Final Report for Research on Magnetized-Target Fusion for Space Propulsion

John F. Santarius and Mohamed E. Sawan

July 2003

UWFDM-1209

FUSION TECHNOLOGY INSTITUTE

UNIVERSITY OF WISCONSIN

MADISON WISCONSIN

Final Report for Research on Magnetized-Target Fusion for Space Propulsion

John F. Santarius and Mohamed E. Sawan

Fusion Technology Institute
University of Wisconsin
1500 Engineering Drive
Madison, WI 53706

http://fti.neep.wisc.edu

July 2003

UWFDM-1209

1 Overview

This *Final Report* summarizes research activities at the University of Wisconsin on using a field-reversed configuration (FRC) as the target in a magnetized-target fusion (MTF) rocket. The MTF rocket is based on a concept invented by Francis Thio, in which plasmas launched by plasma guns implode an FRC or spheromak [1,2].

The four tasks defined for this research project were:

Task 1) Participate in the Revolutionary Aerospace Systems Concepts / Human Outer Planet Exploration (RASC/HOPE) project.

Task 2) Undertake the modeling of the implosion of a magnetized target plasma by a high-velocity plasma liner to achieve thermonuclear fusion reactions.

Task 3) Undertake modeling the fusion burn.

Task 4) Undertake the modeling of the expansion of the fusion plasma against a magnetic field in a magnetic nozzle configuration.

The following sections give a brief account of project activities that address the above tasks. An Appendix in the form of a Mathematica^{tm} [3] notebook gives further details of the calculations and figures appearing in Section 2. Tasks 2 and 3 are very closely related, so they are treated in the same section (2.2).

2 Project activities

2.1 Task 1: Participate in the Revolutionary Aerospace Systems Concepts / Human Outer Planet Exploration (RASC/HOPE) project.

The main objective of the Revolutionary Aerospace Systems Concepts / Human Outer Planet Exploration (RASC/HOPE) program is to enable future NASA missions by the development of revolutionary aerospace systems concepts and related technology. The PI for the this research participated in the April 16-17, 2002 RASC/HOPE meeting at NASA Langley Research Center. At the meeting, he provided expertise in fusion space propulsion, space resources including helium-3 fusion fuel, and advanced-fuel fusion power plant design. He has continued this task in discussions with Dr. Francis Thio during the time frame covered by this report. Advice was also provided to Dr. Slade White, NASA MSFC, regarding neutron interactions with the magnetic nozzle.

2.2 Task 2: Undertake the modeling of the implosion of a magnetized target plasma by a high-velocity plasma liner to achieve thermonuclear fusion reactions and Task 3: Undertake modeling the fusion burn.

2.2.1 Description of the BUCKY computer code

These tasks model plasma-jet magnetized-target fusion (MTF) implosion dynamics and burn dynamics using the University of Wisconsin's 1-D radiation hydrodynamics code, BUCKY [4,5]. The BUCKY code has been ported to the newly acquired fast PC cluster at the University of Wisconsin's Fusion Technology Institute, and the cases of this section were run on those computers. BUCKY is a 1-D Lagrangian radiation-hydrodynamics code which can simulate plasmas in planar, cylindrical, or spherical geometries. It solves single-fluid equations of motion with pressure contributions from electrons, ions, radiation, and fast charged particles. Plasma energy transfer can be treated using either a one-temperature (Te=Ti) or two-temperature model. Both the electrons and ions are assumed to have Maxwellian distributions defined by Ti and Te. Thermal conduction for each species is treated using Spitzer conductivites, with the electron conduction being flux-limited. The temperature equations are coupled by an electron-ion energy exchange term, and each equation has a PdV work term.

In addition to radiation, BUCKY includes a number of other physical processes as source terms in the energy equations: fast ion (beam or target debris) energy deposition; heating due to the deposition of fast charged particles and neutrons during the fusion burn phase; laser energy deposition; and x-ray heating of a cold buffer gas. Fusion burn equations for D-T, D-D, and D-^{3}He reactions are solved, and the charged particle reaction products are transported and slowed using a time-dependent particle tracking algorithm. Neutrons are deposited in the target using an escape probability model. Fast ions from an ion beam or target microexplosion debris are tracked using a time-, energy-, and species-dependent stopping power model. Stopping powers are computed using a Lindhard model at low projectile energies and a Bethe model at high energies. Laser energy is deposited using an inverse bremsstrahlung attenuation model, with a dump of the remaining laser energy at the critical surface.

Results for this task appear in Sec. 2.2.2. The MTF rocket embodiment that served as the starting point for this work was given as paper AIAA-99-2703, by Thio, et al., originally presented at the 35th Joint Propulsion Conference [2]. That paper, in turn, is based on the models proposed in a paper by Thio, et al., published in *Current Trends in International Fusion Research—Proceedings of the Second International Symposium* [1].

