

Principles of Knowledge Representation and Reasoning

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Winter Semester 2015/2016

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Exercise Sheet 2

Due: November 4th, 2015

Exercise 2.1 (Resolution, 2+2)

- (a) Use resolution to show that

$$F = (\neg A \wedge B \wedge C) \vee (A \wedge B) \vee (\neg A \wedge \neg C) \vee \neg B$$

is a tautology (valid).

- (b) Use resolution to show that

$$\{B \wedge \neg C, (A \wedge B) \rightarrow (C \vee \neg A)\} \models \neg A$$

Exercise 2.2 (Horn clauses, 2+2)

- (a) Consider a satisfiable Horn formula ψ . Prove that the interpretation that makes a variable (occurring in ψ) true if and only if the variable is true in all models of ψ is also a model of ψ .
- (b) Apply (a) in order to show that there exists a formula which has no logically equivalent Horn formula.

Exercise 2.3 (Predicate logic, 2+2)

- (a) Classify the following expressions as terms, ground terms, atoms, formulae, sentences, or statements in meta language. If there is more than one possibility for an expression please list them all. The usage of symbols complies with the convention introduced with the syntax of predicate logic.

(a) $P(x, a)$

(d) $\mathcal{I}, \alpha \models P(f(x), f(a))$

(b) $g(a, h(b, c))$

(e) $g(f(y), a)$

(c) $\mathcal{I} \not\models P(a, f(b))$

(f) $Q(b)$ is falsifiable.

(g) $\forall x(P(x, y) \rightarrow Q(x)) \vee \neg P(y, x)$

(h) $\forall x \forall y(P(x, y) \wedge Q(x) \vee P(f(y), x))$

(i) $\forall x(\exists y(P(x, y) \wedge Q(x)) \vee P(x, y))$

(j) $Q(a) \vee P(a, b) \equiv P(b, a) \vee Q(b)$

(b) Consider the following theory:

$$\Theta = \left\{ \begin{array}{l} \forall x \neg P(x, x) \\ \forall x \forall y \forall z (P(x, y) \wedge P(y, z) \rightarrow P(x, z)) \\ \forall x \forall y (P(x, y) \vee x = y \vee P(y, x)) \\ \forall x \exists y P(x, y) \\ \neg \exists y P(y, a) \end{array} \right\}$$

Specify an interpretation $\mathcal{I} = \langle \mathcal{D}, \cdot^{\mathcal{I}} \rangle$ with $\mathcal{I} \models \Theta$ (with proof). Does Θ have a model that is defined on a finite domain D ?