

Fall 2002 exam was a 60 minute exam.

(25) Conversion to Clause Form

- I. (a) Transform the wff  $A$  below into CNF (**clause**) matrix form. For each step required give a brief description of the step and perform the step (if applicable) on the space provided. Failure to follow this format may result in no credit.  
 (b) Rewrite your answer in part (a) as a single (1 line) <wff> simplifying if necessary.  
 (c) Which form is better (matrix form or the 1-line form) and why? {No explanation, No credit}

$$\{\text{wff}\} A: (\forall x)\{ P(x) \rightarrow [ \sim(\forall y)\{ Q(x,y) \rightarrow P(f(z)) \} \wedge (\forall y)\{ Q(x,y) \rightarrow P(x) \} ] \}$$

(1) Step 0: \_\_\_\_\_

Eliminate redundant quantifiers and take the existential closure

$$A: (\exists z)(\forall x)\{ P(x) \rightarrow [ \sim(\forall y)\{ Q(x,y) \rightarrow P(f(z)) \} \wedge (\forall y)\{ Q(x,y) \rightarrow P(x) \} ] \}$$

(1) Step 1: \_\_\_\_\_

Remove implications

$$A: (\exists z)(\forall x)\{ \sim P(x) \vee [ \sim(\forall y)\{ \sim Q(x,y) \vee P(f(z)) \} \wedge (\forall y)\{ \sim Q(x,y) \vee P(x) \} ] \}$$

(1) Step 2: \_\_\_\_\_

Move the Negations down to the Atfs

$$A: (\exists z)(\forall x)\{ \sim P(x) \vee [ (\exists y)\{ Q(x,y) \wedge \sim P(f(z)) \} \wedge (\forall y)\{ \sim Q(x,y) \vee P(x) \} ] \}$$

(1) Step 3: \_\_\_\_\_

Standardize Variables Apart

$$A: (\exists z)(\forall x)\{ \sim P(x) \vee [ (\exists y)\{ Q(x,y) \wedge \sim P(f(z)) \} \wedge (\forall w)\{ \sim Q(x,w) \vee P(x) \} ] \}$$

(1) Step 4: \_\_\_\_\_

Skolemize: Let  $z = h(\cdot) = B$ ;  $y = g(x)$

$$A: (\forall x)\{ \sim P(x) \vee [ \{ Q(x, g(x)) \wedge \sim P(f(B)) \} \wedge (\forall w)\{ \sim Q(x,w) \vee P(x) \} ] \}$$

(1) Step 5: \_\_\_\_\_

Move universal quantifiers to the left.

$$A: (\forall x)(\forall w)\{ \sim P(x) \vee [ \{ Q(x, g(x)) \wedge \sim P(f(B)) \} \wedge \{ \sim Q(x,w) \vee P(x) \} ] \}$$

(1) Step 6: \_\_\_\_\_

Distribute  $\vee$  over  $\wedge$  using  $E_1 \vee (E_2 \wedge E_3) = (E_1 \vee E_2) \wedge (E_1 \vee E_3)$

$$A: (\forall x)(\forall w)\{ \sim P(x) \vee [ Q(x, g(x)) \wedge \sim P(f(B))] \wedge [ \sim P(x) \vee \sim Q(x,w) \vee P(x) ] \}$$

$$A: (\forall x)(\forall w)\{ [ \sim P(x) \vee Q(x, g(x)) ] \wedge [ \sim P(x) \vee \sim P(f(B)) ] \wedge [ \sim P(x) \vee \sim Q(x,w) \vee P(x) ] \}$$

(1) Step 7: \_\_\_\_\_

Write in Matrix Form

$$A: \forall x \forall w [ \sim P(x) \vee Q(x, g(x)) ]$$

$$\wedge \forall x \forall w [ \sim P(x) \vee \sim P(f(B)) ]$$

$$\wedge \forall x \forall w [ \sim P(x) \vee \sim Q(x,w) \vee P(x) ]$$

(1) Step 8: \_\_\_\_\_

Remove Universal Quantifiers

$$A: [ \sim P(x) \vee Q(x, g(x)) ]$$

$$\wedge [ \sim P(x) \vee \sim P(f(B)) ]$$

$$\wedge [ \sim P(x) \vee \sim Q(x,w) \vee P(x) ]$$

**I. Conversion to Clause Form (continued)**

(1) Step 9: \_\_\_\_\_

Rename Variables

$$A: [\sim P(x_1) \vee Q(x_1, g(x_1))]$$

$$[\sim P(x_2) \vee \sim P(f(B))]$$

$$[\sim P(x_3) \vee \sim Q(x_3, w_1) \vee P(x_3)]$$

(1) Step 10: \_\_\_\_\_

Step 10 Remove Tautologies & Simplify

$$A: [\sim P(x_1) \vee Q(x_1, g(x_1))]$$

$$\wedge [\sim P(x_2) \vee \sim P(f(B))]$$

(5) Part (a) Answer:

$$[\sim P(x_1) \vee Q(x_1, g(x_1))]$$

$$[\sim P(x_2) \vee \sim P(f(B))]$$

(5) Part (b) Answer:

$$A: (\forall x)\{\sim P(x) \vee [Q(x, g(x)) \wedge \sim P(f(B))]\} \quad \text{or}$$

$$A: (\forall x)\{P(x) \rightarrow [Q(x, g(x)) \wedge \sim P(f(B))]\}$$

(5) Part (c) Answer:

Part (a) answer is more general than part (b) because if you substitute for x, say  $x=Obj$  in part (b)

You obtain  $\{\sim P(Obj) \vee [Q(Obj, g(Obj)) \wedge \sim P(f(B))]\}$  which is  $[\sim P(Obj) \vee Q(Obj, g(Obj))] \wedge [\sim P(Obj) \vee \sim P(f(B))]$

But substituting in part (a) yields  $[\sim P(Obj) \vee Q(Obj, g(Obj))] \wedge [\sim P(x_2) \vee \sim P(f(B))]$  which is more general.

(25)

**II. Resolution Refutation**

Sam, Clyde and Oscar are elephants. We know the following facts about them:

1. Sam is pink.
2. Clyde is gray and likes Oscar.
3. Oscar is either pink, or gray (but not both) and likes Sam.

Use resolution refutation to prove that a gray elephant likes a pink elephant; that is prove

$$(\exists x)(\exists y)[\text{Gray}(x) \wedge \text{Pink}(y) \wedge \text{Likes}(x,y)]$$

Solve by drawing a Refutation Graph resulting from a **complete** strategy. (Make sure you mark clearly the required substitutions).

[Required: Please note the assigned point values. Each part **MUST** be answered with something. If left blank, then no credit will be assigned]

- (5) **a.** Represent the axioms/goal in the Predicate Calculus.
- (2) **b.** Represent any commonsense knowledge needed to solve the problem using Predicate Calculus,
- (5) **c.** Convert your axioms, goal and commonsense knowledge (if any) to clause form,
- (10) **d.** Draw your Refutation Graph, show substitutions are consistent.
- (3) **e.** Define your strategy, and describe how your graph meets the strategy

(5) **Answers Part a:**

Sam, Clyde and Oscar are elephants. Sam is pink. Clyde is gray and likes Oscar. Oscar is either pink, or gray (but not both) and likes Sam. Prove that a gray elephant likes a pink elephant.

- [1] Pink(Sam)
- [2] Gray(Clyde)
- [3] Likes(Clyde,Oscar)
- [4] Pink(Oscar)  $\vee$  Gray(Oscar)
- [5] Likes(Oscar,Sam)
- [6]  $(\exists x) (\exists y)[\text{Gray}(x) \wedge \text{Pink}(y) \wedge \text{Likes}(x,y)]$  {given}

(2) **Answers Part b:**

None needed

(5) **Answers Part c:**

- [1] Pink(Sam)
- [2] Gray(Clyde)
- [3] Likes(Clyde,Oscar)
- [4] Pink(Oscar)  $\vee$  Gray(Oscar)
- [5] Likes(Oscar,Sam)
- [6']  $\sim\text{Gray}(x) \vee \sim\text{Pink}(y) \vee \sim\text{Likes}(x,y)$

**II. Resolution Refutation**(continued)

(10) Refutation Graph Part d:



$\mathfrak{R}_1$ =[6] with [3]  $\sim$ Gray(x)  $\vee$   $\sim$ Pink(y){ Clyde/x;Oscar/y } or  $\sim$ Gray(Clyde)  $\vee$   $\sim$ Pink(Oscar)

$\mathfrak{R}_2$ = $\mathfrak{R}_1$  with [2]  $\sim$ Pink(Oscar)

$\mathfrak{R}_3$ =[6] with [5]  $\sim$ Gray(x')  $\vee$   $\sim$ Pink(y') { Oscar/x';Sam/y' } or  $\sim$ Gray(Oscar)  $\vee$   $\sim$ Pink(Sam)

$\mathfrak{R}_4$ = $\mathfrak{R}_3$  with [1]  $\sim$ Gray(Oscar)

$\mathfrak{R}_5$ = $\mathfrak{R}_4$  with [4] Pink(Oscar)

$\mathfrak{R}_6$ =Nil with  $\mathfrak{R}_5$  and  $\mathfrak{R}_2$

Consistency Check

$U_1$ =[Clyde,Oscar,Oscar,Sam]  $U_2$ =[x,y,x',y']

$U_1=U_2$ [Clyde/x;Oscar/y;Oscar/x';Sam/y/]

Since  $U_1$  &  $U_2$  unify, then the substitutions are consistent

(3) Answer Part e: My strategy is \_\_\_\_\_ Set of Support \_\_\_\_\_  
Since every resolvent comes from the negation of the goal wff with the base set or one of its descendants

(25)  
**IV. Computation Deduction.**  
Using **Resolution Refutation** deduce the following computation to obtain a value for the goal (3 pts) by drawing the Consistent Solution Graph (17 pts) for the goal and prove (or provide a good argument) its consistency (5 pts). Make sure your graph is clearly marked and it follows a complete strategy. You may assume that the system "knows" how to handle function  $\text{add}(E_1, E_2, E_3)$  such that if  $E_1$  and  $E_2$  are known, then  $E_3$  is set to the sum of  $E_1$  and  $E_2$  automatically thereby removing  $\text{add}(\_, \_, \_)$  from the resolution stack.

Facts:  
F1:  $\text{length}(\text{nil}, 0)$ .

Rules:  
R1:  $\{\text{length}(T, N) \wedge \lambda(\text{add}(N, 1, M))\} \rightarrow \text{length}(\text{cons}(H, T), M)$

Where  $\lambda(y)$  means "Evaluate the argument  $y$ "

Goal:  $(\exists z)\text{length}(\text{cons}(\text{boo}, \text{cons}(\text{on}, \text{cons}(\text{you}, \text{nil}))), z)$

{Note: If you prefer, you may use the notation  $\text{length}([\text{boo}, \text{on}, \text{you}], z)$  or  $\text{length}(\text{boo on you}, z)$ . }

Required: Draw the graph, show the substitutions are consistent, and obtain the value of the goal.

F1:  $\text{length}(\text{nil}, 0)$ .

R1:  $\sim\text{length}(T, N) \vee \sim\lambda(\text{add}(N, 1, M)) \vee \text{length}(\text{cons}(H, T), M)$

$\sim$ Goal:  $\sim\text{length}(\text{cons}(\text{boo}, \text{cons}(\text{on}, \text{cons}(\text{you}, \text{nil}))), z)$

$\mathfrak{R}_1 = \sim$ Goal- $\mathfrak{R}$ -R1:  $\sim\text{depth}(T, N) \vee \sim\lambda(\text{add}(N, 1, M))\{\text{boo}/H, \text{cons}(\text{on}, \text{cons}(\text{you}, \text{nil}))/T, z/M, N+1/M\}$

$\mathfrak{R}_1$ :  $\sim\text{length}(\text{cons}(\text{on}, \text{cons}(\text{you}, \text{nil})), N)$

$\mathfrak{R}_2 = \mathfrak{R}_1$ - $\mathfrak{R}$ -R1':  $\sim\text{depth}(T', N') \vee \sim\lambda(\text{add}(N', 1, M'))\{\text{on}/H', \text{cons}(\text{you}, \text{nil})/T', N'/M'; N'+1/M'\}$

$\mathfrak{R}_2$ :  $\sim\text{length}(\text{cons}(\text{you}, \text{nil}), N')$

$\mathfrak{R}_3 = \mathfrak{R}_2$ - $\mathfrak{R}$ -R1'':  $\sim\text{depth}(T'', N'') \vee \sim\lambda(\text{add}(N'', 1, M''))\{\text{you}/H'', \text{nil}/T''; N''/M''; N''+1/M''\}$

$\mathfrak{R}_3$ :  $\sim\text{length}(\text{nil}, N'')$

$\mathfrak{R}_3 = \mathfrak{R}_2$ - $\mathfrak{R}$ -F1: Nil  $\{0/N''\}$

Therefore  $N''=0$ ;  $M'' = \lambda(0+1)=1$ ;  $N'=1$ ;  $M' = \lambda(1+1)=2$ ;  $N=2$ ;  $M = \lambda(2+1)=3$ ;  $Z=3$

Substitutions will be consistent because I changed variables every time I re-used any rules and all the variables were originally standardized apart. To prove consistency we assemble all the numerators in a set, say  $U_1$  and all the denominators in a set, say,  $U_2$  and see if  $U_1=U_2\sigma$  and  $\sigma \neq \text{null}$ .

Fall 2001 exam was a 90 minute exam.

(25) Conversion to Clause Form

I. Transform the *wff* below into **clause** form. For each step required give a brief description of the step and perform the step (if applicable) on the space provided. Failure to follow this format may result in no credit.

<wff>: A:  $\forall x \forall y [\{P(x,y) \vee Q(x,y)\} \rightarrow R(x,y)]$

(2) Step 0: \_\_\_\_\_  
Eliminate redundant quantifiers and take the existential closure - not needed here

(2) Step 1: \_\_\_\_\_  
Remove implications  
A:  $\forall x \forall y [\sim \{P(x,y) \vee Q(x,y)\} \vee R(x,y)]$

(2) Step 2: \_\_\_\_\_  
Move the Negations down to the *Atfs*  
A:  $\forall x \forall y [\sim P(x,y) \wedge \sim Q(x,y) \vee R(x,y)]$

(2) Step 3: \_\_\_\_\_  
Standardize Variables Apart  
Not Needed here: A:  $\forall x \forall y [\sim P(x,y) \wedge \sim Q(x,y) \vee R(x,y)]$

(2) Step 4: \_\_\_\_\_  
Skolemize: Not needed here  
A:  $\forall x \forall y [\sim P(x,y) \wedge \sim Q(x,y) \vee R(x,y)]$

(2) Step 5: \_\_\_\_\_  
Move universal quantifiers to the left: Not needed here  
A:  $\forall x \forall y [\sim P(x,y) \wedge \sim Q(x,y) \vee R(x,y)]$

(2) Step 6: \_\_\_\_\_  
Distribute  $\vee$  over  $\wedge$  using  $(E1 \wedge E2) \vee E3 = (E1 \vee E3) \wedge (E2 \vee E3)$   
A:  $\forall x \forall y [\{\sim P(x,y) \vee R(x,y)\} \wedge \{\sim Q(x,y) \vee R(x,y)\}]$

(2) Step 7: \_\_\_\_\_  
Write in Matrix Form  
A:  $\forall x \forall y \{ [\sim P(x,y) \vee R(x,y)] \}$   
 $\forall x \forall y \{ [\sim Q(x,y) \vee R(x,y)] \}$

(2) Step 8: \_\_\_\_\_  
Remove Universal Quantifiers  
A:  $\{ [\sim P(x,y) \vee R(x,y)] \}$   
 $\{ [\sim Q(x,y) \vee R(x,y)] \}$

(2) Step 9: \_\_\_\_\_  
Rename Variables  
A:  $\{ [\sim P(x_1,y_1) \vee R(x_1,y_1)] \}$  Step 10  
 $\{ [\sim Q(x_2,y_2) \vee R(x_2,y_2)] \}$  Remove Tautologies & Simplify

(5) Answer:  
 $\{ [\sim P(x_1,y_1) \vee R(x_1,y_1)] \}$   
 $\{ [\sim Q(x_2,y_2) \vee R(x_2,y_2)] \}$

(25)

**II. Resolution Refutation**

If a course is easy, some students are happy. If a course has a final, no students are happy. Use Resolution to show that, if a course has a final, the course is not easy.

Solve by drawing a Refutation Graph resulting from a **complete** strategy. (Make sure you mark clearly the required substitutions).

[Required: Please note the assigned point values. Each part **MUST** be answered with something. If left blank, then no credit will be assigned]

- (5) a. Represent the axioms/goal in the Predicate Calculus.
- (2) b. Represent any commonsense knowledge needed to solve the problem using Predicate Calculus,
- (5) c. Convert your axioms, goal and commonsense knowledge (if any) to clause form,
- (10) d. Draw your Refutation Graph,
- (3) e. Define your strategy, and describe how your graph meets the strategy

(5) Answers Part a:

For every course and every student, if the course has a final and the student is taking the course, then the student is not happy. For every course, if the course is easy, then there is a student taking the course who is not happy.

[1]  $\forall c \forall s \{ [F(c) \wedge T(s,c)] \rightarrow \sim H(s) \}$

[2]  $\forall c E(c) \rightarrow \exists s [T(s,c) \wedge H(s)]$

[3] Goal:  $(\forall c)[F(c) \rightarrow \sim E(c)]$

(2) Answers Part b:

None needed

(5) Answers Part c:

[1]  $\sim F(c) \vee \sim T(s,c) \vee \sim H(s)$

[2a]  $\sim E(c) \vee T(g(c),c)$

[2b]  $\sim E(c) \vee H(g(c))$

[3a]  $F(\text{Crip\_Course})$

[3b]  $E(\text{Crip\_Course})$

$g(c)$  designates the Skolem happy student in each course and  $\text{Crip\_Course}$  designates the Skolem course with a final that is hypothesized to be easy.

**II. Resolution Refutation(continued)**

(10) Refutation Graph Part d:

[1]  $\sim F(c) \vee \sim T(s,c) \vee \sim H(s)$

[2]  $\sim E(c) \vee T(g(c),c)$

[3]  $\sim E(c) \vee H(g(c))$

[4]  $F(\text{Crip\_Course})$

[5]  $E(\text{Crip\_Course})$

[4] [1] [5] [2] [5] [3]

$\mathfrak{R}_3$   $\mathfrak{R}_1$   $\mathfrak{R}_2$

$\mathfrak{R}_4$

nil

$\mathfrak{R}_1=[5]$  with [2]  $T(g(c),c)$  {  $\text{Crip\_Course}/c$  } or  $T(g(\text{Crip\_Course}),\text{Crip\_Course})$

$\mathfrak{R}_2=[5]$  with [3]  $H(g(c))$  {  $\text{Crip\_Course}/c$  } or  $H(g(\text{Crip\_Course}))$

$\mathfrak{R}_3=[4]$  with [1]  $\sim T(s,c) \vee \sim H(s)$  {  $\text{Crip\_Course}/c$  } or  $\sim T(s,\text{Crip\_Course}) \vee \sim H(s)$

$\mathfrak{R}_4=\mathfrak{R}_3$  with  $\mathfrak{R}_1$   $\sim H(s)$  {  $g(\text{Crip\_Course})/s$  } or  $\sim H(g(\text{Crip\_Course}))$

$\mathfrak{R}_5 = \mathfrak{R}_4$  with  $\mathfrak{R}_2$  nil  
Consistency Check

$U_1 = [\text{Crip\_course}, \text{Crip\_course}, \text{Crip\_course}, g(\text{Crip\_course})]$   $U_2 = [c, c, c, s]$

$U_1 = U_2[\text{Crip\_course}/c, g(\text{Crip\_course})/s]$

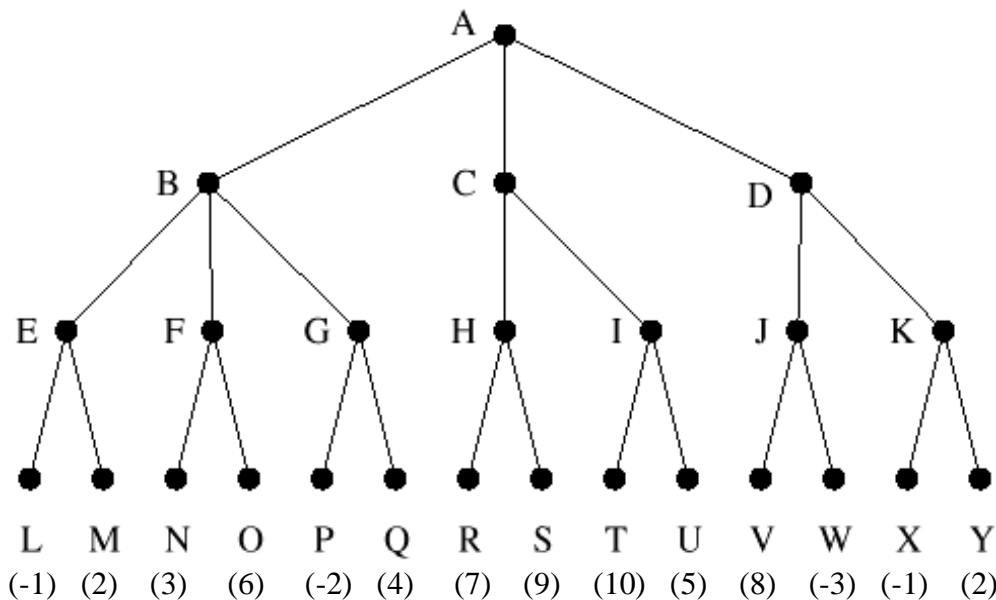
Since  $U_1$  &  $U_2$  unify, then the substitutions are consistent

(3) Answer Part e: My strategy is \_\_\_\_\_ Set of Support \_\_\_\_\_  
Since every resolvent comes from the negation of the goal wff with the base set or one of its descendants

(25)  
III. Adversarial Search

Consider the following game tree in which the static scores (in parentheses at the tip nodes) are all from the first player's point of view.

- (a) Assuming that the first player is the maximizing player, what move should the first player choose?
- (b) Assuming that the first player is the minimizing player, what move should the first player choose?
- (c) What nodes would not need to be examined in part (a) using the alpha-beta algorithm—assuming that the nodes are examined in left-to-right order?
- (d) What nodes would not need to be examined in part (b) using the alpha-beta algorithm—assuming that the nodes are examined in right-to-left order?
- (e) Is the first player's move in parts (a) and (c) or in parts (b) and (d) different? Explain.



Part (a)

E, F, G, H, I, J, K chooses max or E=M(2); F=O(6); G=Q(4); H=S(9); I=T(10); J=V(8); K=Y(2)

B, C, D chooses min or B=E=M(2); C=H=S(9); D=K=Y(2)

A chooses max or A=C=H=S(9)

A chooses C

Part (b)



E,F,G,H,I,J,K chooses min or  $E=L(-1)$ ;  $F=N(3)$ ;  $G=P(-2)$ ;  $H=R(7)$ ;  $I=U(5)$ ;  $J=W(-3)$ ;  $K=X(-1)$   
B,C,D chooses max or  $B=F=N(2)$ ;  $C=H=R(7)$ ;  $D=K=X(-1)$   
A chooses min or  $A=D=K=X(-1)$   
A chooses D

Part (c)

E chooses max evaluating L & M or  $\alpha_E=2(M)$ ; Now B chooses min so  $\beta_B \leq 2(M)$   
Evaluate N; now  $\alpha_F \geq 3(N)$  but  $\beta_B \leq 2(M)$  therefore Beta Cutoff and not evaluate O  
Evaluate P; now  $\alpha_F \geq 2(P)$  but  $\beta_B \leq 2(M)$  therefore evaluate Q, now  $\alpha_G=4(Q)$   
Now B chooses min so  $\beta_B = 2(M)$ , therefore  $\alpha_A \geq 2(M)$   
Evaluate R; now  $\alpha_H \geq 7(R)$  and  $\beta_C \leq 7(R)$  therefore evaluate S, now  $\alpha_H=9(S)$  and  $\beta_C \leq 9(S)$   
Evaluate T; now  $\alpha_H \geq 10(T)$ ; but  $\beta_C \leq 7(R)$ ; stop and do not evaluate U and now  $\beta_C = 9(S)$  and  $\alpha_A \geq 9(S)$   
Evaluate V; now  $\alpha_J \geq 8(V)$ ; and  $\beta_D \leq 8(V)$  Alpha cutoff at D and do not evaluate W,X,Y or K  
A chooses C to H to 9(S)  
Do Not Evaluate {O,U,W,X,Y, and K} [students claim you do have to evaluate W]

Part (d)

K chooses min evaluating Y & X or  $\beta_K=-1(X)$ ; Now D chooses max so  $\alpha_D \geq -1(X)$   
Evaluate W; now  $\beta_J \leq -3(W)$  but  $\alpha_D \geq -1(X)$  therefore Alpha Cutoff and not evaluate V  
 $\alpha_D=-1(X)$ ;  $\beta_A \leq -1(X)$   
Evaluate U; now  $\beta_I \leq 5(U)$  and but  $\alpha_C \geq 5(X)$  therefore Beta Cutoff and do not evaluate T,R,S, or H  
Evaluate Q; now  $\beta_G \leq 4(Q)$  and  $\alpha_B \geq 5(Q)$  therefore Beta Cutoff and do not evaluate P,O,N,M,L or F,E  
A chooses D to K to  $-1(X)$   
Do Not Evaluate {V,T,R,S,H, P,O,N,M,L and F,E }

Part (e)

A chooses C to H to 9(S) in both parts (a) and (c) because Alpha-Beta and Minimax produce the same results for the same problem.  
Similarly, A chooses D to K to  $-1(X)$  in both parts (b) and (d) because Alpha-Beta and Minimax produce the same results for the same problem.  
In my analysis that was indeed the case.