Because there was not sufficient time to add a plasma-jet MTF magnetic-field model to BUCKY on the time frame of this project, the magnetic field was not included in the calculations. The neglect of the magnetic field has the drawback of not accurately simulating the thermal conductivity of the plasma, which must then be parameterized. This approximation was expected to be reasonable, because the jet kinetic energy dominates the magnetic-field pressure in the FRC target (see Appendix). For some constant value of the thermal conductivity, the total heating of the compressed target will be the same as for the case of a thermal conductivity that depends on the magnetic field. Unfortunately, the magnetic field dependence of the thermal conductivity appears to be much more sensitive to details of the compression profile and timing than anticipated, and no suitable reference case near the parameters of Refs. 1 and 2 was found. In particular, for a constant thermal conductivity, unreaslistically little heat is lost from the plasma during the early stages when the magnetic field is low, so the relatively high pressure reduces the compression. Perhaps even more important is that the lack of a magnetic field means that alpha particles generated by D-T reactions will deposit their energy over a much larger distance than if they were constrained by a finite gyroradius.

Progress to date has been good, with a framework set up to examine plasma physics aspects of MTF cases, many parametric cases run and detailed information from these cases available in easily analyzed forms, as shown in the figures of Sec. 2.2.2. Nevertheless, further work, primarily modifying the BUCKY code to include the magnetic field and the dependence of the plasma thermal conductivity on it, will be required to reproduce the base case outputs in detail. In particular, the right balance between compression and heating has not yet been found that would give the same fusion output power as in the base cases. Much more detail for the present calculations and the output for some of the parametric variations appears in the accompanying Mathematica$^{\text{tm}}$ notebook of the Appendix. This notebook was created during the course of this project in order to analyze plasma-jet MTF plasma physics, generate BUCKY input parameters, and post-process the BUCKY results.

2.2.2 Results

2.2.2.1 Reference case

The input parameters that formed the basis of parametric runs searching for a case with output near that of paper Refs. [1] and [2] appear in the Appendix subsection titled *Parameters based on Thio, et al, Current Trends in International Fusion Research 2nd Symposium and Thio, et al, AIAA-99-2703*. Note that v_{jet}=125 km/s for this case, which served as the basis for a search through parameter space for a high-Q (fusion power/input power) case. The difficulties in the search stem from the fact that the timing of the shock waves depends strongly on the plasma parameters and dimensions, while the fusion power produced depends in turn on that timing. As discussed in the previous section, the BUCKY code does not yet treat the magnetic field, and the relation between thermal conductivity and magnetic field is particularly important, as is the effect of the magnetic field on localizing the deposition of alpha particle energy. The cases run so far have parameterized the thermal conductivity in order to model this effect approximately. Also, differences exist between the ideal-gas analytic model of Refs. [1] and [2] compared to the BUCKY calculations, which include the full deuterium and tritium equations of state. The analytic model indicates that an attractive solution exists, but the difficulty in finding this solution with BUCKY shows that improvements in the BUCKY physics must still be made.

Two key characteristics of the physics of magnetized target fusion are the amount of compression and the dwell time during which the target plasma remains compressed. The amount of compression relates to the target plasma density and temperature, of which the fusion power is a strong, nonlinear function. Typical Lagrangian zone boundary radii versus time ($0<t<2\ \mu s$) for the reference case, based on Refs. [1] and [2], appear in Figure 1; the inset shows the key parameters. After the initial shock caused by the jets impacting the target, the jet plasma rebounds and interacts with the somewhat slower incoming buffer plasma. The buffer plasma's role is to mitigate the neutron energy. A rebounding shock in the target sends an outgoing shock through the buffer, starting at ~0.5 μs. The jet plasma rebounds from the buffer and meets the expanding target plasma, causing the target to compress again. For maximum effectiveness, all of the shock waves, which appear as localized compression of lines in the figure, would converge, so this case is clearly not optimized. The evolution of the same shock wave, for times < 0.4 μs and radii < 0.05 m, appears in Figure 2.

