Question 6

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	–6	3	2	8
2	2	–2	–3	0
3	8	7	6	2
6	4	5	3	–1

The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

(b) Let $h(x) = \dfrac{g(x)}{f(x)}$. Find $h'(1)$.

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

(a) $k(3) = f(g(3)) = f(6) = 4$
$k'(3) = f'(g(3)) \cdot g'(3) = f'(6) \cdot 2 = 5 \cdot 2 = 10$

An equation for the tangent line is $y = 10(x - 3) + 4$.

$3 : \begin{cases} 2 : \text{slope at } x = 3 \\ 1 : \text{equation for tangent line} \end{cases}$

(b) $$h'(1) = \frac{f(1) \cdot g'(1) - g(1) \cdot f'(1)}{(f(1))^2} = \frac{(-6) \cdot 8 - 2 \cdot 3}{(-6)^2} = \frac{-54}{36} = -\frac{3}{2}$$

$3 : \begin{cases} 2 : \text{expression for } h'(1) \\ 1 : \text{answer} \end{cases}$

(c) $$\int_1^3 f''(2x)\,dx = \frac{1}{2}\Big[f'(2x)\Big]_1^3 = \frac{1}{2}[f'(6) - f'(2)] = \frac{1}{2}[5 - (-2)] = \frac{7}{2}$$

$3 : \begin{cases} 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

NO CALCULATOR ALLOWED

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	−6	3	2	8
2	2	−2	−3	0
3	8	7	6	2
6	4	5	3	−1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

$k(3) = f(g(3))$
$= f(6)$
$= 4$

$k'(x) = f'(g(x)) \cdot g'(x)$
$k'(3) = f'(g(3)) \cdot g'(3)$
$= f'(6) \cdot 2$
$= 5 \cdot 2$
$= 10$

$\boxed{y - 4 = 10(x-3)}$

Do not write beyond this border.

Continue problem 6 on page 21.

2 of 2

6A

NO CALCULATOR ALLOWED

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

$$h'(x) = \frac{f(x)g'(x) - g(x)f'(x)}{(f(x))^2}$$

$$\boxed{h'(1) = \frac{-3}{2}}$$

$$h'(1) = \frac{f(1)g'(1) - g(1)f'(1)}{(f(1))^2}$$

$$= \frac{(-6)(8) - (2)(3)}{(-6)^2}$$

$$= \frac{-48 - 6}{36}$$

$$= \frac{-54}{36}$$

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

$$\left[\frac{1}{2} f'(2x)\right]_1^3$$

$$\left[\frac{1}{2} f'(6)\right] - \left[\frac{1}{2} f'(2)\right]$$

$$\frac{5}{2} + 1$$

$$\boxed{\frac{7}{2}}$$

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	−6	3	2	8
2	2	−2	−3	0
3	8	7	6	2
6	4	5	3	−1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

$k'(x) = f'(g(x)) \cdot g'(x)$

$k'(3) = f'(g(3)) \cdot g'(3)$

$k'(3) = f'(6) \cdot 2$

$k'(3) = 5 \cdot 2$

$k'(3) = 10$

Do not write beyond this border.

Continue problem 6 on page 21.

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

$h'(x) = \frac{f(x) \cdot g'(x) - g(x) \cdot f'(x)}{(f(x))^2}$

$\frac{-48-6}{36}$

$h'(1) = \frac{(f(1) \cdot g'(1)) - (g(1) \cdot f'(1))}{(f(1))^2}$

$\frac{-54}{36}$

$\frac{(-6 \cdot 8) - (2 \cdot 3)}{(-6)^2}$

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

$u = 2x$

$du = 2dx$

$\frac{du}{2} = dx$

$\int_1^3 f''(u) \frac{du}{2}$

$\frac{1}{2} \int_1^3 f''(u)\,du$

$\frac{1}{2}(f'(3) - f'(1))$

$\frac{1}{2}(7 - 3)$

$\frac{4}{2} = 2$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	−6	3	2	8
2	2	−2	−3	0
3	8	7	6	2
6	4	5	3	−1

6. The functions f and g have continuous second derivatives. The table above gives values of the functions and their derivatives at selected values of x.

(a) Let $k(x) = f(g(x))$. Write an equation for the line tangent to the graph of k at $x = 3$.

$K(3) = f(g(3))$

$K(3) = f(6)$

$K(3) = 4$

$\boxed{y - 4 = 4(x-3)}$

Do not write beyond this border.

Continue problem 6 on page 21.

2 of 2

6C

(b) Let $h(x) = \frac{g(x)}{f(x)}$. Find $h'(1)$.

$$h'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{[f(x)]^2}$$

$$h'(1) = \frac{g'(1)f(1) - g(1)f'(1)}{[f(1)]^2}$$

$$= \frac{8 \cdot -6 - 2 \cdot 3}{36} = \frac{-48 - 6}{36} = \frac{54}{36} = \boxed{\frac{8}{6}}$$

(c) Evaluate $\int_1^3 f''(2x)\,dx$.

$$= f'(2x)\Big]_1^3$$

$$= f'(6) - f'(2)$$

$$= 5 - -2 = \boxed{7}$$

$f(x) = -6$

$f(1) = -6 \quad f'(1) = 3$

$f(x) \cdot -6x^3$

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

Question 6

Overview

In this problem students were given two general functions, f and g, that have continuous second derivatives. A table is presented with values of the functions and their derivatives at selected values of x. In part (a) students needed to find the equation of the line tangent to the graph of k at $x = 3$, where k is defined by $k(x) = f(g(x))$. This required application of the chain rule and use of values from the table to compute $k'(3) = f'(g(3)) \cdot g'(3)$ and $k(3) = f(g(3))$. In part (b) students were given $h(x) = \frac{g(x)}{f(x)}$ and asked to compute $h'(1)$. Students were expected to use the quotient rule and values from the table. Alternately, the product rule and chain rule can be applied to $h(x) = g(x) \cdot (f(x))^{-1}$. In part (c) students needed to evaluate the definite integral $\int_1^3 f''(2x)\,dx$. Using substitution of variables and applying the Fundamental Theorem of Calculus, students were expected to find an antiderivative involving f' and evaluate using values from the table.

Sample: 6A
Score: 9

The response earned all 9 points.

Sample: 6B
Score: 6

The response earned 6 points: 2 points in part (a), 3 points in part (b), and 1 point in part (c). In part (a) the student earned the first 2 points. The student does not present an equation for the tangent line. In part (b) the student's work is correct. In part (c) the student has an error with the substitution. The student earned 1 of the first 2 points and was not eligible for the third point.

Sample: 6C
Score: 3

The response earned 3 points: no points in part (a), 2 points in part (b), and 1 point in part (c). In part (a) the student does not present a value for $k'(3)$. In part (b) the student earned the first 2 points. The student did not earn the third point because of an error in simplification. In part (c) the student has an error with the constant in the antiderivative. The student earned 1 of the first 2 points and was not eligible for the third point.