# Neutron clustering: from Blaise Pascal's ruin theory to the Reactor Critical Facility at RPI

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IRSN
INSTITUT DE RADIOPROTECTION ET DE SÛRETÉ NUCLÉAIRE
*Faire avancer la sûreté nucléaire*

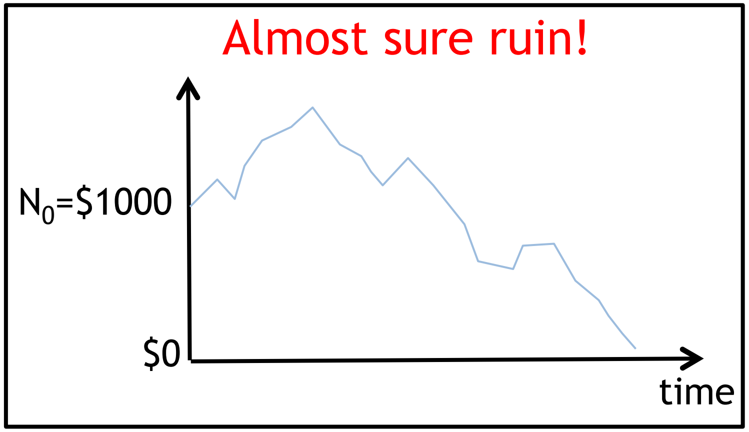
# Foreword on the gambler's ruin

Blaise Pascal (1623-1662)
mathematician & philosopher

Letter (1656)

**"what happens if I have \$1000 at hand and I play a fair game (p=0.5 to win loose) betting \$1 at each trial ?"**

Pierre de Fermat (1605-1665)
mathematician & magistrate



# Outline

Part 1. Initial motivation:
tilts in Monte Carlo criticality simulations

Part 2. Beyond the Boltzmann critical equation:
stochastic modeling of spatial correlations

Part 3. Consequences on eigenvalue calculations:
traveling waves & clustering

Part 4. Consequences on experimental reactor physics:
measuring spatial correlations at RCF

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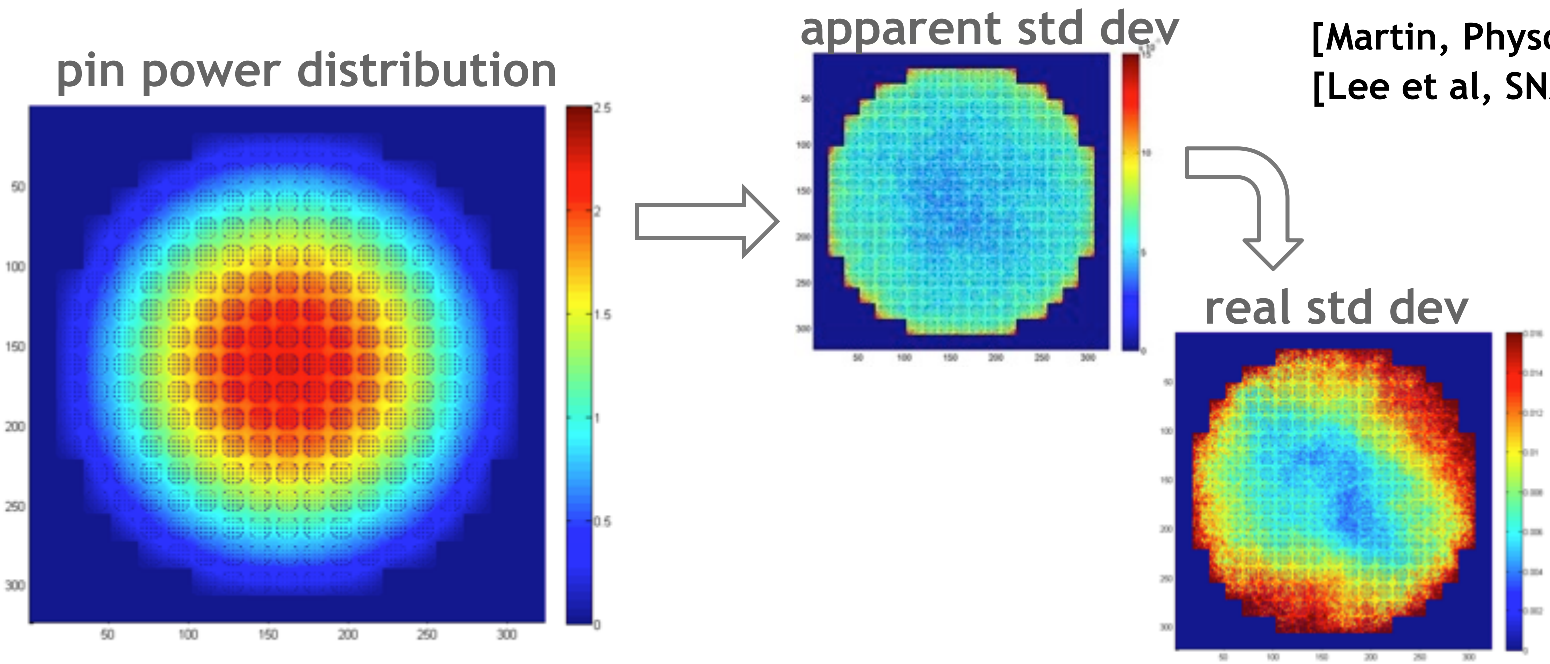
# Initial motivation: numerical tilts?




[Martin, Physor2012]
[Lee et al, SNA+MC2010]

Power tilt in the Monte-Carlo simulation of large reactor cores:

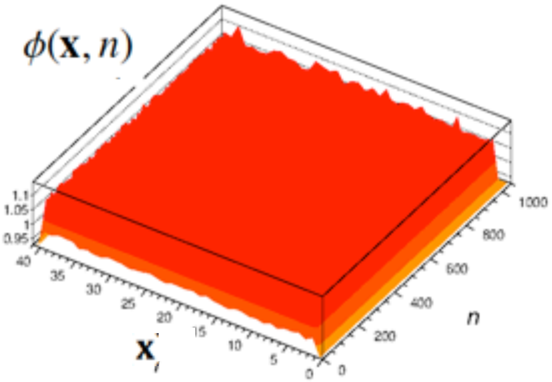
Long standing issue (70's): dedicated publications, expert groups, …
Strong under-estimation of error bars develop

**=> problem for criticality-safety assessment !**

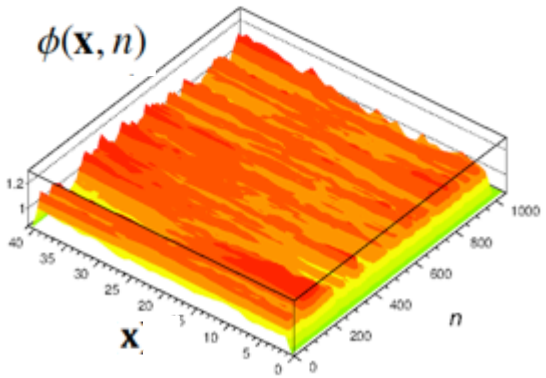
# If we look closer ...

$\phi(\mathbf{x}, n)$ is the "space-time" flux in a pincell

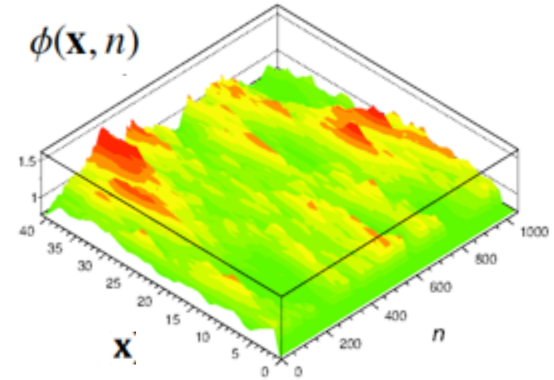
Instead of looking at integrated tallies, can we consider instantaneous tallies?




L = 10 cm




L = 100 cm




L = 400 cm

Strong spatial correlations develop for loosely coupled systems

**"neutron clustering"**

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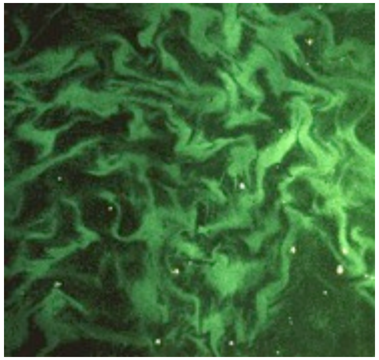
# Clustering in mathematics and in biology

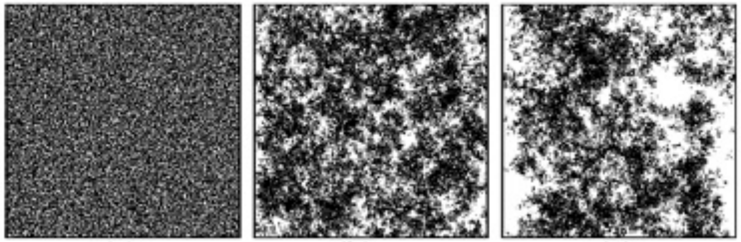
- clustering in **theoretical ecology**

[Dawson, 1972]
[Cox and Griffeath, 1985]

- clustering in **biology** (where it is aka brownian bugs):



  - plankton are any organisms that live in the water column and are incapable of swimming against a current
  - they reproduce, die and are transported by the water (like neutrons!)

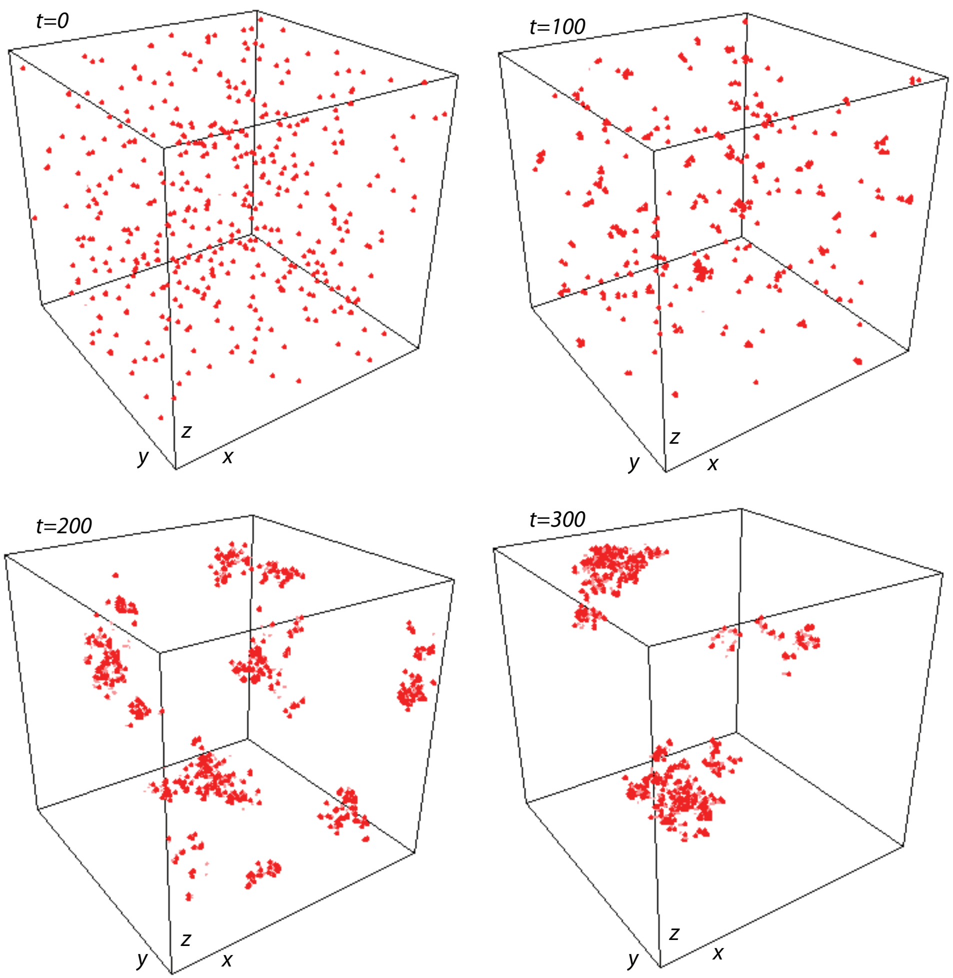


(a) (b) (c)

[Young, Nature 2001]
[Houchmandzadeh, PRE 2008]

- tools to describe clustering in physics: statistical mechanics, in particular ***Branching Brownian Motion (BBM)***

# Neutron clustering




- TRIPOLI-4®
- Exponential flights with
- typical jump size $1/\Sigma_s \to 0$
- to recover the diffusion regime

- Binary branching

$$p(0) = \frac{1}{2} \quad p(2) = \frac{1}{2}$$

- Dimension $d = 3$

- Typical length $L >> l$

**Can we have a quantitative insight into this phenomenon?**

# Branching Brownian motion

<u>simplified model for neutron transport in multiplicative media:</u>

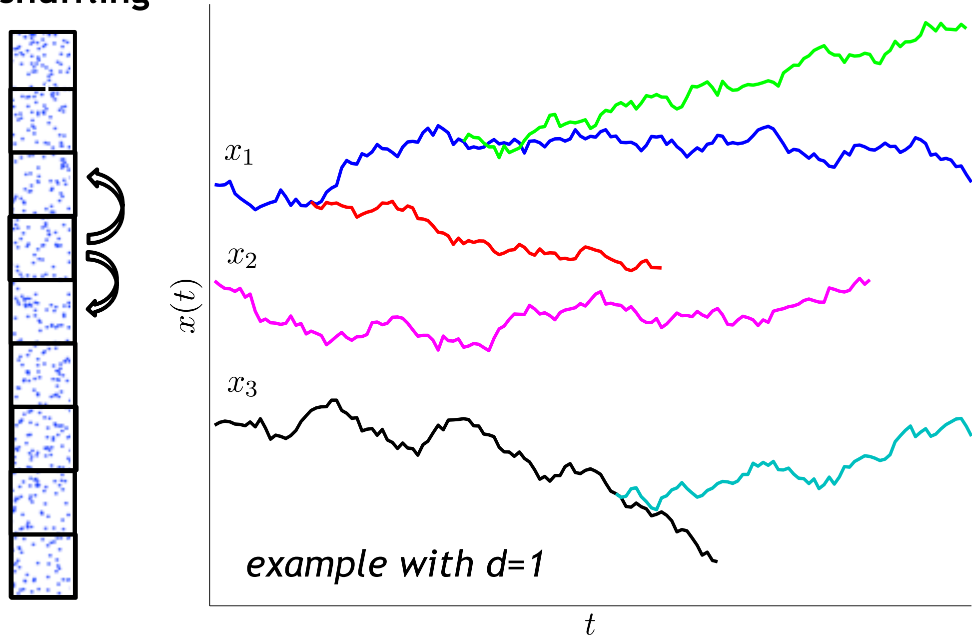
- ❑ $N_0 \to \infty$ neutrons, uniformly distributed at t=0
- ❑ infinite medium ( $L \to \infty$ )

$\Big\} \; c_0 = cte$

- ❑ no energy dependence
- ❑ Brownian motion with diffusion coefficient $D$ [cm²·s⁻¹] $\to \; < x^2(t) >= Dt$
- ❑ undergoes collision at Poissonian times with rate $\lambda$ [s⁻¹]
- ❑ at each collision, $k$ descendants with probability $p(k)$ →
- ❑ dimension *d*

$$p(0) \leftrightarrow \Sigma_c$$
$$p(1) \leftrightarrow \Sigma_s$$
$$p(2), p(3), \ldots \leftrightarrow \Sigma_f$$
$$\nu_1 = \sum_k kp(k)$$

shuffling

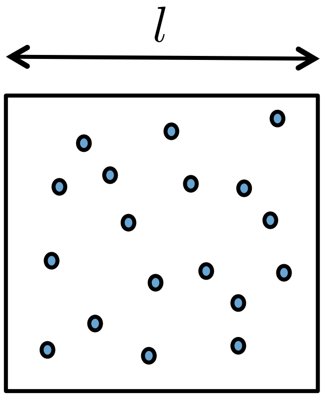



this process couples:

⇒ **Galton-Watson** birth-death process to describe fission and absorption

⇒ **Brownian motion** to simulate neutron transport

# Crash course for clustering in dimension 0




- We consider a "cell" *i* at time *t* with $n$ individuals
- d=0 Branching events with:
  - production rate $\lambda p(2)$
  - disparition rate $\lambda p(0)$
- Proba(n→n+1 in dt): $W^{+}(n)dt = \lambda p(2) n dt$
- Proba(n→n-1 in dt): $W^{-}(n)dt = \lambda p(0) n dt$