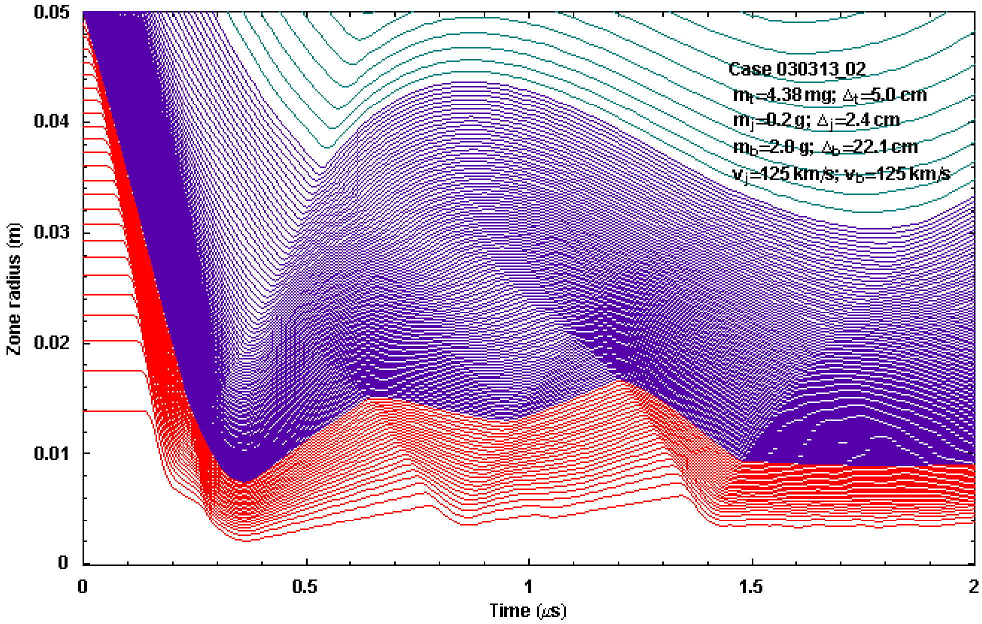


Figure 1. Zone boundary radii vs. time from 0-2 μs for a v_{jet}=125 km/s case. Target zones are red, jet zones are violet, and buffer zones are green.

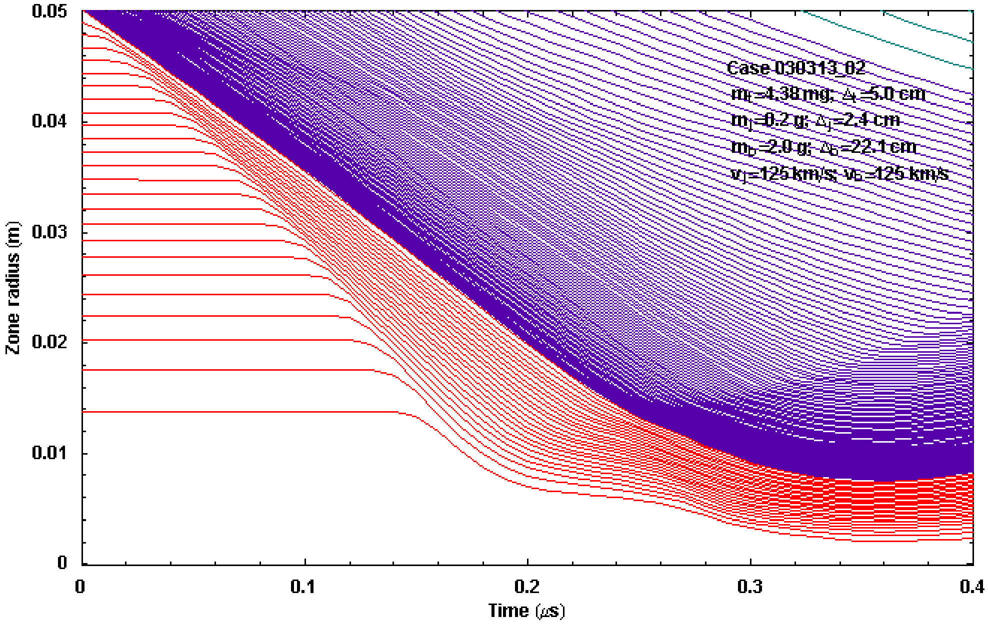


Figure 2. Zone boundary radii vs. time from 0-0.4 μs for a v_{jet}=125 km/s BUCKY case. Target zones are red and jet zones are violet.

In the parametric exploration runs, essentially all of the fusion power is generated during the initial target compression, and only a very small amount is generated during the secondary compressions. This cycle repeats until eventually all three plasma regions expand outward. The density and temperature for each region versus time appear in Figure 3 and Figure 4. BUCKY also can generate a wide variety of parameters versus other parameters at selected times, and some of these cases appear in the Appendix. Cases for various other plasma target and jet conditions also appear in the Appendix. Many cases have been run in an attempt to optimize the parameters, but the reference case [1] has not yet been reproduced. Related considerations are discussed below.

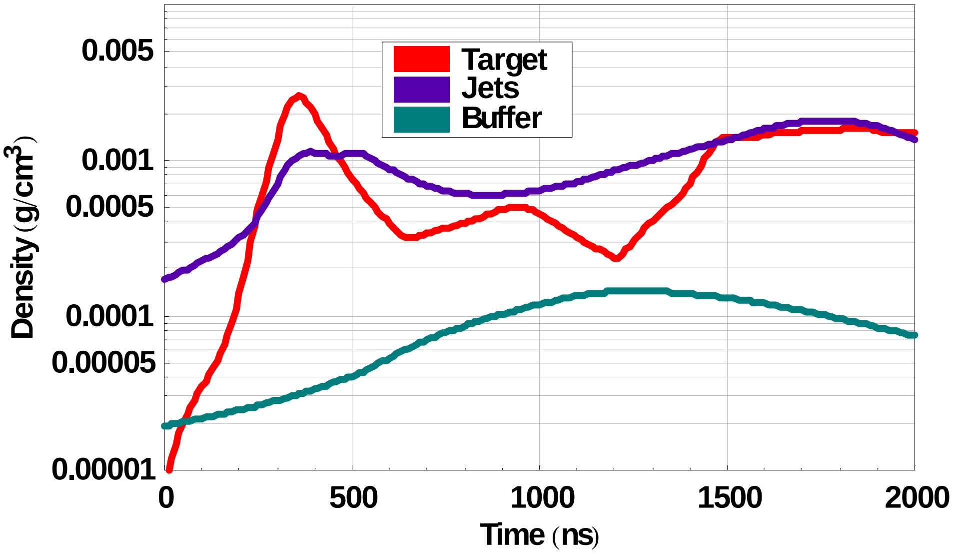


Figure 3. Density vs. time for a v_{jet}=125 km/s BUCKY case.

