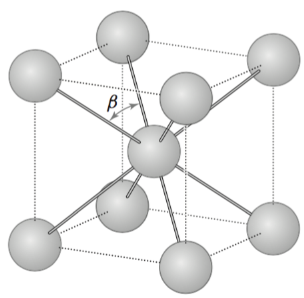
TA: My Huynh — Recitation 3 - Chapters 13.3,13.4

MATH 1920 Sections 209/214 — August 30, 2016

1. Compute $(-\mathbf{k}) \cdot (\mathbf{i} - 2\mathbf{j} + 8\mathbf{k})$.
2. Simplify $(\mathbf{v} + \mathbf{w}) \cdot (\mathbf{v} + \mathbf{w}) - 2\mathbf{v} \cdot \mathbf{w}$..
3. Find the angle between $\mathbf{v}$ and $\mathbf{w}$ if $\mathbf{v} \cdot \mathbf{w} = \frac{1}{2}||\mathbf{v}||\,||\mathbf{w}||$.
4. Assume that $||\mathbf{v}|| = 2, ||\mathbf{w}|| = 3$, and the angle between $\mathbf{v}$ and $\mathbf{w}$ is $\pi/3$. Determine:

   (a) $\mathbf{v} \cdot \mathbf{w}$ (b) $||2\mathbf{v} + w||$
5. Suppose $A = (0, 0, 1), B = (1, 1, 0)$, and $C = (0, 1, 0)$. Find the projection of $\overrightarrow{AB}$ along $\overrightarrow{AC}$.
6. Iron forms a crystal lattice where each central atom appears at the center of a cube, the corners of which correspond to additional iron atoms, as in the figure below. Use the dot product to find the angle β between the line segments from the central atom to two adjacent outer atoms. *Hint:* Take the center atom to be situated at the origin and the corner atoms to occur at $(\pm 1, \pm 1, \pm 1)$.



7. Calculate

   (a) $\begin{vmatrix} 2/3 & 1/6 \\ -5 & 2 \end{vmatrix}$

   (b) $\begin{vmatrix} 1 & 0 & 1 \\ -2 & 0 & 3 \\ 1 & 3 & -1 \end{vmatrix}$

   (c) $(2, 0, 0) \times (-1, 0, 1)$

   (d) $(\mathbf{j} - \mathbf{k}) \times (\mathbf{j} + \mathbf{k})$

   (e) $(\mathbf{u} - 2\mathbf{v}) \times (\mathbf{u} + 2\mathbf{v})$ if $\mathbf{u} \times \mathbf{v} = (1, 1, 0)$

(f) the area spanned by $\mathbf{u} = (1, 1, 1)$ and $\mathbf{v} = (0, 0, 4)$

(g) the volume of the parallelepiped spanned by $\mathbf{u} = (2, 2, 1)$, $\mathbf{v} = (1, 0, 3)$, $\mathbf{w} = (0, -4, 0)$

8. The curves $y = x^3$ and $y = \sqrt{x}$ intersect at $(1, 1)$. Find the "angle between the curves" at this point as follows.

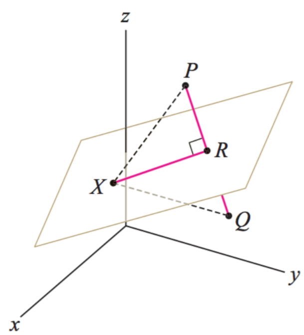
   (a) Compute the slope of the tangent line to both curves at the point in question.

   (b) For each curve, find a vector with the same slope of the curve at that point.

   (c) The angle between the curves is the angle betweent he two "tangent vector". Find this angle.

9. Prove that if $\mathbf{v} \neq 0$, then $\mathbf{v} \times \mathbf{w} = \mathbf{v} \times \mathbf{u}$ if and only if $\mathbf{u} = \mathbf{w} + \lambda \mathbf{v}$ for some scalar λ.

10. **(bonus)** The set of all points $X = (x, y, z)$ equidistant from two points P, Q in $\mathbb{R}^3$ is a plane.



   (a) Show that X lies on this plane if

$$\overrightarrow{PQ} \cdot \overrightarrow{OX} = \frac{1}{2}\left(\left| |\overrightarrow{OQ}| \right|^2 - \left| |\overrightarrow{OP}| \right|^2 \right)$$

   Hint: If R is the midpoint of $\overrightarrow{PQ}$, then X is equidistant from P and Q if and only if $\overrightarrow{XR}$ is orthogonal to $\overrightarrow{PQ}$.

   (b) Sketch the plane consisting of all points $X = (x, y, z)$ equidistant from the points $P = (0, 1, 0)$ and $Q = (0, 0, 1)$. Use the equation from part (a) to show that X lies on this plane if and only if $y = z$.

   (c) Use the equation from part (a) to find the equation of the plane consisting of all points $X = (x, y, z)$ equidistant from $P = (2, 1, 1)$ and $Q = (1, 0, 2)$.

Partial answers:

1. -8

2. $||\mathbf{v}||^2 + ||\mathbf{w}||^2$

3. $\pi/3$

4. (a) 3 (b) $\sqrt{37}$

5. $\overrightarrow{AC}$ or $(0, 1, -1)$.

6. $\beta = \arccos(1/3) \approx 1.2310$ radians $\approx 70.5288°$.

7. (a) $13/6$, (b) -15, (c) $(0, -2, 0)$, (d) $2\mathbf{i}$, (e) $(4, 4, 0)$, (f) $4\sqrt{2}$, (g) 20

8. (a) 3 and $1/2$, (b) $(1, 3)$ and $2, 1)$, (c) $\pi/4$

9 Hint: Recall that for vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{a} \times \mathbf{b} = 0$ if and only if $\mathbf{a} = 0$ or $\mathbf{b} = \lambda\mathbf{a}$ for some scalar λ.