(25)  
**IV. Computation Deduction.**  
Using **Resolution Refutation** deduce the following computation to obtain a value for the goal (3 pts) by drawing the Consistent Solution Graph (17 pts) for the goal and prove (or provide a good argument) its consistency (5 pts). Make sure your graph is clearly marked and it follows a complete strategy. You may assume that the system "knows" how to handle function  $\max(E_1, E_2, E_3)$  such that if  $E_1$  and  $E_2$  are known, then  $E_3$  is set to the maximum of  $E_1$  and  $E_2$  automatically thereby removing  $\max(\_, \_, \_)$  from the resolution stack. Alternatively, your answers can consist of unevaluated calls to the built-in function  $\max(\_, \_, \_)$ .

Facts:

F1:  $\text{depth}(\text{nil}, 1)$ .

Rules:

R1:  $\text{atomic}(S) \rightarrow \text{depth}(S, 0)$

R2:  $\text{depth}(H, A_1) \wedge \text{depth}(T, A_2) \wedge \max(1+A_1, A_2, A_3) \rightarrow \text{depth}(\text{cons}(H, T), A_3)$

Goal:  $(\exists z)\text{depth}(\text{cons}(\text{cons}(a, \text{nil}), \text{cons}(b, \text{nil})), z)$

{Note: If you prefer, you may use the notation  $\text{depth}([a, b], z)$  or  $\text{depth}((a) (b), z)$ . }

Required: Draw the graph, show the substitutions are consistent, and obtain the value of the goal.

F1:  $\text{depth}(\text{nil}, 1)$ .

R1:  $\text{atomic}(S) \rightarrow \text{depth}(S, 0)$

R2:  $\sim\text{depth}(H, A_1) \vee \sim\text{depth}(T, A_2) \vee \sim\max(1+A_1, A_2, A_3) \vee \text{depth}(\text{cons}(H, T), A_3)$

$\sim$ Goal:  $\sim\text{depth}(\text{cons}(\text{cons}(a, \text{nil}), \text{cons}(b, \text{nil})), z)$ ; fact1:  $\text{atomic}(a)$ ; fact2:  $\text{atomic}(b)$

$\mathfrak{R}_1 = \sim$ Goal- $\mathfrak{R}$ -R2:  $\sim\text{depth}(H, A_1) \vee \sim\text{depth}(T, A_2) \vee \sim\max(1+A_1, A_2, A_3) \{ \text{cons}(a, \text{nil})/H, \text{cons}(b, \text{nil})/T, z/A_3 \}$

$\mathfrak{R}_1$ :  $\sim\text{depth}(\text{cons}(a, \text{nil}), A_1) \vee \sim\text{depth}(\text{cons}(b, \text{nil}), A_2) \vee \sim\max(1+A_1, A_2, z)$

$\mathfrak{R}_2 = \mathfrak{R}_1 - \mathfrak{R}$ -R2':  $\sim\text{depth}(H', A_1') \vee \sim\text{depth}(T', A_2') \vee \sim\max(1+A_1', A_2', A_3') \vee \sim\text{depth}(\text{cons}(b, \text{nil}), A_2)$

$\vee \sim\max(1+A_1, A_2, z) \{ a/H', \text{nil}/T', A_1/A_3' \}$

$\mathfrak{R}_2$ :  $\sim\text{depth}(a, A_1') \vee \sim\text{depth}(\text{nil}, A_2') \vee \sim\max(1+A_1', A_2', A_1) \vee \sim\text{depth}(\text{cons}(b, \text{nil}), A_2) \vee \sim\max(1+A_1, A_2, z)$

$\mathfrak{R}_3 = \mathfrak{R}_2 - \mathfrak{R}$ -R1:  $\sim\text{atomic}(S) \vee \sim\text{depth}(\text{nil}, A_2') \vee \sim\max(1+A_1', A_2', A_1) \vee \sim\text{depth}(\text{cons}(b, \text{nil}), A_2)$

$\vee \sim\max(1+A_1, A_2, z) \{ a/S, 0/A_1' \}$

$\mathfrak{R}_3$ :  $\sim\text{atomic}(a) \vee \sim\text{depth}(\text{nil}, A_2') \vee \sim\max(1+0, A_2', A_1) \vee \sim\text{depth}(\text{cons}(b, \text{nil}), A_2) \vee \sim\max(1+A_1, A_2, z)$

$\mathfrak{R}_4$ :  $\mathfrak{R}_3 - \mathfrak{R}$ -fact1:  $\sim\text{depth}(\text{nil}, A_2') \vee \sim\max(1+0, A_2', A_1) \vee \sim\text{depth}(\text{cons}(b, \text{nil}), A_2) \vee \sim\max(1+A_1, A_2, z)$

$\mathfrak{R}_5$ :  $\mathfrak{R}_4 - \mathfrak{R}$ -F1:  $\sim\max(1+0, A_2', A_1) \vee \sim\text{depth}(\text{cons}(b, \text{nil}), A_2) \vee \sim\max(1+A_1, A_2, z) \{ 1/A_2' \}$

$\mathfrak{R}_5$ :  $\sim\max(1+0, 1, A_1) \vee \sim\text{depth}(\text{cons}(b, \text{nil}), A_2) \vee \sim\max(1+A_1, A_2, z)$

Evaluate  $\sim\max(1+0, 1, A_1)$  with  $\{ 1/A_1 \}$

$\mathfrak{R}_2''$ :  $\sim\text{depth}(H'', A_1'') \vee \sim\text{depth}(T'', A_2'') \vee \sim\max(1+A_1'', A_2'', A_3'') \vee \text{depth}(\text{cons}(H'', T''), A_3'')$

$\mathfrak{R}_6$ :  $\mathfrak{R}_5 - \mathfrak{R}$ -R2''  $\sim\text{depth}(H'', A_1'') \vee \sim\text{depth}(T'', A_2'') \vee \sim\max(1+A_1'', A_2'', A_3'')$

$\nu \sim \max(1+1, A_2, z) \{b/H', nil/T' A_2/A_3'\}$

$\mathfrak{R}_6: \sim \text{depth}(b, A_1') \nu \sim \text{depth}(nil, A_2') \nu \sim \max(1+A_1'', A_2'', A_2) \nu \sim \max(1+1, A_2, z)$

$\mathfrak{R}_7: \mathfrak{R}_6 - \mathfrak{R} - R3'': \sim \text{atomic}(S') \nu \sim \text{depth}(nil, A_2'') \nu \sim \max(1+A_1'', A_2'', A_2) \nu \sim \max(1+1, A_2, z) \{b/S', 0/A_1''\}$

$\mathfrak{R}_7: \sim \text{atomic}(b) \nu \sim \text{depth}(nil, A_2'') \nu \sim \max(1+0, A_2'', A_2) \nu \sim \max(1+1, A_2, z)$

$\mathfrak{R}_8: \mathfrak{R}_7 - \mathfrak{R} - \text{fact2}: \sim \text{depth}(nil, A_2'') \nu \sim \max(1+0, A_2'', A_2) \nu \sim \max(1+1, A_2, z)$

$\mathfrak{R}_9: \mathfrak{R}_8 - \mathfrak{R} - F1: \sim \max(1+0, A_2'', A_2) \nu \sim \max(1+1, A_2, z) \{1/A_2''\}$

$\max(1+0, 1, A_2) \nu \sim \max(1+1, A_2, z)$

Evaluate  $\sim \max(1+0, 1, A_2)$  with  $\{1/A_2\}$

Evaluate  $\sim \max(1+1, 1, z)$  with  $\{2/z\}$

$z=2$

Goal:  $\text{depth}(\text{cons}(\text{cons}(a, \text{nil}), \text{cons}(b, \text{nil})), 2)$

Substitutions will be consistent because I changed variables every time I re-used any rules and all the variables were originally standardized apart. To prove consistency we assemble all the numerators in a set, say  $U_1$  and all the denominators in a set, say,  $U_2$  and see if  $U_1 = U_2\sigma$  and  $\sigma \neq \text{null}$ .

Fall 1999 exam was a 90 minute exam.

(25) Conversion to Clause Form

I. Transform the <wff> below into **clause** form. For each of the steps required give a brief description of the step and perform the step (if applicable) on the space provided. Failure to follow this format may result in zero credit.

<wff>:  $(\forall x)[(\forall y)[P(x, y)] \rightarrow \sim\{(\forall y)[Q(x, y) \rightarrow R(x, y)]\}]$

II. Resolution Refutation

Given the following axioms, "Show there is something Green on the table" by drawing a Refutation Graph resulting from a Set-of-Support strategy. (Make sure you mark clearly the required substitutions).

Axioms:

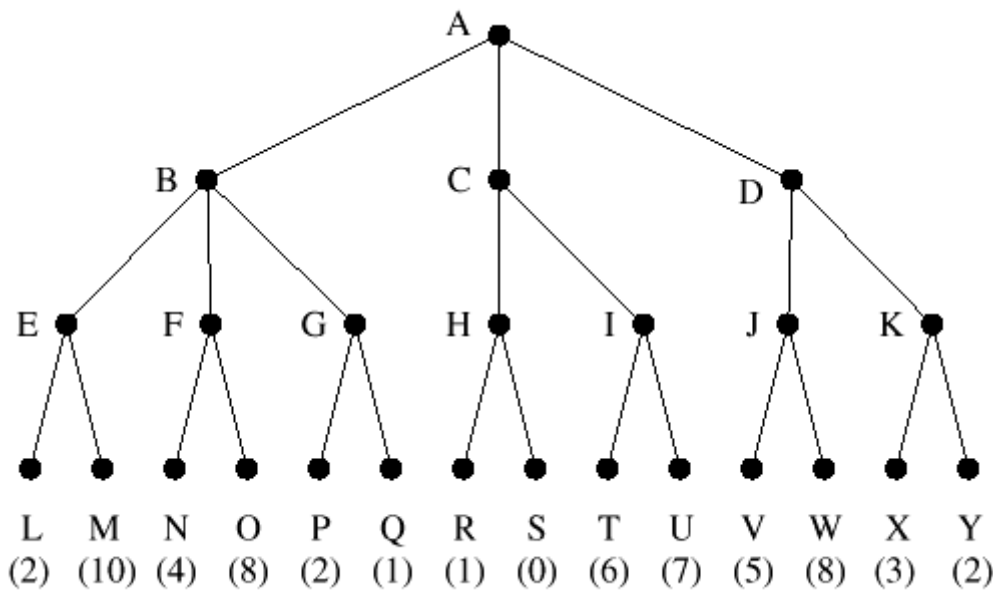
1. Block-1 is on the Table.
2. Block-2 is on the Table.
3. The Color of Block-1 or the Color of Block-2 is Green.

Solve by drawing a Refutation Graph resulting from a **complete** strategy. (Make sure you mark clearly the required substitutions).

[Required: Please note the assigned point values

- (7) a. Represent the axioms/goal in the Predicate Calculus.
- (3) b. Represent any commonsense knowledge needed to solve the problem using Predicate Calculus,
- (7) c. Convert your axioms, goal and commonsense knowledge (if any) to clause form,
- (10) d. Draw your Refutation Graph,
- (3) e. Describe how your graph meets the strategy

- (30)
- III. Adversarial Search**
- Consider the following game tree in which the static scores (in parentheses at the tip nodes) are all from the first player's point of view.
- Assuming that the first player is the maximizing player, what move should the first player choose?
  - Assuming that the first player is the minimizing player, what move should the first player choose?
  - What nodes would not need to be examined in part (a) using the alpha-beta algorithm—assuming that the nodes are examined in left-to-right order?
  - What nodes would not need to be examined in part (b) using the alpha-beta algorithm—assuming that the nodes are examined in right-to-left order?
  - Is the first player's move in parts (a) and (c) or in parts (b) and (d) different? Explain.



- (30)
- IV. Computation Deduction.**
- Using **Resolution Refutation** deduce the following computation to obtain a value for the goal (3 pts) by drawing the Consistent Solution Graph (21 pts) for the goal and prove its consistency (6 pts). Make sure your graph is clearly marked and it follows a complete strategy.

Facts:

- F1:  $\text{member}(X, \text{cons}(X, Y))$ .
- F2:  $\text{subset}(\text{nil}, Z)$ .

Rules:

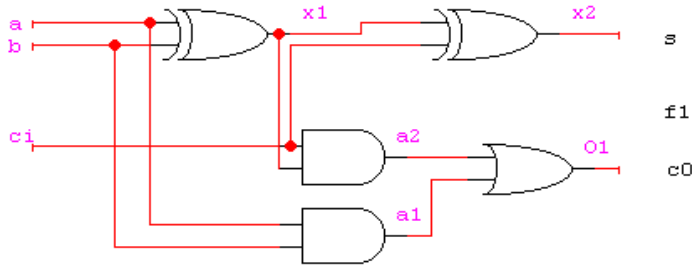
- R1:  $\text{member}(X2, Y2) \rightarrow \text{member}(X2, \text{cons}(U, Y2))$ .
- R2:  $\text{member}(X3, Y3) \wedge \text{subset}(Z3, Y3) \rightarrow \text{subset}(\text{cons}(X3, Z3), Y3)$ .

Goal:  $\text{subset}(\text{cons}(3, \text{cons}(2, \text{nil})), \text{cons}(1, \text{cons}(2, \text{cons}(3, \text{cons}(4, \text{nil}))))))$ .

{Note: If you prefer, you may use the notation  $\text{subset}([3,2], [1,2,3,4])$  or  $\text{subset}((3\ 2), (1\ 2\ 3\ 4))$ .}

Required: Draw the graph, show the substitutions are consistent, and obtain the value of the goal.

(35)  
**V. Resolution Applications.**  
 The following full adder in an EEL-3701 lab with asserted inputs {1,0,1} for {a,b,c1} has asserted outputs {0,1} for {s,c0}, respectively. This means that if you assert A1, A2 and A3 you will deduce A4 and A5 using plain Resolution {not Resolution Refutation}. However, Jason Gates obtains outputs {1,1} and requests your (TA) help in figuring out what is wrong. Using resolution refutation find out what is wrong with the circuit. (Bonus: 5 additional points if you tell me which IC is defective. 5 more points if you give me the IC number, e.g., 74LSXX]. Indicate any commonsense knowledge needed to solve the problem using Predicate Calculus.



- Let {f1, x1, x2, a1, a2, o1} designate the six components.
- Adder(x) means that x is an adder.
- Xorg(x) means that x is an xor gate.
- Andg(x) means that x is an and gate.
- Org(x) means that x is an or gate.
- I(i,x) designates the i<sup>th</sup> input port of device x.
- O(i,x) designates the i<sup>th</sup> output port of device x.
- Conn(x,y) means that port x is connected to port y.
- V(x,z) means that the value on port x is z.
- 1 and 0 designate high and low voltages, respectively.

Now:

Adder(f1)

1. Xorg(x1)
2. Xorg(x2)
3. Andg(a1)
4. Andg(a2)
5. Org(o1)
6. Conn(I(1,f1),I(1,x1))
7. Conn(I(2,f1),I(2,x1))
8. Conn(I(1,f1),I(1,a1))
9. Conn(I(2,f1),I(2,a1))
10. Conn(I(3,f1),I(2,x2))
11. Conn(I(3,f1),I(1,a2))
12. Conn(O(1,x1),I(1,x1))
13. Conn(O(1,x1),I(2,a2))
14. Conn(O(1,a2),I(1,o1))
15. Conn(O(1,a1),I(2,o1))
16. Conn(O(1,x2),O(1,f1))
17. Conn(O(1,o1),O(2,f1))
- A1. V(I(1,f1),1)
- A2. V(I(2,f1),0)
- A3. V(I(3,f1),1)
- A4. V(O(1,f1),0)
- A5. V(O(2,f1),1)

18.  $\forall x (Andg(x) \wedge V(I(1, x),1) \wedge V(I(2, x),1) \rightarrow V(O(1, x),1))$
19.  $\forall x \forall n (Andg(x) \wedge V(I(n, x),0) \rightarrow V(O(1, x),0))$
20.  $\forall x \forall n (Org(x) \wedge V(I(n, x),1) \rightarrow V(O(1, x),1))$
21.  $\forall x (Org(x) \wedge V(I(1, x),0) \wedge V(I(2, x),0) \rightarrow V(O(1, x),0))$
22.  $\forall x \forall z (Xorg(x) \wedge V(I(1, x), z) \wedge V(I(2, x), z) \rightarrow V(O(1, x),0))$
23.  $\forall x \forall y \forall z (Xorg(x) \wedge V(I(1, x), y) \wedge V(I(2, x), z) \wedge y \neq z \rightarrow V(O(1, x),1))$
24.  $\forall x \forall y \forall z (Conn(x, y) \wedge V(x, z) \rightarrow V(y, z))$

Fall 2000 exam was a 60 minute exam.

(25) Conversion to Clause Form

I. Transform the *wff* below into **clause** form. For each step required give a brief description of the step and perform the step (if applicable) on the space provided. Failure to follow this format may result in no credit.

<wff>: A:  $(\forall x)\{P(x) \rightarrow \exists z\{\sim \forall y[Q(x,y) \rightarrow P(f(z))]\} \wedge \forall y[Q(x,y) \rightarrow P(z)]\}$

(2) Step 0: \_\_\_\_\_

Eliminate redundant quantifiers and take the existential closure - not needed here

(2) Step 1: \_\_\_\_\_

Remove implications

A:  $(\forall x)\{\sim P(x) \vee \exists z\{\sim \forall y[\sim Q(x,y) \vee P(f(z))]\} \wedge \forall y[\sim Q(x,y) \vee P(z)]\}$

(2) Step 2: \_\_\_\_\_

Move the Negations down to the Atfs

A:  $(\forall x)\{\sim P(x) \vee \exists z\{\exists y[Q(x,y) \wedge \sim P(f(z))]\} \wedge \forall y[\sim Q(x,y) \vee P(z)]\}$

(2) Step 3: \_\_\_\_\_

Standardize Variables Apart

A:  $(\forall x)\{\sim P(x) \vee \exists z\{\exists w[Q(x,w) \wedge \sim P(f(z))]\} \wedge \forall y[\sim Q(x,y) \vee P(z)]\}$

(2) Step 4: \_\_\_\_\_

Skolemize: Let  $z=g(x)$  and  $w=h(x)$

A:  $(\forall x)\{\sim P(x) \vee \{[Q(x,h(x)) \wedge \sim P(f(g(x)))]\} \wedge \forall y[\sim Q(x,y) \vee P(g(x))]\}$

(2) Step 5: \_\_\_\_\_

Move universal quantifiers to the left

A:  $(\forall x)(\forall y)\{\sim P(x) \vee \{[Q(x,h(x)) \wedge \sim P(f(g(x)))]\} \wedge [\sim Q(x,y) \vee P(g(x))]\}$

(2) Step 6: \_\_\_\_\_

Distribute  $\vee$  over  $\wedge$  using  $E1 \vee (E2 \wedge E3) = (E1 \vee E2) \wedge (E1 \vee E3)$

A:  $(\forall x)(\forall y)\{\sim P(x) \vee [Q(x,h(x)) \wedge \sim P(f(g(x)))]\} \wedge \{\sim P(x) \vee [\sim Q(x,y) \vee P(g(x))]\}$

A:  $(\forall x)(\forall y)\{\sim P(x) \vee Q(x,h(x))\} \wedge \{\sim P(x) \vee \sim P(f(g(x)))\} \wedge \{\sim P(x) \vee [\sim Q(x,y) \vee P(g(x))]\}$

(2) Step 7: \_\_\_\_\_

Write in Matrix Form {already in matrix form}

A:  $(\forall x)(\forall y)\{\sim P(x) \vee Q(x,h(x))\} \wedge \{\sim P(x) \vee \sim P(f(g(x)))\} \wedge \{\sim P(x) \vee \sim Q(x,y) \vee P(g(x))\}$

(2) Step 8: \_\_\_\_\_

Remove Universal Quantifiers

A:  $\{\sim P(x) \vee Q(x,h(x))\} \wedge \{\sim P(x) \vee \sim P(f(g(x)))\} \wedge \{\sim P(x) \vee \sim Q(x,y) \vee P(g(x))\}$

(2) Step 9: \_\_\_\_\_

Rename Variables

A:  $\{\sim P(x_1) \vee Q(x_1,h(x_1))\} \wedge \{\sim P(x_2) \vee \sim P(f(g(x_2)))\} \wedge \{\sim P(x_3) \vee \sim Q(x_3,y_3) \vee P(g(x_3))\}$

(5) Answer:

$\{\sim P(x_1) \vee Q(x_1,h(x_1))\}$

$\{\sim P(x_2) \vee \sim P(f(g(x_2)))\}$

$\{\sim P(x_3) \vee \sim Q(x_3,y_3) \vee P(g(x_3))\}$

(25)

**II. Resolution Refutation**

Bill has been murdered, and AL, Ralph, and George are suspects. AL says he did not do it. He says that Ralph was the victim's friend but that George hated the victim. Ralph says that he was out of town on the day of the murder, and besides he didn't even know the guy. George says he is innocent and that he saw AL and Ralph with the victim just before the murder. Assuming that everyone—except possibly for the murderer—is telling the truth, using Resolution Refutation, solve the crime.

Solve by drawing a Refutation Graph resulting from a **complete** strategy. (Make sure you mark clearly the required substitutions).

[Required: Please note the assigned point values

- (5) a. Represent the axioms/goal in the Predicate Calculus.
- (3) b. Represent any commonsense knowledge needed to solve the problem using Predicate Calculus,
- (5) c. Convert your axioms, goal and commonsense knowledge (if any) to clause form,
- (10) d. Draw your Refutation Graph,
- (2) e. Define your strategy, and describe how your graph meets the strategy
- (5) **Answers Part a:**

Let  $I(x)$  mean  $x$  is innocent;  $F(x,y)$  mean  $x$  is a friend of  $y$ ;  $Hate(x,y)$   $x$  hates  $y$ ;  $Out(x)$   $x$  is out of town  
 $With(x,y)$   $x$  is with  $y$ ;  $Knows(x,y)$   $x$  knows  $y$

[1]  $I(AL) \rightarrow F(Ralph,Bill) \wedge Hate(George,Bill)$

[2]  $I(Ralph) \rightarrow Out(Ralph) \wedge \sim Knows(Ralph,Bill)$

[3]  $I(George) \rightarrow With(AL,Bill) \wedge With(Ralph,Bill)$

[4] Goal:  $(\exists z)\sim I(z)$

(3) **Answers Part b:**

[5]  $(\forall x)(\forall y)Hate(x,y) \rightarrow \sim F(x,y)$  If  $x$  hates  $y$  then  $x$  is not a friend of  $y$ ; also  $(\forall x)(\forall y)F(x,y) \rightarrow \sim Hate(x,y)$

[6]  $(\forall x)(\forall y)F(x,y) \rightarrow Knows(x,y)$  If  $x$  is a friend of  $y$  then  $x$  knows  $y$ .

[7]  $(\forall x) Out(x) \rightarrow \sim With(x,Bill)$  If  $x$  is out of town then  $x$  cannot be with Bill

[8]  $\sim Knows(x,y) \rightarrow \sim With(x,y) \wedge \sim F(x,y)$  If  $x$  does not know  $y$  then  $x$  is not a friend of  $y$  nor can  $x$  be with  $y$

[9]  $\sim Knows(x,Bill) \rightarrow I(x)$  If  $x$  does not know Bill, then  $x$  must be innocent.

[10]  $I(AL) \vee I(Ralph)$  Either AL or Ralph are innocent (i.e., George is not innocent)

[11]  $I(George) \vee I(Ralph)$  Either George or Ralph are innocent (i.e., AL is not innocent)

[12]  $I(AL) \vee I(George)$  Either AL or George are innocent (i.e., Ralph is not innocent)

(5) **Answers Part c:**

[1a]  $\sim I(AL) \vee F(Ralph,Bill)$

[1b]  $\sim I(AL) \vee Hate(George,Bill)$

[2a]  $\sim I(Ralph) \vee Out(Ralph)$

[2b]  $\sim I(Ralph) \vee \sim Knows(Ralph,Bill)$

[3a]  $\sim I(George) \vee With(AL,Bill)$

[3b]  $\sim I(George) \vee With(Ralph,Bill)$

[4']  $I(z)$  {The negation of the goal in clause form}

[5]  $\sim Hate(x,y) \vee \sim F(x,y)$

[6]  $\sim F(x,y) \vee Knows(x,y)$

[7]  $\sim Out(x) \vee \sim With(x,Bill)$

[8a]  $Knows(x,y) \vee \sim With(x,y)$

[8b]  $Knows(x,y) \vee \sim F(x,y)$

[9]  $Knows(x,y) \vee I(x)$

[10]  $I(AL) \vee I(Ralph)$

[11]  $I(George) \vee I(Ralph)$

[12]  $I(AL) \vee I(George)$

**II. Resolution Refutation**(continued)

(10) Refutation Graph Part **d**:

- [1a]  $\sim I(AL) \vee F(\text{Ralph}, \text{Bill})$
- [1b]  $\sim I(AL) \vee \text{Hate}(\text{George}, \text{Bill})$
- [2a]  $\sim I(\text{Ralph}) \vee \text{Out}(\text{Ralph})$
- [2b]  $\sim I(\text{Ralph}) \vee \sim \text{Knows}(\text{Ralph}, \text{Bill})$
- [3a]  $\sim I(\text{George}) \vee \text{With}(AL, \text{Bill})$
- [3b]  $\sim I(\text{George}) \vee \text{With}(\text{Ralph}, \text{Bill})$
- [4']  $I(z)$  {The negation of the goal in clause form}
- [5]  $\sim \text{Hate}(x, y) \vee \sim F(x, y)$
- [6]  $\sim F(x, y) \vee \text{Knows}(x, y)$
- [7]  $\sim \text{Out}(x) \vee \sim \text{With}(x, \text{Bill})$
- [8a]  $\text{Knows}(x, y) \vee \sim \text{With}(x, y)$
- [8b]  $\text{Knows}(x, y) \vee \sim F(x, y)$
- [9]  $\text{Knows}(x, \text{Bill}) \vee I(x)$
- [10]  $I(AL) \vee I(\text{Ralph})$
- [11]  $I(\text{George}) \vee I(\text{Ralph})$
- [12]  $I(AL) \vee I(\text{George})$

- $\mathfrak{R}_1 = [4']$  with [2b]  $\sim \text{Knows}(\text{Ralph}, \text{Bill})$  {Ralph/z}
- $\mathfrak{R}_2 = \mathfrak{R}_1$  with [6]  $\sim F(\text{Ralph}, \text{Bill})$
- $\mathfrak{R}_3 = \mathfrak{R}_2$  with [1a]  $\sim I(AL)$
- $\mathfrak{R}_4 = \mathfrak{R}_3$  with [10]  $I(\text{Ralph})$
- $\mathfrak{R}_5 = \mathfrak{R}_4$  with [2a]  $\text{Out}(\text{Ralph})$
- $\mathfrak{R}_6 = \mathfrak{R}_5$  with [7]  $\sim \text{With}(\text{Ralph}, \text{Bill})$
- $\mathfrak{R}_7 = \mathfrak{R}_6$  with [10]  $\sim I(\text{George})$
- $\mathfrak{R}_8 = \mathfrak{R}_7$  with [12]  $I(AL)$
- $\mathfrak{R}_9 = \mathfrak{R}_8$  with  $\mathfrak{R}_3$  nil

Since  $z = \text{Ralph}$  then  $\sim I(\text{Ralph})$

(2) Answer Part **e**: My strategy is \_\_\_\_\_ **Ancestry Filetered** \_\_\_\_\_  
Since every resolvent uses a parent from the base or one that is an ancestor of the other parent

(25)  
**III. Heuristic Search**

You are to place 6 Queens on a 6x6 board so no two Queens can attack each other. Use a 6-tuple to represent the global database, such that each  $x_i$  in the tuple stands for the column number of the queen in row $_i$ . Give a heuristic function  $h(n)$  that takes into account such things as: (1) two queens cannot occupy the same row or column, (2) queens cannot be in adjacent rows and columns, and (3) a position  $(i, j)$  is preferred over position  $(n, m)$  if  $\text{diag}(i, j) < \text{diag}(n, m)$  where  $\text{diag}(i, j)$  is defined to be the length of the longest



diagonal passing through position  $(i,j)$ . Give the  $A^*$  tree for at least the first 4 levels. Is your  $h(n)$  a lower bound of  $h^*(n)$ ? NO JUSTIFICATION  $\iff$  NO CREDIT

(25)

**IV. Computation Deduction.**

Using **Resolution Refutation** deduce the following computation to obtain a value for the goal (3 pts) by drawing the Consistent Solution Graph (17 pts) for the goal and prove its consistency (5 pts). Make sure your graph is clearly marked and it follows a complete strategy.