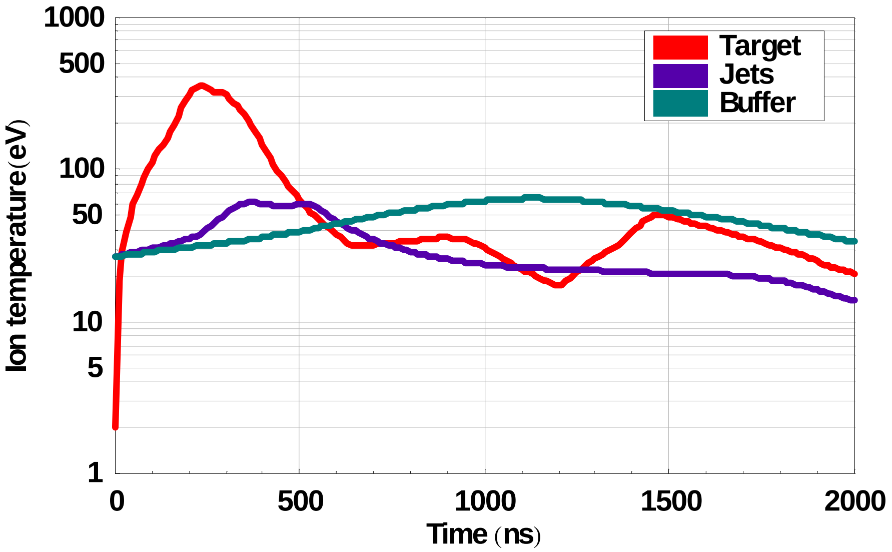


Figure 4. Ion temperature vs. time for a v_{jet}=125 km/s BUCKY case.

2.2.2.2 Case with jet velocity increased to 398 km/s

Because it proved difficult to reproduce Thio, et al.'s reference case without a magnetic field in the BUCKY code, the decision was made to explore a high-velocity plasma-jet case, nominally 400 km/s. A jet velocity of 398 km/s was used in order to satisfy a "perfect acoustic matching" condition [1]. When this condition applies, the velocity of the contact surface between the jets and the target equals the ion-acoustic velocity in the jets, resulting in no outward radial shock propagation in the jets.

Typical Lagrangian zone boundary radii versus time for this case appear in Figure 5, which shows the first 0.2 μs for radii < 0.05 m. The momentum of the jets now suffices to overwhelm most of the shocks, giving good inertial confinement for ~100 ns. The compression can be seen to be a maximum at ≈0.15 μs and to expand slightly beyond that time. The evolution of the same shocks appears in Figure 6 with the y axis expanded so that $r < 0.02$ m; it shows more numerous shocks with smaller amplitudes than in the reference case, but shocks have not been completely eliminated.