$\lambda p(0), \lambda p(2) [s^{-}1]$

$n\ [\#]$

$dt\ [s]$

## Forward master equation

$$\frac{dP(n,t)}{dt} = W^{-}(n+1)P(n+1,t) - W^{+}(n)P(n,t) + W^{+}(n-1)P(n-1,t) - W^{-}(n)P(n,t)$$

$$< n(t) >= \sum_n nP(n,t)$$

$$< n^2(t) >= \sum_n n^2 P(n,t)$$

$$< n(t) >= n_0 e^{\lambda(p(2)-p(0))t}$$

$$< V(t) >=< n^2(t) > - < n(t) >^2 = \lambda(p(0)+p(2))n_0 t$$

**Critical:**

$$\lambda p(0) = \lambda p(2)$$

$$< n(t) >= n_0$$

$$< V(t) >= \lambda n_0 t$$

# From gambler's ruin to critical catastrophy…

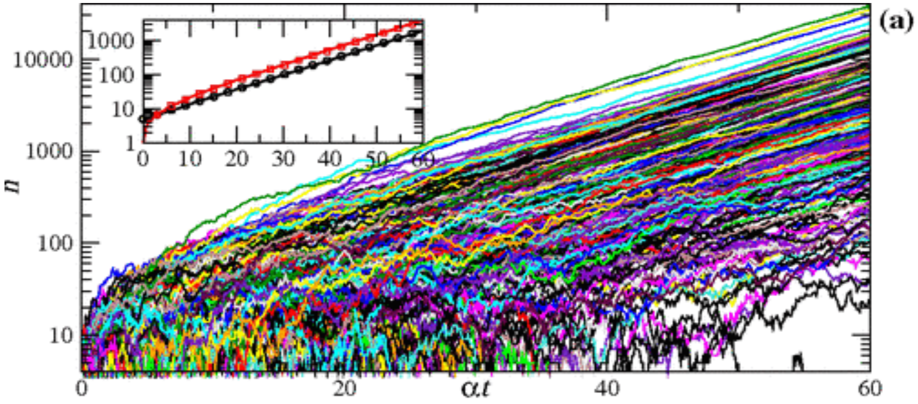
Ultimate fate of this population?
Controlled by $\nu_1 = \sum_k k p(k)$
(mean number of part/collision)

$\nu_1 > 1$ population grows unbounded
$\nu_1 < 1$ population becomes extinct
$\nu_1 = 1$ population constant on average: critical condition

N neutrons in a critical spatial cell which undergo fission or capture events

⇕

N $1 coins in a box which are played in a fair game

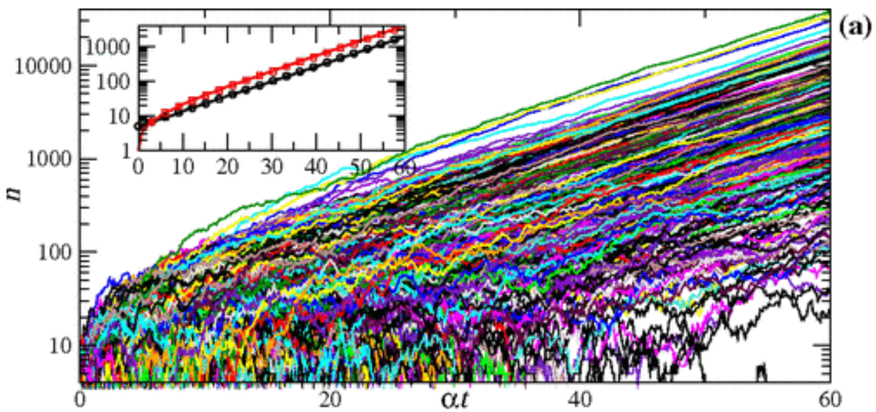



Fair game in neutron transport = criticality
Gambler's ruin = critical catastrophe!

[Williams, 1974]

# … and from critical catastrophy to neutron clustering

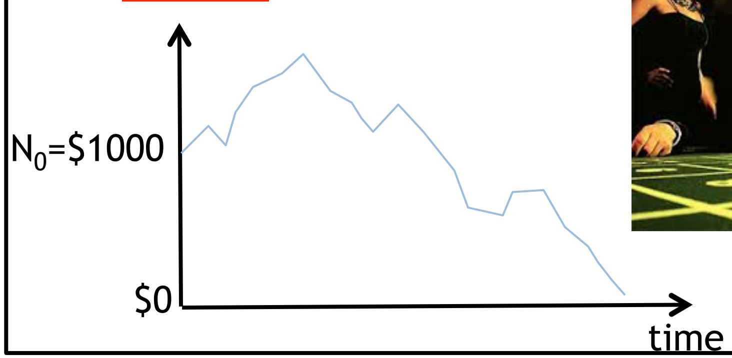
[Houchmandzadeh, PRE 2008]

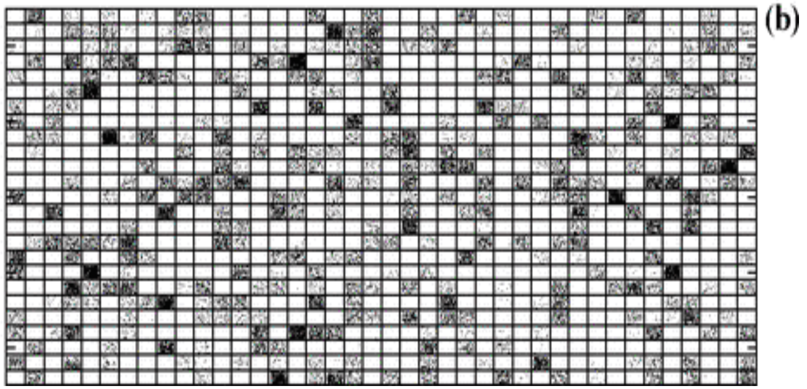



(b)

$$< n(t) >= n_0$$
$$< V(t) >= \lambda n_0 t$$

Almost sure ruin!



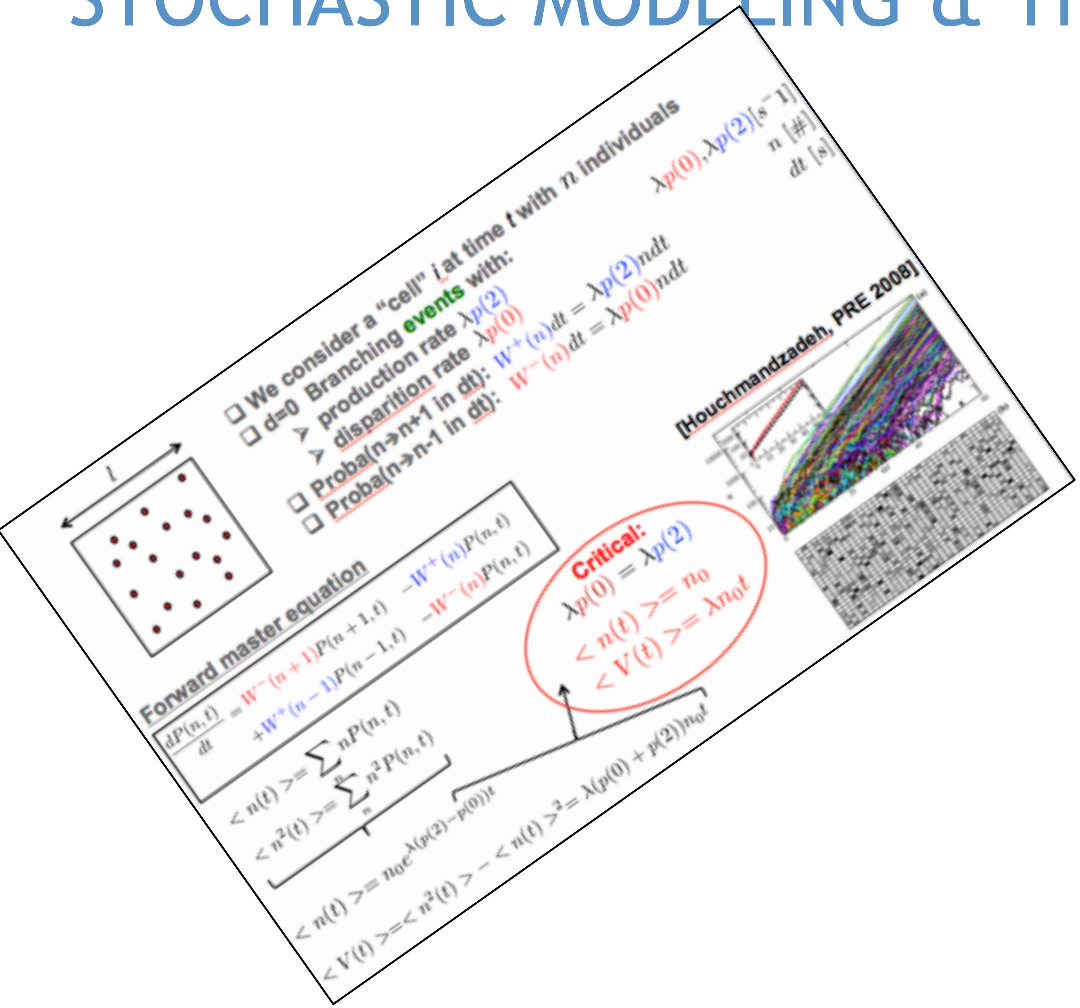
# From d=0 to d=2

d=0 => Critical castastrophy ⇔ Gambler's ruin
d>0 => Neutron clustering

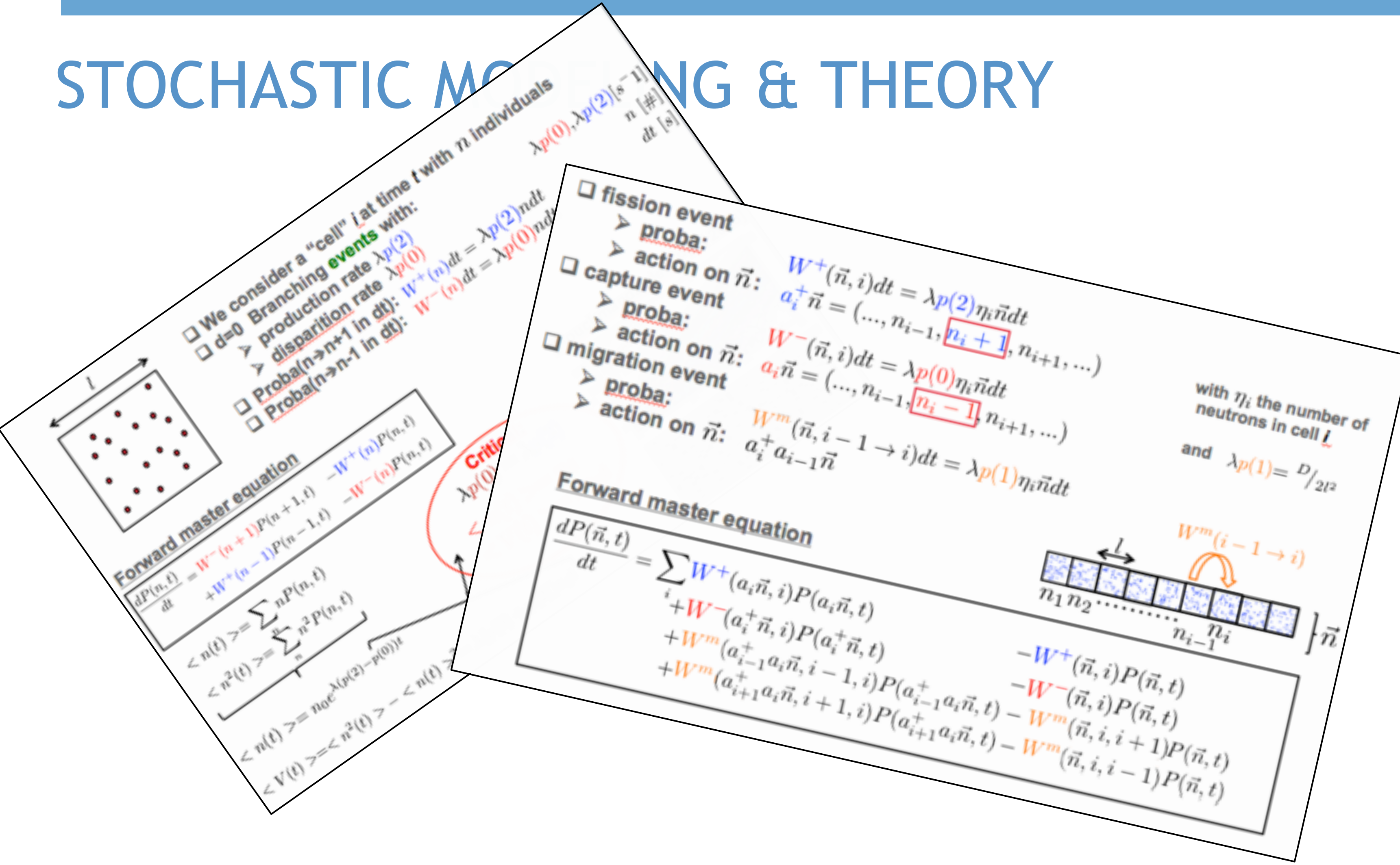
but here the cells where totally decoupled
"fake" d=2

