or decreasing, the second point was not earned.

Sample: 2C
Score: 3

The response earned 3 points: 2 points in part (a), no points in part (b), 1 point in part (c), and no points in part (d). In part (a) the first point was earned for the integral $\int_0^{\frac{\pi}{2}} (1 + \sin\theta\cos 2\theta)^2\, d\theta$. The second point was earned for the answer of .648. In part (b) the first point was not earned because the student does not present a correct integral expression where one of the limits of integration is k. The student is not eligible to earn the second point without earning the first point. In part (c) the first point was earned for the expression $w(\theta) = g(\theta) - f(\theta)$. The second point was not earned because the student's integral expression $\int_0^{\frac{\pi}{2}} \frac{1}{2}(2\cos\theta)^2\, d\theta - \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 + \sin\theta\cos\theta)^2\, d\theta$ is incorrect. The third point was not earned because the student's answer of .359 is incorrect. The third point can only be earned with the average value answer 0.485. In part (d) the first point was not earned because the student does not solve $w(\theta) = .359$. Without a value for θ, the student is not eligible for the second point.