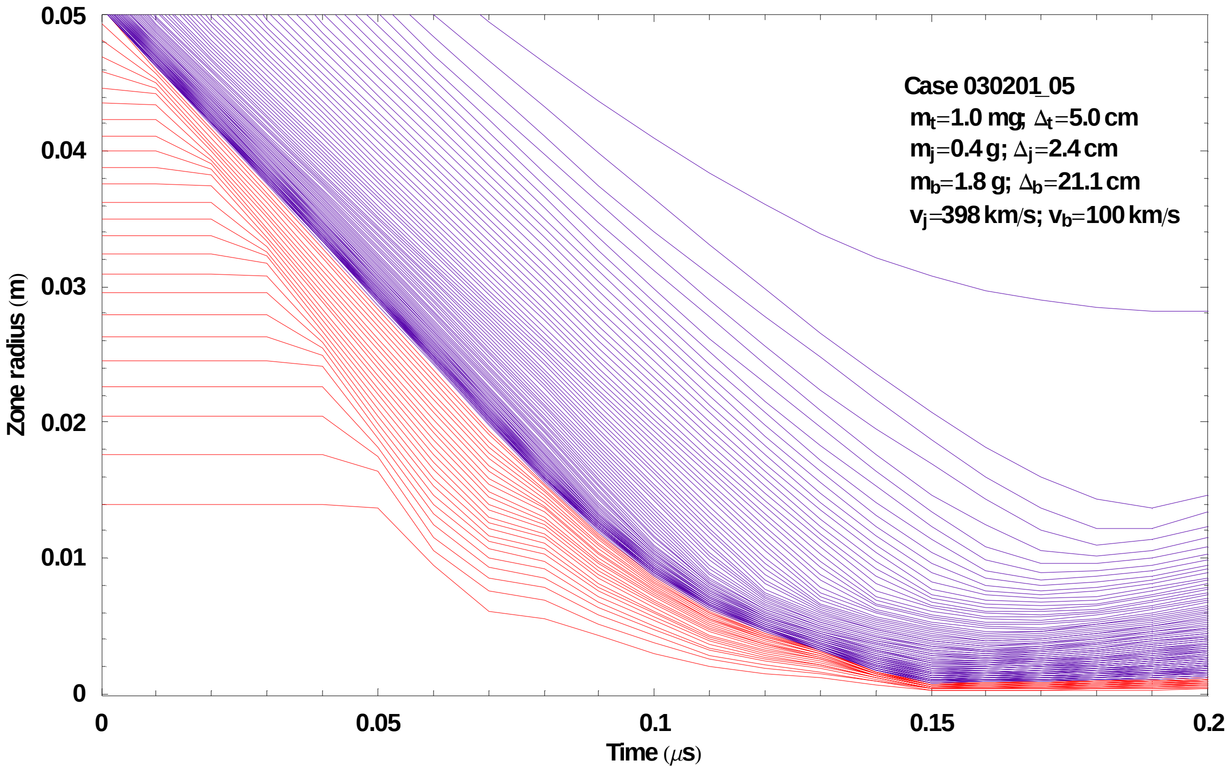


Figure 5. Zone boundary radii ($r<0.05$ m) vs. time (0-0.2 μs) for a v_{jet}=398 km/s case. Target zones are red and jet zones are violet.

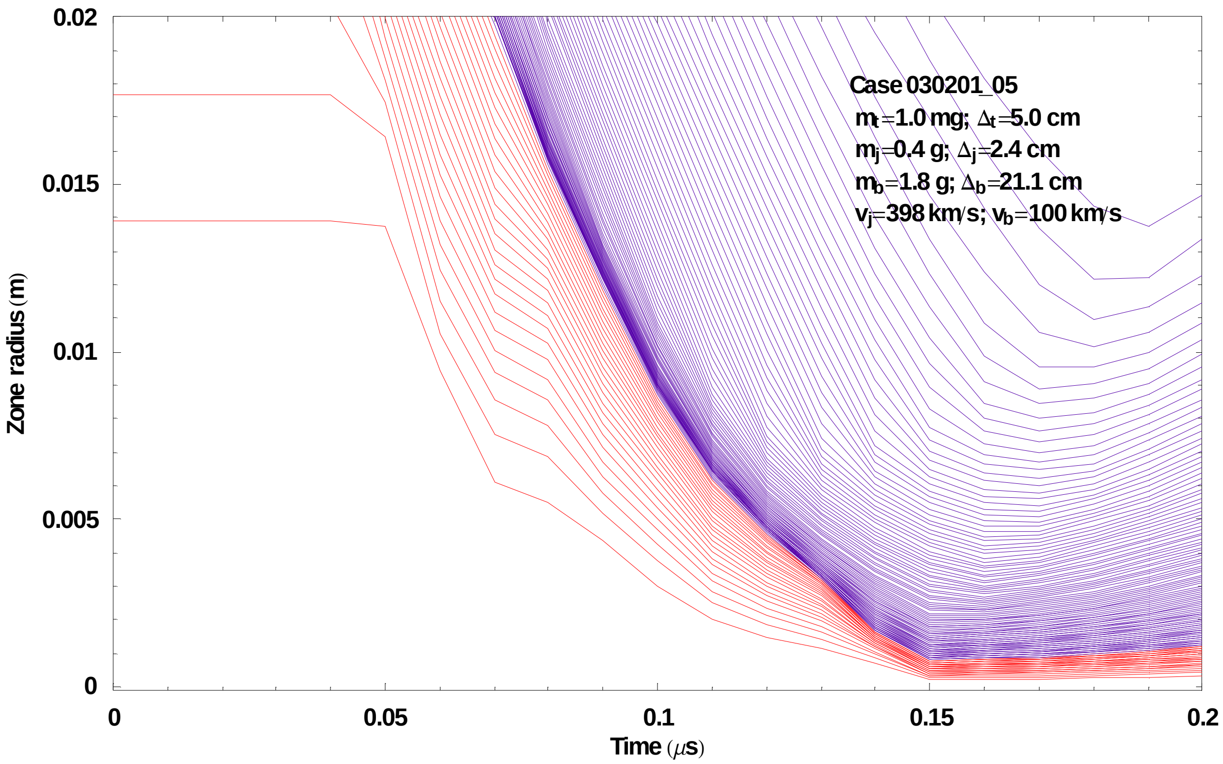


Figure 6. Zone boundary radii (r<0.02 m) vs. time (0-0.2 μs) for a v_{jet}=398 km/s BUCKY case. Target zones are red and jet zones are violet.

The neutron production averaged over the various regions versus time appears in Figure 7, which shows that most of the neutron production takes place over the relatively short time span of ~30 ns. The total fusion power produced for this case was only about 700 kJ, much less than expected from Refs. 1 and 2. The explanation for this difference appears to be the timing of the density and temperature peaks for each region as illustrated in Figure 8 and Figure 9 (averaged over each region). The density peaks about 30 ns later than the ion temperature. This seems likely to be due to the differences between the analytic and computer-code models, probably the lack of dependence of the thermal conductivity and alpha particle energy deposition on magnetic field in the BUCKY computer code. This leads to less compression of the plasma at early times when the magnetic field is low in the computer calculations, because the assumption of high, constant thermal conductivity leads to a higher temperature and, therefore, higher pressure than in the analytic model. The mass density, ion temperature, and specific fusion power for the individual Lagrangian zones appear in Figure 10 for times from 60-160 ns at 20 ns intervals. The enhancement of the alpha particle energy deposition by magnetic fields of the order of 500 T to 1000 T over the size of the compressed target of a few millimeters can be significant due to the small gyro-radius of the alpha particles. This enhancement is not incorporated in the BUCKY calculation at this point. The enhanced alpha energy deposition could potentially maintain the target temperature at the elevated level or even ramp up the temperature.