We have to take into account the
diffusion of neutrons

# STOCHASTIC MODELING & THEORY



# STOCHASTIC MO NG & THEORY



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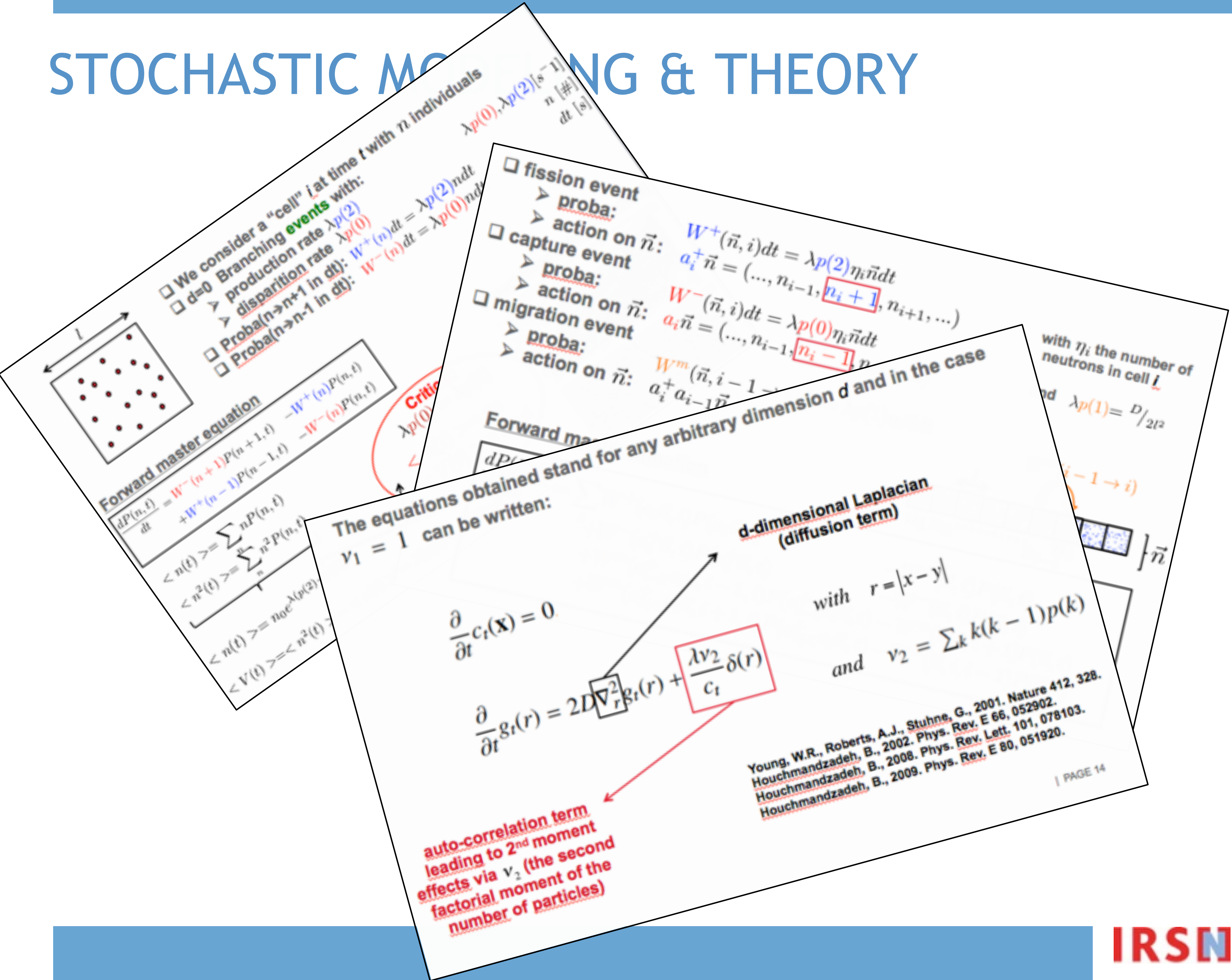
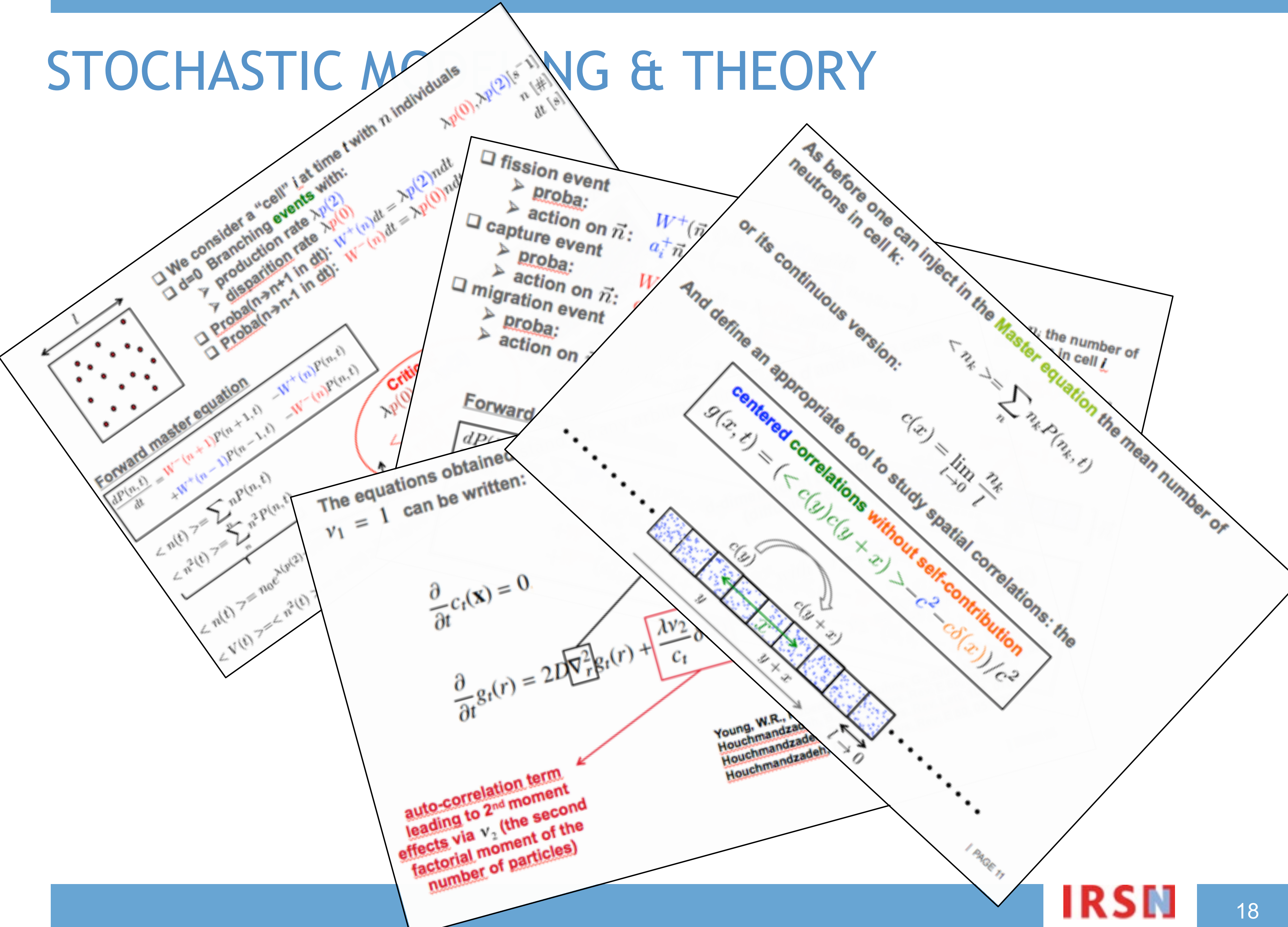
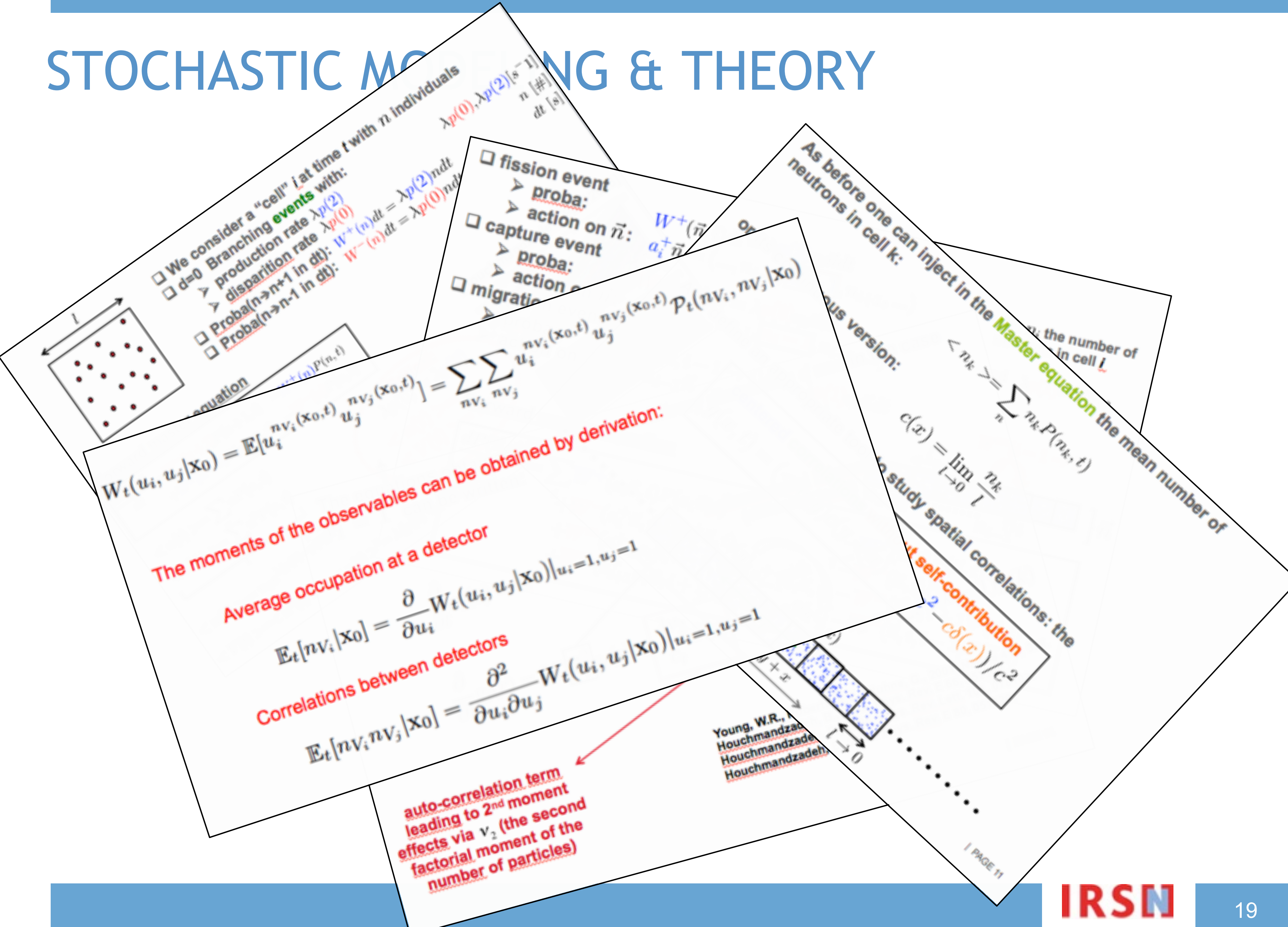


# STOCHASTIC MO… NG & THEORY



# STOCHASTIC MO[…]NG & THEORY

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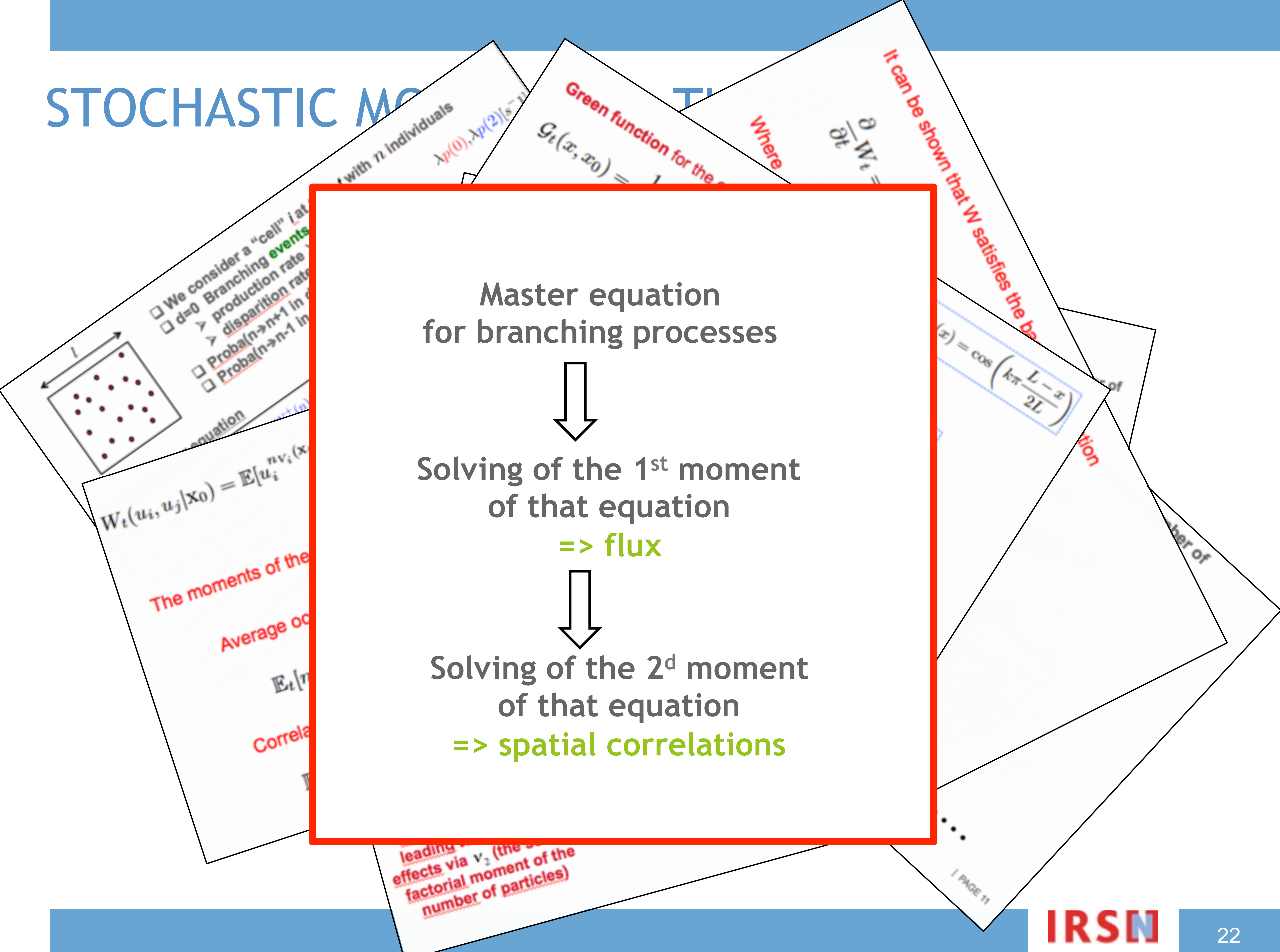
# STOCHASTIC MO… NG & TH…

# STOCHASTIC MO… THEO…

# STOCHASTIC MO… THEORY




Master equation
for branching processes

⇩

Solving of the 1st moment
of that equation
=> flux

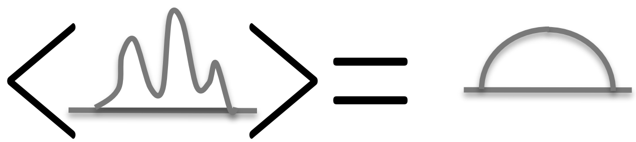
⇩

Solving of the 2d moment
of that equation
=> spatial correlations

# No 1-dimensional nuclear reactor

All those equations model the neutron transport in fissile medium
(not only the criticality mode of MC codes)

The solution to the 2-points function when
dimension *d* = 1 or *d* = 2
diverges with time…



…a purely 1d infinite system systematicaly develops power peaks at arbitrary places!

The typical amplitude of those peaks is controlled by $\frac{\nu_2}{c_0}$

- $\nu_2$ → fission process
- $c_0$ → different in reactor physics and MC simu

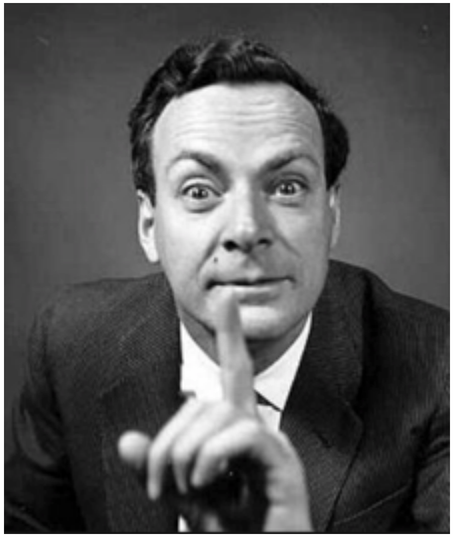
Challenge in MC criticality simulations: $c_0$ << Less than in reality!

# Beyond the Boltzmann equation: Feynman-Kac & Master equations

Feynman
1918-1988

Kac
1914-1984

Boltzmann
1844-1906



Ulam
1909-1984

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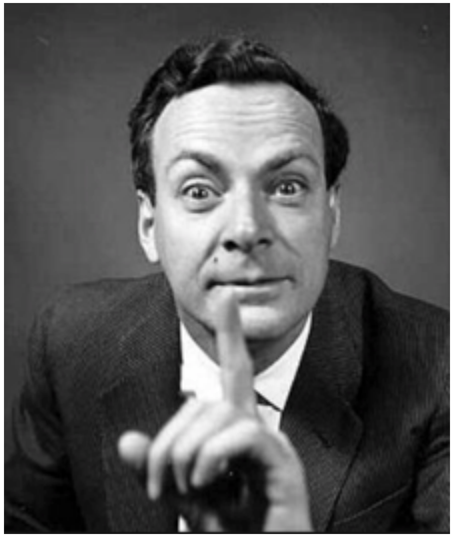
# Beyond the Boltzmann equation: Feynman-Kac & Master equations

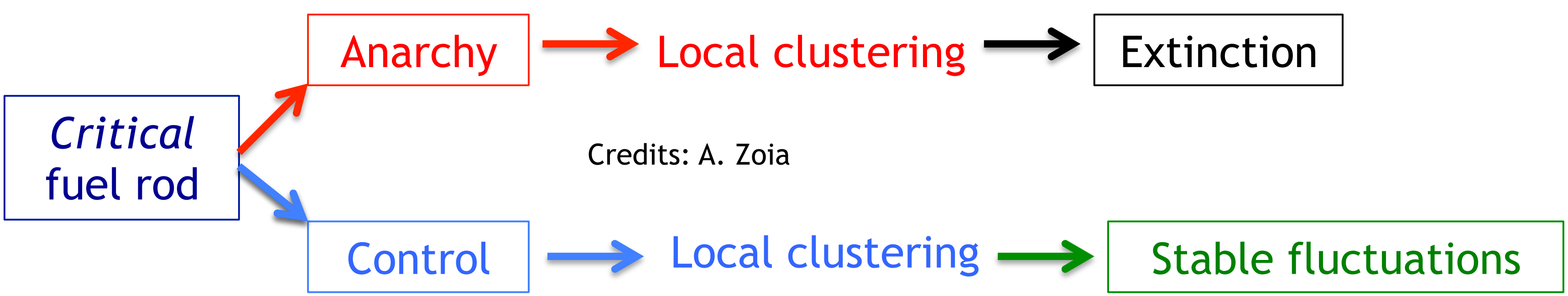
- The **Boltzmann critical equation** calculates **mean quantities**
- The **Feynman-Kac path integral approach** (backward equations) or Fokker-Planck type equations are equations for the **probability => mean + variance/correlations +** …