Facts:

- F1: appended(nil,A,A).
- F2: appended(B,nil,B).
- F3: squash(nil,nil)

Rules:

- R1: Appended( $X_2, Y_2, Z_2$ )  $\rightarrow$  Appended( $\text{cons}(U_2, X_2), Y_2, \text{cons}(U_2, Z_2)$ ).
- R2: atomic(S)  $\rightarrow$  squash(S,  $\text{cons}(S, \text{nil})$ )
- R3: squash(H,  $A_1$ )  $\wedge$  squash(T,  $A_2$ )  $\wedge$  appended( $A_1, A_2, A_3$ )  $\rightarrow$  squash( $\text{cons}(H, T), A_3$ )

Goal:  $(\exists z) \text{ squash}(\text{cons}(\text{cons}(a, \text{nil}), \text{cons}(b, \text{nil})), z)$

{Note: If you prefer, you may use the notation  $\text{squash}([\text{a}], \text{b}, z)$  or  $\text{squash}(((a) b), z)$ .}

$\sim$ Goal:  $\sim \text{squash}(\text{cons}(\text{cons}(a, \text{nil}), \text{cons}(b, \text{nil})), z)$ ; fact1:atomic(a); fact2:atomic(b)

$\mathfrak{R}_1 = \sim$ Goal- $\mathfrak{R}3$ :  $\sim \text{squash}(H, A_1) \vee \sim \text{squash}(T, A_2) \vee \sim \text{append}(A_1, A_2, A_3)$  {  $\text{cons}(a, \text{nil})/H, \text{cons}(b, \text{nil})/T, z/A_3$  }  
 $\mathfrak{R}_1$ :  $\sim \text{squash}(\text{cons}(a, \text{nil}), A_1) \vee \sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(A_1, A_2, z)$

$\mathfrak{R}_2 = \mathfrak{R}_1$ - $\mathfrak{R}3'$ :  $\sim \text{squash}(H', A_1') \vee \sim \text{squash}(T', A_2') \vee \sim \text{append}(A_1', A_2', A_3') \vee \sim \text{squash}(\text{cons}(b, \text{nil}), A_2)$   
 $\vee \sim \text{append}(A_1, A_2, z)$  {  $a/H', \text{nil}/T', A_1/A_3'$  }

$\mathfrak{R}_2$ :  $\sim \text{squash}(a, A_1') \vee \sim \text{squash}(\text{nil}, A_2') \vee \sim \text{append}(A_1', A_2', A_1) \vee \sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(A_1, A_2, z)$

$\mathfrak{R}_3 = \mathfrak{R}_2$ - $\mathfrak{R}2$ :  $\sim \text{atomic}(S) \vee \sim \text{squash}(\text{nil}, A_2') \vee \sim \text{append}(A_1', A_2', A_1) \vee \sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(A_1, A_2, z)$   
{  $a/S, \text{cons}(S, \text{nil})/A_1'$  }

$\mathfrak{R}_3$ :  $\sim \text{atomic}(a) \vee \sim \text{squash}(\text{nil}, A_2') \vee \sim \text{append}(\text{cons}(a, \text{nil}), A_2', A_1) \vee \sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(A_1, A_2, z)$

$\mathfrak{R}_4$ :  $\mathfrak{R}_3$ - $\mathfrak{R}$ -fact1:  $\sim \text{squash}(\text{nil}, A_2') \vee \sim \text{append}(\text{cons}(a, \text{nil}), A_2', A_1) \vee \sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(A_1, A_2, z)$

$\mathfrak{R}_5$ :  $\mathfrak{R}_4$ - $\mathfrak{R}$ -F3:  $\sim \text{append}(\text{cons}(a, \text{nil}), A_2', A_1) \vee \sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(A_1, A_2, z)$  {  $\text{nil}/A_2'$  }

$\mathfrak{R}_5$ :  $\sim \text{append}(\text{cons}(a, \text{nil}), \text{nil}, A_1) \vee \sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(A_1, A_2, z)$

$\mathfrak{R}_6$ :  $\mathfrak{R}_5$ - $\mathfrak{R}$ -F2:  $\sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(A_1, A_2, z)$  {  $\text{cons}(a, \text{nil})/A_1$  }

$\mathfrak{R}_6$ :  $\sim \text{squash}(\text{cons}(b, \text{nil}), A_2) \vee \sim \text{append}(\text{cons}(a, \text{nil}), A_2, z)$

$\mathfrak{R}_7$ :  $\mathfrak{R}_6$ - $\mathfrak{R}3''$ :  $\sim \text{squash}(H'', A_1'') \vee \sim \text{squash}(T'', A_2'') \vee \sim \text{append}(A_1'', A_2'', A_3'') \vee \sim \text{append}(\text{cons}(a, \text{nil}), A_2, z)$   
{  $b/H'', \text{nil}/T'', A_2/A_3''$  }

$\mathfrak{R}_7$ :  $\sim \text{squash}(b, A_1'') \vee \sim \text{squash}(\text{nil}, A_2'') \vee \sim \text{append}(A_1'', A_2'', A_2) \vee \sim \text{append}(\text{cons}(a, \text{nil}), A_2, z)$

**IV. Computation Deduction.** (continued)

$\mathfrak{R}_7$ :  $\mathfrak{R}_7$ - $\mathfrak{R}$ -R2':  $\sim$ atomic(S')  $\vee$   $\sim$ squash(nil,A<sub>2</sub>')  $\vee$   $\sim$ append(A<sub>1</sub>',A<sub>2</sub>',A<sub>2</sub>)  $\vee$   $\sim$ append(cons(a,nil),A<sub>2</sub>,z) {b/S',cons(S',nil)/A<sub>1</sub>'}

$\mathfrak{R}_8$ :  $\sim$ atomic(b)  $\vee$   $\sim$ squash(nil,A<sub>2</sub>')  $\vee$   $\sim$ append(cons(b,nil),A<sub>2</sub>',A<sub>2</sub>)  $\vee$   $\sim$ append(cons(a,nil),A<sub>2</sub>,z)

$\mathfrak{R}_9$ :  $\mathfrak{R}_8$ - $\mathfrak{R}$ -fact2:  $\sim$ squash(nil,A<sub>2</sub>')  $\vee$   $\sim$ append(cons(b,nil),A<sub>2</sub>',A<sub>2</sub>)  $\vee$   $\sim$ append(cons(a,nil),A<sub>2</sub>,z)

$\mathfrak{R}_{10}$ = $\mathfrak{R}_9$ - $\mathfrak{R}$ -F3:  $\sim$ append(cons(b,nil),A<sub>2</sub>',A<sub>2</sub>)  $\vee$   $\sim$ append(cons(a,nil),A<sub>2</sub>,z) {nil/A<sub>2</sub>'}

$\mathfrak{R}_{10}$ =  $\sim$ append(cons(b,nil),nil,A<sub>2</sub>)  $\vee$   $\sim$ append(cons(a,nil),A<sub>2</sub>,z)

$\mathfrak{R}_{11}$ = $\mathfrak{R}_{10}$ - $\mathfrak{R}$ -F2:  $\sim$ append(cons(a,nil),A<sub>2</sub>,z) {cons(b,nil)/A<sub>2</sub>}

$\mathfrak{R}_{11}$ :  $\sim$ append(cons(a,nil),cons(b,nil),z)

$\mathfrak{R}_{12}$ = $\mathfrak{R}_{11}$ - $\mathfrak{R}$ -R1:  $\sim$ append(X<sub>2</sub>,Y<sub>2</sub>,Z<sub>2</sub>) {a/U<sub>2</sub>,nil/X<sub>2</sub>,cons(b,nil)/Y<sub>2</sub>,cons(a,Z<sub>2</sub>)/z}

$\mathfrak{R}_{12}$ =  $\sim$ append(nil,cons(b,nil),Z<sub>2</sub>)

$\mathfrak{R}_{13}$ = $\mathfrak{R}_{12}$ - $\mathfrak{R}$ -F1: nil {cons(b,nil),Z<sub>2</sub>}

z=cons(a,cons(b,nil))

Goal: squash(cons(cons(a,nil),cons(b,nil)),cons(a,cons(b,nil)))

Substitutions will be consistent because I changed variables every time I re-used any rules and all the variables were originally standardized apart. To prove consistency we assemble all the numerators in a set, say U1 and all the denominators in a set, say, U2 and see if U1σ=U2 and σ≠null.

Fall 2003 Exam 2

(20) Conversion to Clause Form

I. Transform the wff A below into CNF (**clause form**) matrix form. For each of the steps required give a brief description of the step and perform the step or write N/A{not applicable} on the space provided. Failure to follow this format will result in no credit. In wff A the set {x,y,z} are variables, the set {A,B,C,D,E} are functions and I is a constant.

{wff A}:  $(\forall x)\{[A(x) \wedge B(x)] \rightarrow [C(x,I) \wedge (\exists y)((\exists z)[C(y,z)] \rightarrow D(x,y))]\} \vee (\forall x)[E(x)]$

(2) Step 0: \_\_\_\_\_

Eliminate redundant quantifiers and take the existential closure (not applicable)

A<sub>0</sub>:  $(\forall x)\{[A(x) \wedge B(x)] \rightarrow [C(x,I) \wedge (\exists y)((\exists z)[C(y,z)] \rightarrow D(x,y))]\} \vee (\forall x)[E(x)]$

(2) Step 1: \_\_\_\_\_

Remove implications

A<sub>1</sub>:  $(\forall x)\{\sim[A(x) \wedge B(x)] \vee [C(x,I) \wedge (\exists y)((\exists z)[\sim C(y,z)] \vee D(x,y))]\} \vee (\forall x)[E(x)]$

(2) Step 2: \_\_\_\_\_

Move the Negations down to the Atfs

A<sub>2</sub>:  $(\forall x)\{[\sim A(x) \vee \sim B(x)] \vee [C(x,I) \wedge (\exists y)((\exists z)[\sim C(y,z)] \vee D(x,y))]\} \vee (\forall x)[E(x)]$

(2) Step 3: \_\_\_\_\_

Standardize Variables Apart

A<sub>3</sub>:  $(\forall x)\{[\sim A(x) \vee \sim B(x)] \vee [C(x,I) \wedge (\exists y)((\exists z)[\sim C(y,z)] \vee D(x,y))]\} \vee (\forall w)[E(w)]$

(2) Step 4: \_\_\_\_\_

Skolemize: Let  $y = f(x)$ ;  $z = g(x)$

$$A_4: (\forall x)\{[\sim A(x) \vee \sim B(x)] \vee [C(x,I) \wedge (\sim C(f(x),g(x)) \vee D(x,f(x)))]\} \vee (\forall w)[E(w)]$$

(2) Step 5: \_\_\_\_\_

Move universal quantifiers to the left.

$$A_5: (\forall x)(\forall w)\{[\sim A(x) \vee \sim B(x)] \vee [C(x,I) \wedge (\sim C(f(x),g(x)) \vee D(x,f(x)))]\} \vee E(w)$$

(2) Step 6: \_\_\_\_\_

Distribute  $\vee$  over  $\wedge$  using  $E_1 \vee (E_2 \wedge E_3) = (E_1 \vee E_2) \wedge (E_1 \vee E_3)$

$$A_{6a}: (\forall x)(\forall w)\{\sim A(x) \vee \sim B(x) \vee C(x,I) \vee E(w)\} \wedge$$

$$A_{6b}: (\forall x)(\forall w)\{\sim A(x) \vee \sim B(x) \vee \sim C(f(x),g(x)) \vee D(x,f(x)) \vee E(w)\}$$

(2) Step 7: \_\_\_\_\_

Write in Matrix Form

$$A_7: (\forall x)(\forall w)[\sim A(x) \vee \sim B(x) \vee C(x,I) \vee E(w)]$$

$$(\forall x)(\forall w)[\sim A(x) \vee \sim B(x) \vee \sim C(f(x),g(x)) \vee D(x,f(x)) \vee E(w)]$$

(2) Step 8: \_\_\_\_\_

Remove Universal Quantifiers

$$A_8: [\sim A(x) \vee \sim B(x) \vee C(x,I) \vee E(w)]$$

$$[\sim A(x) \vee \sim B(x) \vee \sim C(f(x),g(x)) \vee D(x,f(x)) \vee E(w)]$$

I. Conversion to Clause Form (continued)

(2) Step 9: \_\_\_\_\_

Rename Variables

$$A_9: [\sim A(x_1) \vee \sim B(x_1) \vee C(x_1, I) \vee E(w_1)]$$

$$[\sim A(x_2) \vee \sim B(x_2) \vee \sim C(f(x_2), g(x_2)) \vee D(x_2, f(x_2)) \vee E(w_2)]$$

(2) Step 10: \_\_\_\_\_

Step 10 Remove Tautologies & Simplify (not applicable)

$$A_{10}: [\sim A(x_1) \vee \sim B(x_1) \vee C(x_1, I) \vee E(w_1)]$$

$$[\sim A(x_2) \vee \sim B(x_2) \vee \sim C(f(x_2), g(x_2)) \vee D(x_2, f(x_2)) \vee E(w_2)]$$

(25)

**II. Resolution Refutation**

EXCITING LIFE

ALL PEOPLE WHO ARE NOT POOR AND ARE SMART ARE HAPPY. THOSE PEOPLE WHO READ ARE NOT STUPID. JOHN CAN READ AND IS WEALTHY. HAPPY PEOPLE HAVE EXCITING LIVES. CAN ANYONE BE FOUND WITH AN EXCITING LIFE?

Solve by drawing a Refutation Graph resulting from a **complete** strategy. (Make sure you mark clearly the required substitutions).

[Required: Please note the assigned point values. Each part **MUST** be answered with something. If left blank, then no credit will be assigned]

- (5) a. Represent the axioms/goal in the Predicate Calculus.
- (2) b. Represent any commonsense knowledge needed to solve the problem using Predicate Calculus,
- (5) c. Convert your axioms, goal and commonsense knowledge (if any) to clause form,
- (10) d. Draw your Refutation Graph, show substitutions are consistent.
- (3) e. Define your strategy, and describe how your graph meets the strategy

(5) Answers Part a:

ALL PEOPLE WHO ARE NOT POOR AND ARE SMART ARE HAPPY. THOSE PEOPLE WHO READ ARE NOT STUPID.  
JOHN CAN READ AND IS WEALTHY. HAPPY PEOPLE HAVE EXCITING LIVES. CAN ANYONE BE FOUND WITH AN EXCITING LIFE?

- [1]  $(\forall x)[\{\sim\text{poor}(x) \wedge \text{smart}(x)\} \rightarrow \text{happy}(x)]$
- [2]  $(\forall y)[\text{read}(y) \rightarrow \text{smart}(y)]$
- [3]  $\text{read}(\text{John}) \wedge \sim\text{poor}(\text{John})$
- [4]  $(\forall z)[\text{happy}(z) \rightarrow \text{exciting}(z)]$
- [5]  $(\exists w)[\text{exciting}(w)]$  {goal}

(2) Answers Part b:

None needed

(5) Answers Part c:

- [1]  $\text{poor}(x) \vee \sim\text{smart}(x) \vee \text{happy}(x)$
- [2]  $\sim\text{read}(y) \vee \text{smart}(y)$
- [3a]  $\text{read}(\text{John})$
- [3b]  $\sim\text{poor}(\text{John})$
- [4]  $\sim\text{happy}(z) \vee \text{exciting}(z)$
- [5']  $\sim\text{exciting}(w)$

**II. Resolution Refutation**(continued)

(10) Refutation Graph Part d:



- $\mathfrak{R}_1 = [5']$  with [4]  $\sim$ happy(z) {z/w}
- $\mathfrak{R}_2 = \mathfrak{R}_1$  with [1]  $\text{poor}(x) \vee \sim$ smart(x) {x/z}
- $\mathfrak{R}_3 = \mathfrak{R}_2$  with [2]  $\text{poor}(y) \vee \sim$ read(y) {y/x}
- $\mathfrak{R}_4 = \mathfrak{R}_3$  with [3a]  $\text{poor}(\text{John})$  {John/y}
- $\mathfrak{R}_5 = \text{Nil}$  with  $\mathfrak{R}_4$  and [3b]

Consistency Check

$$U_1 = [z, x, y, \text{John}] \quad U_2 = [w, z, x, y]$$

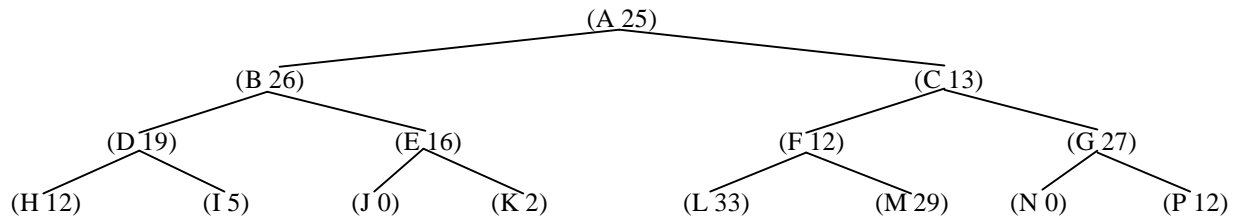
$$U_1 = U_2[z/w, x/z, y/x, \text{John}/y]$$

Since  $U_1$  &  $U_2$  unify, then the substitutions are consistent

(3) Answer Part e: My strategy is \_\_\_\_\_ Set of Support \_\_\_\_\_

Since every resolvent  $\mathfrak{R}_1$ - $\mathfrak{R}_5$  comes from the negation of the goal wff with the base set or one of its descendants

(30)  
**III. Heuristic Search**  
The following figure shows a search tree with the state indicated by the tuple inside parentheses. A letter indicates the state name and the integer indicates the estimated cost for finding a solution from that state (a cost of 0 indicates a goal state). Using the Graph-Search algorithm discussed in class, give the solution tree or steps using depth-first search. How many nodes did depth-first expand? Repeat using breadth-first search. How many nodes did breadth-first expand? Repeat using heuristic search. How many nodes did heuristic search expand? Repeat using A\* search. How many nodes did A\* expand? You must clearly justify your answer(s). "Feelings" or "intuition" are not good/sound reasons. NO JUSTIFICATION  $\Leftrightarrow$  NO CREDIT. You must give me the details of the algorithm in order to receive any credit for each case. Can any of these algorithms ever find N as a solution? Explain



**ALGORITHM DETAILS: YOU CAN USE ALGORITHM GRAPHSEARCH FOR EVERYTHING**  
START: OPEN={A} CLOSED={} G={} M={}  $f(n)=g(n)+h(n)$  where  $g(n)=depth(n)$  &  $h(n)=heuristic\ fcn$

**BREADTH FIRST: USE THE FUNCTION  $f(n)=CAR(OPEN)$  AND APPEND M AT THE END OF THE OPEN LIST.**

1. The algorithm selects A and expands A (applies  $\Gamma$ ) in order to obtain  $M=\{B,C\}$   
 $n_1=B; n_2=C; Open=\{B,C\}, Closed=\{A\}, G=\{A\}, f(n_1)=1; f(n_2)=1$
2. The algorithm expands B in order to obtain  $M=\{D,E\}$   
 $n_3=D; n_4=E; Open=\{C,D,E\}, Closed=\{A,B\}, G=\{A,B\}, f(n_3)=1; f(n_4)=1$
3. The algorithm expands C in order to obtain  $M=\{F,G\}$   
 $n_5=F; n_6=G; Open=\{D,E,F,G\}, Closed=\{A,B,C\}, G=\{A,B,C\}, f(n_5)=1; f(n_6)=1$
4. The algorithm expands D in order to obtain  $M=\{H,I\}$   
 $n_7=H; n_8=I; Open=\{E,F,G,H,I\}, Closed=\{A,B,C,D\}, G=\{A,B,C,D\}, f(n_7)=1; f(n_8)=1$
5. The algorithm expands E in order to obtain  $M=\{J,K\}$   
 $n_9=J; n_{10}=K; Open=\{F,G,H,I,J,K\}, Closed=\{A,B,C,D,E\}, G=\{A,B,C,D,E\}, f(n_9)=1; f(n_{10})=1$
6. The algorithm expands F in order to obtain  $M=\{L,M\}$   
 $n_{11}=L; n_{12}=M; Open=\{G,H,I,J,K,L,M\}, G=Closed=\{A,B,C,D,E,F\}, f(n_{11})=1; f(n_{12})=1$
7. The algorithm expands G in order to obtain  $M=\{N,P\}$   
 $n_{13}=N; n_{14}=P; Open=\{H,I,J,K,L,M,N,P\}, G=Closed=\{A,B,C,D,E,F,G\}, f(n_{13})=1; f(n_{14})=1$
8. The algorithm expands H in order to obtain  $M=\{\}$   
 $Open=\{I,J,K,L,M,N,P\}, G=Closed=\{A,B,C,D,E,F,G,H\}$
9. The algorithm expands I in order to obtain  $M=\{\}$   
 $Open=\{J,K,L,M,N,P\}, G=Closed=\{A,B,C,D,E,F,G,H,I\}$
10. The algorithm expands J in order to obtain  $M=\{\}$   
J is a solution and the algorithm terminates. BFS expands  $\{A,B,C,D,E,F,G,H,I,J\}$  10 nodes

**III. Heuristic Search. (continued)**

**DEPTH-FIRST: USE THE FUNCTION  $f(n)=\text{DEPTH}(N)$  AND APPEND  $M$  AT THE FRONT OF THE OPEN LIST.**

1. The algorithm selects  $A$  and expands  $A$  (applies  $\Gamma$ ) in order to obtain  $M=\{B,C\}$   
 $n_1=B; n_2=C; \text{Open}=\{B,C\}, \text{Closed}=\{A\}, G=\{A\}, f(n_1)=1; f(n_2)=1$
2. The algorithm expands  $B$  in order to obtain  $M=\{D,E\}$   
 $n_3=D; n_4=E; \text{Open}=\{D,E,C\}, \text{Closed}=\{A,B\}, G=\{A,B\}, f(n_3)=2; f(n_4)=2$
3. The algorithm expands  $D$  in order to obtain  $M=\{H,I\}$   
 $n_4=H; n_5=I; \text{Open}=\{H,I,E,C\}, \text{Closed}=\{A,B,D\}, G=\{A,B,D\}, f(n_5)=3; f(n_6)=3$
4. The algorithm expands  $H$  in order to obtain  $M=\{ \}$   
 $\text{Open}=\{I,E,C\}, G=\text{Closed}=\{A,B,D,H\}$
5. The algorithm expands  $I$  in order to obtain  $M=\{ \}$   
 $\text{Open}=\{E,C\}, G=\text{Closed}=\{A,B,D,H,I\}$
6. The algorithm expands  $E$  in order to obtain  $M=\{J,K\}$   
 $n_6=J; n_7=K; \text{Open}=\{J,K,C\}, \text{Closed}=\{A,B,D,H,I,E\}, G=\{A,B,D,H,I,E\}, f(n_9)=3; f(n_{10})=3$
7. The algorithm expands  $J$  in order to obtain  $M=\{ \}$   
 $J$  is a solution and the algorithm terminates. DFS expands  $\{A,B,D,H,I,E,J\}$  7 nodes