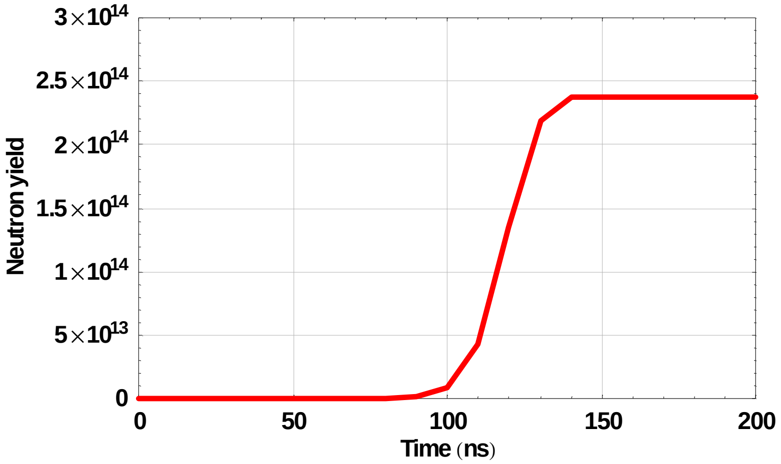


Figure 7. Neutron production vs. time for a v_{jet}=398 km/s BUCKY case.

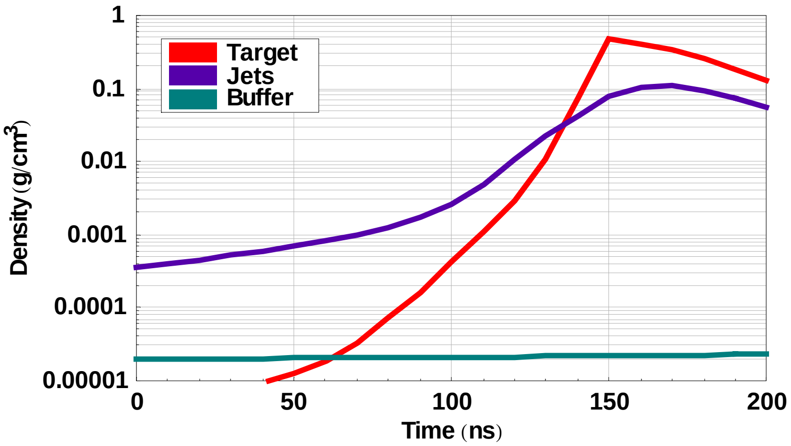


Figure 8. Density vs. time for a v_{jet}=398 km/s BUCKY case.

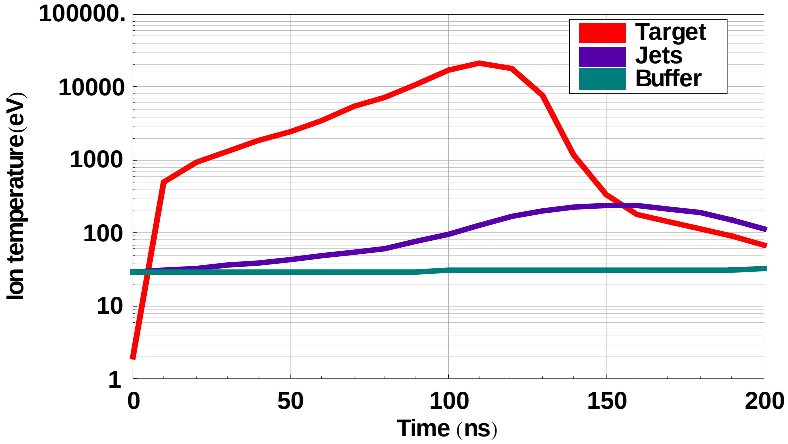


Figure 9. Ion temperature vs. time for a v_{jet}=398 km/s BUCKY case.