**And surprisingly variance & correlations take the lead over mean statistics !**

# Advanced modeling

- ❑ Dimensionality (3d vs. 1d) — Dumonteil, E. et al, Annals of Nuclear Energy 63, 612-618 (2014)
- ❑ Finite-speed effects (transport vs. diffusion) — Zoia, A. et al, Physical Review E, 90, 042118 (2014)
- ❑ Vacuum boundary conditions (absorbing BC vs. reflecting BC)
- ❑ Delayed neutrons (two time scales vs. single time scale) — Houchmandzadeh et al, Phys. Rev. E 92 (5), 052114 (2015)
- ❑ Population control (N does not depend on time) — De Mulatier et al, J. Stat. Mech., 15 , P08021, 1742-5468 (2015)
- ❑ Clustering and entropy — Nowak et al, Ann. Nuc. Ener. 94, 856-868 (2015)
- ❑ Bias modeling — Dumonteil et al, Nuc. Eng. Tech., 10.1016/j.net.2017.07.011 (2017)
- ❑ Time => generations — Sutton and Mittal, Nuc. Eng. Tech., 10.1016/j.net.2017.07.008 (2017)




Credits: A. Zoia

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# Consequence 3:

under-sampling biases

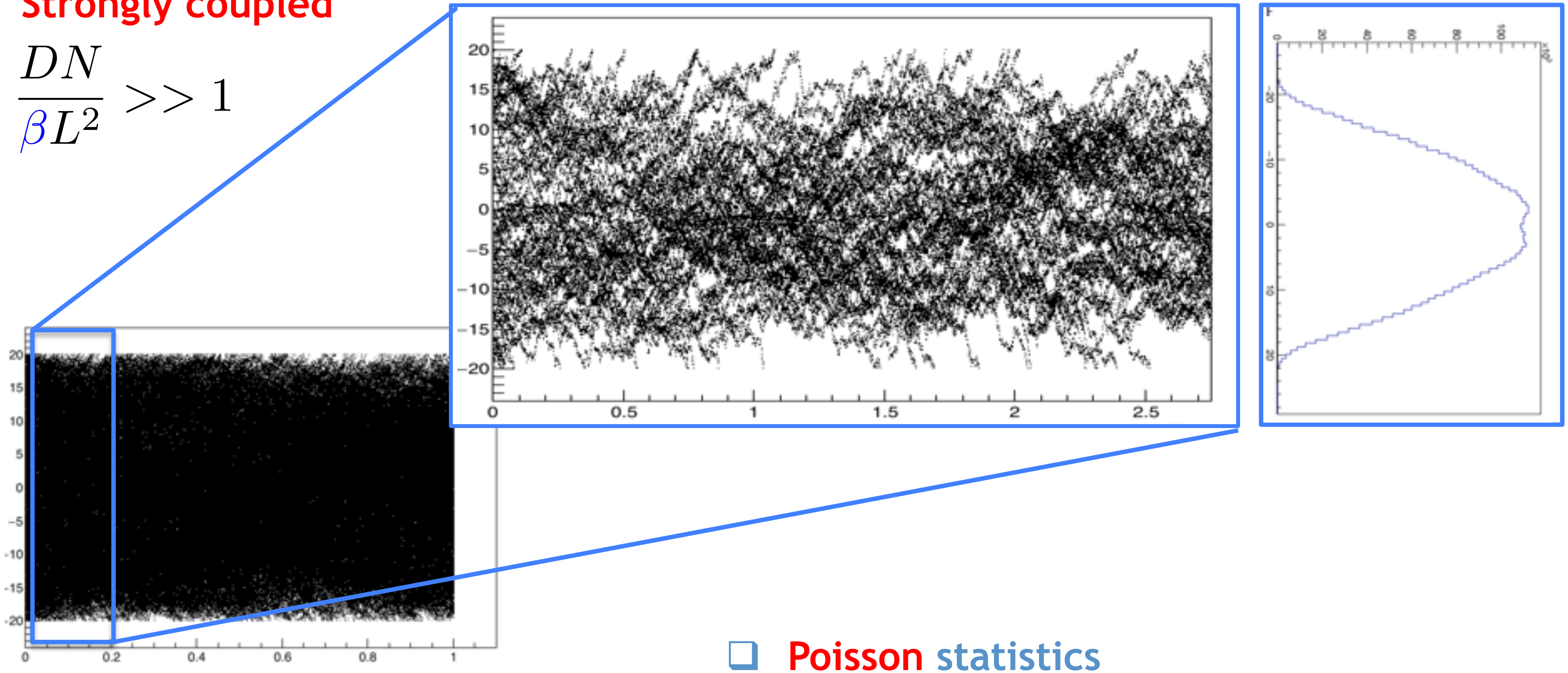
& clustering

& traveling waves

- 1-D BBM with population control
- Uniform initial distribution
- 50 neutrons
- [-L,L] Dirichlet

**Strongly coupled**

$$\frac{DN}{\beta L^2} >> 1$$



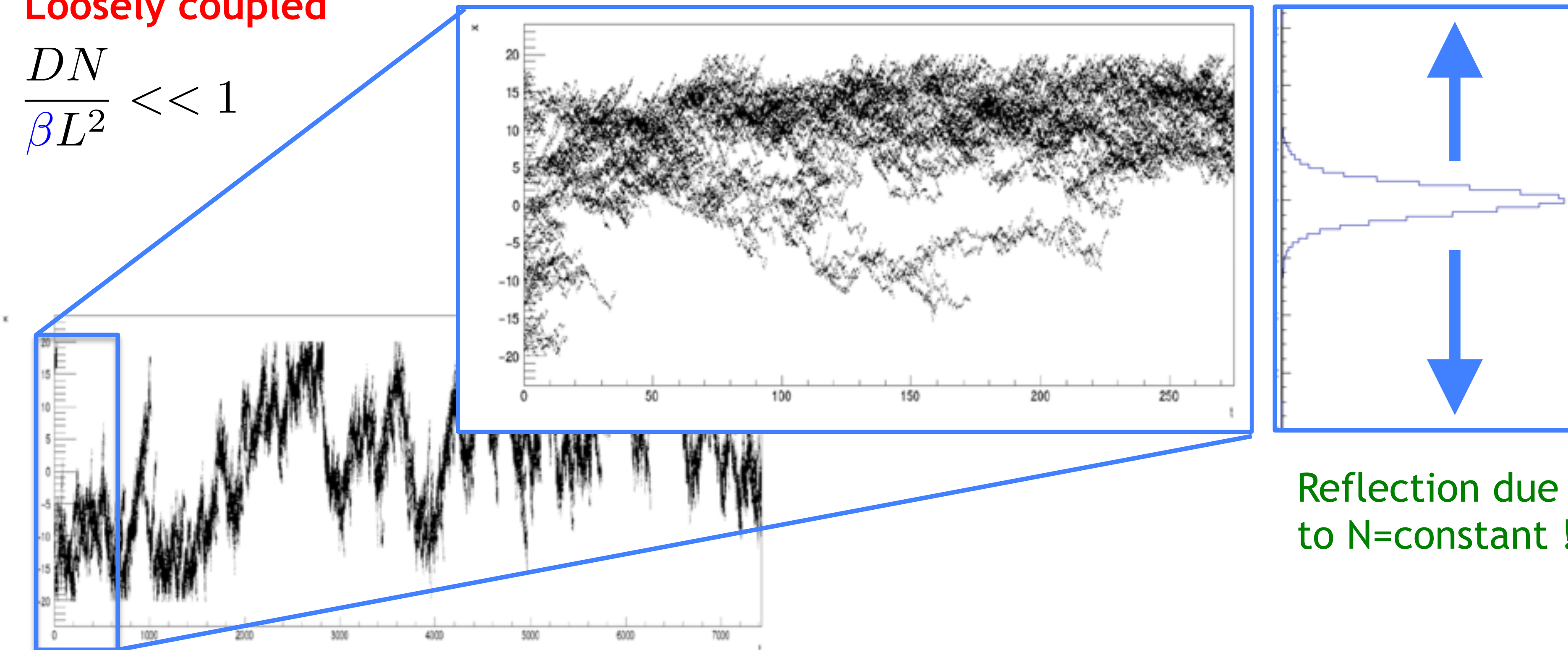
- Poisson statistics
- Cosine shape

- 1-D BBM with population control
- Uniform initial distribution
- 50 neutrons
- [-L,L] Dirichlet

Reflection due to N=constant !

**Loosely coupled**

$$\frac{DN}{\beta L^2} << 1$$



Reflection due to N=constant !

- **Clustering**
- **Only one** cluster after some time
- **Reflected** albeit leaking boundaries !

# Population control & traveling waves

$$\partial_t \phi = D\nabla^2\phi + (\beta - \gamma)\ \phi + \left( \frac{-\beta + \gamma - D\ \partial_x \phi(x,t)\big|_{x=\pm L}}{\int_{-L}^{+L} dx \int_{-L}^{+L} dx\ \phi(x,t)^2} \right) \phi(x,t)^2$$

- Non-linear equation with $\phi^2$ term
- Can be simplified under some assumptions

Fisher, Ann. Eugenics 7:353-369 (1937)

$$\partial_t \phi = D\nabla^2\phi + (\beta - \gamma)\ \phi(1 - \phi)$$

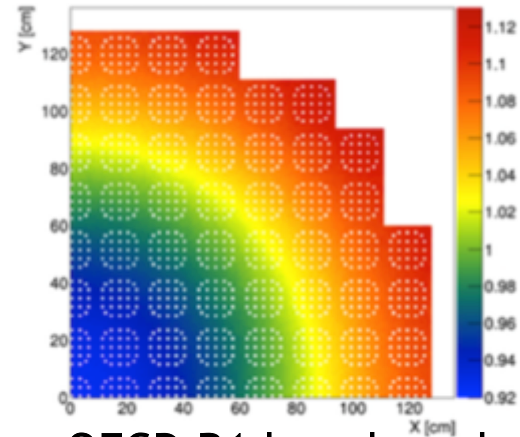
$$\phi(x,t) = \frac{1}{\left(1 + C \exp^{\pm\frac{1}{6}\sqrt{6(\beta-\gamma)}x - \frac{5}{6}(\beta-\gamma)t}\right)^2}$$

Dumonteil et al, Nuc. Eng. Tech., 10.1016/j.net.2017.07.011 (2017)

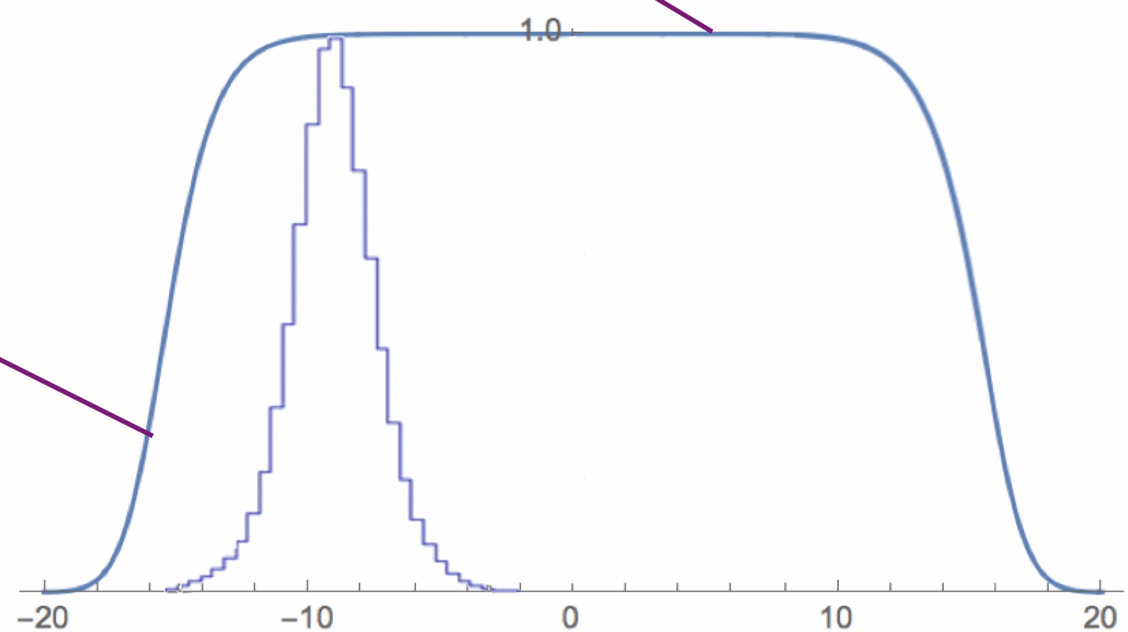
- F-KPP equation with traveling waves solutions
- Counter-reaction depending on the sign of $1 - \phi$

# Traveling wave & solitons

- Fux profile => comes from the averaging through time of the cluster displacement
- Connection between clustering & solitons
  - Clustering typical of branching processes
  - Solitons typical of non-linear equations
- Qualitative & Quantitative scheme to explain under-sampling biases on local tallies



OECD R1 benchmark



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# Is it possible to observe/characterize clustering effects through experiments?

- Clustering **should be measurable**, if certain conditions are gathered:

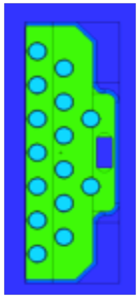
$$\frac{\tau_D}{\tau_E} \simeq \left(\frac{L^2}{D}\right) / \left(\frac{N}{\lambda}\right) = \frac{1}{N}\frac{L^2}{\ell_m^2}$$

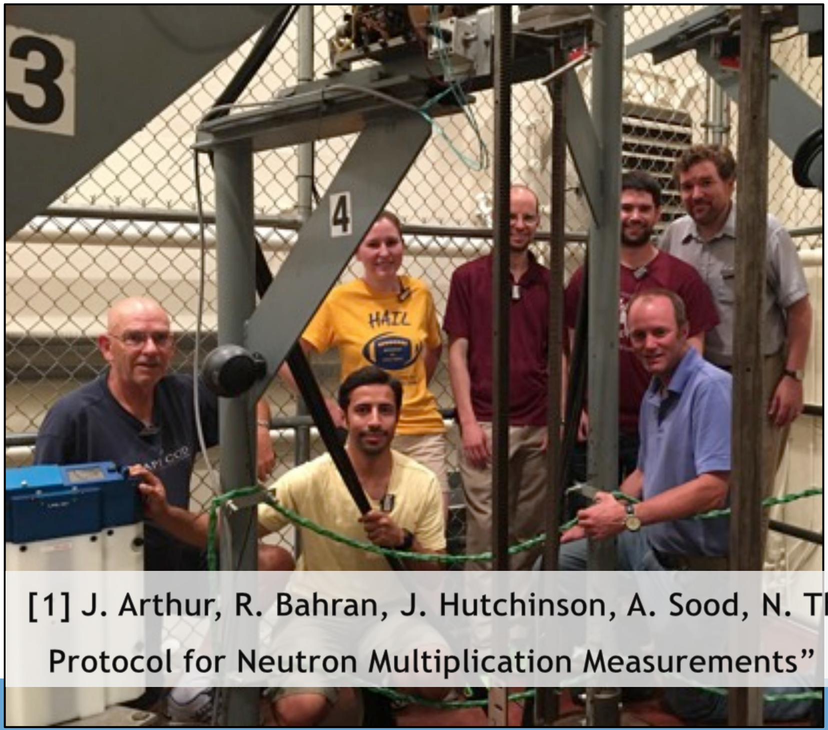
$$\ell_m^2 = \frac{D}{\lambda}$$

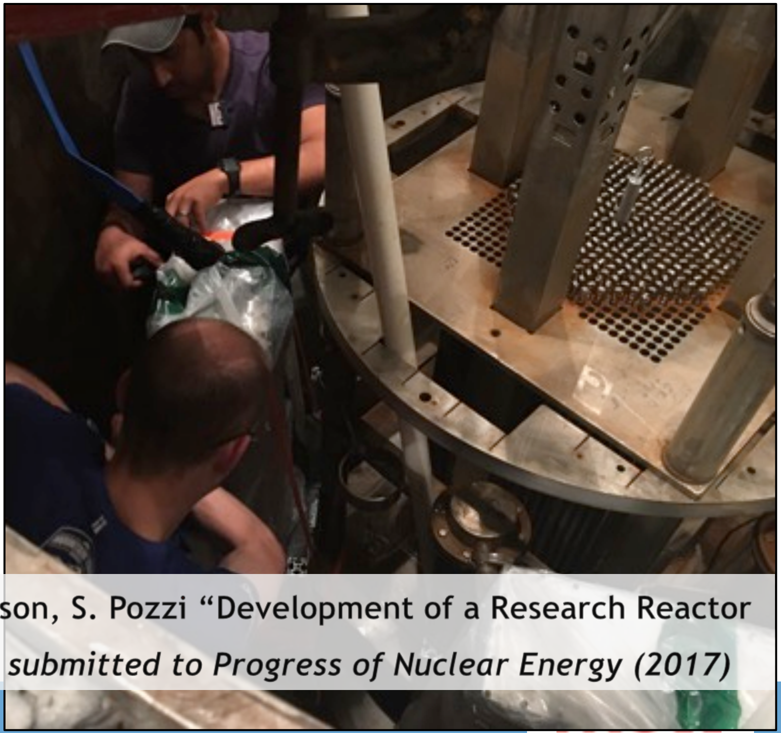
Neutron **migration area**

# In 2016, LANL/UMich Performed Subcritical Measurements at the RPI-RCF with LANL Neutron Multiplicity Detectors

- Two important goals achieved:
  - ✓ established a protocol for subcritical neutron multiplication measurements at a research reactor [1]
  - ✓ did not drown very expensive state-of-the-art LANL multiplicity detectors aka MC-15 detectors (15 He-3 tubes encased in poly)