**HEURISTIC-SEARCH: USE THE FUNCTION  $f(n)=h(n)$  AND SORT THE OPEN LIST USING  $f$  VALUES.**

1. The algorithm selects  $A$  and expands  $A$  (applies  $\Gamma$ ) in order to obtain  $M=\{B,C\}$   
 $n_1=B; n_2=C; \text{Open}=\{B,C\}, \text{Closed}=\{A\}, G=\{A\}, f(n_1)=26; f(n_2)=13$
2. The algorithm expands  $C$  in order to obtain  $M=\{F,G\}$   
 $n_3=F; n_4=G; \text{Open}=\{F,B,G\}, \text{Closed}=\{A,C\}, G=\{A,C\}, f(n_3)=12; f(n_4)=27$
3. The algorithm expands  $F$  in order to obtain  $M=\{L,M\}$   
 $n_5=L; n_6=M; \text{Open}=\{B,G,M,L\}, G=\text{Closed}=\{A,C,F\}, f(n_5)=33; f(n_6)=29$
4. The algorithm expands  $B$  in order to obtain  $M=\{D,E\}$   
 $n_7=D; n_8=E; \text{Open}=\{E,D,G,M,L\}, G=\text{Closed}=\{A,C,F,B\}, f(n_7)=19; f(n_8)=16$
5. The algorithm expands  $E$  in order to obtain  $M=\{J,K\}$   
 $n_9=J; n_{10}=K; \text{Open}=\{J,K,D,G,M,L\}, G=\text{Closed}=\{A,C,F,B,E\}, f(n_9)=0; f(n_{10})=2$
6. The algorithm expands  $J$  in order to obtain  $M=\{ \}$   
 $J$  is a solution and the algorithm terminates. Heuristic search expands  $\{A,C,F,B,E,J\}$  6 nodes

**A\* SEARCH: USES  $f(n)=g(n)+h(n)$  where  $g(n)=\text{depth}(n)$  &  $h(n)=\text{cost}$  AND SORT THE OPEN LIST USING  $f$**

1. The algorithm selects  $A$  and expands  $A$  (applies  $\Gamma$ ) in order to obtain  $M=\{B,C\}$   
 $n_1=B; n_2=C; \text{Open}=\{B,C\}, \text{Closed}=\{A\}, G=\{A\}, f(n_1)=1+26; f(n_2)=1+13$
2. The algorithm expands  $C$  in order to obtain  $M=\{F,G\}$   
 $n_3=F; n_4=G; \text{Open}=\{F,B,G\}, \text{Closed}=\{A,C\}, G=\{A,C\}, f(n_3)=2+12; f(n_4)=2+27$
3. The algorithm expands  $F$  in order to obtain  $M=\{L,M\}$   
 $n_5=L; n_6=M; \text{Open}=\{B,G,M,L\}, G=\text{Closed}=\{A,C,F\}, f(n_5)=3+33; f(n_6)=3+29$
4. The algorithm expands  $B$  in order to obtain  $M=\{D,E\}$   
 $n_7=D; n_8=E; \text{Open}=\{E,D,G,M,L\}, G=\text{Closed}=\{A,C,F,B\}, f(n_7)=2+19; f(n_8)=2+16$
5. The algorithm expands  $E$  in order to obtain  $M=\{J,K\}$   
 $n_9=J; n_{10}=K; \text{Open}=\{J,K,D,G,M,L\}, G=\text{Closed}=\{A,C,F,B,E\}, f(n_9)=3+0; f(n_{10})=3+2$
6. The algorithm expands  $J$  in order to obtain  $M=\{ \}$   
 $J$  is a solution and the algorithm terminates. Heuristic search expands  $\{A,C,F,B,E,J\}$  6 nodes

**$N$  WILL NOT BE FOUND BY ANY OF THE ALGORITHMS BECAUSE PATH  $\{A,B,E,J\}$  IS CONSIDERED BEFORE  $\{A,C,G,N\}$  due to the fact that  $h(E)=16$  and  $h(G)=27$  and  $h(E)<h(G)$ .**



(25)  
**IV. Computation Deduction.**  
We wish to separate the sheep from the goats. We define the predicate  $\text{herd}(L,S,G)$  which is *true* if  $S$  is a list of all the sheep in  $L$ , and  $G$  is a list of all the goats in  $L$ . Using **Resolution Refutation** deduce the following computation to obtain a value for the goal (3 pts) by drawing the Consistent Solution Refutation Tree (17 pts) for the goal and *prove (or provide a good argument)* its consistency (5 pts.) Make sure your resolution refutation tree is clearly marked and it follows a complete strategy.

Facts:

$F_1: \text{herd}(\text{nil}, \text{nil}, \text{nil}).$

Rules:

$R_1: \text{herd}(T,S,G) \rightarrow \text{herd}(\text{cons}(\text{sheep}, T), \text{cons}(\text{sheep}, S), G)$

$R_2: \text{herd}(T,S,G) \rightarrow \text{herd}(\text{cons}(\text{goat}, T), S, \text{cons}(\text{goat}, G))$

Goal:  $(\exists z)(\exists w) \text{herd}(\text{cons}(\text{sheep}, \text{cons}(\text{goat}, \text{cons}(\text{goat}, \text{nil}))), w, z)$

{Note: If you prefer, you may use the notation  $\text{herd}([\text{sheep}, \text{goat}, \text{goat}], w, z)$  or  $\text{herd}((\text{sheep goat goat}), w, z).$ }

Required: Draw the graph, show the substitutions are consistent, and obtain the value of the goal.

(1 pts) I am using Set-of-Support which is a complete strategy

$F_1: \text{herd}(\text{nil}, \text{nil}, \text{nil}).$

$R_1: \sim \text{herd}(T,S,G) \vee \text{herd}(\text{cons}(\text{sheep}, T), \text{cons}(\text{sheep}, S), G)$

$R_2: \sim \text{herd}(T', S', G') \vee \text{herd}(\text{cons}(\text{goat}, T'), S', \text{cons}(\text{goat}, G'))$

$\sim \text{Goal}: \sim \text{herd}(\text{cons}(\text{sheep}, \text{cons}(\text{goat}, \text{cons}(\text{goat}, \text{nil}))), w, z)$

(3 pts)  $\mathfrak{R}_1 = \mathfrak{R}\{\sim \text{Goal}, R_1\}: \sim \text{herd}(T,S,G) \{ \text{cons}(\text{goat}, \text{cons}(\text{goat}, \text{nil}))/T, \text{cons}(\text{sheep}, S)/w, G/z \}$

$\mathfrak{R}_1: \sim \text{herd}(\text{cons}(\text{goat}, \text{cons}(\text{goat}, \text{nil})), S, G)$

(3 pts)  $\mathfrak{R}_2 = \mathfrak{R}\{\mathfrak{R}_1, R_2\}: \sim \text{herd}(T', S', G') \{ \text{cons}(\text{goat}, \text{nil})/T', S'/S; \text{cons}(\text{goat}, G')/G \}$

$\mathfrak{R}_2: \sim \text{herd}(\text{cons}(\text{goat}, \text{nil}), S', G')$

(3 pts)  $\mathfrak{R}_3 = \mathfrak{R}_2 - \mathfrak{R} - R_2'': \sim \text{herd}(\text{nil}, T'', G'') \{ \text{nil}/T''; S''/S', \text{cons}(\text{goat}, G'')/G' \}$

$\mathfrak{R}_3: \sim \text{herd}(\text{nil}, S'', G'')$

(3 pts)  $\mathfrak{R}_4 = \mathfrak{R}_3 - \mathfrak{R} - F_1: \text{nil} \{ \text{nil}/S'', \text{nil}/G'' \}$

(4 pts) Therefore  $G'' = \text{nil}; G' = \text{cons}(\text{goat}, G'') = \text{cons}(\text{goat}, \text{nil}); G = \text{cons}(\text{goat}, G') = \text{cons}(\text{goat}, (\text{cons}(\text{goat}, \text{nil})));$

$z = G = \text{cons}(\text{goat}, (\text{cons}(\text{goat}, \text{nil}))); S'' = \text{nil}; S' = S'' = \text{nil}; S = S' = \text{nil}; w = \text{cons}(\text{sheep}, s) = \text{cons}(\text{sheep}, \text{nil}).$

(3) Answer:  $(\exists z)(\exists w) \text{herd}(\text{cons}(\text{sheep}, \text{cons}(\text{goat}, \text{cons}(\text{goat}, \text{nil}))), w, z)$  is true  
with  $w = \text{cons}(\text{sheep}, \text{nil})$  and  $z = \text{cons}(\text{goat}, (\text{cons}(\text{goat}, \text{nil})))$

(5) Substitutions will be consistent because I changed variables every time I re-used any rules and all the variables were originally standardized apart. To prove consistency we assemble all the numerators in a set, say,  $U_1$ , and all the denominators in a set, say,  $U_2$  and show that  $U_1 = U_2\sigma$  and  $\sigma \neq \text{null}$ .

Fall 2004 Exam was 90 minutes

(25) Conversion to Clause Form

I. Transform the wff  $A$  below into CNF (**clause form**) matrix form. For each of the steps required give a brief description of the step and perform the step or write N/A{not applicable} on the space provided. Failure to follow this format will result in no credit. In wff  $A$  the set  $\{x,y,z\}$  are variables, the set  $\{P,Q,S\}$  are functions and there are no constants.

$$\{wff A\}: (\forall x \exists y) \{ [P(x,y) \rightarrow Q(y,z)] \wedge [Q(y,x) \rightarrow S(x,y)] \} \rightarrow (\exists x \forall y) [P(x,y) \rightarrow S(x,y)]$$

(2) Step 0: **Eliminate redundant quantifiers and take the existential closure** \_\_\_\_\_

$$A_0: \exists z (\forall x \exists y) \{ [P(x,y) \rightarrow Q(y,z)] \wedge [Q(y,x) \rightarrow S(x,y)] \} \rightarrow (\exists x \forall y) [P(x,y) \rightarrow S(x,y)]$$

(2) Step 1: **Remove implications** \_\_\_\_\_

$$A_1: \exists z \sim (\forall x \exists y) \{ [\sim P(x,y) \vee Q(y,z)] \wedge [\sim Q(y,x) \vee S(x,y)] \} \vee (\exists x \forall y) [\sim P(x,y) \vee S(x,y)]$$

(2) Step 2: **Move the Negations down to the Atfs** \_\_\_\_\_

$$A_2: \exists z (\exists x \forall y) \{ [P(x,y) \wedge \sim Q(y,z)] \vee [Q(y,x) \wedge \sim S(x,y)] \} \vee (\exists x \forall y) [\sim P(x,y) \vee S(x,y)]$$

(2) Step 3: **Standardize Variables Apart** \_\_\_\_\_

$$A_3: \exists z (\exists x_1 \forall y_1) \{ [P(x_1, y_1) \wedge \sim Q(y_1, z)] \vee [Q(y_1, x_1) \wedge \sim S(x_1, y_1)] \} \vee (\exists x_2 \forall y_2) [\sim P(x_2, y_2) \vee S(x_2, y_2)]$$

(2) Step 4: **Skolemize: Let  $z=C_1$ ,  $x_1 = C_2$ ;  $x_2=C_3$**  \_\_\_\_\_

$$A_4: (\forall y_1) \{ [P(C_2, y_1) \wedge \sim Q(y_1, C_1)] \vee [Q(y_1, C_2) \wedge \sim S(C_2, y_1)] \} \vee (\forall y_2) [\sim P(C_3, y_2) \vee S(C_3, y_2)]$$

(2) Step 5: **Move universal quantifiers to the left** \_\_\_\_\_

$$A_5: (\forall x)(\forall y) \{ [P(C_2, x) \wedge \sim Q(x, C_1)] \vee [Q(x, C_2) \wedge \sim S(C_2, x)] \} \vee [\sim P(C_3, y) \vee S(C_3, y)]$$

(6) Step 6: **Multiply out & distribute  $\vee$  over  $\wedge$  using  $E_1 \vee (E_2 \wedge E_3) = (E_1 \vee E_2) \wedge (E_1 \vee E_3)$**  \_\_\_\_\_

$$A_{6a}: (\forall x)(\forall y) \{ [P(C_2, x) \vee Q(x, C_2)] \wedge [P(C_2, x) \vee \sim S(C_2, x)] \wedge [\sim Q(x, C_1) \vee Q(x, C_2)] \wedge [\sim Q(x, C_1) \vee \sim S(C_2, x)] \} \vee [\sim P(C_3, y) \vee S(C_3, y)]$$

$$A_{6b}: (\forall x)(\forall y) \{ [\sim P(C_3, y) \vee S(C_3, y)] \vee [P(C_2, x) \vee Q(x, C_2)] \wedge [\sim P(C_3, y) \vee S(C_3, y)] \vee [P(C_2, x) \vee \sim S(C_2, x)] \wedge [\sim P(C_3, y) \vee S(C_3, y)] \vee [\sim Q(x, C_1) \vee Q(x, C_2)] \wedge [\sim P(C_3, y) \vee S(C_3, y)] \vee [\sim Q(x, C_1) \vee \sim S(C_2, x)] \}$$

(2) Step 7: **Write in Matrix Form** \_\_\_\_\_

$$A_7: (\forall x)(\forall y) [\sim P(C_3, y) \vee S(C_3, y) \vee P(C_2, x) \vee Q(x, C_2)] \\ (\forall x)(\forall y) [\sim P(C_3, y) \vee S(C_3, y) \vee P(C_2, x) \vee \sim S(C_2, x)] \\ (\forall x)(\forall y) [\sim P(C_3, y) \vee S(C_3, y) \vee \sim Q(x, C_1) \vee Q(x, C_2)] \\ (\forall x)(\forall y) [\sim P(C_3, y) \vee S(C_3, y) \vee \sim Q(x, C_1) \vee \sim S(C_2, x)]$$

(2) Step 8: **Eliminate Universal Quantifiers** \_\_\_\_\_

$$A_8: [\sim P(C_3, y) \vee S(C_3, y) \vee P(C_2, x) \vee Q(x, C_2)] \\ [\sim P(C_3, y) \vee S(C_3, y) \vee P(C_2, x) \vee \sim S(C_2, x)] \\ [\sim P(C_3, y) \vee S(C_3, y) \vee \sim Q(x, C_1) \vee Q(x, C_2)] \\ [\sim P(C_3, y) \vee S(C_3, y) \vee \sim Q(x, C_1) \vee \sim S(C_2, x)]$$

I. Conversion to Clause Form (continued)

(2) Step 9: **Rename Variables** \_\_\_\_\_

$$A_9: [\sim P(C_3, y_1) \vee S(C_3, y_1) \vee P(C_2, x_1) \vee Q(x_1, C_2)]$$
$$[\sim P(C_3, y_2) \vee S(C_3, y_2) \vee P(C_2, x_2) \vee \sim S(C_2, x_2)]$$
$$[\sim P(C_3, y_3) \vee S(C_3, y_3) \vee \sim Q(x_3, C_1) \vee Q(x_3, C_2)]$$
$$[\sim P(C_3, y_4) \vee S(C_3, y_4) \vee \sim Q(x_4, C_1) \vee \sim S(C_2, x_4)]]$$

(1) Step 10: **Remove Tautologies & Simplify (not applicable)** \_\_\_\_\_

$$A_{10}: [\sim P(C_3, y_1) \vee S(C_3, y_1) \vee P(C_2, x_1) \vee Q(x_1, C_2)]$$
$$[\sim P(C_3, y_2) \vee S(C_3, y_2) \vee P(C_2, x_2) \vee \sim S(C_2, x_2)]$$
$$[\sim P(C_3, y_3) \vee S(C_3, y_3) \vee \sim Q(x_3, C_1) \vee Q(x_3, C_2)]$$
$$[\sim P(C_3, y_4) \vee S(C_3, y_4) \vee \sim Q(x_4, C_1) \vee \sim S(C_2, x_4)]]$$

(25)

**II. Resolution Refutation**

CONSIDER THE FOLLOWING DATABASE ABOUT ZEBRAS

ZEBRAS ARE MAMMALS, STRIPPED, AND MEDIUM SIZE. MAMMALS ARE ANIMALS AND WARM-BLOODED. STRIPED THINGS ARE NON-SOLID AND NON-SPOTTED. THINGS OF MEDIUM SIZE ARE NEITHER SMALL NOR LARGE. IF ZEKE IS A ZEBRA, IS ZEKE NON-LARGE?

Solve by drawing a Refutation Graph resulting from the **Breadth-First** strategy. (Make sure you mark clearly the required substitutions).

[Required: Please note the assigned point values. Each part **MUST** be answered with something. If left blank, then no credit will be assigned]

- (5) a. Represent the axioms/goal in the Predicate Calculus.
- (2) b. Represent any commonsense knowledge needed to solve the problem using Predicate Calculus,
- (5) c. Convert your axioms, goal and commonsense knowledge (if any) to clause form,
- (10) d. Draw your Refutation Graph, show substitutions are consistent.
- (3) e. Describe how your graph meets the strategy. What other strategy could you have used and why?

(5) Answers Part a:

ZEBRAS ARE MAMMALS, STRIPPED, AND MEDIUM SIZE. MAMMALS ARE ANIMALS AND WARM-BLOODED. STRIPED THINGS ARE NON-SOLID AND NON-SPOTTED. THINGS OF MEDIUM SIZE ARE NEITHER SMALL NOR LARGE. IF ZEKE IS A ZEBRA, IS ZEKE NON-LARGE?

- [1]  $(\forall x)[zebra(x) \rightarrow mammal(x)]$
- [2]  $(\forall x)[zebra(x) \rightarrow striped(x)]$
- [3]  $(\forall x)[zebra(x) \rightarrow medium(x)]$
- [4]  $(\forall x)[mammal(x) \rightarrow animal(x)]$
- [5]  $(\forall x)[mammal(x) \rightarrow warm(x)]$
- [6]  $(\forall x)[striped(x) \rightarrow nonsolid(x)]$
- [7]  $(\forall x)[striped(x) \rightarrow nonspotted(x)]$
- [8]  $(\forall x)[medium(x) \rightarrow nonsmall(x)]$
- [9]  $(\forall x)[medium(x) \rightarrow nonlarge(x)]$
- [10]  $[zebra(zeke) \rightarrow nonlarge(zeke)]$

(2) Answers Part b:

None needed

(5) Answers Part c:

- [1]  $\sim zebra(x_1) \vee mammal(x_1)$
- [2]  $\sim zebra(x_2) \vee striped(x_2)$
- [3]  $\sim zebra(x_3) \vee medium(x_3)$
- [4]  $\sim mammal(x_4) \vee animal(x_4)$
- [5]  $\sim mammal(x_5) \vee warm(x_5)$
- [6]  $\sim striped(x_6) \vee nonsolid(x_6)$
- [7]  $\sim striped(x_7) \vee nonspotted(x_7)$
- [8]  $\sim medium(x_8) \vee nonsmall(x_8)$
- [9]  $\sim medium(x_9) \vee nonlarge(x_9)$
- [10a]  $zebra(zeke)$
- [10b]  $\sim nonlarge(zeke)$

**II. Resolution Refutation(continued)**

(10) Refutation Graph Part d:

[10a] [1] [2] [3] [4] [5] [6] [7] [8] [10b] [9]

$\mathfrak{R}_1$   $\mathfrak{R}_2$   $\mathfrak{R}_3$   $\mathfrak{R}_5$   $\mathfrak{R}_6$   $\mathfrak{R}_7$   $\mathfrak{R}_8$   $\mathfrak{R}_9$   $\mathfrak{R}_4$   $\mathfrak{R}_{10}$

nil

nil

$\mathfrak{R}_1$ =[10a'] with [1] mammal(zeke) {zeke/ $x_1$ }

$\mathfrak{R}_2$ =[10a'] with [2] striped(zeke) {zeke/ $x_2$ }

$\mathfrak{R}_3$ =[10a'] with [3] medium(zeke) {zeke/ $x_3$ }

$\mathfrak{R}_4$ =[10b'] with [9] ~medium(zeke) {zeke/ $x_9$ }

$\mathfrak{R}_5$ =[1] with [4] animal( $x_1$ ) { $x_1/x_4$ }

$\mathfrak{R}_6$ =[1] with [5] warm( $x_1$ ) { $x_1/x_4$ }

$\mathfrak{R}_7$ =[2] with [6] nonsolid( $x_2$ ) { $x_2/x_6$ }

$\mathfrak{R}_8$ =[2] with [7] nospotted( $x_2$ ) { $x_2/x_7$ }

$\mathfrak{R}_9$ =[3] with [8] nosmall( $x_3$ ) { $x_3/x_8$ }

$\mathfrak{R}_{10}$ =[3] with [9] nonlarge( $x_3$ ) { $x_3/x_9$ }

$\mathfrak{R}_{11}$ =Nil with  $\mathfrak{R}_3$  and  $\mathfrak{R}_4$

$\mathfrak{R}_{12}$ =Nil with [10b'] and  $\mathfrak{R}_{10}$

Consistency Check

$U_1$ =[zeke, zeke, zeke, zeke]  $U_2$ =[ $x_1, x_2, x_3, x_9$ ]

$U_1=U_2$ [zeke/ $x_1$ , zeke/ $x_2$ , zeke/ $x_3$ , zeke/ $x_9$ ]

Since  $U_1$  &  $U_2$  unify, then the substitutions are consistent

(3) Answer Part e: My strategy is **Breadth First**

(1) Since every 1<sup>st</sup> level resolvent  $\mathfrak{R}_1$ - $\mathfrak{R}_{10}$  comes from the base set + negation of the wff to be proved.

Nil came from two first level resolvents  $\mathfrak{R}_3$  and  $\mathfrak{R}_4$  or from a 1<sup>st</sup> level resolvent  $\mathfrak{R}_{10}$  and from a base set [10b']

(2) Since Nil came from a 1<sup>st</sup> level resolvent  $\mathfrak{R}_{10}$  which came from the negation of the wff

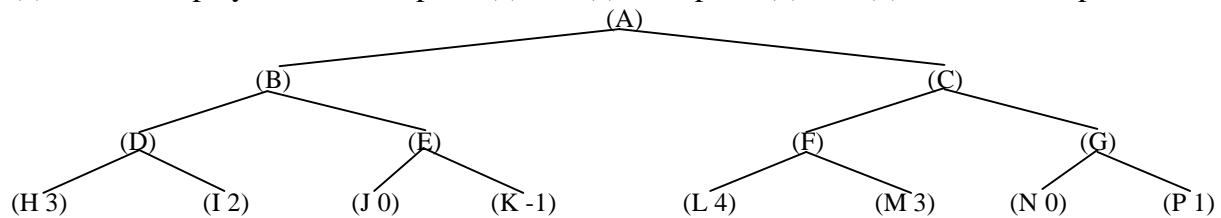
and from it and another member of the negation of the wff [10b'], this represents a set of support strategy

(2  
5)

**III. Adversarial Search**

Consider the following game tree in which the static scores (in parentheses at the tip nodes) are all from the first player's point of view.

- Assuming that the first player is the maximizing player, what move should the first player choose?
- Assuming that the first player is the minimizing player, what move should the first player choose?
- What nodes would not need to be examined in part (a) using the alpha-beta algorithm—assuming that the nodes are examined in left-to-right order?
- What nodes would not need to be examined in part (b) using the alpha-beta algorithm—assuming that the nodes are examined in left-to-right order?
- Is the first player's move in parts (a) and (c) or in parts (b) and (d) different? Explain.



**Part (a)**

D, E, F, G choose max or  $D=(H\ 3); E=(J\ 0); F=(L\ 4); G=(P\ 1)$   
 B, C chooses min or  $B= E=(J\ 0); C=(P\ 1)$   
 A chooses max or  $A=C=G=(P\ 1)$   
 A chooses C toward solution  $A \rightarrow C \rightarrow G \rightarrow P$

**Part (b)**

D, E, F, G choose min or  $D=(I\ 2); E=(K\ -1); F=(M\ 3); G=(N\ 0)$   
 B, C chooses max or  $B=D=(I\ 2); C=F=(M\ 3)$   
 A chooses min or  $A=B=D=(I\ 2)$   
 A chooses B toward solution  $A \rightarrow B \rightarrow D \rightarrow I$

**Part (c)**

D chooses max evaluating (H 3) & (I 2) or  $\alpha_D=3$  (H 3); Now B chooses min so  $\beta_B \leq 3$  (H 3)  
 Evaluate (J 0); now  $\alpha_E \geq 0$  (J 0) and  $\beta_B \leq 3$  (H 3) therefore no Beta Cutoff and continue  
 Evaluate (K -1); now  $\alpha_E=0$  (J 0) now  $\beta_B = 0$  (J 0)  
 Now B chooses min so  $\beta_B = 0$  (J 0), therefore  $\alpha_A \geq 0$  (J 0)  
 Evaluate (L 4); now  $\alpha_F \geq 4$  (L 4) and  $\beta_C \leq 4$  (L 4) therefore no Alpha cutoff & continue  
 Evaluate (M 3); now  $\alpha_F=4$  (L 4); and  $\beta_C \leq 4$  (L 4)  
 Evaluate (N 0); now  $\alpha_G \geq 0$  (N 0); no cutoff & continue  
 Evaluate (P 1); now  $\alpha_G=1$  (P 1);  $\beta_C = 1$  (P 1) no cutoff & continue  
 A chooses C to G to P (P 1)  
 Alpha-Beta had Pruning resulted in no advantage

**III. Adversarial Search. (continued)**

**Part (d)**

D chooses min evaluating (H 3) & (I 2) or  $\beta_D=2$  (I 2); Now B chooses max so  $\alpha_E \geq 2$  (I 2)  
Evaluate (J 0); now  $\beta_F \leq 0$  (J 0) and  $\alpha_E \geq 2$  (I 2) Alpha Cutoff at E and continue  $\alpha_E=2$  (I 2);  $\beta_A \leq 2$  (I 2)  
Evaluate (L 4); now  $\beta_D \leq 4$  (L 4) and  $\alpha_C \geq 4$  (L 4) therefore no Alpha cutoff & continue  
Evaluate (M 3); now  $\alpha_F=4$  (L 4); and  $\beta_C \leq 4$  (L 4) and  $\beta_A \leq 2$  (I 2) Beta Cutoff at C and  $\beta_A=2$  (I 2)  
A chooses B to D to I (I 2)  
Do Not Evaluate {K, G, N, P}

**Part (e)**

A chooses C toward solution  $A \rightarrow C \rightarrow G \rightarrow P$  in both parts (a) and (c) because Alpha-Beta and Minimax produce the same results for the same problem.  
Similarly, A chooses B toward solution  $A \rightarrow B \rightarrow E \rightarrow J$  in both parts (b) and (d) because Alpha-Beta and Minimax produce the same results for the same problem.  
In my analysis that was indeed the case.