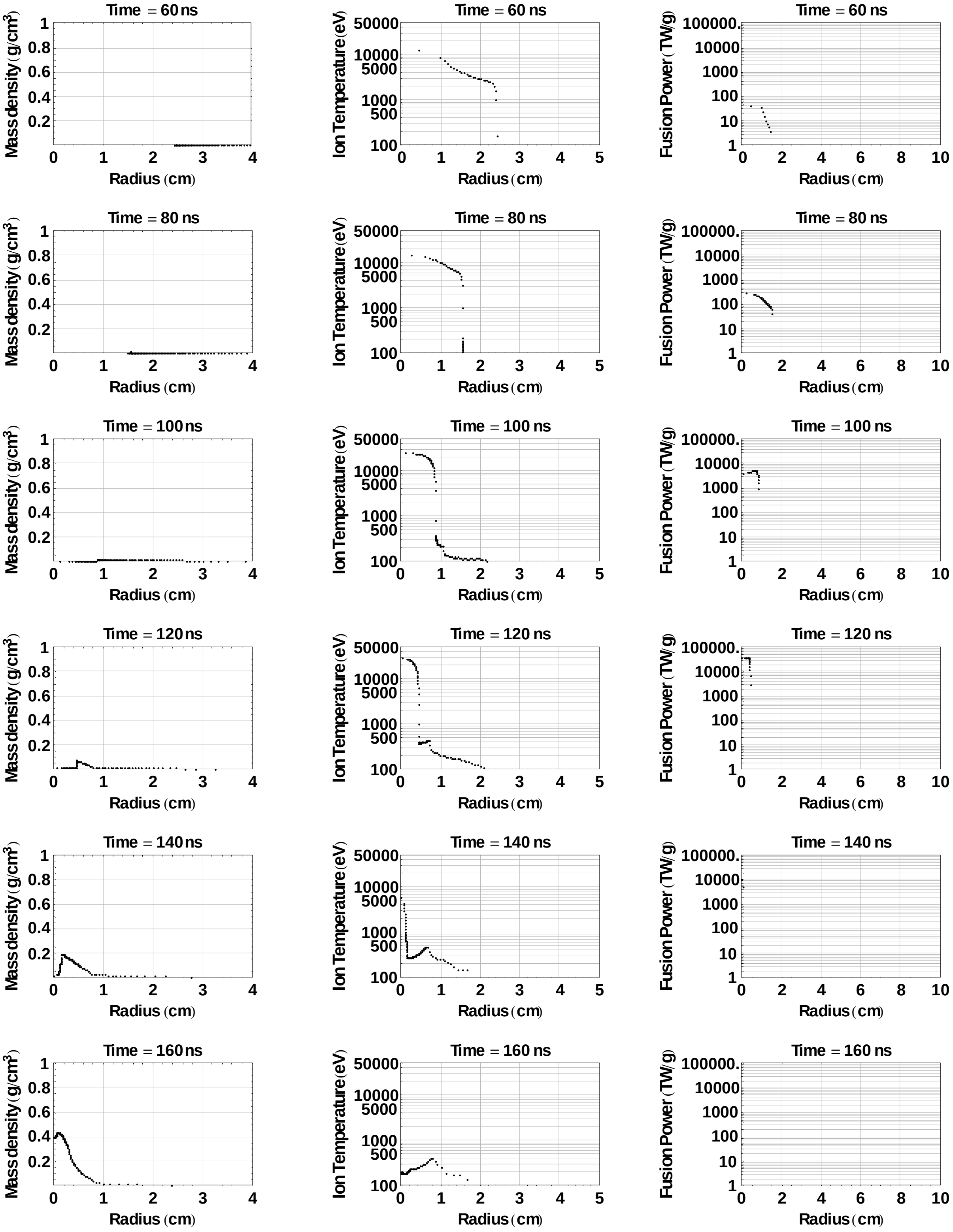


Figure 10. Mass density, ion temperature, and specific fusion power vs. radius near the origin from 60-160 ns for a v_{jet}=398 km/s BUCKY case. Each dot represents a Lagrangian zone.

2.3 Task 4: Undertake the modeling of the expansion of the fusion plasma against a magnetic field in a magnetic nozzle configuration.

The key foci of this activity were: (1) searching the literature for support of the 80% predicted nozzle efficiency; (2) calculating the energy produced in the nozzle magnets and structure by neutrons and gamma rays (Appendix subsection *Neutronics/MES ONEDANT cases)*; and (3) setting up functions to calculate the neutron moderation by the buffer for use when a suitable BUCKY case has been found (Appendix subsection *Neutronics/Calculation of neutron slowing down in deuterium*).

Investigating the engineering and scientific literature related to magnetic nozzles for pulsed plasma propulsion led to the conclusion that the predicted nozzle efficiencies (up to ~80%) appear to be achievable [6,7,8,9,10,11,12,13,14,15,16]. Some detailed inertial-confinement fusion calculations predict 65% efficiency [8,13]. The projected specific powers of plasma MTF propulsion systems are so high that even a reduction of the predicted 80% nozzle efficiency by a factor of two would leave sufficient margin to perform well for most missions of interest.

The key nozzle questions from the fusion technology standpoint are:

1. What fraction of the fusion neutron energy would be deposited in the nozzle?
2. What would be the consequent heating of the structure?

Dr. Mohamed E. Sawan of the UW Fusion Technology Institute agreed to perform some quick calculations, using the ONEDANT module of the DANTSYS 3.0 discrete ordinates particle transport code system [17] to perform calculations to indicate the approximate amount of the heating starting with various neutron energies. The most recent version of the International Fusion Energy Nuclear Data Library, FENDL-2 [18], was utilized in a 175 neutron energy group structure. For this purpose, 1-D geometry and a 5-cm thick slab of 316 stainless steel served to approximate the magnetic coil and structure of the nozzle. The cross sections for reactions with neutrons of most candidate materials are sufficiently close to those of steel that the results should be broadly applicable. Materials also can generally be tailored to reduce heating by specific isotopes by avoiding those isotopes or reducing their amount.