[1] J. Arthur, R. Bahran, J. Hutchinson, A. Sood, N. Thompson, S. Pozzi "Development of a Research Reactor Protocol for Neutron Multiplication Measurements" *to be submitted to Progress of Nuclear Energy (2017)*

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$$\ell_m^2 = \frac{D}{\lambda}$$

Neutron **migration area**

- Ideal conditions for an experiment that could characterize clustering?
  - ✓ Zero power reactor
  - ✓ Fresh fuel, no burn-up effects
  - ✓ As big as possible

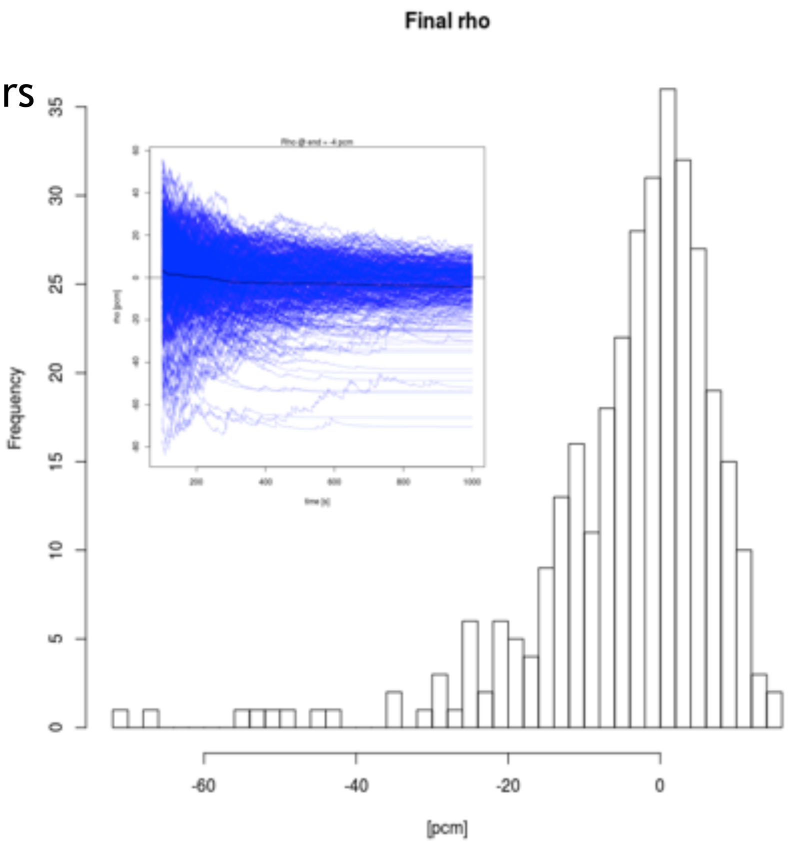
  } **RCF@RPI**

  - ✓ Find a way to do spatial measurements — **MC15 detectors & He3 tubes**

# MORET 5 simulations to design the experiment

- **MORET 5 code with all Random Noise options activated => dynamic + analog**
  - Data library: Endfb71
  - Fission sampling:
    - ✓ Freya
    - ✓ discrete Zucker and Holden tabulated
    - ✓ Pn distributions and corresponding nubars
    - ✓ Only Spontaneous fissions

- **Highly parallel simulations:**
  - Simulated signal = 1000 s (prompt+delayed)
  - Number of independent simulations = 330
  - Number of neutrons per simulation = $2.4\ 10^4$

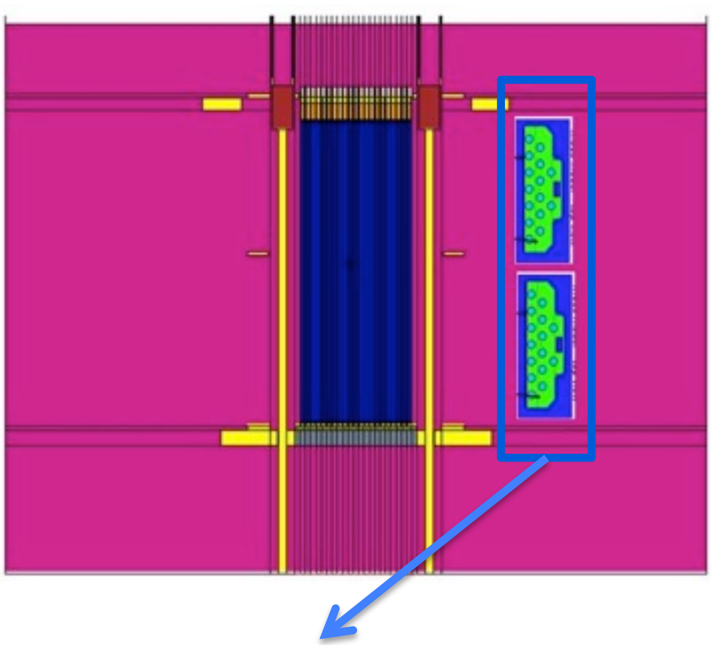



**Excellent reactivity: Rho = -4 pcm**
**+**
**Up to 10 mW of simulated power!**

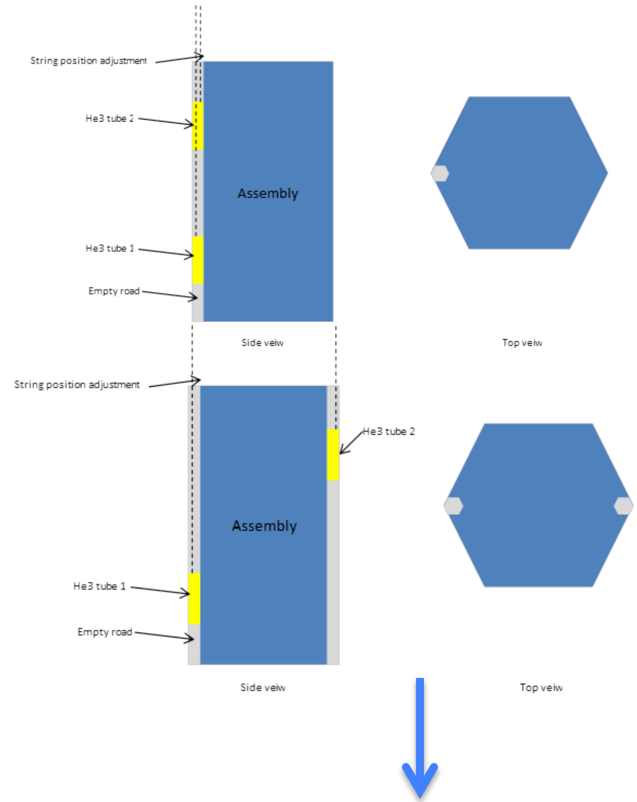
# Preliminary results of RCF simulation

- Ideal scenario@RCF => 1st question: are there spatial correlations in the reactor ?

  => 2nd question: if yes, are there measurable?



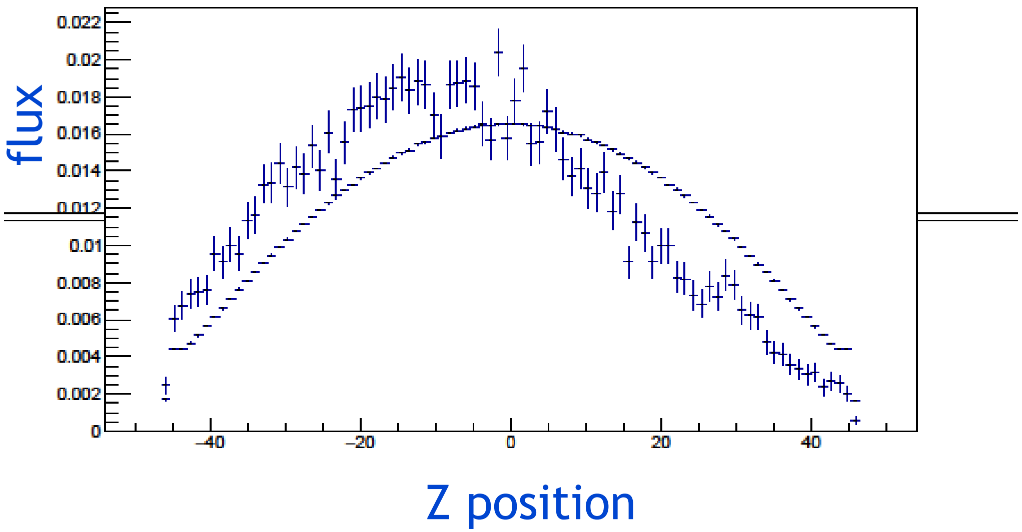
Simulation of expected signal in the MC15 detectors



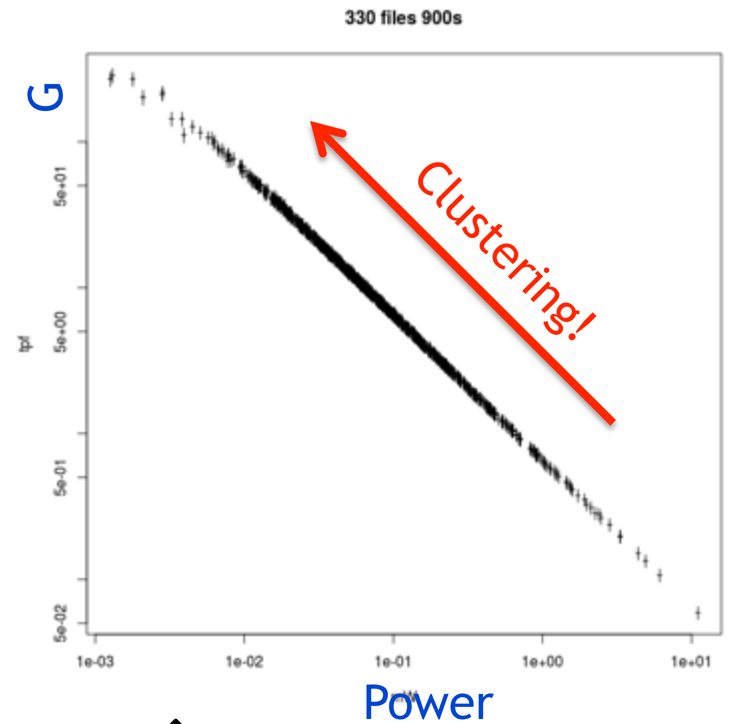

Simulation of in-core effects with tallies defined over He3 tubes

# Simulation of RCF in-core effects

Simulation of in-core effects with $He^3$ tallies




Simulation of expected signal in the MC15 detectors




- ❑ Experimental program should include:
  - ✓ Power scan
  - ✓ PuBe source effects
- ❑ **RCF has the potential to be conclusive regarding the neutron clustering theory!**

$$G_P(n,m) = \left.\frac{\langle nm\rangle - \langle n\rangle\langle m\rangle}{\langle n\rangle\langle m\rangle}\right|_P$$

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# Partial conclusions (see N. Thompson's talk!)

- Stochastic modelling is used to characterize the behavior of loosely coupled systems and predicts a:
  - **clustering phenomenon**…
  - …**obeying traveling waves** equations

- Analog Monte Carlo simulations (with **MCNP** and/or **MORET**) were used to design such an experiment, using **LANL MC15 detectors** and the **RCF@RPI reactor**

- This experiment happened in August 2017 and showed that….

  see Nick Thompson's talk!

- Nick Thompson is currently working @ LANL and will rejoin IRSN in June to **improve the analyses of the data**

Thank you !

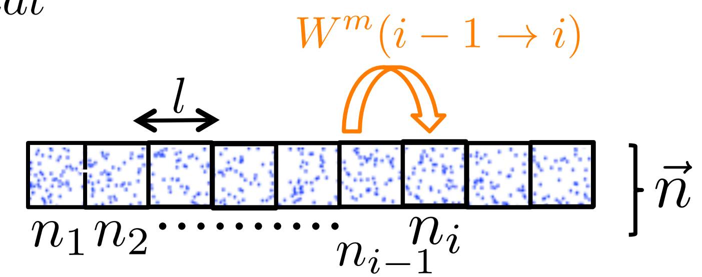
# Clustering theory

# A little bit of field theory

- fission event
  - proba: $W^{+}(\vec{n},i)dt = \lambda p(2)\eta_i \vec{n} dt$
  - action on $\vec{n}$: $a_i^{+}\vec{n} = (\dots, n_{i-1}, \boxed{n_i + 1}, n_{i+1}, \dots)$
- capture event
  - proba: $W^{-}(\vec{n},i)dt = \lambda p(0)\eta_i \vec{n} dt$
  - action on $\vec{n}$: $a_i\vec{n} = (\dots, n_{i-1}, \boxed{n_i - 1}, n_{i+1}, \dots)$
- migration event
  - proba: $W^{m}(\vec{n}, i-1 \rightarrow i)dt = \lambda p(1)\eta_i \vec{n} dt$
  - action on $\vec{n}$: $a_i^{+} a_{i-1}\vec{n}$

with $\eta_i$ the number of neutrons in cell *i*

and $\lambda p(1) = {}^{D}/_{2l^2}$




**Forward master equation**

$$
\begin{aligned}
\frac{dP(\vec{n},t)}{dt} = \sum_i & W^{+}(a_i\vec{n},i)P(a_i\vec{n},t) && -W^{+}(\vec{n},i)P(\vec{n},t) \\
& +W^{-}(a_i^{+}\vec{n},i)P(a_i^{+}\vec{n},t) && -W^{-}(\vec{n},i)P(\vec{n},t) \\
& +W^{m}(a_{i-1}^{+}a_i\vec{n},i-1,i)P(a_{i-1}^{+}a_i\vec{n},t) && -W^{m}(\vec{n},i,i+1)P(\vec{n},t) \\
& +W^{m}(a_{i+1}^{+}a_i\vec{n},i+1,i)P(a_{i+1}^{+}a_i\vec{n},t) && -W^{m}(\vec{n},i,i-1)P(\vec{n},t)
\end{aligned}
$$

# And a little bit more

As before one can inject in the Master equation the mean number of neutrons in cell k:

$$< n_k >= \sum_n n_k P(n_k, t)$$

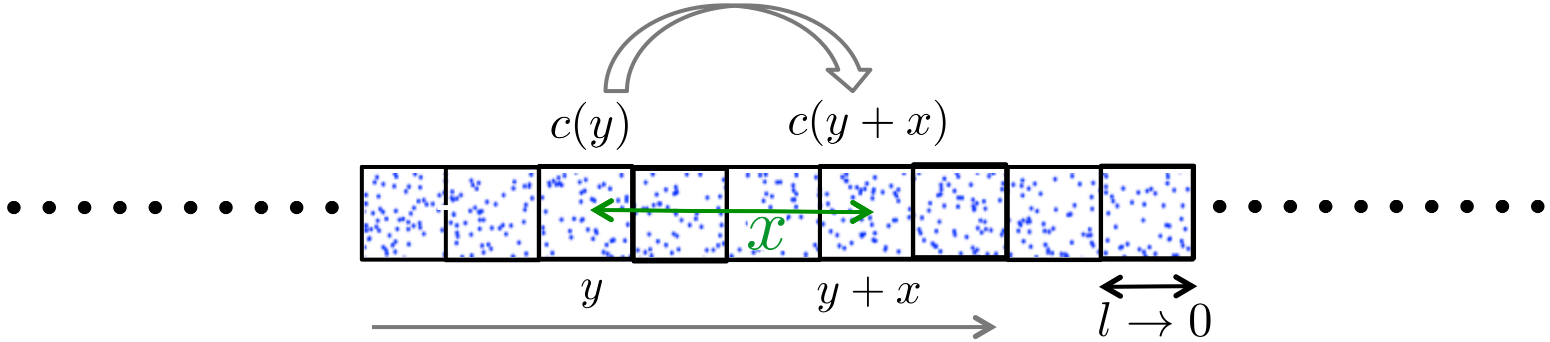
or its continuous version:

$$c(x) = \lim_{l \to 0} \frac{n_k}{l}$$

And define an appropriate tool to study spatial correlations:

the centered correlations without self-contribution

$$g(x,t) = (< c(y)c(y+x) > - c^2 - c\delta(x))/c^2$$



# Equation for the 2-points correlation function

**The equations obtained stand for any arbitrary dimension *d* and in the case $\nu_1 = 1$ can be written:**

$$\frac{\partial}{\partial t} c_t(\mathbf{x}) = 0$$

$$\frac{\partial}{\partial t} g_t(r) = 2D\nabla_r^2 g_t(r) + \frac{\lambda \nu_2}{c_t}\delta(r)$$