(25)  
**IV. Computation Deduction.**  
 We wish to replace Ron Zook with Bob Stoops in a short list of ex-Gator coaches. Using **Resolution Refutation** deduce the following computation to obtain a value for the goal (3 pts) by performing a consistent Refutation Trace (17 pts) for the goal and prove (or provide a good argument) its consistency (5 pts.) Make sure your resolution refutation trace is clearly marked and it follows a complete strategy.

Facts:

F<sub>1</sub>: swap(X,Y,nil,nil).

Rules:

R<sub>1</sub>: swap(S<sub>1</sub>,S<sub>2</sub>,Y,Z) → swap(S<sub>1</sub>,S<sub>2</sub>,cons(S<sub>1</sub>,Y),cons(S<sub>2</sub>,Z))

R<sub>2</sub>: {swap(S<sub>1</sub>,S<sub>2</sub>,Y,Z) ∧ X≠S<sub>1</sub>} → swap(S<sub>1</sub>,S<sub>2</sub>,cons(X,Y),cons(X,Z))

Goal: (∃z) swap(ron, bob, cons(steve, cons(ron, cons(galen,nil))), z)

{Note: If you prefer, you may use the notation swap(ron, bob, (steve ron galen), z). }

Required: Give the resolution trace, show the substitutions are consistent, and obtain the value of the goal.

(1 pts) I am using Set-of-Support which is a complete strategy

F<sub>1</sub>: swap(A,B,nil,nil).

R<sub>1</sub>: ~ swap(S<sub>1</sub>,S<sub>2</sub>,Y<sub>1</sub>, Z<sub>1</sub>) ∨ swap (S<sub>1</sub>,S<sub>2</sub>,cons(S<sub>1</sub>,Y<sub>1</sub>),cons(S<sub>2</sub>,Z<sub>1</sub>))

R<sub>2</sub>: ~ swap(S<sub>1</sub>,S<sub>2</sub>,Y<sub>2</sub>, Z<sub>2</sub>) ∨ ~X≠S<sub>1</sub> ∨ swap(S<sub>1</sub>,S<sub>2</sub>,cons(X,Y<sub>2</sub>),cons(X,Z<sub>2</sub>))

~Goal: ~swap(ron, bob, (steve ron galen), z)

(3 pts)  $\mathfrak{R}_1 = \mathfrak{R}_1 \{ \sim \text{Goal}, R_2 \} : \sim \text{swap}(S_1, S_2, Y_2, Z_2) \vee \sim X \neq S_1 \{ \text{ron}/S_1, \text{bob}/S_2, \text{steve}/X, (\text{ron galen})/Y_2, \text{cons}(X, Z_2)/z \}$

$\mathfrak{R}_1 : \sim \text{swap}(\text{ron}, \text{bob}, (\text{ron galen}), Z_2) \vee \sim \text{steve} \neq \text{ron} \{ \text{this evaluates to nil} \}$

(3 pts)  $\mathfrak{R}_2 = \mathfrak{R}_1 \{ \mathfrak{R}_1, R_1 \} : \sim \text{swap}(S_1, S_2, Y_1, Z_1) \{ \text{ron}/S_1, \text{bob}/S_2, (\text{galen})/Y_1, \text{cons}(S_2, Z_1)/Z_2 \}$

$\mathfrak{R}_2 : \sim \text{swap}(\text{ron}, \text{bob}, \text{cons}(\text{galen}, \text{nil}), Z_1)$

(3 pts)  $\mathfrak{R}_3 = \mathfrak{R}_2 \{ \mathfrak{R}_2, R_2' \} : \sim \text{swap}(S_1, S_2, Y_2', Z_2') \vee \sim X' \neq S_1 \{ \text{ron}/S_1, \text{bob}/S_2, \text{galen}/X', \text{nil}/Y_2', \text{cons}(X', Z_2')/Z_1 \}$

$\mathfrak{R}_3 : \sim \text{swap}(\text{ron}, \text{bob}, \text{nil}, Z_2') \vee \sim \text{galen} \neq \text{ron} \{ \text{this evaluates to nil} \}$

(3 pts)  $\mathfrak{R}_4 = \mathfrak{R}_3 \{ \mathfrak{R}_3, F_1 \} : \text{nil} \{ \text{ron}/S_1, \text{bob}/S_2, \text{nil}/Z_2' \}$

(4 pts) Therefore Z<sub>2</sub>'=nil; Z<sub>1</sub>=cons(galen,nil)=(galen); Y<sub>2</sub>'=nil; Z<sub>2</sub>=cons(bob,(galen))=(bob galen);

z=cons(steve, cons(bob, cons(galen,nil)))=(steve bob galen)

(3) Answer: (∃z)swap(ron, bob, cons(steve, cons(ron,cons(galen,nil))), z) is true  
with z = cons (steve, cons(bob, cons(galen,nil))) = (steve bob galen)

(5) Substitutions will be consistent because I changed variables every time I re-used any rules and all the variables were originally standardized apart. To prove consistency we assemble all the numerators in a set, say, U<sub>1</sub>, and all the denominators in a set, say, U<sub>2</sub> and show that U<sub>1</sub>=U<sub>2</sub>σ and σ≠null.



Fall 2005 Exam 2 Periods

(20) Conversion to Clause Form

I. Transform the *wff*  $A$  below into CNF (**clause form**) matrix form. For each of the 10 “official steps” required give a brief description of the step and perform the step or write N/A{not applicable} on the space provided. Failure to follow this format will result in no credit. In *wff*  $A$  the set  $\{w,x,y,t\}$  are variables, the set  $\{P,Q,R,A,B\}$  are functions and there are no constants.

$$\{wff A\}: (\forall x)\{P(x) \rightarrow (A(x) \wedge B(x) \vee \sim C(x, w))\} \vee (\forall y) (\exists u) [Q(y,t) \vee ((\forall x) R(x) \rightarrow \sim B(y))]$$

(2) Step 0: **Eliminate redundant quantifiers and take the existential closure** \_\_\_\_\_

$$A_0: (\exists w) (\exists t) (\forall x) \{P(x) \rightarrow (A(x) \wedge B(x) \vee \sim C(x, w))\} \vee (\forall y) [Q(y,t) \vee ((\forall x) R(x) \rightarrow \sim B(y))]$$

(2) Step 1: **Remove implications** \_\_\_\_\_

$$A_1: (\exists w) (\exists t) (\forall x) \{\sim P(x) \vee (A(x) \wedge B(x) \vee \sim C(x, w))\} \vee (\forall y) [Q(y,t) \vee (\sim(\forall x) R(x) \vee \sim B(y))]$$

(2) Step 2: **Move the Negations down to the Atfs** \_\_\_\_\_

$$A_2: (\exists w) (\exists t) (\forall x) \{\sim P(x) \vee (A(x) \wedge B(x) \vee \sim C(x, w))\} \vee (\forall y) [Q(y,t) \vee ((\exists x)\sim R(x) \vee \sim B(y))]$$

(1) Step 3: **Standardize Variables Apart** \_\_\_\_\_

$$A_3: (\exists w) (\exists t) (\forall x) \{\sim P(x) \vee (A(x) \wedge B(x) \vee \sim C(x, w))\} \vee (\forall y) [Q(y,t) \vee ((\exists z)\sim R(z) \vee \sim B(y))]$$

(2) Step 4: **Skolemize: Let  $w=W, t=T; z=f(y)$**  \_\_\_\_\_

$$A_4: (\forall x)\{\sim P(x) \vee (A(x) \wedge B(x) \vee \sim C(x, W))\} \vee (\forall y) [Q(y,T) \vee \sim R(f(y)) \vee \sim B(y)]$$

(1) Step 5: **Move universal quantifiers to the left** \_\_\_\_\_

$$A_5: (\forall x) (\forall y) \{\sim P(x) \vee (A(x) \wedge B(x) \vee \sim C(x, W))\} \vee [Q(y,T) \vee \sim R(f(y)) \vee \sim B(y)]$$

(4) Step 6: **Multiply out & distribute  $\vee$  over  $\wedge$  using  $E_1 \vee (E_2 \wedge E_3) = (E_1 \vee E_2) \wedge (E_1 \vee E_3)$**  \_\_\_\_\_

$$A_{6a}: (\forall x)(\forall y)\{(\sim P(x) \vee Q(y,T) \vee \sim R(f(y)) \vee \sim B(y)) \vee [(\sim C(x,W) \vee A(x)) \wedge (\sim C(x,W) \vee B(x))]\}$$

$$A_{6b}: (\forall x)(\forall y)\{[\sim P(x) \vee Q(y,T) \vee \sim R(f(y)) \vee \sim B(y) \vee \sim C(x,W) \vee A(x)] \wedge [\sim P(x) \vee Q(y,T) \vee \sim R(f(y)) \vee \sim B(y) \vee \sim C(x,W) \vee B(x)]\}$$

**I. Conversion to Clause Form (continued)**

(1) **Step 7: Write in Matrix Form** \_\_\_\_\_

$$A_7: \quad (\forall x)(\forall y)[ \sim P(x) \vee Q(y, T) \vee \sim R(f(y)) \vee \sim B(y) \vee \sim C(x, W) \vee A(x) ]$$
$$(\forall x)(\forall y)[ \sim P(x) \vee Q(y, T) \vee \sim R(f(y)) \vee \sim B(y) \vee \sim C(x, W) \vee B(x) ]$$

(1) **Step 8: Eliminate Universal Quantifiers** \_\_\_\_\_

$$A_8: \quad [ \sim P(x) \vee Q(y, T) \vee \sim R(f(y)) \vee \sim B(y) \vee \sim C(x, W) \vee A(x) ]$$
$$[ \sim P(x) \vee Q(y, T) \vee \sim R(f(y)) \vee \sim B(y) \vee \sim C(x, W) \vee B(x) ]$$

(2) **Step 9: Rename Variables** \_\_\_\_\_

$$A_9: \quad [ \sim P(x_1) \vee Q(y_1, T) \vee \sim R(f(y_1)) \vee \sim B(y_1) \vee \sim C(x_1, W) \vee A(x_1) ]$$
$$[ \sim P(x_2) \vee Q(y_2, T) \vee \sim R(f(y_2)) \vee \sim B(y_2) \vee \sim C(x_2, W) \vee B(x_2) ]$$

(2) **Step 10: Remove Tautologies & Simplify: 2<sup>nd</sup> row drops out since  $\{ \sim B(y_2) \vee B(x_2) \} = \text{True}$**  \_\_\_\_\_

$$A_{10}: \quad [ \sim P(x_1) \vee Q(y_1, T) \vee \sim R(f(y_1)) \vee \sim B(y_1) \vee \sim C(x_1, W) \vee A(x_1) ]$$

(25)

## II. Resolution Refutation

THE MEMBERS OF THE ELM ST. BRIDGE CLUB ARE JOE, SALLY, BILL, AND ELLEN. JOE IS MARRIED TO SALLY. BILL IS ELLEN'S BROTHER. THE SPOUSE OF EVERY MARRIED PERSON IN THE CLUB IS ALSO IN THE CLUB. THE LAST MEETING OF THE CLUB WAS AT JOE'S HOUSE. PROVE THAT (1) THE LAST MEETING OF THE CLUB WAS AT SALLY'S HOUSE & (2) ELLEN IS NOT MARRIED.

Solve by drawing a Refutation Graph resulting from your choice of strategy. (Make sure you mark clearly the required substitutions).

[Required: Please note the assigned point values. Each part MUST be answered with something. If left blank, then no credit will be assigned]

- (5) a. Represent the axioms/goal in the Predicate Calculus.
- (2) b. Represent any commonsense knowledge needed to solve the problem using Predicate Calculus,
- (5) c. Convert your axioms, goal and commonsense knowledge (if any) to clause form,
- (10) d. Draw your Refutation Graph, show substitutions are consistent.
- (3) e. Describe how your graph meets the strategy. What other strategy could you have used and why?

(5) Answers Part a:

[1] is\_member(Joe)  
[2] is\_member(Sally)  
[3] is\_member(Bill)  
[4] is\_member(Allen)  
[5] married(Joe,Sally)  
[6] sibling(Bill,Allen)  
[7]  $(\forall x)(\forall y)\{[\text{married}(x,y) \wedge \text{is\_member}(x)] \rightarrow \text{is\_member}(y)\}$   
[8] last\_meeting(Joe)  
G1: last\_meeting(Sally)  
G2:  $\sim(\exists y)\text{married}(\text{Allen},y)$

(2) Answers Part b:

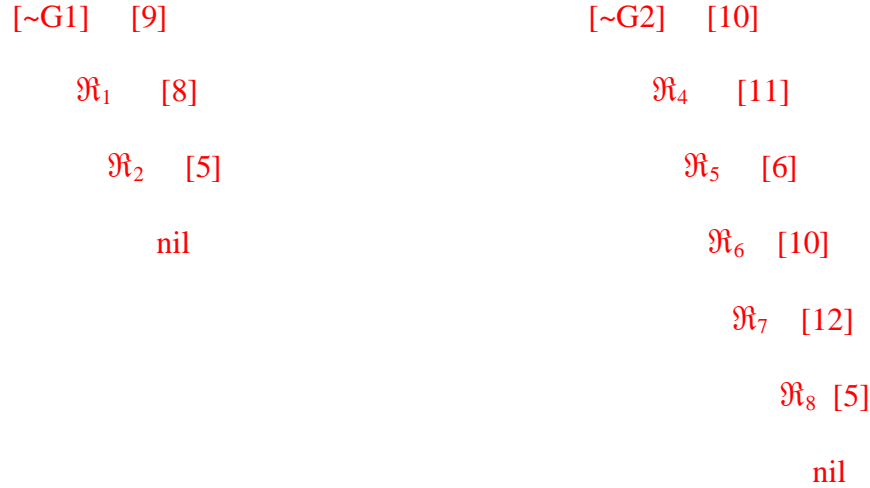
[9]  $(\forall x)(\forall y)\{[\text{married}(x,y) \wedge \text{last\_meeting}(x)] \rightarrow \text{last\_meeting}(y)\}$   
[10]  $(\forall x)(\forall y)\{\text{married}(x,y) \rightarrow \text{married}(y,x)\}$   
[11]  $(\forall x)(\forall y)\{\text{married}(x,y) \rightarrow \sim \text{sibling}(x,y)\}$   
[12]  $(\forall x)(\forall y)(\forall z)\{\text{married}(x,y) \rightarrow \sim \text{married}(x,z)\}$

(5) Answers Part c:

[1] is\_member(Joe)  
[2] is\_member(Sally)  
[3] is\_member(Bill)  
[4] is\_member(Allen)  
[5] married(Joe,Sally)  
[6] sibling(Bill,Allen)  
[7]  $\sim \text{married}(x_7,y_7) \vee \sim \text{is\_member}(x_7) \vee \text{is\_member}(y_7)$   
[8] last\_meeting(Joe)  
[9]  $\sim \text{married}(x_9,y_9) \vee \sim \text{last\_meeting}(x_9) \vee \text{last\_meeting}(y_9)$   
[10]  $\sim \text{married}(x_{10},y_{10}) \vee \text{married}(y_{10},x_{10})$   
[11]  $\sim \text{married}(x_{11},y_{11}) \vee \sim \text{sibling}(x_{11},y_{11})$   
[12]  $\sim \text{married}(x_{12},y_{12}) \vee \sim \text{married}(x_{12},z_{12})$   
~G1:  $\sim \text{last\_meeting}(Sally)$   
~G2:  $\text{married}(\text{Allen},Joe) \vee \text{married}(\text{Allen},Bill)$

**II. Resolution Refutation(continued)**

(10) Refutation Graph Part d:



ℱ<sub>1</sub>=[~G1] with [9]    ~married(x<sub>9</sub>,Sally) v ~last\_meeting(x<sub>9</sub>) {Sally/y<sub>9</sub>}  
 ℱ<sub>2</sub>=ℱ<sub>1</sub> with [8]    ~married(Joe,Sally) {Joe/x<sub>9</sub>}  
 ℱ<sub>3</sub>=ℱ<sub>2</sub> with 5        nil

ℱ<sub>4</sub>=[~G2] with [10]    married(Bill,Ellen) v married(Ellen,Joe) {Ellen/x<sub>10</sub>, Bill/y<sub>10</sub>}  
 ℱ<sub>5</sub>=ℱ<sub>4</sub> with [11]    ~sibling(Bill,Ellen) v married(Ellen,Joe) {Bill/x<sub>11</sub>, Ellen/y<sub>11</sub>}  
 ℱ<sub>6</sub>=ℱ<sub>5</sub> with [6]        married(Ellen,Joe)  
 ℱ<sub>7</sub>=ℱ<sub>6</sub> with [10]        married(Joe,Ellen) {Ellen/x<sub>10</sub>' , Joe/y<sub>10</sub>' }  
 ℱ<sub>8</sub>=ℱ<sub>7</sub> with [12]        ~married(Joe,z<sub>12</sub>) {Joe/x<sub>12</sub>, Ellen/y<sub>12</sub>}  
 ℱ<sub>9</sub>=ℱ<sub>8</sub> with [5]        nil {Sally/z<sub>12</sub>}

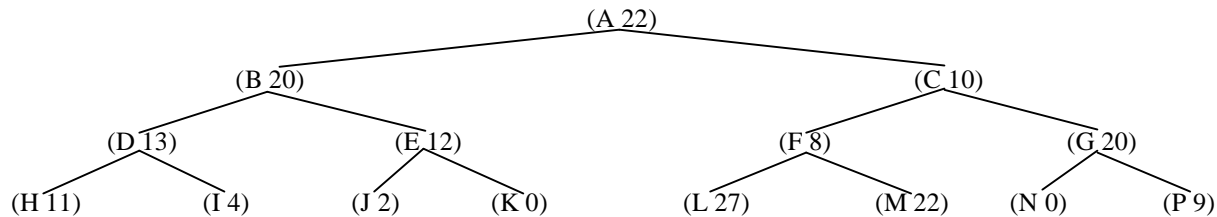
Consistency Check

U<sub>1</sub>=[Sally,Joe,Ellen,Bill,Bill,Ellen,Ellen,Joe,Joe,Ellen,Sally] U<sub>2</sub>=[y<sub>9</sub>,x<sub>9</sub>,x<sub>10</sub>,y<sub>10</sub>,x<sub>11</sub>,y<sub>11</sub>,x<sub>10</sub>' ,y<sub>10</sub>' ,x<sub>12</sub>,y<sub>12</sub>,z<sub>12</sub>]  
 U<sub>1</sub>=U<sub>2</sub>[Sally/y<sub>9</sub>,Joe/x<sub>9</sub>,Ellen/x<sub>10</sub>,Bill/y<sub>10</sub>,Bill/x<sub>11</sub>,Ellen/y<sub>11</sub>,Ellen/x<sub>10</sub>' ,Joe/y<sub>10</sub>' ,Joe/x<sub>12</sub>,Ellen/y<sub>12</sub>,Sally/z<sub>12</sub>]  
 Since U<sub>1</sub> & U<sub>2</sub> unify, then the substitutions are consistent

(3) Answer Part e: My strategy is Set-of-Support

Every resolvent ℱ<sub>1</sub>-ℱ<sub>9</sub> comes from the negation of the wff to be proved. Note ~G2 is not Horne.  
 I could have used ancestry-filtered or breadth-first because they are complete strategies.

(30)  
**III. Heuristic Search**  
The following figure shows a search tree with the state indicated by the tuple inside parentheses. A letter indicates the state name and the integer indicates the estimated cost for finding a solution from that state (a cost of 0 indicates a goal state). Using the Graph-Search algorithm discussed in class, **give the algorithm steps** using (1) **breadth-first search**. How many nodes did breadth-first expand? Repeat using (2) **depth-first search**. How many nodes did depth-first expand? Repeat using (3) **heuristic search** (you **MUST** specify a rule to break ties). How many nodes did heuristic search expand? Repeat using (4) **A\* search**. How many nodes did A\* expand? You must clearly justify your answer(s). "Feelings" or "intuition" are not good/sound reasons. **NO JUSTIFICATION <=> NO CREDIT**. You must give me the details of each step of the algorithm in order to receive any credit for each case. Can any of these algorithms ever find N as a solution? Explain



**ALGORITHM DETAILS: YOU CAN USE ALGORITHM GRAPHSEARCH FOR EVERYTHING**  
**START: OPEN={A} CLOSED={ } G={ } M={ }  $f(n)=g(n)+h(n)$  where  $g(n)=depth(n)$  &  $h(n)=heuristic\ fcn$**

**BREADTH FIRST: APPEND M AT THE END OF THE OPEN LIST &  $f(n)=null$ .**

1. The algorithm selects A and expands A (applies  $\Gamma$ ) in order to obtain  $M=\{B,C\}$   
 $n_1=B; n_2=C; Open=\{B,C\}, Closed=\{A\}, G=\{A,B,C\}, f(n_1)=1; f(n_2)=1$
2. The algorithm expands B in order to obtain  $M=\{D,E\}$   
 $n_3=D; n_4=E; Open=\{C,D,E\}, Closed=\{A,B\}, G=\{A,B,C,D,E\}, f(n_3)=1; f(n_4)=1$
3. The algorithm expands C in order to obtain  $M=\{F,G\}$   
 $n_5=F; n_6=G; Open=\{D,E,F,G\}, Closed=\{A,B,C\}, G=\{A,B,C,D,E,F,G\}, f(n_5)=1; f(n_6)=1$
4. The algorithm expands D in order to obtain  $M=\{H,I\}$   
 $n_7=H; n_8=I; Open=\{E,F,G,H,I\}, Closed=\{A,B,C,D\}, G=\{A,B,C,D,E,F,G,H,I\}, f(n_7)=1; f(n_8)=1$
5. The algorithm expands E in order to obtain  $M=\{J,K\}, n_9=J; n_{10}=K$   
 $Open=\{F,G,H,I,J,K\}, Closed=\{A,B,C,D,E\}, G=\{A,B,C,D,E,F,G,H,I,J,K\}, f(n_9)=1; f(n_{10})=1$
6. The algorithm expands F in order to obtain  $M=\{L,M\}, n_{11}=L; n_{12}=M, \}; f(n_{11})=1; f(n_{12})=1$   
 $Open=\{G,H,I,J,K,L,M\}, Closed=\{A,B,C,D,E,F\}, G=\{A,B,C,D,E,F,G,H,I,J,K,L,M$
7. The algorithm expands G in order to obtain  $M=\{N,P\}, n_{13}=N; n_{14}=P; f(n_{13})=1; f(n_{14})=1$   
 $Open=\{H,I,J,K,L,M,N,P\}, Closed=\{A,B,C,D,E,F,G\}, G=\{A,B,C,D,E,F,G,H,I,J,K,L,M,P\}$
8. The algorithm expands H in order to obtain  $M=\{ \}$   
 $Open=\{I,J,K,L,M,N,P\}, Closed=\{A,B,C,D,E,F,G,H\}, G=\{A,B,C,D,E,F,G,H,I,J,K,L,M,P\}$
9. The algorithm expands I in order to obtain  $M=\{ \}$   
 $Open=\{J,K,L,M,N,P\}, G=\{A,B,C,D,E,F,G,H,I,J,K,L,M,P\}, Closed=\{A,B,C,D,E,F,G,H,I\}$
10. The algorithm expands J in order to obtain  $M=\{ \}$   
 $Open=\{K,L,M,N,P\}, G=\{A,B,C,D,E,F,G,H,I,J,K,L,M,P\}, Closed=\{A,B,C,D,E,F,G,H,I,J\}$
11. The algorithm expands K in order to obtain  $M=\{ \}, G=\{A,B,C,D,E,F,G,H,I,J,K,L,M,P\}$   
 K is a solution exit w/ success. BFS expands  $Closed=\{A,B,C,D,E,F,G,H,I,J,K\}$  11 nodes

### III. Heuristic Search. (continued)

**DEPTH-FIRST:** APPEND M AT THE FRONT OF THE OPEN LIST WITH  $f(n) = \text{NULL}$  (ALTERNATIVELY USE  $f(n) = \text{DEPTH}(n)$ ).

