

Circular Motion



Equations

T - the time taken for one complete circuit (s)

$$F = \frac{1}{T} \quad (\text{Hz})$$

angular displacement $s = \theta r$ (m)

linear velocity $v = \frac{2\pi r}{T}$ (ms⁻¹)

tangent to the radius

angular velocity $\omega = \frac{2\pi}{T}$ (rad s⁻¹)

$$v = \omega^2 r$$

centripetal acceleration $a_c = \frac{v^2}{r} = \omega^2 r$ (ms⁻²)

toward the center of the circle

centripetal force $\vec{F}_c = m\vec{a}_c$ net force (N)

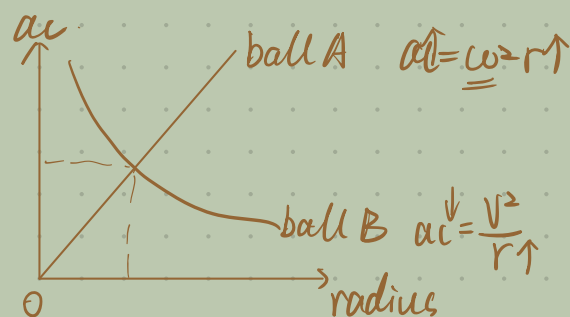
$$F_c = m \frac{v^2}{r} = m \omega^2 r$$

toward the center of the circle



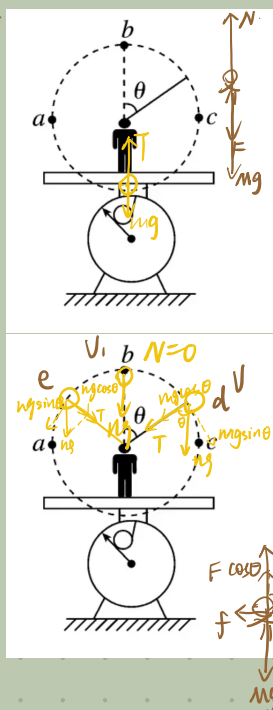
3. Example

Ball A & B are doing uniform circular motion. The graph shows the relationship between their centripetal acceleration and radius.

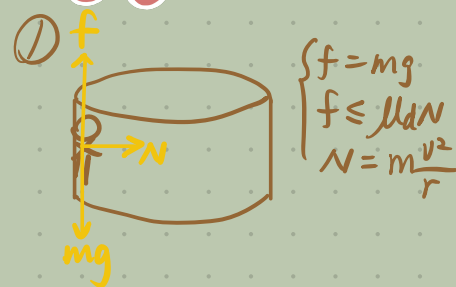


Ball B: v constant

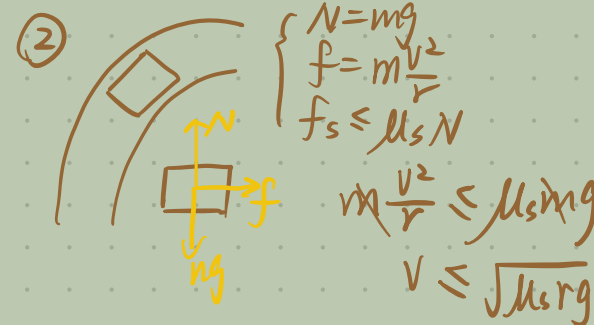
Ball A: ω constant



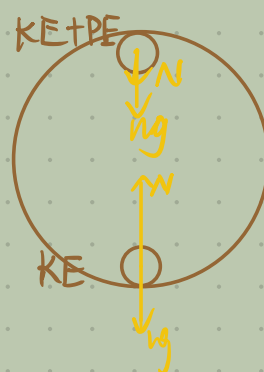
Models



$$\begin{cases} f = mg \\ f \leq \mu_s N \\ N = m \frac{v^2}{r} \end{cases}$$



B. In Vertical Direction



① min speed at top

$$F_c = N + mg \quad N = 0$$

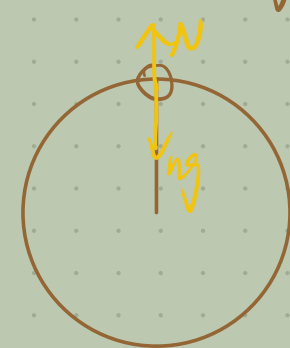
$$F_c = mg$$

$$m \frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

$v > \sqrt{rg}$ fly out of the track

$v < \sqrt{rg}$ fall down



$$F_c = mg - N$$

$$m \frac{v^2}{r} = mg - N \quad v \uparrow \quad N \downarrow \quad N \text{ upwards}$$

$$m \frac{v^2}{r} = mg \quad v = \sqrt{rg} \quad N = 0 \quad F_c = mg$$

$$v > \sqrt{rg} \quad F_c = mg + N \quad N \text{ downwards}$$

$$v < \sqrt{rg} \quad F_c < mg \quad \text{ball falling down}$$

② $N = ?$ at bottom if min speed at top?

conservation of mechanical energy

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_2^2 + mgh$$

$$m \frac{5rg}{r} = N - mg$$

$$\frac{1}{2} v_1^2 = \frac{1}{2} v_2^2 + g \cdot 2r$$

$$N = 5mg + mg$$

$$\frac{1}{2} v_1^2 = \frac{1}{2} rg + 2rg$$

$$N = 6mg$$

$$v_1 = \sqrt{5rg}$$

A man standing on the balance. mass = M ball mass = m
He is holding a string with a ball on one end. string length = R
The ball is doing circular motion. The ball can just past the highest point.
 $v = \sqrt{rg}$ low $v = \sqrt{5rg}$

① At lowest point, the value on the scale $N_1 = 6mg + mg$

$$N = F + Mg \quad \text{inward} +$$

$$F = T \quad F_c = T - mg \quad F = T = F_c + mg$$

$$N = F_c + mg + Mg = m \frac{v^2}{r} + mg + Mg = m \frac{5rg}{R} + mg + mg = 6mg + mg$$

② At highest point, the minimum value on the scale $N_2 = Mg$

③ At point e, d, the minimum value on the scale $N_3 = Mg - \frac{3}{4}mg$

$$F_c = T + mg \cos \theta \quad T = F_c - mg \cos \theta = m \frac{v^2}{r} - mg \cos \theta$$

$$N + F \cos \theta = Mg \quad N = Mg - F \cos \theta = Mg - (m \frac{v^2}{r} - mg \cos \theta) \cos \theta$$

$$v_1 > v: mg \cdot 2R + \frac{1}{2} m \cdot Rg = mg(R + \cos R) + \frac{1}{2} m v_1^2$$

$$\frac{3}{2} Rg + \frac{1}{2} Rg = Rg + \cos R \cdot g + \frac{1}{2} v_1^2$$

$$v_1^2 = 3Rg - 2 \cos R \cdot g$$

At a, b, c, $N_4 = Mg$

highest \rightarrow lowest $N \uparrow$

$$\begin{aligned} N_3 &= Mg - (m \cdot 3g - 3mg \cos \theta) \cos \theta \\ &= Mg - m \cos \theta (3g - 3g \cos \theta) \\ &= Mg - m \cdot 3g (\cos \theta - \cos^2 \theta) \rightarrow \text{max} \\ \cos \theta &= -\frac{b}{2a} \text{ quadratic} \\ \cos \theta &= \frac{-1}{-1 \times 2} = \frac{1}{2} \quad \theta = 60^\circ \\ &= mg - \frac{3}{4} mg \end{aligned}$$