**d-dimensional Laplacian (diffusion term)**

*with* $r = |x - y|$

*and* $\nu_2 = \sum_k k(k-1)p(k)$

**auto-correlation term leading to 2nd moment effects ($\nu_2$ is the mean number of pairs)**


Young, W.R., Roberts, A.J., Stuhne, G., Nature 412, 328 (2001)
Houchmandzadeh, B., Phys. Rev. E 66, 052902 (2002)
Houchmandzadeh, B., Phys. Rev. Lett. 101, 078103 (2008)
Houchmandzadeh, B., Phys. Rev. E 80, 051920 (2009)
Dumonteil, E. et al, Annals of Nuclear Energy 63, 612-618 (2014)

# Analytical solution to this equation

With initial condition $c_0(\mathbf{x}) = c_0$ the solution to the 1st equation is:

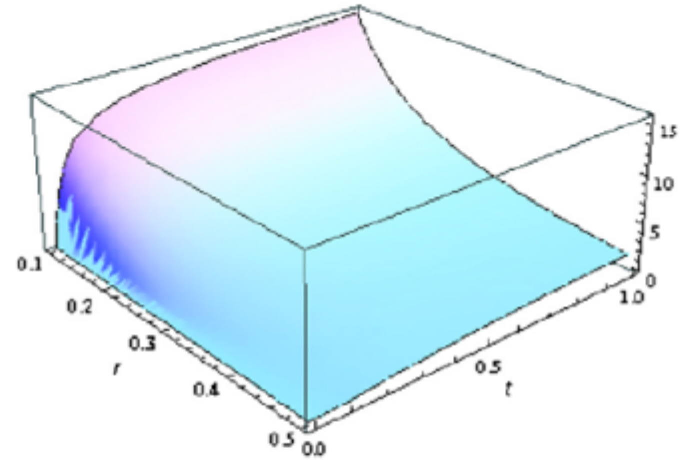
$$c_t(\mathbf{x}) = c_0 \quad \text{(for all } t\text{)}$$

And the solution to the 2-points function is, **taking dimension $d = 3$**:

$$g_t(r) = \frac{\lambda \nu_2}{8 D c_0 \pi^{3/2} r} \Gamma\left(\frac{1}{2}, \frac{r^2}{8Dt}\right)$$

where $\Gamma(a, z)$ stands for the incomplete Gamma function

$$\text{Amplitude} \propto \frac{\lambda \nu_2}{D c_0}$$




g can be interpreted as the probability to find a neutron next to another

IRSN

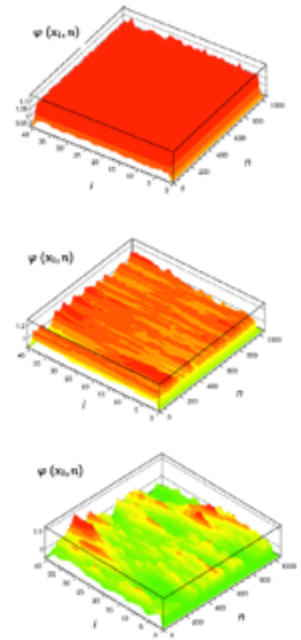
# Consequence 1: Convergence criteria

Typical separation between particles: $\ell = \sqrt{\langle r_a^2 \rangle}$

Number of particles to suppress clustering: $N_0$ => $N_0 \gg (L/\ell)^3$

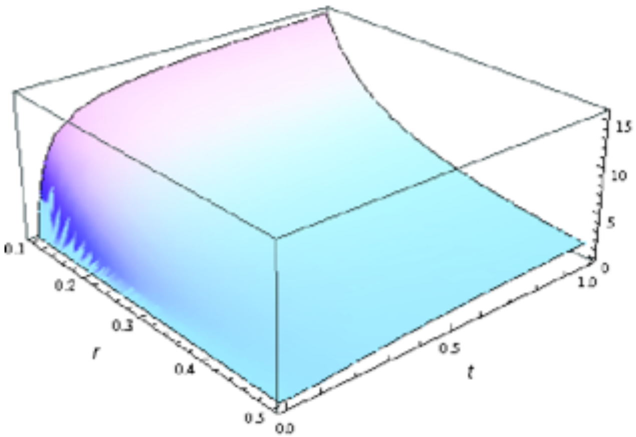
Let's go back to the pincell test-case: $\ell \simeq 6$ cm and $N = 10^4$ (# particles simulated)

| | | |
|---|---|---|
| $L = 10$ cm | $N_0 \simeq 4$ | $N \gg N_0$ |
| $L = 100$ cm | $N_0 \simeq 5 \cdot 10^3$ | $N \simeq N_0$ |
| $L = 400$ cm | $N_0 \simeq 3 \cdot 10^5$ | $N \ll N_0$ |

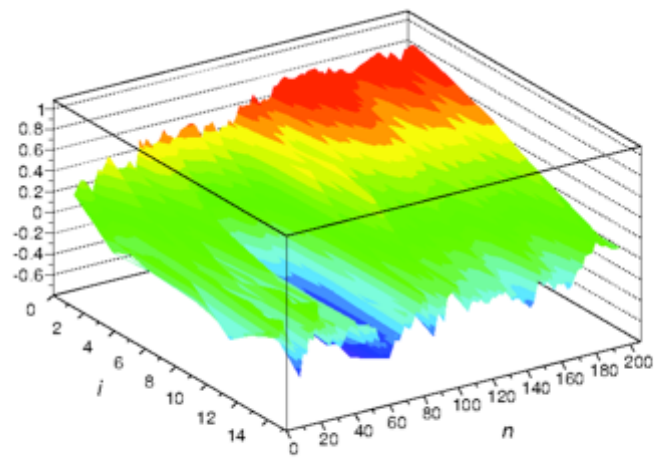


# Consequence 2: Diagnostic tool

**2-points correlation function versus $(r,t)$ for the 3-d analytical function**
**$(i,n)$ for the TRIPOLI-4® simulation of the pincell ($i$ is the bin number)**




« theory »




« MC criticality simulation »
clustering diagnostic tool in TRIPOLI-4®:
histogram of inter-collisions distances

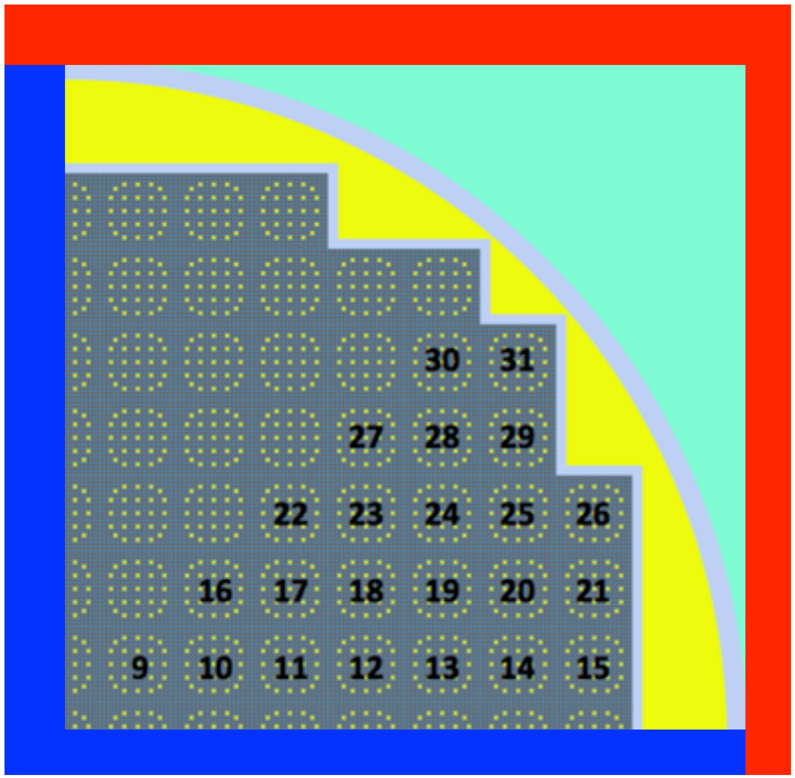
$\Rightarrow$ very good agreement
$\Rightarrow$ saturation of the 2-points estimator in the MC simulation

# Traveling waves

# OECD/NEA R1 Benchmark

- Expert Group on Advanced Monte-Carlo Techniques @ OECD/NEA
- R1 **Benchmark** = ¼ PWR-type reactor core
- Designed to understand **biases on local tallies** estimates (+uncertainties)

Reflective boundary Conditions (Neumann)

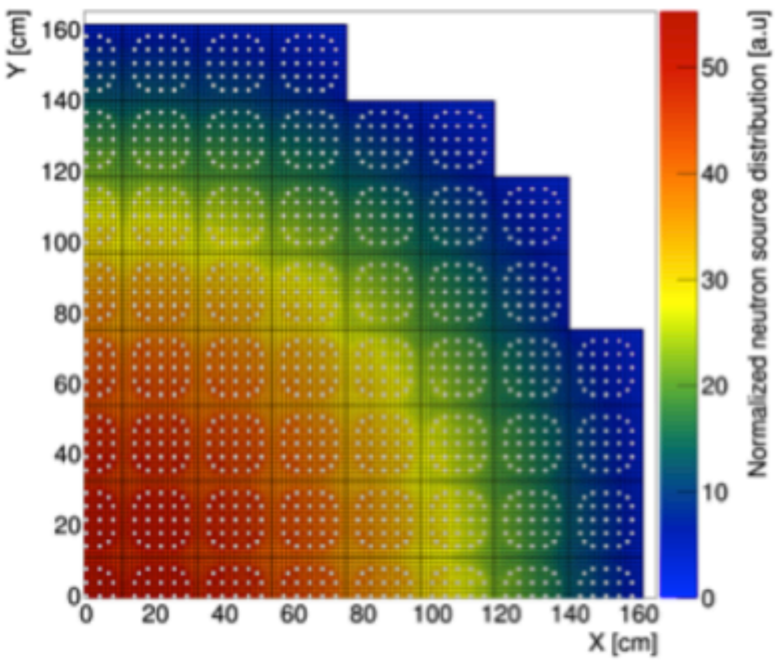


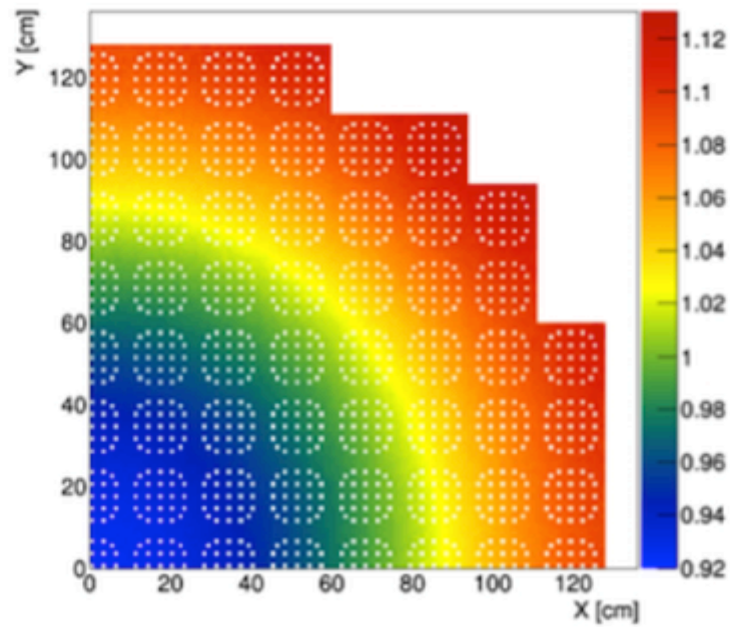

Absorbing boundary Conditions (Dirichlet)

# MORET Simulation of the R1 benchmark

Fluxes ($10^4$ active cycles of $10^4$ neutrons)

$$\frac{\text{Fluxes } (10^6 \text{ active cycles of } 10^2 \text{ neutrons})}{\text{Fluxes } (10^2 \text{ active cycles of } 10^6 \text{ neutrons})}$$




**Under-estimation inside the core, over-estimation for the outer assemblies**

# 1-D binary branching Brownian motion

- Uniform material, mono-energy, leakage bc
- Brownian motion with diffusion coefficient ***D*** [cm2.s-1]
- undergoes collision at Poissonian times with rate $\beta + \gamma + \lambda$ [s-1]
- at each collision, ***k*** descendants with probability $p(k)$
- total number of particles ***N*** kept constant

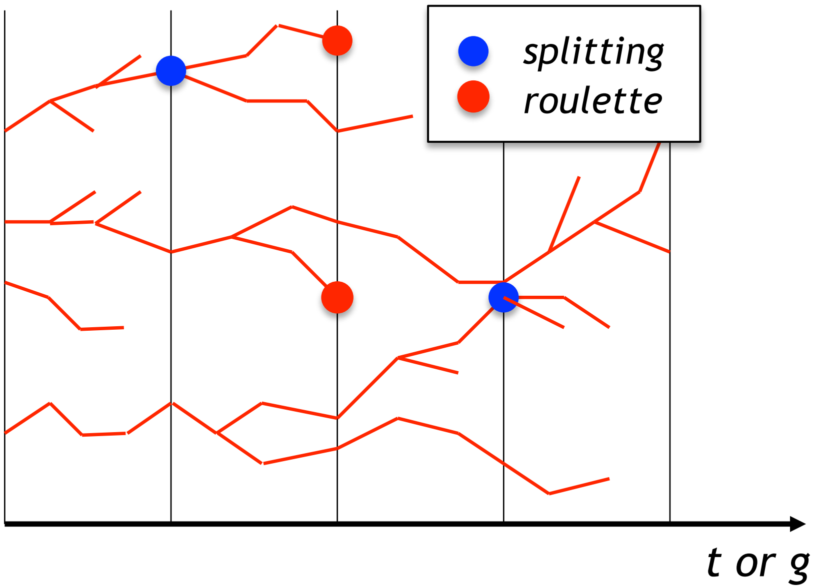
$$< x^2(t) >= Dt$$

$$p(0) \propto \gamma$$

$$p(1) \propto \lambda$$

$$p(2) \propto \beta$$

*Population control algo. to keep N constant*




Branching Brownian motion with population control couples:

⇒ **Galton-Watson** birth-death process to describe fission and absorption

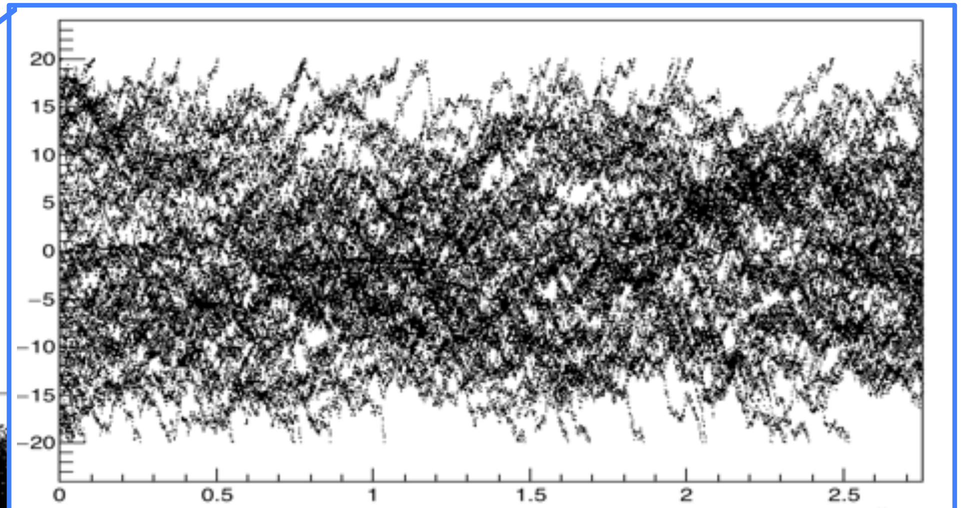
⇒ **Brownian** motion to simulate neutron transport

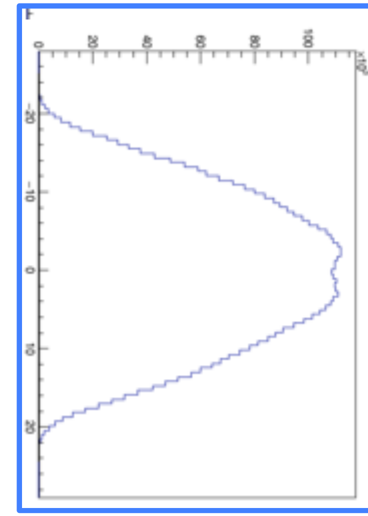
⇒ **Population control** that reproduces the end of cycle renormalization of MC criticality codes