1. The algorithm selects A and expands A (applies  $\Gamma$ ) in order to obtain  $M = \{B, C\}$   
 $n_1 = B; n_2 = C; \text{Open} = \{B, C\}, \text{Closed} = \{A\}, G = \{A, B, C\}, f(n_1) = 1; f(n_2) = 1$
2. The algorithm expands B in order to obtain  $M = \{D, E\}$   
 $n_3 = D; n_4 = E; \text{Open} = \{D, E, C\}, \text{Closed} = \{A, B\}, G = \{A, B, C, D, E\}, f(n_3) = 2; f(n_4) = 2$
3. The algorithm expands D in order to obtain  $M = \{H, I\}$   
 $n_5 = H; n_6 = I; \text{Open} = \{H, I, E, C\}, \text{Closed} = \{A, B, D\}, G = \{A, B, C, D, E, H, I\}, f(n_5) = 3; f(n_6) = 3$
4. The algorithm expands H in order to obtain  $M = \{\}$ ,  $G = \{A, B, C, D, E, H, I\}$   
 $\text{Open} = \{I, E, C\}, G = \text{Closed} = \{A, B, D, H\}$
5. The algorithm expands I in order to obtain  $M = \{\}$ ,  $G = \{A, B, C, D, E, H, I\}$   
 $\text{Open} = \{E, C\}, G = \text{Closed} = \{A, B, D, H, I\}$
6. The algorithm expands E in order to obtain  $M = \{J, K\}$   
 $n_7 = J; n_8 = K; \text{Open} = \{J, K, C\}, \text{Closed} = \{A, B, D, H, I, E\}, G = \{A, B, C, D, E, H, I, J, K\}, f(n_7) = 3; f(n_8) = 3$
7. The algorithm expands J in order to obtain  $M = \{\}$   
 $\text{Open} = \{K, C\}, \text{Closed} = \{A, B, D, H, I, E, J\}, G = \{A, B, C, D, E, H, I, J, K\}$
8. The algorithm expands K in order to obtain  $M = \{\}$   
K is a solution and the algorithm terminates. DFS expands  $\text{Closed} = \{A, B, D, H, I, E, J, K\}$  8 nodes

**HEURISTIC-SEARCH:** USE THE FUNCTION  $f(n) = h(n)$  AND SORT THE OPEN LIST USING  $f$  VALUES, FIFO.

1. The algorithm selects A and expands A (applies  $\Gamma$ ) in order to obtain  $M = \{B, C\}$   
 $n_1 = B; n_2 = C; \text{Open} = \{B, C\}, \text{Closed} = \{A\}, G = \{A, B, C\}, f(n_1) = 20; f(n_2) = 10, \text{Open} = \{C_{10}, B_{20}\}$
2. The algorithm expands C in order to obtain  $M = \{F, G\}$   
 $n_3 = F; n_4 = G; \text{Open} = \{F, G, B\}, \text{Closed} = \{A, C\}, G = \{A, B, C, F, G\}, f(n_3) = 8; f(n_4) = 20, \text{Open} = \{F_8, B_{20}, G_{20}\}$
3. The algorithm expands F in order to obtain  $M = \{L, M\}$ ,  $G = \{A, B, C, F, G, L, M\}$   
 $n_5 = L; n_6 = M; \text{Open} = \{M, L, G, B\}, \text{Closed} = \{A, C, F\}, f(n_5) = 27; f(n_6) = 22, \text{Open} = \{B_{20}, G_{20}, M_{22}, L_{27}\}$
4. The algorithm expands B in order to obtain  $M = \{D, E\}$ ,  $G = \{A, B, C, F, G, L, M, D, E\}$   
 $n_7 = D; n_8 = E; \text{Open} = \{E, D, G, M, L\}, \text{Closed} = \{A, C, F, B\}, f(n_7) = 13; f(n_8) = 12, \text{Open} = \{E_{12}, D_{13}, M_{22}, L_{27}\}$
5. The algorithm expands E in order to obtain  $M = \{J, K\}$ ,  $G = \{A, B, C, F, G, L, M, D, E, J, K\}$   
 $n_9 = J; n_{10} = K; \text{Open} = \{J, K, D, G, M, L\}, \text{Closed} = \{A, C, F, B, E\}, f(n_9) = 2; f(n_{10}) = 0$
6. The algorithm expands K in order to obtain  $M = \{\}$ ,  $G = \{A, B, C, F, G, L, M, D, E, J, K\}$   
K is a solution and the algorithm terminates. Heuristic search expands  $\text{Closed} = \{A, C, F, B, E, K\}$  6 nodes

**HEURISTIC-SEARCH:** USE THE FUNCTION  $f(n) = h(n)$  AND SORT THE OPEN LIST USING  $f$  VALUES, LIFO.

1. The algorithm selects A and expands A (applies  $\Gamma$ ) in order to obtain  $M = \{B, C\}$   
 $n_1 = B; n_2 = C; \text{Open} = \{B, C\}, \text{Closed} = \{A\}, G = \{A, B, C\}, f(n_1) = 20; f(n_2) = 10, \text{Open} = \{C_{10}, B_{20}\}$
2. The algorithm expands C in order to obtain  $M = \{F, G\}$   
 $n_3 = F; n_4 = G; \text{Open} = \{F, G, B\}, \text{Closed} = \{A, C\}, G = \{A, B, C, F, G\}, f(n_3) = 8; f(n_4) = 20, \text{Open} = \{F_8, G_{20}, B_{20}\}$
3. The algorithm expands F in order to obtain  $M = \{L, M\}$ ,  $G = \{A, B, C, F, G, L, M\}$ ;  $n_5 = L; n_6 = M$   
 $\text{Open} = \{M, L, G, B\}, \text{Closed} = \{A, C, F\}, f(n_5) = 27; f(n_6) = 22, \text{Open} = \{G_{20}, B_{20}, M_{22}, L_{27}\}$
4. The algorithm expands G in order to obtain  $M = \{N, P\}$ ;  $G = \{A, B, C, F, G, L, M, N, P\}$ ;  $n_7 = N; n_8 = P$   
 $\text{Open} = \{N, P, B, M, L\}, \text{Closed} = \{A, C, F, G\}, f(n_7) = 0; f(n_8) = 9, \text{Open} = \{N_0, P_9, B_{20}, M_{22}, L_{27}\}$
5. The algorithm expands N in order to obtain  $M = \{\}$ ,  $G = \{A, B, C, F, G, L, M, N, P\}$   
N is a solution and the algorithm terminates. Heuristic search expands  $\text{Closed} = \{A, C, F, G, N\}$  5 nodes

**A\* SEARCH:** USES  $f(n) = g(n) + h(n)$  where  $g(n) = \text{depth}(n)$  &  $h(n) = \text{cost}$  AND SORT THE OPEN LIST USING  $f$

1. The algorithm selects A and expands A (applies  $\Gamma$ ) in order to obtain  $M = \{B, C\}$   
 $n_1 = B; n_2 = C; \text{Open} = \{B, C\}, \text{Closed} = \{A\}, G = \{A, B, C\}, f(n_1) = 1 + 20; f(n_2) = 1 + 10, \text{Open} = \{C_{11}, B_{21}\}$
2. The algorithm expands C in order to obtain  $M = \{F, G\}$   
 $n_3 = F; n_4 = G; \text{Open} = \{F, B, G\}, \text{Closed} = \{A, C\}, G = \{A, B, C, F, G\}, f(n_3) = 2 + 8; f(n_4) = 2 + 20, \text{Open} = \{F_{10}, B_{21}, G_{22}\}$
3. The algorithm expands F in order to obtain  $M = \{L, M\}$ ;  $G = \{A, B, C, F, G, L, M\}$ ,  $n_5 = L; n_6 = M$   
 $\text{Open} = \{B, G, M, L\}, \text{Closed} = \{A, C, F\}, f(n_5) = 3 + 27; f(n_6) = 3 + 22, \text{Open} = \{B_{21}, G_{22}, M_{25}, L_{30}\}$
4. The algorithm expands B in order to obtain  $M = \{D, E\}$ ;  $G = \{A, B, C, F, G, L, M, D, E\}$ ,  $n_7 = D; n_8 = E$   
 $\text{Open} = \{E, D, G, M, L\}, \text{Closed} = \{A, C, F, B\}, f(n_7) = 2 + 13; f(n_8) = 2 + 12, \text{Open} = \{E_{14}, D_{15}, G_{22}, M_{25}, L_{30}\}$
5. The algorithm expands E in order to obtain  $M = \{J, K\}$ ;  $G = \{A, B, C, F, G, L, M, D, E, J, K\}$ ,  $n_9 = J; n_{10} = K$   
 $\text{Open} = \{J, K, D, G, M, L\}, \text{Closed} = \{A, C, F, B, E\}, f(n_9) = 3 + 2; f(n_{10}) = 3 + 0; \text{Open} = \{K_3, J_5, D_{15}, G_{22}, M_{25}, L_{30}\}$
6. The algorithm expands K in order to obtain  $M = \{\}$ ,  $G = \{A, B, C, F, G, L, M, D, E, J, K\}$   
K is a solution and the algorithm terminates. Heuristic search expands  $\text{Closed} = \{A, C, F, B, E, K\}$  6 nodes

N is found by heuristic search with LIFO:  $\{A, C, F, G\}$  comes before  $\{A, C, F, B\}$  &  $h(B) = h(G) = 20$  & LIFO orders G before B.

(25)  
**IV. Computation Deduction.**  
 We wish to find the last coach in a short list of UF coaches. Using **Resolution Refutation** deduce the following computation to obtain a value for the goal (3 pts) by performing a consistent Refutation Trace (17 pts) for the goal and prove (or provide a good argument) its consistency (5 pts.) Make sure your resolution refutation trace is clearly marked and it follows a complete strategy.

Facts:

$F_1: \text{last}(\text{cons}(\text{U}, \text{nil}), \text{U}).$

Rules:

$R_1: \text{last}(\text{X}, \text{Y}) \rightarrow \text{last}(\text{cons}(\text{W}, \text{X}), \text{Y})$

Goal:  $(\exists z) \text{last}(\text{cons}(\text{steve}, \text{cons}(\text{ron}, \text{cons}(\text{urban}, \text{nil}))), z)$

{Note: If you prefer, you may use the notation  $\text{last}(\text{ (steve ron urban), } z).$ }

Required: Give the resolution trace (17 pts), show the substitutions are consistent (5pts), and obtain the value of the goal (3 pts).

(1 pts) I am using Set-of-Support which is a complete strategy

$F_1: \text{last}(\text{cons}(\text{U}, \text{nil}), \text{U}).$

$R_1: \sim \text{last}(\text{X}, \text{Y}) \vee \text{last}(\text{cons}(\text{W}, \text{X}), \text{Y})$

$\sim \text{Goal}: \sim \text{last}(\text{ (steve ron urban), } z)$

(4 pts)  $\mathfrak{R}_1 = \mathfrak{R}\{\sim \text{Goal}, R_1\}: \sim \text{last}(\text{X}, \text{Y}) \{ \text{steve}/\text{W}, (\text{ron urban})/\text{X}, z/\text{Y} \}$

$\mathfrak{R}_1: \sim \text{last}(\text{ (ron urban), } z)$

(4 pts)  $\mathfrak{R}_2 = \mathfrak{R}\{\mathfrak{R}_1, R_1\}: \sim \text{last}(\text{X}', \text{Y}') \{ \text{ron}/\text{W}', (\text{urban})/\text{X}', z/\text{Y}' \}$

$\mathfrak{R}_2: \sim \text{last}(\text{(urban), } z)$

(4 pts)  $\mathfrak{R}_3 = \mathfrak{R}\{\mathfrak{R}_2, F_1\}: \text{nil} \{ \text{urban}/\text{U}, \text{U}/z \}$

(4 pts) Therefore  $z = \text{U} = \text{Y}' = \text{Y} = \text{urban}; \text{X}' = (\text{urban}); \text{W}' = \text{ron}; \text{X} = (\text{ron urban}); \text{W} = \text{steve}$

(3) Answer:  $(\exists z) \text{last}(\text{cons}(\text{steve}, \text{cons}(\text{ron}, \text{cons}(\text{urban}, \text{nil}))), z)$  is true  
with  $z = \text{urban}$

(5) Substitutions will be consistent because I changed variables every time I re-used any rules and all the variables were originally standardized apart. To prove consistency we assemble all the numerators in a set, say,  $U_1$ , and all the denominators in a set, say,  $U_2$  and show that  $U_1 = U_2\sigma$  and  $\sigma \neq \text{null}$ .

$U_1 = [\text{steve}, (\text{ron}, \text{urban}), z, \text{ron}, (\text{urban}), z, \text{urban}, \text{U}]$ ,  $U_2 = [\text{W}, \text{X}, \text{Y}, \text{W}', \text{X}', \text{Y}', \text{U}, z]$  and  $U_1 = U_2\sigma$

$\sigma = \{ \text{steve}/\text{W}, (\text{ron urban})/\text{X}, z/\text{Y}, \text{ron}/\text{W}', (\text{urban})/\text{X}', z/\text{Y}', \text{urban}/\text{U}, \text{U}/z \}$  and  $\sigma \neq \text{null}$ .

Fall 2006 was a Two-Period Exam

(20) Conversion to Clause Form

I. Transform the *wff*  $A$  below into CNF (**clause form**) matrix form. For each of the 10 “**official steps**” required give a brief description of the step and perform the step or write N/A {not applicable} on the space provided. Failure to follow this format will result in no credit. In *wff*  $A$  the set  $\{w,x,y\}$  are variables, the set  $\{E\}$  are functions and there are no constants.

$$\{wff A\}: (\forall x) \{ \sim E(x,v) \rightarrow [ (\exists y) (\exists w) ( E(y,w) \wedge (\forall x) \{ E(x,w) \rightarrow E(y,x) \} ) ] \}$$

(2) Step 0: **Eliminate redundant quantifiers and take the existential closure** \_\_\_\_\_

$$A_0: (\exists v) (\forall x) \{ \sim E(x,v) \rightarrow [ (\exists y) (\exists w) ( E(y,w) \wedge (\forall x) \{ E(x,w) \rightarrow E(y,x) \} ) ] \}$$

(2) Step 1: **Remove implications** \_\_\_\_\_

$$A_1: (\exists v) (\forall x) \{ \sim \sim E(x,v) \vee [ (\exists y) (\exists w) ( E(y,w) \wedge (\forall x) \{ \sim E(x,w) \vee E(y,x) \} ) ] \}$$

(2) Step 2: **Move the Negations down to the Atfs** \_\_\_\_\_

$$A_2: (\exists v) (\forall x) \{ E(x,v) \vee [ (\exists y) (\exists w) ( E(y,w) \wedge (\forall x) \{ \sim E(x,w) \vee E(y,x) \} ) ] \}$$

(1) Step 3: **Standardize Variables Apart** \_\_\_\_\_

$$A_3: (\exists v) (\forall x) \{ E(x,v) \vee [ (\exists y) (\exists w) ( E(y,w) \wedge (\forall z) \{ \sim E(z,w) \vee E(y,z) \} ) ] \}$$

(2) Step 4: **Skolemize: Let  $v=f(\cdot)=V$ ,  $y=f(x)$ ,  $w=g(x)$**  \_\_\_\_\_

$$A_4: (\forall x) \{ E(x,V) \vee [ (E(f(x),g(x))) \wedge (\forall z) \{ \sim E(z,g(x)) \vee E(f(x),z) \} ) ] \}$$

(1) Step 5: **Move universal quantifiers to the left** \_\_\_\_\_

$$A_5: (\forall x) (\forall z) \{ E(x,V) \vee [ (E(f(x),g(x))) \wedge \{ \sim E(z,g(x)) \vee E(f(x),z) \} ) ] \}$$

(4) Step 6: **Multiply &  $\vee$  over  $\wedge$  using  $P_1 \wedge (P_2 \vee P_3) \equiv (P_1 \wedge P_2) \vee (P_1 \wedge P_3)$  or  $P_1 \vee (P_2 \wedge P_3) \equiv (P_1 \vee P_2) \wedge (P_1 \vee P_3)$**  \_\_\_\_\_

Let  $P_1=E(x,V)$   $P_2=E(f(x),g(x))$   $P_3=\sim E(z,g(x))$   $P_4=E(f(x),z)$   $P_5=P_3 \vee P_4$

$$A_5: \{ P_1 \vee [(P_2 \wedge \{ P_3 \vee P_4 \})] \} \equiv \{ P_1 \vee [(P_2 \wedge P_5)] \} \equiv \{ [P_1 \vee P_2] \wedge [P_1 \vee P_5] \}$$

$$A_6: (\forall x) (\forall z) \{ [E(x,V) \vee E(f(x),g(x))] \wedge [E(x,V) \vee \sim E(z,g(x)) \vee E(f(x),z)] \}$$



**I. Conversion to Clause Form (continued)**

(1) Step 7: **Write in Matrix Form** \_\_\_\_\_

$$A_7: (\forall x) [ E(x, V) \vee E(f(x), g(x)) ] \\ (\forall x)(\forall z) [ E(x, V) \vee \sim E(z, g(x)) \vee E(f(x), z) ]$$

(1) Step 8: **Eliminate Universal Quantifiers** \_\_\_\_\_

$$A_8: [ E(x, V) \vee E(f(x), g(x)) ] \\ [ E(x, V) \vee \sim E(z, g(x)) \vee E(f(x), z) ]$$

(2) Step 9: **Rename Variables** \_\_\_\_\_

$$A_9: [ E(x_1, V) \vee E(f(x_1), g(x_1)) ] \\ [ E(x_2, V) \vee \sim E(z, g(x_2)) \vee E(f(x_2), z) ]$$

(2) Step 10: **Remove Tautologies & Simplify: None** \_\_\_\_\_

$$A_{10}: [ E(x_1, V) \vee E(f(x_1), g(x_1)) ] \\ [ E(x_2, V) \vee \sim E(z, g(x_2)) \vee E(f(x_2), z) ]$$

Fall 2006  
(25)

**II. Resolution Refutation**

THE CUSTOM OFFICIALS SEARCHED EVERYONE WHO ENTERED THIS COUNTRY WHO WAS NOT A VIP. SOME OF THE DRUG PUSHERS ENTERED THIS COUNTRY AND THEY WERE ONLY SEARCHED BY DRUG PUSHERS. NO DRUG PUSHER WAS A VIP. PROVE THAT SOME OF THE CUSTOM OFFICIALS WERE DRUG PUSHERS.

Solve by drawing a Refutation Graph resulting from **your choice of** strategy. (Make sure you mark clearly the required substitutions).

[Required: Please note the assigned point values. Each part **MUST** be answered with something. If left blank, then no credit will be assigned]

- (5) a. Represent the axioms/goal in the Predicate Calculus. Let  $E(x)$  mean “ $x$  entered this country,”  $V(x)$  mean “ $x$  was a VIP,”  $S(x,y)$  mean “ $y$  searched  $x$ ,”  $C(x)$  mean “ $x$  was a custom official” and  $P(x)$  mean “ $x$  was a drug pusher.”
- (2) b. Represent any commonsense knowledge needed to solve the problem using Predicate Calculus,
- (5) c. Convert your axioms, goal and commonsense knowledge (if any) to clause form,
- (10) d. Draw your Refutation Graph, show substitutions are consistent.
- (3) e. Describe how your graph meets the strategy. What other strategy could you have used and why?

(5) Answers Part a:

[1]  $(\forall x)([E(x) \wedge \sim V(x)] \rightarrow (\exists y)\{S(x,y) \wedge C(y)\})$  THE CUSTOM OFFICIALS SEARCHED EVERYONE WHO ENTERED THIS COUNTRY WHO WAS NOT A VIP

[2]  $(\exists x)[P(x) \wedge E(x) \wedge (\forall y)\{S(x,y) \rightarrow P(y)\}]$  SOME OF THE DRUG PUSHERS ENTERED THIS COUNTRY & THEY WERE ONLY SEARCHED BY DRUG PUSHERS

[3]  $(\forall x)[P(x) \rightarrow \sim V(x)]$  NO DRUG PUSHER WAS A VIP

Goal:  $(\exists x)[P(x) \wedge C(x)]$  SOME OF THE CUSTOM OFFICIALS WERE DRUG PUSHERS

(2) Answers Part b:

None

(5) Answers Part c:

[1]  $\sim E(x_1) \vee V(x_1) \vee S(x_1, f(x_1))$

[2]  $\sim E(x_2) \vee V(x_2) \vee C(f(x_2))$

[3]  $P(a)$

[4]  $E(a)$

[5]  $\sim S(a,y) \vee P(y)$

[6]  $\sim P(x_3) \vee \sim V(x_3)$

$\sim$ Goal:  $\sim P(z) \vee \sim C(z)$

**II. Resolution Refutation**(continued)

(10) Refutation Graph Part d:

[~Goal] [3]

$\mathfrak{R}_1$  [2]

$\mathfrak{R}_2$  [4]

$\mathfrak{R}_3$  [6]

$\mathfrak{R}_4$  [3]

nil

$\mathfrak{R}_1 = [\sim\text{Goal}]$  with [3]  $\sim C(a) \{a/z\}$   
 $\mathfrak{R}_2 = \mathfrak{R}_1$  with [2]  $\sim E(x_2) \vee V(x_2) \{a/f(x_2)\}$   
 $\mathfrak{R}_3 = \mathfrak{R}_2$  with [4]  $V(a) \{a/x_2\}$   
 $\mathfrak{R}_4 = \mathfrak{R}_3$  with [6]  $\sim P(a) \{a/x_3\}$   
 $\mathfrak{R}_5 = \mathfrak{R}_4$  with [3] nil

Consistency Check

$U_1 = [a, a, a, a]$   $U_2 = [z, f(x_2), x_2, x_3, ]$

$U_1 = U_2\sigma$  with  $\sigma = [a/z, a/f(x_2), a/x_2, a/x_3]$

Since  $U_1$  &  $U_2$  unify with a non-nil substitution  $\sigma$ , then the substitutions are consistent

(3) Answer Part e: My strategy is Set-of-Support

Every resolvent  $\mathfrak{R}_1$ - $\mathfrak{R}_5$  comes from the negation of the wff to be proved.

In logic, a Horn clause is a clause (a disjunction of literals) with at most one positive literal.

The clauses are not Horn because there is more than 1 positive literal in clauses [1] and [2].

I could have used ancestry-filtered or breadth-first because they are complete strategies.

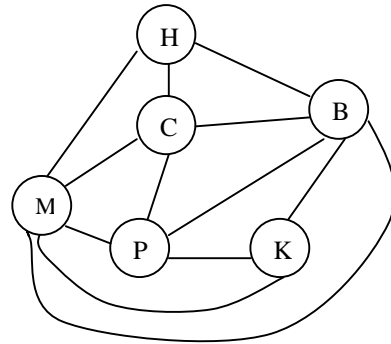
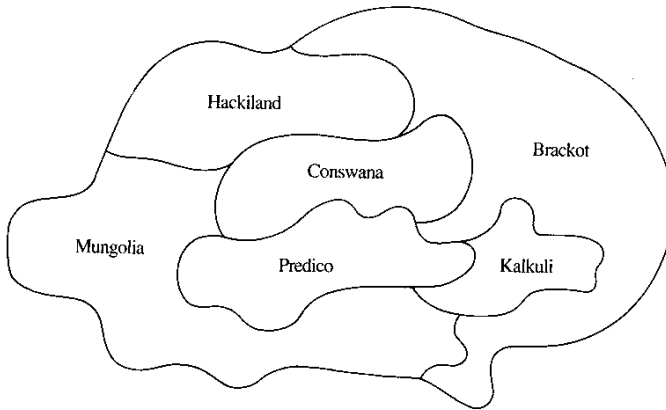
I cannot use linear-input or unit preference because they are incomplete strategies.

Fall 2006  
(30)

**III. Heuristic Search**

A map is to be colored with a set of  $n$  distinct colors, such that no two adjacent countries have the same color. If you can use colors {yellow, red, white and green} what is a legal coloring for the following map? Colorings are represented as lists of pairs: ((country color) (country color)...) ( (country color) (country color)...) )

- a. Suppose  $Sol_1$  represents the use of the A\* algorithm with heuristic function  $h_1(n)$ =number of uncolored countries.
- b. Suppose  $Sol_2$  represents the use of the A\* algorithm with heuristic function  $h_2(n)$ =Of two states with the same number of uncolored countries, the one with more options open is better. The number of options of a partial coloring might be measured by finding the uncolored country with the fewest possible colors, and returning the number of possible colors for that country.
- c. Give the A\* results for  $Sol_1$  and for  $Sol_2$  if the countries are always picked in {H C P K B M} order and the colors are picked in {Y R W G} order. How much better is  $Sol_2$  over  $Sol_1$ ?



Hidden Solution {send back}  
{Y,R,W,G}  
[H,C,P,K,B,M]  
17 Nodes  
 $f(n)=h_1(n)$  = number of uncolored countries

EEL-5840  
Fall 2009

Class Exam 2  
Sample Questions

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(Name)

### III. Heuristic Search (continued)

Suppose  $Sol_2$  represents the use of the A\* algorithm with heuristic function  $h_2(n)$ . Of two states with the same number of uncolored countries, the one with more options open is better. The number of options of a partial coloring might be measured by finding the uncolored country with the fewest possible colors, and returning the number of possible colors for that country

Hidden Solution {send back}

{Y,R,W,G}

[H,C,P,K,B,M]

8 Nodes

$f(n)=h_2(n)$ =the number of colors for the  
uncolored country with fewest possible colors  
Solution two is about 7/15 or about 50% better

Fall 2006  
(25)

**IV. Computation Deduction.**

We wish to make a set of UF basketball centers from a list of tall players. Using **Resolution Refutation** deduce the following computation to obtain a value for the goal (2 pts) by performing a consistent Refutation Trace (19 pts) for the goal and *prove (or provide a good argument for)* its consistency (4 pts.) Make sure your resolution refutation trace is clearly marked and it follows a complete strategy. Assume that the evaluation of member is built-in, e.g., member(a,(a b)) returns true, and member (c,(a b)) returns nil.

Facts:

F<sub>1</sub>: makeset(nil,nil).

Rules:

R<sub>1</sub>: [ member(X<sub>1</sub>,Y<sub>1</sub>) ∧ makeset(Y<sub>1</sub>,Z<sub>1</sub>) ] → makeset(cons(X<sub>1</sub>,Y<sub>1</sub>),Z<sub>1</sub>).

R<sub>2</sub>: [ ~member(X<sub>2</sub>,Y<sub>2</sub>) ∧ makeset(Y<sub>2</sub>,Z<sub>2</sub>) ] → makeset(cons(X<sub>2</sub>,Y<sub>2</sub>),cons(X<sub>2</sub>,Z<sub>2</sub>)).

Goal: (∃z)(makeset(cons(AL, cons(JOAKIM, cons(AL,nil))), z))

{ Note: If you prefer, you may use the notation makeset( (AL JOAKIM AL), z) }

Required: Give the entire resolution trace (18 pts) using a complete strategy (tell me what strategy (1)), show the substitutions are consistent (4pts), and obtain the value of the goal (2 pts).

(1 pts) I am using Set-of-Support which is a complete strategy

F<sub>1</sub>: makeset(nil,nil).

R<sub>1</sub>: ~member(X<sub>1</sub>,Y<sub>1</sub>) ∨ ~makeset(Y<sub>1</sub>,Z<sub>1</sub>) ∨ makeset(cons(X<sub>1</sub>,Y<sub>1</sub>),Z<sub>1</sub>).

R<sub>2</sub>: member(X<sub>2</sub>,Y<sub>2</sub>) ∨ ~makeset(Y<sub>2</sub>,Z<sub>2</sub>) ∨ makeset(cons(X<sub>2</sub>,Y<sub>2</sub>),cons(X<sub>2</sub>,Z<sub>2</sub>)).

~Goal: ~makeset(cons(al, cons(joakim, cons(al,nil))), z)

(3 pts)  $\mathfrak{R}_1 = \mathfrak{R} \{ \sim\text{Goal}, R_1 \} : \sim\text{member}(X_1, Y_1) \vee \sim\text{makeset}(Y_1, Z_1) \{ AL/X_1, (JOAKIM AL)/Y_1, z/Z_1 \}$

$\mathfrak{R}_1 : \sim\text{member}(AL, (JOAKIM AL)) \vee \sim\text{makeset}((JOAKIM AL), z)$  with  $\sim\text{member}(AL, (JOAKIM AL))$  returning nil

$\mathfrak{R}_1 : \sim\text{makeset}((JOAKIM AL), z)$

(3 pts)  $\mathfrak{R}_2 = \mathfrak{R} \{ \mathfrak{R}_1, R_1 \} : \sim\text{member}(X'_1, Y'_1) \vee \sim\text{makeset}(Y'_1, Z'_1) \{ JOAKIM/X'_1, (AL)/Y'_1, z/Z'_1 \}$

$\mathfrak{R}_2 : \sim\text{member}(JOAKIM, (AL)) \vee \sim\text{makeset}((AL), z)$  with  $\sim\text{member}(JOAKIM, (AL))$  returning true i.e., inconsistent!

(3 pts)  $\mathfrak{R}_3 = \mathfrak{R} \{ \mathfrak{R}_1, R_2 \} : \text{member}(X_2, Y_2) \vee \sim\text{makeset}(Y_2, Z_2) \{ JOAKIM/X_2, (AL)/Y_2, \text{cons}(JOAKIM, Z_2)/z \}$

$\mathfrak{R}_3 : \text{member}(JOAKIM, (AL)) \vee \sim\text{makeset}((AL), Z_2)$  with  $\text{member}(JOAKIM, (AL))$  returning nil

$\mathfrak{R}_3 : \sim\text{makeset}((AL), Z_2)$

**IV. Computation Deduction.** (continued)

(3 pts)  $\mathfrak{R}_4 = \mathfrak{R}\{\mathfrak{R}_3, R''_1\}$ :  $\sim\text{member}(X''_1, Y''_1) \vee \sim\text{makeset}(Y''_1, Z''_1) \{AL/X''_1, nil/Y''_1, Z_2/Z''_1\}$   
 $\mathfrak{R}_4$ :  $\sim(\text{member}(AL, nil)) \vee \sim\text{makeset}(nil, Z_2)$  with  $\sim\text{member}(AL, nil)$  returning true i.e., inconsistent!

(3 pts)  $\mathfrak{R}_5 = \mathfrak{R}\{\mathfrak{R}_3, R''_2\}$ :  $\text{member}(X''_2, Y''_2) \vee \sim\text{makeset}(Y''_2, Z''_2) \{AL/X''_2, nil/Y''_2, \text{cons}(AL, Z''_2)/Z_2\}$   
 $\mathfrak{R}_5$ :  $\text{member}(AL, nil) \vee \sim\text{makeset}(nil, Z''_2)$  with  $\text{member}(AL, nil)$  returning nil  
 $\mathfrak{R}_5$ :  $\sim\text{makeset}(nil, Z''_2)$

(3 pts)  $\mathfrak{R}_6 = \mathfrak{R}\{\mathfrak{R}_5, F_1\}$ :  $nil \{nil/Z''_2\}$   
Therefore,  $Z_2 = \text{cons}(AL, nil)$ , and  $z = \text{cons}(JOAKIM, Z_2) = \text{cons}(JOAKIM, \text{cons}(AL, nil)) = (JOAKIM AL)$

(4) Substitutions will be consistent because I changed variables every time I re-used any rules and all the variables were originally standardized apart. To prove consistency we assemble all the numerators in a set, say,  $U_1$ , and all the denominators in a set, say,  $U_2$  and show that  $U_1 = U_2\sigma$  and  $\sigma \neq \text{null}$ .

$U_1 = [AL, (JOAKIM, AL), z, JOAKIM, (AL), z, JOAKIM, (AL), \text{cons}(JOAKIM, Z_2), AL, nil, Z_2, AL, nil, \text{cons}(AL, Z''_2), nil]$ ,  
 $U_2 = [X_1, Y_1, Z_1, X'_1, Y'_1, Z'_1, X_2, Y_2, z, X''_1, Y''_1, Z''_1, X''_2, Y''_2, Z_2, Z''_2]$  and  $U_1 = U_2\sigma$   
 $\sigma = \{AL/X_1, (JOAKIM AL)/Y_1, z/Z_1, JOAKIM/X'_1, (AL)/Y'_1, z/Z'_1, JOAKIM/X_2, (AL)/Y_2, \text{cons}(JOAKIM, Z_2)/z,$   
 $AL/X''_1, nil/Y''_1, Z_2/Z''_1, AL/X''_2, nil/Y''_2, \text{cons}(AL, Z''_2)/Z_2, nil/Z''_2\}$  and  $\sigma \neq \text{null}$ .

(2) Answer:  $(\exists z) (\text{makeset}(\text{cons}(AL, \text{cons}(JOAKIM, \text{cons}(AL, nil))), z))$  is true  
with  $z = \text{cons}(JOAKIM, \text{cons}(AL, nil))$  or  $z = (JOAKIM AL)$



Fall 2007

(20) Conversion to Clause Form

I. Transform the *wff*  $A$  below into **clause form**. For each of the 10 “official steps” {the order is important!} required give a brief description of the step and perform the step or write N/A {not applicable} on the space provided. Failure to follow this format will result in **no credit**. In *wff*  $A$  the set  $\{v, x, y, z\}$  are variables, the set  $\{P, Q, R\}$  are functions and there are no constants.

$$\{wff A\}: (\forall x)(P(x) \rightarrow \{ \sim \forall y [ \sim Q(x, y) \rightarrow P(y) ] \wedge \forall y \exists z [ R(x, y) \rightarrow P(x) ] \})$$

(2) Step 0: **Eliminate redundant quantifiers and take the existential closure.** \_\_\_\_\_

The variable  $v$  is free, put a  $\exists v$  in front of the entire *wff*. Remove  $\exists z$  since it is redundant

$$A_0: (\exists v)(\forall x)(P(x) \rightarrow \{ \sim \forall y [ \sim Q(x, y) \rightarrow P(v) ] \wedge \forall y [ R(x, y) \rightarrow P(x) ] \})$$

(2) Step 1: **Remove implications** \_\_\_\_\_

$$A_1: (\exists v)(\forall x)(\sim P(x) \vee \{ \sim \forall y [ Q(x, y) \vee P(v) ] \wedge \forall y [ \sim R(x, y) \vee P(x) ] \})$$

(2) Step 2: **Move the Negations down to the *Atoms*** \_\_\_\_\_

$$A_2: (\exists v)(\forall x)(\sim P(x) \vee \{ \exists y [ \sim Q(x, y) \wedge \sim P(v) ] \wedge \forall y [ \sim R(x, y) \vee P(x) ] \})$$

(1) Step 3: **Standardize Variables Apart** \_\_\_\_\_

$$A_3: (\exists v)(\forall x)(\sim P(x) \vee \{ \exists y [ \sim Q(x, y) \wedge \sim P(v) ] \wedge \forall z [ \sim R(x, z) \vee P(x) ] \})$$

(2) Step 4: **Skolemize: Let  $v=f(\cdot)=V$ ,  $y=f(x)$**  \_\_\_\_\_

$$A_4: (\forall x)(\sim P(x) \vee \{ [\sim Q(x, f(x)) \wedge \sim P(V)] \wedge \forall z [ \sim R(x, z) \vee P(x) ] \})$$

(1) Step 5: **Move universal quantifiers to the left** \_\_\_\_\_

$$A_5: (\forall x) (\forall z) (\sim P(x) \vee \{ [\sim Q(x, f(x)) \wedge \sim P(V)] \wedge [\sim R(x, z) \vee P(x)] \})$$

(4) Step 6: **Multiply &  $d^n$   $\vee$  over  $\wedge$  using  $E_1 \vee (E_2 \wedge E_3) \equiv (E_1 \vee E_2) \wedge (E_1 \vee E_3)$**  \_\_\_\_\_

Let  $E_1 = \sim P(x)$ ;  $E_2 = \sim Q(x, f(x)) \wedge \sim P(V)$ ;  $E_3 = \sim R(x, z) \vee P(x)$ ; thus  $A: E_1 \vee (E_2 \wedge E_3) \equiv (E_1 \vee E_2) \wedge (E_1 \vee E_3)$

$$A_6: \{ (E_1 \vee E_2) \wedge (E_1 \vee E_3) \} \equiv \{ [E_1 \vee \sim Q(x, f(x))] \wedge [E_1 \vee \sim P(V)] \} \wedge [E_1 \vee E_3]$$

$$A_6': (\forall x) (\forall z) \{ [\sim P(x) \vee \sim Q(x, f(x))] \wedge [\sim P(x) \vee \sim P(V)] \wedge [\sim P(x) \vee \sim R(x, z) \vee P(x)] \}$$

**I. Conversion to Clause Form (continued)**

(1) Step 7: **Write in Matrix Form** \_\_\_\_\_

$$\begin{aligned} A_7: & (\forall x) [\sim P(x) \vee \sim Q(x, f(x))] \\ & (\forall x) [\sim P(x) \vee \sim P(V)] \\ & (\forall x)(\forall z) [\sim P(x) \vee \sim R(x, z) \vee P(x)] \end{aligned}$$

(1) Step 8: **Eliminate Universal Quantifiers** \_\_\_\_\_

$$\begin{aligned} A_8: & [\sim P(x) \vee \sim Q(x, f(x))] \\ & [\sim P(x) \vee \sim P(V)] \\ & [\sim P(x) \vee \sim R(x, z) \vee P(x)] \end{aligned}$$

(2) Step 9: **Rename Variables** \_\_\_\_\_

$$\begin{aligned} A_9: & [\sim P(x_1) \vee \sim Q(x_1, f(x_1))] \\ & [\sim P(x_2) \vee \sim P(V)] \\ & [\sim P(x_3) \vee \sim R(x_3, z) \vee P(x_3)] \end{aligned}$$

(2) Step 10: **Remove Tautologies & Simplify:** \_\_\_\_\_

$$\begin{aligned} & [\sim P(x_3) \vee \sim R(x_3, z) \vee P(x_3)] = [true \vee \sim R(x_3, z)] = true. \text{ Also } row_1 \wedge row_2 \wedge true = row_1 \wedge row_2 \\ A_{10}: & [\sim P(x_1) \vee \sim Q(x_1, f(x_1))] \\ & [\sim P(x_2) \vee \sim P(V)] \\ & \text{But since } (\forall x_2) [\sim P(x_2) \vee \sim P(V)] \equiv \sim P(x_2), \text{ i.e., } \sim P(x_2) \text{ subsumes } \sim P(V) \\ & [\sim P(x_1) \vee \sim Q(x_1, f(x_1))] \wedge \sim P(x_2) \equiv \sim P(x) \end{aligned}$$

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**II. Resolution Refutation**

The mathematical definition of the factorial function is: (i)  $\text{Fact}(0)=1$ , (ii)  $\text{Fact}(k)=k*\text{Fact}(k-1)$

Some suitable axioms for factorial are: (i)  $\text{Fact}(0)=1$   
(ii)  $[k-1=j \wedge \text{Fact}(j)=m \wedge k*m=n] \rightarrow [\text{Fact}(k)=n]$   
(iii)  $(\forall x)(\forall y)[x=y]$  with side effect  $\{\text{eval}(x)/y\}$

Using the axioms find the value of  $2!$  by using **Resolution Refutation** and **answer extraction**. Solve by drawing a **Refutation Graph** resulting from **your choice of** strategy. (Make sure you indicate clearly the required substitutions). Note: the function  $x=y$  evaluates the left argument and unifies it (equates it) with the right argument, e.g.,  $4-2=q$  evaluates  $4-2$  to  $2$  and sets  $q=2$  (i.e., it stores the substitution  $\{4-2/x, \text{eval}(4-2)/y, q/y, 2/q\}$  or  $\{2/q\}$  in the system.)

[Required: Please note the assigned point values. Each subpart MUST be answered with something. If left blank, then **zero** credit]

- (4) a. Represent the axioms/goal in clause form.
- (2) b. Is any commonsense knowledge needed to solve the problem using Predicate Calculus? Explain.
- (14) c. Give the Resolvents with the required substitutions.
- (5) d. Draw your Refutation Graph.
- (3) e. Prove formally that your substitutions are consistent.
- (2) f. Describe how your graph meets the strategy. What other strategy could you have used and why?

(4) Answers Part a:

[1]  $\text{Fact}(0)=1$

[2]  $\sim k-1=j \vee \sim \text{Fact}(j)=m \vee \sim k*m=n \vee \text{Fact}(k)=n$  i.e.,  $k-1 \neq j \vee \text{Fact}(j) \neq m \vee k*m \neq n \vee \text{Fact}(k)=n$

[3]  $[x=y]$  with side effect  $\{\text{eval}(x)/y\}$

Goal:  $[(\exists n)n=\text{Fact}(2)]$  or  $[4] \sim \text{Goal}: [\text{Fact}(2) \neq n \vee \text{Ans}(n)]$

(2) Answer Part b:

None

(14) Answers (Resolvents & required substitutions) Part c:

$\mathfrak{R}_1=[\sim \text{Goal}]$  with [2]  $k_1-1 \neq j_1 \vee \text{Fact}(j_1) \neq m_1 \vee k_1*m_1 \neq n_1 \vee \text{Ans}(n) \{n/n_1, 2/k_1\}$

$\mathfrak{R}_1:$   $2-1 \neq j_1 \vee \text{Fact}(j_1) \neq m_1 \vee 2*m_1 \neq n \vee \text{Ans}(n)$

$\mathfrak{R}_2=[\mathfrak{R}_1]$  with [3]  $\text{Fact}(j_1) \neq m_1 \vee 2*m_1 \neq n \vee \text{Ans}(n) \{1/j_1\}$

$\mathfrak{R}_2:$   $\text{Fact}(1) \neq m_1 \vee 2*m_1 \neq n \vee \text{Ans}(n)$

$\mathfrak{R}_3=[\mathfrak{R}_2]$  with [2]  $k_2-1 \neq j_2 \vee \text{Fact}(j_2) \neq m_2 \vee k_2*m_2 \neq n_2 \vee 2*m_1 \neq n \vee \text{Ans}(n) \{1/k_2, m_1/n_2\}$

$\mathfrak{R}_3:$   $1-1 \neq j_2 \vee \text{Fact}(j_2) \neq m_2 \vee 1*m_2 \neq m_1 \vee 2*m_1 \neq n \vee \text{Ans}(n)$

$\mathfrak{R}_4=[\mathfrak{R}_3]$  with [3]  $\text{Fact}(j_2) \neq m_2 \vee 1*m_2 \neq m_1 \vee 2*m_1 \neq n \vee \text{Ans}(n) \{0/j_2\}$

$\mathfrak{R}_4:$   $\text{Fact}(0) \neq m_2 \vee 1*m_2 \neq m_1 \vee 2*m_1 \neq n \vee \text{Ans}(n)$

$\mathfrak{R}_5=[\mathfrak{R}_4]$  with [1]  $1*m_2 \neq m_1 \vee 2*m_1 \neq n \vee \text{Ans}(n) \{1/m_2\}$

$\mathfrak{R}_5:$   $1*1 \neq m_1 \vee 2*m_1 \neq n \vee \text{Ans}(n)$

$\mathfrak{R}_6=[\mathfrak{R}_5]$  with [3]  $2*m_1 \neq n \vee \text{Ans}(n) \{1/m_1\}$

$\mathfrak{R}_6:$   $2*1 \neq n \vee \text{Ans}(n)$

$\mathfrak{R}_7=[\mathfrak{R}_6]$  with [3]  $\text{nil} \vee \text{Ans}(n) \{2/n\}$

$\mathfrak{R}_7:$   $\text{Ans}(2)$

**II. Resolution Refutation(continued)**

(5) Refutation Graph Part d:

[~Goal] [2]

$\mathfrak{R}_1$  [3]

$\mathfrak{R}_2$  [2]

$\mathfrak{R}_3$  [3]

$\mathfrak{R}_4$  [1]

$\mathfrak{R}_5$  [3]

$\mathfrak{R}_6$  [3]

nil

(3) Consistency Check Part e:

Consistency Check  $\{n/n_1, 2/k_1, 1/j_1, 1/k_2, m_1/n_2, 0/j_2, 1/m_2, 1/m_1, 2/n\}$

$U_1=[n,2,1,1,m_1,0,1,1,2]$   $U_2=[n_1,k_1,j_1,k_2,n_2,j_2,m_2,m_1,n]$

$U_1=U_2\sigma$  with  $\sigma = [n/n_1, 2/k_1, 1/j_1, 1/k_2, m_1/n_2, 0/j_2, 1/m_2, 1/m_1, 2/n]$

Since  $U_1$  &  $U_2$  unify with a non-nil substitution  $\sigma$ , then the substitutions are consistent

(2) Answer Part f: My strategy is Set-of-Support

What other strategy could you have used and why? Explain:

Every resolvent  $\mathfrak{R}_1$ - $\mathfrak{R}_7$  comes from the negation of the wff to be proved.

In logic, a Horn clause is a clause (a disjunction of literals) with at most one positive literal.

The clauses are Horne because there is no more than 1 positive literal in all clauses.

I could have used ancestry-filtered or breadth-first because they are complete strategies.

I could use linear-input or unit preference because they are complete strategies for Horne Clauses.

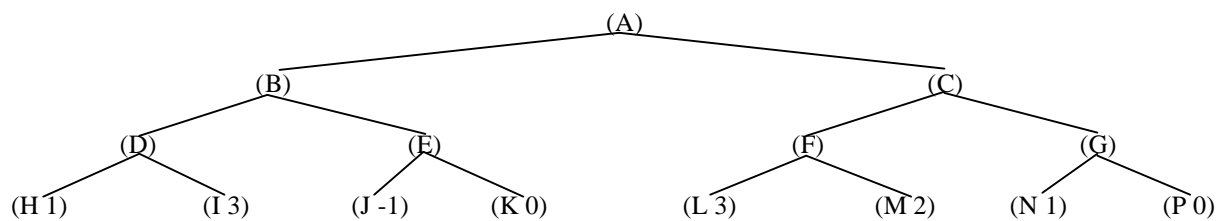
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**III. Adversarial Search**

Consider the following game tree in which the static scores (in parentheses at the tip nodes) are all from the first player's point of view.

- (5) a. Assuming that the first player is the maximizing player, what move should the first player choose?
- (5) b. Assuming that the first player is the minimizing player, what move should the first player choose?
- (5) c. What nodes would not need to be examined in part (a) using the alpha-beta algorithm—assuming that the nodes are examined in left-to-right order?
- (5) d. What nodes would not need to be examined in part (b) using the alpha-beta algorithm—assuming that the nodes are examined in left-to-right order?
- (5) e. Is the first player's move in parts (a) and (c) or in parts (b) and (d) different? Explain.



(5) Part (a):

D, E, F, G choose max or  $D \rightarrow (I\ 3)$ ;  $E \rightarrow (K\ 0)$ ;  $F \rightarrow (L\ 3)$ ;  $G \rightarrow (N\ 1)$   
 B, C chooses min of  $B \rightarrow E \rightarrow (K\ 0)$ ;  $C \rightarrow G \rightarrow (N\ 1)$   
 A chooses max or  $A \rightarrow C \rightarrow G \rightarrow (N\ 1)$   
 A chooses C toward solution  $A \rightarrow C \rightarrow G \rightarrow N$

(5) Part (b):

D, E, F, G choose min or  $D \rightarrow (H\ 1)$ ;  $E \rightarrow (J\ -1)$ ;  $F \rightarrow (M\ 2)$ ;  $G \rightarrow (P\ 0)$   
 B, C chooses max of  $B \rightarrow D \rightarrow (H\ 1)$ ;  $C \rightarrow F \rightarrow (M\ 2)$   
 A chooses min or  $A \rightarrow B \rightarrow D \rightarrow (H\ 1)$   
 A chooses B toward solution  $A \rightarrow B \rightarrow D \rightarrow H$

(5) Part (c):

Evaluate (H 1) & (I 3) D chooses max or  $\alpha_D = 3$  from (I 3); Now B chooses min so  $\beta_B \leq 3$  from (I 3)  
 Evaluate (J -1); now  $\alpha_E \geq -1$  from (J -1) and  $\beta_B \leq 3$  (H 3) therefore no Beta Cutoff and continue  
 Evaluate (K 0); now  $\alpha_E = 0$  from (K 0) and  $\beta_B = 0$  from (K 0)  
 Now B chooses min so  $\beta_B = 0$  from (K 0), therefore  $\alpha_A \geq 0$  from (K 0)  
 Evaluate (L 3) and  
 Evaluate (M 2); now  $\alpha_F = 3$  from (L 3); and  $\beta_C \leq 3$  from (L 3)  
 Evaluate (N 1); now  $\alpha_G \geq 1$  from (N 1); no cutoff & continue  
 Evaluate (P 0); now  $\alpha_G = 1$  from (N 1);  $\beta_C = 1$  from (N 1) no cutoff & continue  
 A chooses C to G to N (N 1) i.e.,  $A \rightarrow C \rightarrow G \rightarrow N$   
 Alpha-Beta had Pruning resulted in no advantage

**III. Adversarial Search. (continued)**

(5) Part (d):

Evaluate (H 1) & (I 3) and D chooses min or  $\beta_D=1$  from (H 1)  
Now B chooses max so  $\alpha_E \geq 1$  from (H 1)  
Evaluate (J -1); now  $\beta_F \leq -1$  from (J -1) but  $\alpha_E \geq 1$  from (H 1)  
Alpha Cutoff at E, do not evaluate (K 0) and continue  $\alpha_E=1$  from (H 1);  $\beta_A \leq 1$  from (H 1)  
Evaluate (L 3)  
Evaluate (M 2) now  $\beta_F=2$  (M 2) and  $\alpha_C \geq 2$  (M 2)  
Beta cutoff at C.  $A \rightarrow B \rightarrow D \rightarrow H$   
Do Not Evaluate {K, G, N, P}

(5) Part (e):

A chooses C toward solution  $A \rightarrow C \rightarrow G \rightarrow N$  in both parts (a) and (c) because Alpha-Beta and Minimax produce the same results for the same problem.  
Similarly, A chooses B toward solution  $A \rightarrow B \rightarrow D \rightarrow H$  in both parts (b) and (d) because Alpha-Beta and Minimax produce the same results for the same problem.  
In my analysis that was indeed the case.

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**IV. Computation Deduction.**

The following facts and rules accomplish the evaluation of the inner product of two vectors. Note that  $\{A, As, B, Bs, N, Z\}$  are variables

Fact:

$F_1: \text{inner}(\text{nil}, \text{nil}, 0).$

$F_2: \text{is}(X, Y)$  with side effect  $\{\text{eval}(X)/Y\}.$

Rule:

$R_1: [\text{inner}(As, Bs, Ns) \wedge \text{is}(Ns + A * B, N)] \rightarrow \text{inner}(\text{cons}(A, As), \text{cons}(B, Bs), N).$

Goal:  $(\exists Z)(\text{inner}(\text{cons}(1, \text{cons}(2, \text{nil})), \text{cons}(3, \text{cons}(4, \text{nil})), Z))$

{ Note: If you prefer, you may use the notation  $\text{inner}((1\ 2), (3\ 4), Z)$  }

Required: Tell me what your strategy is (1 pt). Give the clause form (4 pts) of the axiom set & the negation of the goal. Give me the Resolution resolvents (15 pts) using a complete strategy. Prove the substitutions are consistent (4 pts). Obtain the value of the goal (1 pt). Note: the function  $\text{is}(X, Y)$  evaluates the left argument and unifies it (equates it) with the right argument, e.g.,  $\text{is}(4+2, Q)$  evaluates  $4+2$  to 6 and sets  $Q=6$  (i.e., it stores the substitution  $\{4+2/X, \text{eval}(4+2)/Y, Q/Y, 6/Q\}$  or  $\{6/Q\}$  in the system.)

(1) Tell me your strategy I am using Set-of-Support which is a complete strategy

(4) Give me your axioms & negation of the goal in clause form

$F_1: \text{inner}(\text{nil}, \text{nil}, 0).$

$F_2: \text{is}(X, Y)$  with side effect  $\{\text{eval}(X)/Y\}.$

$R: \sim \text{inner}(As, Bs, Ns) \vee \sim \text{is}(Ns + A * B, N) \vee \text{inner}(\text{cons}(A, As), \text{cons}(B, Bs), N).$

$\sim \text{Goal}: \sim \text{inner}(\text{cons}(1, \text{cons}(2, \text{nil})), \text{cons}(3, \text{cons}(4, \text{nil})), Z)$

(15) Give me the resolution resolvents

(3 pts)  $\mathfrak{R}_1 = \mathfrak{R}\{\sim \text{Goal}, R\}: \sim \text{inner}(As, Bs, Ns) \vee \sim \text{is}(Ns + A * B, N) \{1/A, \text{cons}(2, \text{nil})/As, 3/B, \text{cons}(4, \text{nil})/Bs, Z/N\}$

$\mathfrak{R}_1: \sim \text{inner}(\text{cons}(2, \text{nil}), \text{cons}(4, \text{nil}), Ns) \vee \sim \text{is}(Ns + 1 * 3, Z)$

(3 pts)  $\mathfrak{R}_2 = \mathfrak{R}\{\mathfrak{R}_1, R'\}: \sim \text{inner}(As', Bs', Ns') \vee \sim \text{is}(Ns' + A' * B', N') \vee \sim \text{is}(Ns + 1 * 3, Z) \{2/A', \text{nil}/As', 4/B', \text{nil}/Bs', Ns/N'\}$

$\mathfrak{R}_2: \sim \text{inner}(\text{nil}, \text{nil}, Ns') \vee \sim \text{is}(Ns' + 2 * 4, N') \vee \sim \text{is}(Ns + 1 * 3, Z)$

(3 pts)  $\mathfrak{R}_3 = \mathfrak{R}\{\mathfrak{R}_2, F_1\}: \sim \text{is}(Ns' + 2 * 4, N') \vee \sim \text{is}(Ns + 1 * 3, N) \{0/Ns\}$

$\mathfrak{R}_3: \sim \text{is}(0 + 2 * 4, N') \vee \sim \text{is}(N' + 1 * 3, Z)$

**IV. Computation Deduction.** (continued)

(3 pts)  $\mathfrak{R}_4 = \mathfrak{R}\{\mathfrak{R}_3, F_2\}$ :  $\sim is(N'+1*3, Z) \{0+2*4/X, N'/Y, 8/N'\}$   
 $\mathfrak{R}_4$ :  $\sim is(8+1*3, Z)$

(3 pts)  $\mathfrak{R}_5 = \mathfrak{R}\{\mathfrak{R}_4, F_2'\}$ :  $nil \{8+1*3/X', Z/Y', 11/Z\}$

Therefore,  $\{11/Z\}$  yields that the inner product of (1 2) and (3 4) is 11 or  $inner((1\ 2), (3\ 4), 11)$  is true

(4) Prove the substitutions are consistent.