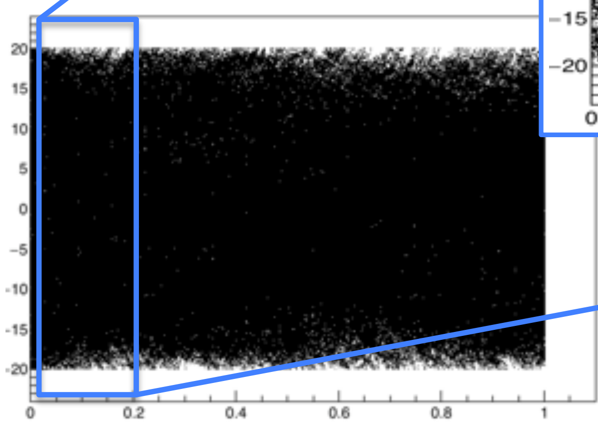
- 1-D BBM with population control
- Uniform initial distribution
- 50 neutrons
- [-L,L] Dirichlet

**Strongly coupled**

$$\frac{DN}{\beta L^2} >> 1$$





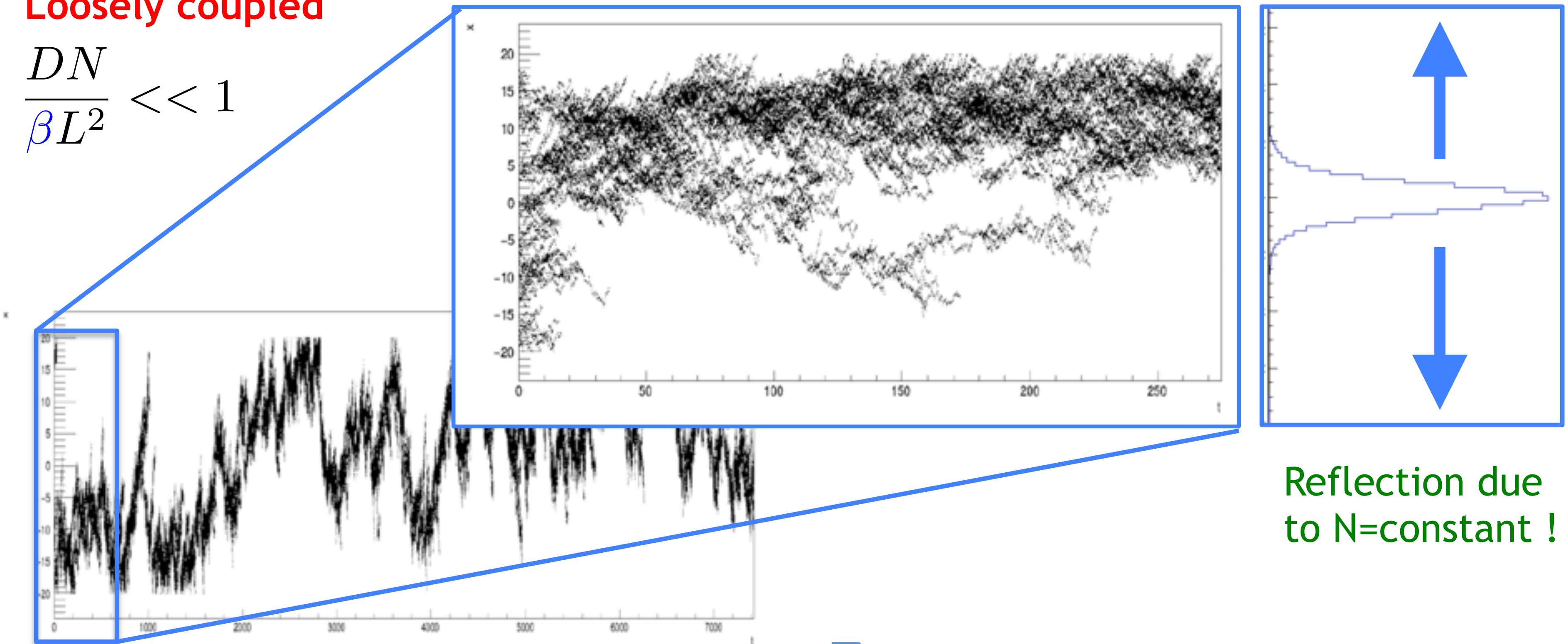

- Poisson statistics
- Cosine shape

- 1-D BBM with population control
- Uniform initial distribution
- 50 neutrons
- [-L,L] Dirichlet

**Loosely coupled**

$$\frac{DN}{\beta L^2} << 1$$

Reflection due to N=constant !

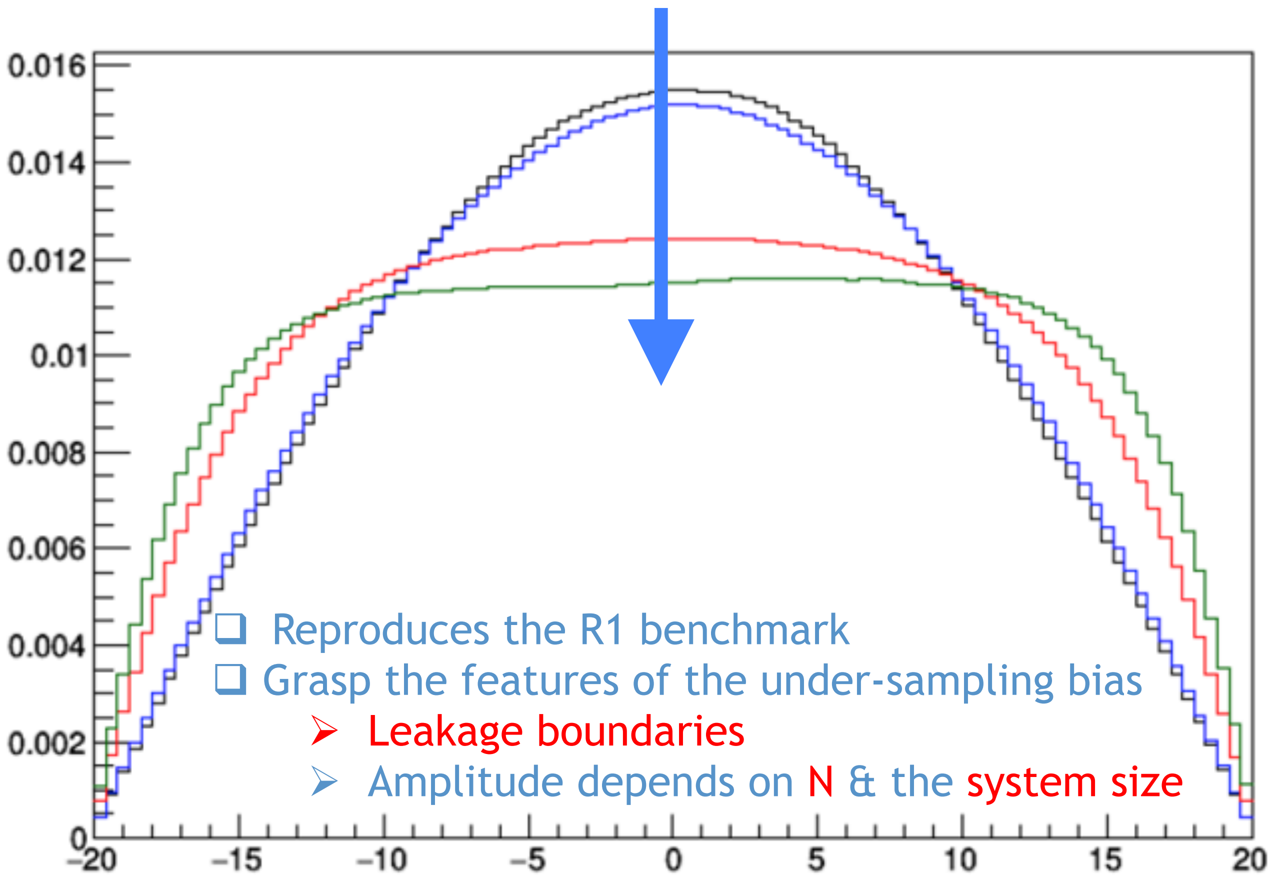


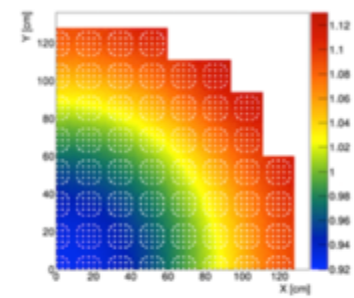

Reflection due to N=constant !

- Clustering
- Only one cluster after some time
- Reflected albeit leaking boundaries !

# How do these processes average through time ?

From strongest to lousiest coupled systems



- ❑ Reproduces the R1 benchmark
- ❑ Grasp the features of the under-sampling bias
  - ➢ Leakage boundaries
  - ➢ Amplitude depends on N & the system size

# Diffusion equation with population control

- ❑ Monte-Carlo criticality codes = Boltzmann equation **+ population control**
- ❑ **Population control** = Weight Watching techniques (i.e. **splitting+roulette**) played at end of cycles to ensure that **N~cte**

Can we build an equation for what MC criticality codes actually solve ?

Fission rate

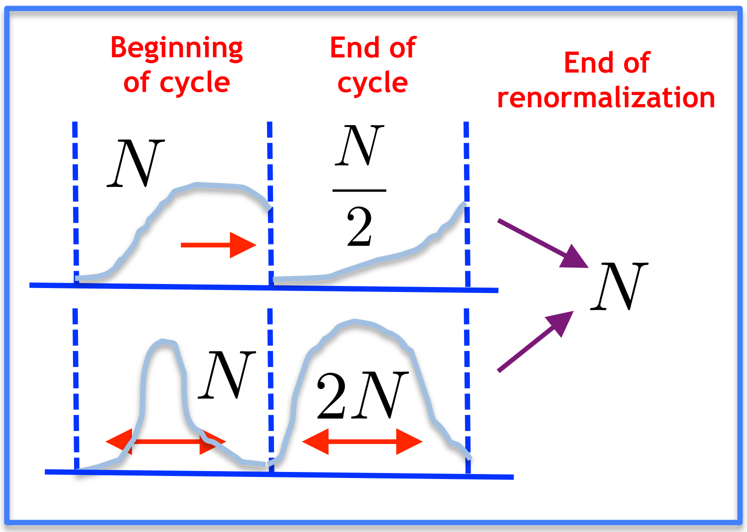
$$\partial_t \phi = D\nabla^2\phi + (\beta - \gamma)\ \phi \quad + ?$$

Diffusion operator
(same demonstration for transport)

Capture rate

# Fission/Capture vs Splitting/Russian Roulette

Probability for a given neutron to be splitted/captured depends on the overall # of neutrons



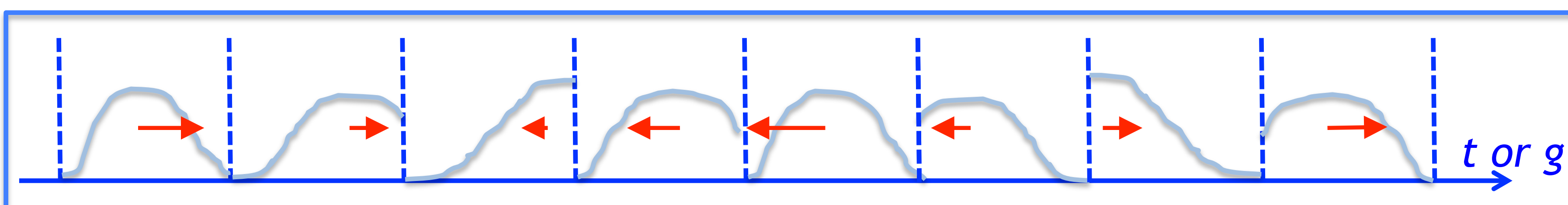

$\beta^* N \leftarrow \beta N$

$\gamma^* N \leftarrow \gamma N$

renormalization rate depends on N and *t/g*

$f(N)N$

$\lambda(t)N$



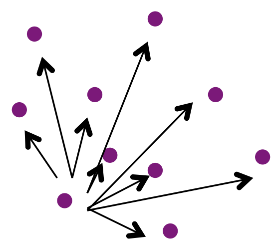
# Pair interactions

**But how many neutrons do we remove/split at the end of each cycle and how to select them ?**

**renormalization rate depends on time and N !**

$$\lambda(t) \underbrace{f(N)}_{(N-1)N} N$$

$$(N-1)N \approx N^2$$



Generalization # neutrons captured in $x \pm dx$ if k>1

$$\lambda(t) \int dy \, \underbrace{G(x,y,t)}_{\text{number of pairs}}$$

number of pairs

Birch et al, Theoretical Population Biology, 70, 26-42 (2006)

- Combinatorial interactions ! ~ $N^2$ at first order (# pairs)
- Depends on the total mass N
- Depends on the local mass N(x)

# Diffusion with pair interactions

$$\partial_t \phi = D\nabla^2 \phi + (\beta - \gamma)\ \phi$$

+ pair interactions

rate of renormalization

$$\partial_t \phi = D\nabla^2 \phi + (\beta - \gamma)\ \phi + \lambda(t) \int dy\ G(x,y,t)$$

number of pairs

$$G(x,y,t) = \Big[1 + g(x,y,t)\Big]\phi(x)\phi(y)$$

$g(x,y,t)$ spatial correlation function


- "Hierarchy horror" (2d order moment pops back in the mean field equation!)
- Clustering = spatial correlations => Bias induced on the flux wrt pure diffusion

# Small population size

$$\partial_t \phi = D\nabla^2 \phi + (\beta - \gamma)\ \phi + \left( \frac{-\beta + \gamma - D\ \partial_x \phi(x,t)\big|_{x=\pm L}}{\int_{-L}^{+L} dx \int_{-L}^{+L} dx\ \phi(x,t)^2} \right) \phi(x,t)^2$$

- Non-linear equation with $\phi^2$ term
- Can be simplified under some assumptions

Fisher, Ann. Eugenics 7:353-369 (1937)

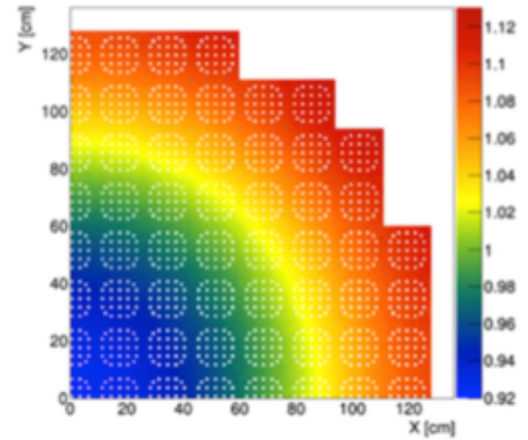
$$\partial_t \phi = D\nabla^2 \phi + (\beta - \gamma)\ \phi(1 - \phi)$$

$$\phi(x,t) = \frac{1}{\left(1 + C \exp^{\pm \frac{1}{6}\sqrt{6(\beta-\gamma)}x - \frac{5}{6}(\beta-\gamma)t}\right)^2}$$

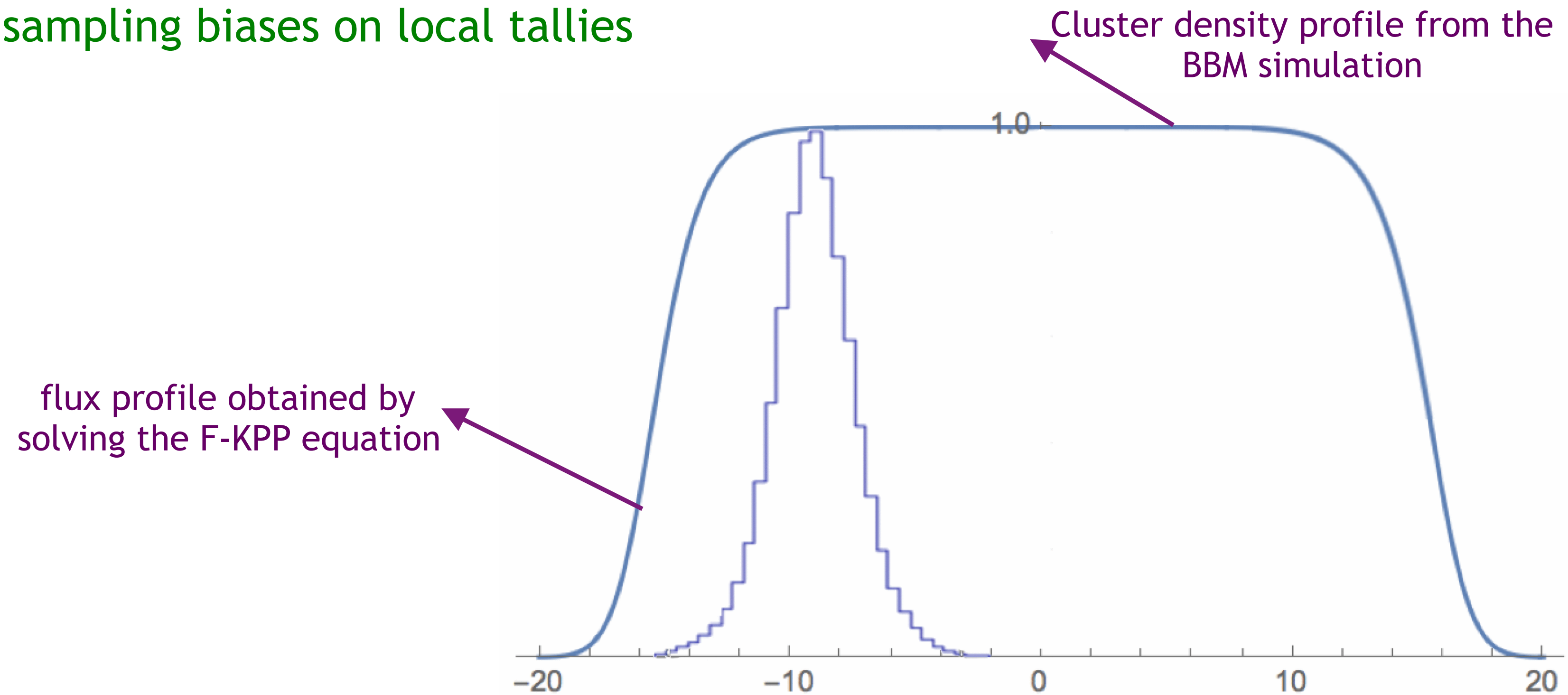
Dumonteil et al, Nuc. Eng. Tech., 10.1016/j.net.2017.07.011 (2017)