Substitutions will be consistent because I changed variables every time I re-used any rules and all the variables were originally standardiZed apart. To prove consistency we assemble all the numerators in a set, say,  $U_1$ , and all the denominators in a set, say,  $U_2$  and show that  $U_1 = U_2\sigma$  and  $\sigma \neq null$ .

$\{1/A, cons(2, nil)/As, 3/B, cons(4, nil)/Bs, Z/N\} \{0/Ns\} \{0+2*4/X, N'/Y, 8/N'\}$   
 $\{2/A', nil/As', 4/B', nil/Bs', Ns/N'\} \{8+1*3/X', Z/Y', 11/Z\}$   
 $U_1 = [1, cons(2, nil), 3, cons(4, nil), Z, 0, (0+2*4), N', 8, 2, nil, 4, nil, ns, (8+1*3), Z, 11],$   
 $U_2 = [A, As, B, Bs, N, Ns, X, Y, N', A', As', B', Bs', N', X', Y', Z]$  and  $U_1 = U_2\sigma$   
 $\sigma = \{ \{1/A, cons(2, nil)/As, 3/B, cons(4, nil)/Bs, Z/N, 0/Ns, 0+2*4/X, N'/Y, 8/N',$   
 $2/A', nil/As', 4/B', nil/Bs', Ns/N', 8+1*3/X', Z/Y', 11/Z\}$  and  $\sigma \neq null$ .

(1) Give me the solved goal, i.e., the answer:

*Answer:*  $(\exists Z) (inner(cons(1, cons(2, nil)), cons(3, cons(4, nil)), Z))$  is true  
with  $Z = 11$



Fall 2008

(20) Conversion to Clause Form

I. Transform the *wff*  $A$  below into **clause form**. For each of the 10 “official steps” {the order is important!} required give a brief description of the step and perform the step or write N/A {not applicable} on the space provided. Failure to follow this format will result in **no credit**. In *wff*  $A$  the set  $\{w, x, y, z\}$  are variables, the set  $\{\text{Animal, Loves}\}$  are functions and there are no constants.

$$\{\text{wff } A\}: (\forall x)(\exists w)[(\forall y)\{\text{Animal}(y) \rightarrow \text{Loves}(x,y)\} \rightarrow \{(\forall z)(\exists y)\text{Loves}(y,x)\}]$$

(2) Step 0: **Eliminate redundant quantifiers and take the existential closure.** \_\_\_\_\_  
 Remove  $(\exists w)$  &  $(\forall z)$  since they are redundant  
 $A_0: (\forall x)[(\forall y)\{\text{Animal}(y) \rightarrow \text{Loves}(x,y)\} \rightarrow \{(\exists y)\text{Loves}(y,x)\}]$

(2) Step 1: **Remove implications** \_\_\_\_\_  
 $A_1: (\forall x)[\sim (\forall y)\{\sim \text{Animal}(y) \vee \text{Loves}(x,y)\} \vee \{(\exists y)\text{Loves}(y,x)\}]$

(2) Step 2: **Move the Negations down to the Atfs** \_\_\_\_\_  
 $A_2: (\forall x)[(\exists y)\{\text{Animal}(y) \wedge \sim \text{Loves}(x,y)\} \vee \{(\exists y)\text{Loves}(y,x)\}]$

(1) Step 3: **Standardize Variables Apart** \_\_\_\_\_  
 $A_3: (\forall x)[(\exists y)\{\text{Animal}(y) \wedge \sim \text{Loves}(x,y)\} \vee \{(\exists z)\text{Loves}(z,x)\}]$

(2) Step 4: **Skolemize: Let  $y=f(x)$ ,  $z=g(x)$**  \_\_\_\_\_  
 $A_4: (\forall x)[\{\text{Animal}(f(x)) \wedge \sim \text{Loves}(x, f(x))\} \vee \{\text{Loves}(g(x),x)\}]$

(1) Step 5: **Move universal quantifiers to the left** \_\_\_\_\_  
 $A_5: (\forall x)[\{\text{Animal}(f(x)) \wedge \sim \text{Loves}(x, f(x))\} \vee \{\text{Loves}(g(x),x)\}]$

(4) Step 6: **Multiply &  $d^n \vee$  over  $\wedge$  using  $(E_2 \wedge E_3) \vee E_1 \equiv E_1 \vee (E_2 \wedge E_3) \equiv (E_1 \vee E_2) \wedge (E_1 \vee E_3)$**  \_\_\_\_\_  
 Let  $E_1 = \text{Loves}(g(x),x)$ ;  $E_2 = \text{Animal}(f(x))$ ;  $E_3 = \sim \text{Loves}(x, f(x))$ ; thus  $A: E_1 \vee (E_2 \wedge E_3) \equiv (E_1 \vee E_2) \wedge (E_1 \vee E_3)$   
 $A_6: \{ (E_1 \vee E_2) \wedge (E_1 \vee E_3) \} \equiv \{ [\text{Loves}(g(x),x) \vee \text{Animal}(f(x))] \wedge [\text{Loves}(g(x),x) \vee \sim \text{Loves}(x, f(x))]\}$   
 or  $A_6: \{ (E_1 \vee E_2) \wedge (E_1 \vee E_3) \} \equiv \{ [\text{Animal}(f(x)) \vee \text{Loves}(g(x),x)] \wedge [\sim \text{Loves}(x, f(x)) \vee \text{Loves}(g(x),x)] \}$

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Class Exam 2  
Sample Questions

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(Name)

I. Conversion to Clause Form (continued)

(1) Step 7: **Write in Matrix Form** \_\_\_\_\_

$$A_7: (\forall x) [ \text{Animal}(f(x)) \vee \text{Loves}(g(x),x) ] \\ (\forall x) [ \sim\text{Loves}(x, f(x)) \vee \text{Loves}(g(x),x) ]$$

(1) Step 8: **Eliminate Universal Quantifiers** \_\_\_\_\_

$$A_8: [ \text{Animal}(f(x)) \vee \text{Loves}(g(x),x) ] \\ [ \sim\text{Loves}(x, f(x)) \vee \text{Loves}(g(x),x) ]$$

(2) Step 9: **Rename Variables** \_\_\_\_\_

$$A_9: [ \text{Animal}(f(x_1)) \vee \text{Loves}(g(x_1),x_1) ] \\ [ \sim\text{Loves}(x_2, f(x_2)) \vee \text{Loves}(g(x_2),x_2) ]$$

(2) Step 10: **Remove Tautologies & Simplify:** \_\_\_\_\_

$$A_{10}: [ \text{Animal}(f(x_1)) \vee \text{Loves}(g(x_1),x_1) ] \\ [ \sim\text{Loves}(x_2, f(x_2)) \vee \text{Loves}(g(x_2),x_2) ] \\ A_{10}': [ \text{Loves}(g(x),x) \vee \{ \text{Animal}(f(x)) \wedge \sim\text{Loves}(x, f(x)) \} ]$$

**II. Resolution Refutation (30)**

*The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is an American. Is Colonel West is a criminal?*

Prove that West is a criminal by using **Resolution Refutation**. Draw a **Refutation Graph** resulting from **your choice of** strategy. **(Indicate clearly the required substitutions).**

[Required: Please note the assigned point values. Each subpart **MUST** be answered with something. If left blank, then **zero** credit]

- (5) a. Represent the axioms/goal in the Predicate Calculus. {If you cannot do this, I will give it to you for the 5 points}
- (4) b. Represent the axioms/goal in clause form.
- (2) c. Is any commonsense knowledge needed to solve the problem? Explain. {If you can't do it, I will give it to you for 2 pts}
- (10) d. Give the Resolvents with the required substitutions.
- (5) e. Draw your Refutation Graph.
- (2) f. Prove formally that your substitutions are consistent.
- (2) g. Describe how your graph meets the strategy. What other strategy could you have used and why?

(5) Answers Part a:  
 [1]  $(\forall x)(\forall y)(\forall z) [\{ \text{American}(x) \wedge \text{Weapon}(y) \wedge \text{Sells}(x,y,z) \wedge \text{Hostile}(z) \} \rightarrow \text{Criminal}(x)]$   
 [2]  $(\exists x)[\text{Owns}(\text{Nono},x) \wedge \text{Missile}(x)]$   
 [3]  $(\forall x)\{[\text{Missile}(x) \wedge \text{Owns}(\text{Nono},x)] \rightarrow \text{Sells}(\text{West},x,\text{Nono})\}$   
 [4]  $\text{Enemy}(\text{Nono},\text{America})$   
 [5]  $\text{American}(\text{West})$   
 Goal:  $\text{Criminal}(\text{West})$  or [8]  $\sim\text{Goal: } \sim\text{Criminal}(\text{West})$

(4) Answer(s) Part b:  
 [1]  $\sim\text{American}(x_1) \vee \sim\text{Weapon}(y_1) \vee \sim\text{Sells}(x_1,y_1,z_1) \vee \sim\text{Hostile}(z_1) \vee \text{Criminal}(x_1)$   
 [2a]  $\text{Owns}(\text{Nono},M_1)$   
 [2b]  $\text{Missile}(M_1)$   
 [3]  $\sim\text{Missile}(x_2) \vee \sim\text{Owns}(\text{Nono},x_2) \vee \text{Sells}(\text{West},x_2,\text{Nono})$   
 [4]  $\text{Enemy}(\text{Nono},\text{America})$   
 [5]  $\text{American}(\text{West})$   
 Goal:  $\text{Criminal}(\text{West})$  or [8]  $\sim\text{Goal: } \sim\text{Criminal}(\text{West})$

(2) Answer(s) Part c: All enemies of America are also hostile to America and all missiles are weapons.  
 [6]  $(\forall x)[\text{Enemy}(x,\text{America}) \rightarrow \text{Hostile}(x)] \equiv \sim\text{Enemy}(x_3,\text{America}) \vee \text{Hostile}(x_3)$   
 [7]  $(\forall x)[\text{Missile}(x) \rightarrow \text{Weapon}(x)] \equiv \sim\text{Missile}(x_4) \vee \text{Weapon}(x_4)$

(10) Answers (Resolvents & required substitutions) Part d:

$\mathfrak{R}_1 = [\sim\text{Goal}]$  with [1]  $\sim\text{American}(x_1) \vee \sim\text{Weapon}(y_1) \vee \sim\text{Sells}(x_1,y_1,z_1) \vee \sim\text{Hostile}(z_1) \{ \text{West}/x_1 \}$   
 $\mathfrak{R}_1:$   $\sim\text{American}(\text{West}) \vee \sim\text{Weapon}(y_1) \vee \sim\text{Sells}(\text{West},y_1,z_1) \vee \sim\text{Hostile}(z_1)$

$\mathfrak{R}_2 = [\mathfrak{R}_1]$  with [5]  $\sim\text{Weapon}(y_1) \vee \sim\text{Sells}(\text{West},y_1,z_1) \vee \sim\text{Hostile}(z_1) \{ \}$

$\mathfrak{R}_3 = [\mathfrak{R}_2]$  with [7]  $\sim\text{Missile}(x_4) \vee \sim\text{Sells}(\text{West},y_1,z_1) \vee \sim\text{Hostile}(z_1) \{ y_1/x_4 \}$   
 $\mathfrak{R}_3:$   $\sim\text{Missile}(y_1) \vee \sim\text{Sells}(\text{West},y_1,z_1) \vee \sim\text{Hostile}(z_1)$

$\mathfrak{R}_4 = [\mathfrak{R}_3]$  with [2b]  $\sim\text{Sells}(\text{West},M_1,z_1) \vee \sim\text{Hostile}(z_1) \{ M_1/y_1 \}$

$\mathfrak{R}_5 = [\mathfrak{R}_4]$  with [3]  $\sim\text{Missile}(x_2) \vee \sim\text{Owns}(\text{Nono},x_2) \vee \sim\text{Hostile}(z_1) \{ M_1/x_2, \text{Nono}/z_1 \}$   
 $\sim\text{Missile}(M_1) \vee \sim\text{Owns}(\text{Nono},M_1) \vee \sim\text{Hostile}(\text{Nono})$

$\mathfrak{R}_6 = [\mathfrak{R}_5]$  with [2b]  $\sim\text{Owns}(\text{Nono},M_1) \vee \sim\text{Hostile}(\text{Nono})$

$\mathfrak{R}_7 = [\mathfrak{R}_6]$  with [2a]  $\sim\text{Hostile}(\text{Nono}) \{ \}$

$\mathfrak{R}_8 = [\mathfrak{R}_7]$  with [6]  $\sim\text{Enemy}(\text{Nono},\text{America}) \{ \text{Nono}/x_3 \}$

$\mathfrak{R}_9 = [\mathfrak{R}_8]$  with [4] nil

**II. Resolution Refutation(continued)**

(5) Refutation Graph Part d:

[~Goal] [1]

$\mathfrak{R}_1$  [5]

$\mathfrak{R}_2$  [7]

$\mathfrak{R}_3$  [2b]

$\mathfrak{R}_4$  [3]

$\mathfrak{R}_5$  [2b]

$\mathfrak{R}_6$  [2a]

$\mathfrak{R}_7$  [6]

$\mathfrak{R}_8$  [4]

nil

(2) Consistency Check Part e:

Consistency Check { West/ $x_1$ ,  $y_1/x_4$ ,  $M_1/y_1, M_1/x_2, \text{Nono}/z_1$ ,  $\text{Nono}/x_3$  }

$U_1 = [\text{West}, y_1, M_1, M_1, \text{Nono}, \text{Nono}]$   $U_2 = [x_1, x_4, y_1, x_2, z_1, x_3]$

$U_1 = U_2\sigma$  with  $\sigma = [\text{West}/x_1, y_1/x_4, M_1/y_1, M_1/x_2, \text{Nono}/z_1, \text{Nono}/x_3]$

Since  $U_1$  &  $U_2$  unify with a non-nil substitution  $\sigma$ , then the substitutions are consistent

Since I changed variable names in all clauses and used clauses once in each resolution, then all substitutions are consistent

(2) Answer Part f: My strategy is: Set-of-Support

What other strategy could you have used and why? Explain:

Every resolvent  $\mathfrak{R}_1$ - $\mathfrak{R}_8$  comes from the negation of the wff to be proved.

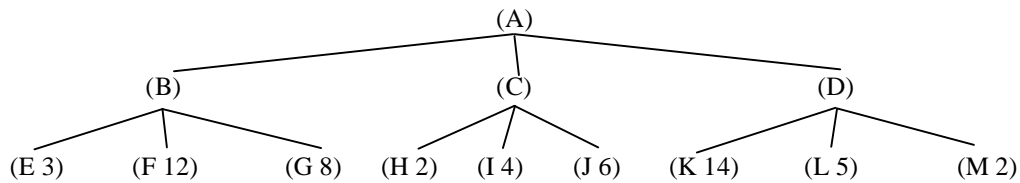
In logic, a Horn clause is a clause (a disjunction of literals) with at most one positive literal.

The clauses are Horn because there is no more than 1 positive literal in all clauses.

I could have used ancestry-filtered or breadth-first because they are complete strategies.

I could use linear-input because it is a complete strategy for Horn Clauses.

- (25)  
**III. Adversarial Search**  
 Consider the following game tree in which the static scores (in parentheses at the tip nodes) are all from the first player's point of view.
- (5) a. Assuming that the first player is the maximizing player, what move should the first player choose?
  - (5) b. Assuming that the first player is the minimizing player, what move should the first player choose?
  - (5) c. What nodes would not need to be examined in part (a) using the alpha-beta algorithm—assuming that the nodes are examined in left-to-right order?
  - (5) d. What nodes would not need to be examined in part (b) using the alpha-beta algorithm—assuming that the nodes are examined in left-to-right order?
  - (5) e. Is the first player's move in parts (a) and (c) or in parts (b) and (d) different? Explain.



(5) Part (a):

B, C, D choose min or B→(E 3); C→(H 2); D→(M 2)  
 A chooses max or A→B→(E 3)  
 A chooses B toward solution A→B→E and all nodes were evaluated

(5) Part (b):

B, C, D choose max or B→(F 12); C→(J 6); D→(K 14)  
 A chooses min or A→C→(J 6)  
 A chooses C toward solution A→C→J and all nodes were evaluated

(5) Part (c):

Evaluate (E 3) & (F 12) & (G 8); B chooses min so  $\beta_B = 3$  and  $\alpha_a \geq 3$  from (E 3)  
 Evaluate (H 2); now  $\beta_c \leq 2$  therefore Alpha Cutoff and do not evaluate I & J  
 Evaluate (K 14); now  $\beta_d \leq 14$ ; evaluate (L 5) now  $\beta_d \leq 5$ ; evaluate (M 2) and  $\beta_d = 2$  from (M 2)  
 A chooses max or A→B→(E 3)  
 A chooses B toward solution A→B→E  
 Alpha-Beta Pruning saved two nodes I and J

**III. Adversarial Search. (continued)**

(5) Part (d):

Evaluate (E 3) & (F 12) & (G 8); B chooses max so  $\alpha_b=12$  and  $\beta_a \leq 12$  from (F 12)

Evaluate (H 2) with  $\alpha_c \geq 2$ ; evaluate (I 4) with  $\alpha_c \geq 4$ ; evaluate (J 6) therefore with  $\alpha_c = 6$ ; now  $\beta_a \leq 6$

Evaluate (K 14); now  $\alpha_d \geq 14$ ; Beta Cutoff and do not evaluate L and M and  $\alpha_d = 14$

A chooses min or  $A \rightarrow C \rightarrow (J 6)$

Alpha-Beta Pruning saved two nodes L and M

(5) Part (e):

A chooses B toward solution  $A \rightarrow B \rightarrow E$  in both parts (a) and (c) because Alpha-Beta and Minimax produce the same results for the same problem.

Similarly, A chooses C toward solution  $A \rightarrow C \rightarrow J$  in both parts (b) and (d) because Alpha-Beta and Minimax produce the same results for the same problem.

In my analysis that was indeed the case.

(25)

**IV. Computation Deduction.**

In EEL-5840 Exam 1 we have a **TAIL RECURSIVE LISP** function COUNT-TOP-ATOMS (CTA for short) to count the number of top level atoms in a given list expression. Here are fact(s) and rule(s) to define the equivalent predicate IS\_CTA(LIS,N). IS\_CTA(LIS,N) is true when N equals the count of the number of top level atoms in LIS.

F<sub>1</sub>: IS\_CTA(NIL, 0).

R<sub>1</sub>: [ATOM(U) ∧ IS\_CTA(T,N) ∧ IS(N+1,ANS)] → IS\_CTA(CONS(U,T),ANS)

R<sub>2</sub>: [LISTP(U) ∧ IS\_CTA(T,ANS)] → IS\_CTA(CONS(U,T),ANS)

Evaluate  $(\exists Z)IS\_CTA(CONS(CONS(A,NIL),CONS(B,CONS(CONS(C,NIL),NIL))),Z)$  using computation deduction.

{ Note: If you prefer, you may use the notation IS\_CTA(((A) B (C)),Z), and ATOM and LISTP are the built-in LISP functions we already know }

Required: Tell me what your strategy is (1 pt). Give the clause form (4 pts) of the axiom set & the negation of the goal. Give me the Resolution resolvents (16 pts) using a complete strategy. Prove the substitutions are consistent (3 pts). Obtain the value of the goal (1 pt). Note: the function IS(X,Y) evaluates the left argument and unifies it (equates it) with the right argument, e.g., IS(4+2,Q) evaluates 4+2 to 6 and sets Q=6 (i.e., it stores the substitution {4+2/X, eval(4+2)/Y, Q/Y, 6/Q} or {6/Q} in the system.)

(1) Tell me your strategy I am using Set-of-Support which is a complete strategy

(4) Give me your axioms & negation of the goal in clause form

F<sub>1</sub>: IS\_CTA(nil,0).

R<sub>1</sub>: ~ATOM(V) ∨ ~IS\_CTA(R,N) ∨ ~IS(N+1,ANS<sub>1</sub>) ∨ IS\_CTA(CONS(V,R),ANS<sub>1</sub>).

R<sub>2</sub>: ~LISTP(U) ∨ ~IS\_CTA(T,ANS<sub>2</sub>) ∨ IS\_CTA(CONS(U,T),ANS<sub>2</sub>)

~Goal: ~IS\_CTA(CONS(CONS(A,NIL),CONS(B,CONS(CONS(C,NIL),NIL))),Z)

(16) Give me the resolution resolvents

(4 pts)  $\mathfrak{R}_1 = \mathfrak{R}\{\sim\text{Goal}, R_2\}: \sim\text{LISTP}(U) \vee \sim\text{IS\_CTA}(T, \text{ANS}) \{ \text{CONS}(A, \text{NIL})/U, \text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}))/T, z/\text{ANS}_2 \}$

$\mathfrak{R}_1: \sim\text{LISTP}(\text{CONS}(A, \text{NIL})) \vee \sim\text{IS\_CTA}(\text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL})), z)$

$\mathfrak{R}_1: \sim t \vee \sim\text{IS\_CTA}(\text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL})), z) \{ \text{EVALUATE LISTP}(\text{CONS}(A, \text{NIL})) \text{ TO } t \}$

$\mathfrak{R}_1: \sim\text{IS\_CTA}(\text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL})), z)$

(4 pts)  $\mathfrak{R}_2 = \mathfrak{R}\{\mathfrak{R}_1, R_1\}: \sim\text{ATOM}(V) \vee \sim\text{IS\_CTA}(R, N) \vee \sim\text{IS}(N+1, \text{ANS}_1) \{ B/V, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL})/R, z/\text{ANS}_1 \}$

$\mathfrak{R}_2: \sim\text{ATOM}(B) \vee \sim\text{IS\_CTA}(\text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}), N) \vee \sim\text{IS}(N+1, Z) \{ \text{EVALUATE ATOM}(B) \text{ TO } t \}$

$\mathfrak{R}_2: \sim t \vee \sim\text{IS\_CTA}(\text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}), N) \vee \sim\text{IS}(N+1, Z)$

$\mathfrak{R}_2: \sim\text{IS\_CTA}(\text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}), N) \vee \sim\text{IS}(N+1, Z)$



**IV. Computation Deduction.** (continued)

(4 pts)  $\mathfrak{R}_3 = \mathfrak{R}\{\mathfrak{R}_2, R_2\}$ :  $\sim\text{LISTP}(U') \vee \sim\text{IS\_CTA}(T', \text{ANS}_2')$  {CONS(C,NIL)/U', NIL/T', N/ANS\_2' }  
 $\mathfrak{R}_3$ :  $\sim\text{LISTP}(\text{CONS}(C, \text{NIL})) \vee \sim\text{IS\_CTA}(\text{NIL}, N) \vee \sim\text{IS}(N+1, \text{ANS})$  {EVALUATE LISTP(CONS(C,NIL) TO t}  
 $\mathfrak{R}_3$ :  $\sim t \vee \sim\text{IS\_CTA}(\text{NIL}, N) \vee \sim\text{IS}(N+1, Z)$   
 $\mathfrak{R}_3$ :  $\sim\text{IS\_CTA}(\text{NIL}, N) \vee \sim\text{IS}(N+1, Z)$

(4 pts)  $\mathfrak{R}_4 = \mathfrak{R}\{\mathfrak{R}_3, F_1\}$ :  $\sim\text{IS}(N+1, Z)$  {0/N}  
 $\mathfrak{R}_4 = \text{nil}$  {1/Z}

Therefore, {0/N, 1/Z} yields that  $\text{IS\_CTA}(\text{CONS}(\text{CONS}(A, \text{NIL}), \text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}))), 1)$  is true

(3) Prove the substitutions are consistent.

Substitutions will be consistent because I changed variables every time I re-used  $R_2$  and all the variables were originally standardized apart. To prove consistency we assemble all the numerators in a set, say,  $U_1$ , and all the denominators in a set, say,  $U_2$  and show that  $U_1 = U_2\sigma$  and  $\sigma \neq \text{null}$ .

$\{s\} = \{\text{CONS}(A, \text{NIL})/U, \text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}))/T, Z/\text{ANS}_2, B/V, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL})/R, Z/\text{ANS}_1, \text{CONS}(C, \text{NIL})/U', \text{NIL}/T', N/\text{ANS}_2'\}$

$U_1 = [\text{CONS}(A, \text{NIL}), \text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL})), Z, B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}), Z, \text{CONS}(C, \text{NIL}), \text{NIL}, N],$

$U_2 = [U, T, \text{ANS}_2, V, R, \text{ANS}_1, U', T', \text{ANS}_2']$  and  $U_1 = U_2\sigma$

$\sigma = \{\text{CONS}(A, \text{NIL})/U, \text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}))/T, Z/\text{ANS}_2, B/V, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL})/R, Z/\text{ANS}_1, \text{CONS}(C, \text{NIL})/U', \text{NIL}/T', N/\text{ANS}_2'\}$  and  $\sigma \neq \text{null}$ .

(1) Give me the solved goal, i.e., the answer:

*Answer:*  $(\exists z)\text{IS\_CTA}(\text{CONS}(\text{CONS}(A, \text{NIL}), \text{CONS}(B, \text{CONS}(\text{CONS}(C, \text{NIL}), \text{NIL}))), Z)$  is true with  $Z=1$