- F-KPP equation with traveling waves solutions
- Counter-reaction depending on the sign of $1 - \phi$

# Traveling wave & solitons

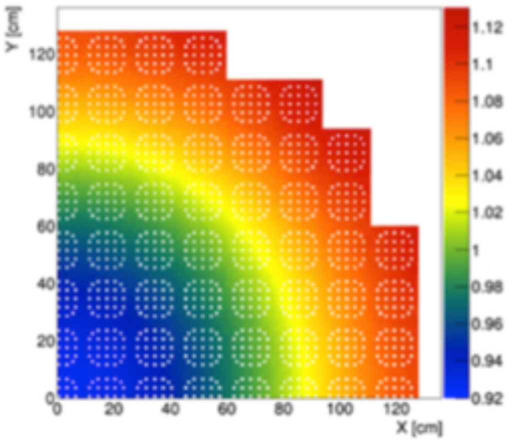



- Fux profile => comes from the averaging through time of the cluster displacement
- Connection between clustering & solitons
  - Clustering typical of branching processes
  - Solitons typical of non-linear equations
- Qualitative & Quantitative scheme to explain under-sampling biases on local tallies



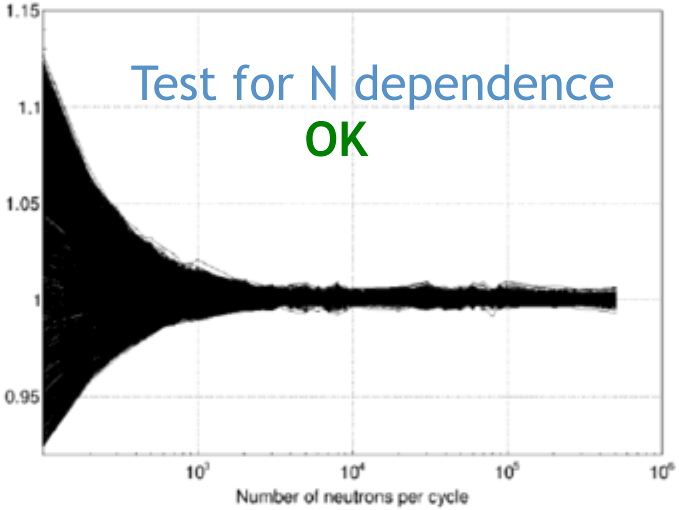
# Back to the under-sampling bias

- Under-sampling bias due to combination between **clustering + population control + bc**
- Parameters controlling the amplitude of the under-sampling bias are linked to the spatial correlation function:




$$|g_c(x_i, x_j, t)| \leqslant \frac{\lambda \nu_2}{N} \frac{2}{3} \frac{L^2}{D}$$

De Mulatier et al, J. Stat. Mech., 15, P08021, 1742-5468 (2015)




- N
- Total reaction rate
- Typical size of the system
- Diffusion coefficient
- Second moment of the descending factorial of p(z)

# Population control

- N has to be kept constant : $\int_{-L}^{L} dx \ \phi(x,t) = 1$
- $\lambda$ depends on time!
- Injecting the normalization relation in our equation, we can calculate $\lambda(t)$

$$\lambda(t) = \frac{-\beta + \gamma - D \int_{-L}^{L} dx \ \nabla^2 \phi(x,t)}{\int_{-L}^{L} dx \int_{-L}^{L} dy \ G(x,y,t)}$$

Newman et al, Phys. Rev. Lett., 92, 228103 (2004)

# What equation do MC codes solve ?

$$\lambda(t) = \frac{-\beta + \gamma - D \int_{-L}^{L} dx \; \nabla^2 \phi(x,t)}{\int_{-L}^{L} dx \int_{-L}^{L} dy \; G(x,y,t)}$$

Probability that one neutron in x is captured

$$\partial_t \phi = D \nabla^2 \phi + (\beta - \gamma) \; \phi + \lambda(t) \int_{-L}^{L} dy \; (1 + g(x,y,t)) \phi(y,t) \phi(x,t)$$

Flux factorized out of the integral

Large population size

$$g(x,y,t) \to 0$$

Small population size

$$g(x,y,t) \to g_N^{\infty}(x,y) >> 1$$

De Mulatier et al, J. Stat. Mech., 15, P08021, 1742-5468 (2015)

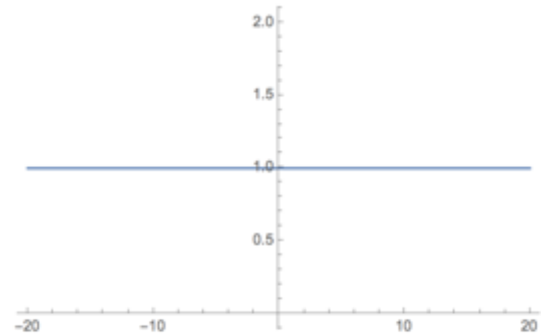
# Large population size

$$\nabla^2\phi - \left(\int_{-L}^{L} dx\ \nabla^2\phi(x)\right)\ \phi = 0$$

$$\partial_x \phi(x)\Big|_{x=\pm L}$$

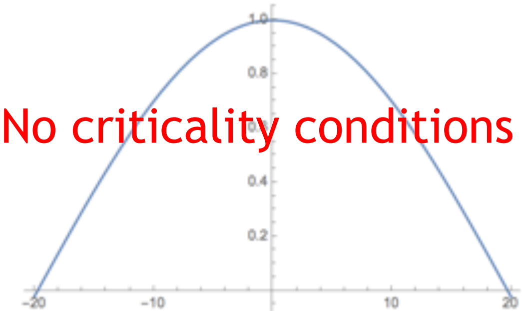
Neumann/Reflective bc

$$\nabla^2\phi = 0$$




Dirichlet/Absorbing bc

$$\nabla^2\phi + \frac{\pi^2}{2L^2}\phi = 0$$




No criticality conditions ;)

# Experimental design

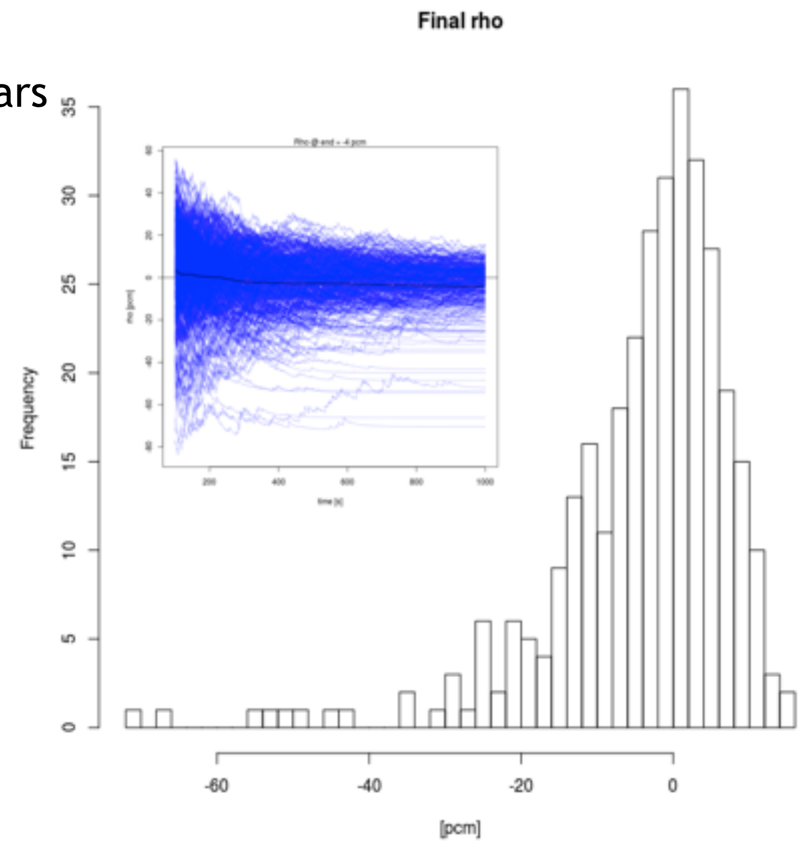
# In more details

- Size of the reactor (the bigger, the better) => control rod insertion matters
- Power of the reactor (the lower, the better) => ideally different run at different power. Ability to differentiate the power “signal” (fission chains) and the following “noise” sources:
  - (alpha,n) reactions  have to be simulated
  - Spontaneous fission level has to be simulated
  - Inhibition of triggering sources as much as possible (PuBe)
- Define the time gate width (analysis) to reveal the non-Poissonian effects
- Spatial extension of the measurement => detector with a spatial resolution over more than few 10 cm, or at least being able to move the detector

# MORET Simulations  to design the experiment

- **MORET5 code with all Random Noise options activated:**
  - Data library: Endfb71
  - Fission sampling:
    - ✓ Freya
    - ✓ discrete Zucker and Holden tabulated
    - ✓ Pn distributions and corresponding nubars
    - ✓ Only Spontaneous fissions
- **Highly parallel simulations:**
  - Simulated signal = 1000 s (prompt+delayed)
  - Number of independent simulations = 330
  - Number of neutrons per simulation = 2.4 $10^4$

**Excellent reactivity: Rho = -4 pcm**
**+**
**Up to 10 mW of simulated power!**

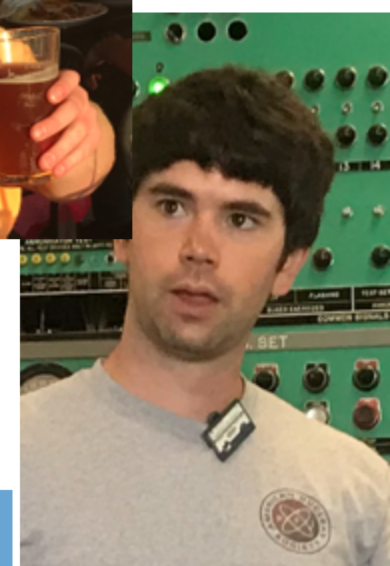


# RPI Measurements 2017: Neutron clustering

# Featuring



IRSN : Eric Dumonteil, Wilfried Monange
LANL: Rian Bahran, Jesson Hutchinson,
Geordy McKenzie, Mark Nelson
RPI: Peter Caracappa, Nick Thompson,
Glenn Winters



# Surprise #1
# Hotel View And Rendering

# Surprise #2
# De Gaulle


THE TIMES RECORD

HOME EDITION

TROY, N.Y., WEDNESDAY EVENING, JUNE 4, 1958

PRICE SEVEN CENTS

RPI Gets $2,500,000 Atom Facility

*Throngs Roar Welcome To Premier De Gaulle In Algiers*

General To Confer With Insurgents

Brown Leads Knowland In Popular Vote

New Air Mail Stamp Unveiled

AEC Will Provide Equipment

Ike Urges Middies To Seek Peace

U.S. Studies Russian Trade Bid

Dulles Washes Dishes For Wife At Vacation Retreat

Bodies Of 45 Dead In Plane Crash Looted

Hard Ridden Winner In Epsom Derby

Mansfield Wins Easily

Six Airmen Die In Crash

Death Car Driver Drunk

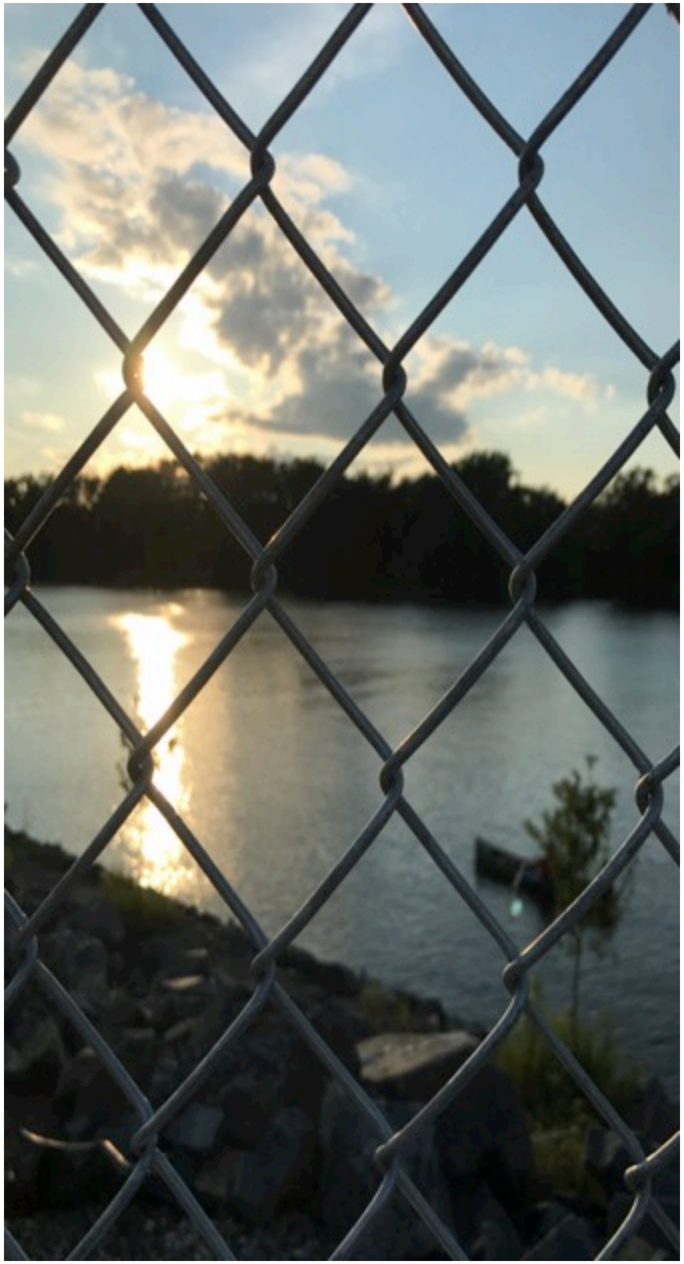
IRSN
LANL
& RPI

# Good moments


# Beautiful RCF outside views

Rian's

mine



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**Clustering in biology**

Young, W.R., et al, 2001. Nature, 412, 328.
Houchmandzadeh, B., 2002. Phys. Rev. E 66, 052902.
Houchmandzadeh, B., 2008. Phys. Rev. Lett. 101, 078103.
Houchmandzadeh, B., 2009. Phys. Rev. E 80, 051920.

**Clustering in neutronics**

[Dumonteil, E., Courau, T., 2010. Nuclear Technology 172, 120.] → Observation of clusters in MC criticality simulations

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Zoia, A. et al, Physical Review E, 90, 042118 (2014). → Confined geometries

De Mulatier et al, J. Stat. Mech., 15 , P08021, 1742-5468 (2015) → Population control

Nowak et al, Ann. Nuc. Ener. 94, 856-868 (2015) → Consequences on MC criticality source convergence (with MIT)

Houchmandzadeh et al, Phys. Rev. E 92 (5), 052114 (2015) → Effects of delayed neutrons

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Dumonteil et al, Nuc. Eng. Tech., 10.1016/j.net.2017.07.011 (2017) → Traveling waves and biases (Jeju best papers)

Sutton, T., Mittal, A., Proceedings of M&C 2017 (Jeju) → cycles/time (Jeju best papers)