The 5 cm magnet coil thickness stems from assuming an average coil current density for the superconductor, stabilizer, and structure of 50-100 MA/m^2. Performance of 50 MA/m^2 has been achieved for a twisted, 6.9 T magnetic coil of inner dimension 3.9 m [19], so this assumption seems reasonable for a zero order estimate.

The crucial neutronics issue facing magnetic nozzle designers is that the absorption of even very low energy neutrons can lead to (n,γ) reactions that produce several MeV of energy per reaction. Fortunately, the mean free path for neutrons in materials can be much larger than the structure thickness, depending on the neutron's energy, so most high-energy neutrons will pass through the few cm's of structure. The deposited neutron and gamma-ray energy as a function of the initial neutron energy are shown in Figure 11, and the resulting neutron energy multiplication appears in Figure 12. The energy absorbed by the nozzle magnet and structure as a fraction of the initial fusion energy appears in Figure 13. As Figure 13 illustrates, there exists a broad minimum in the absorbed energy by the nozzle of less than 1% of the fusion energy over a fairly wide range of initial neutron energies, 200-855 keV. A buffer plasma of properly chosen parameters should be able to limit almost all of the neutrons exiting it to this energy range.

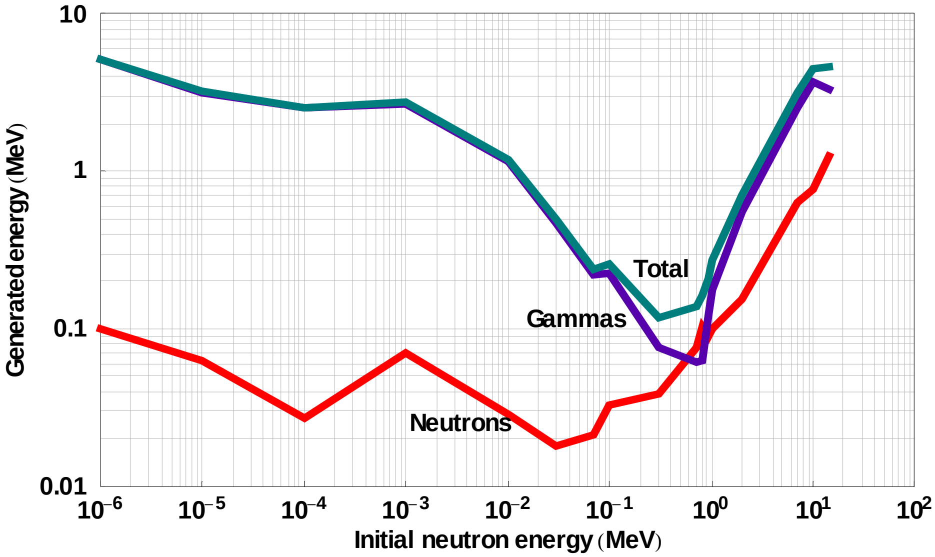


Figure 11. Energy deposited in 5 cm of 316 stainless steel versus initial neutron energy. Energy from neutrons in red, from gamma rays in violet, and total in green.

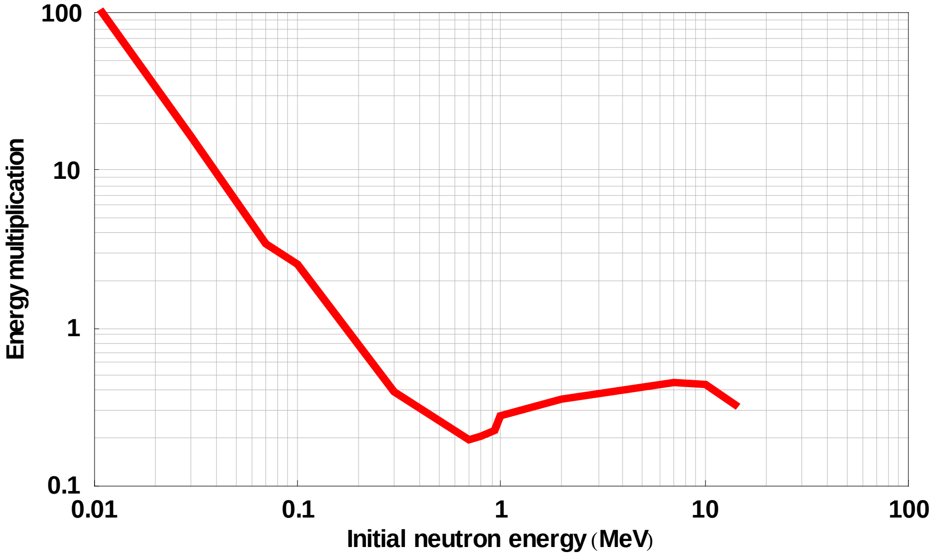


Figure 12. Energy multiplication in 5 cm of 316 stainless steel as a function of initial neutron energy.

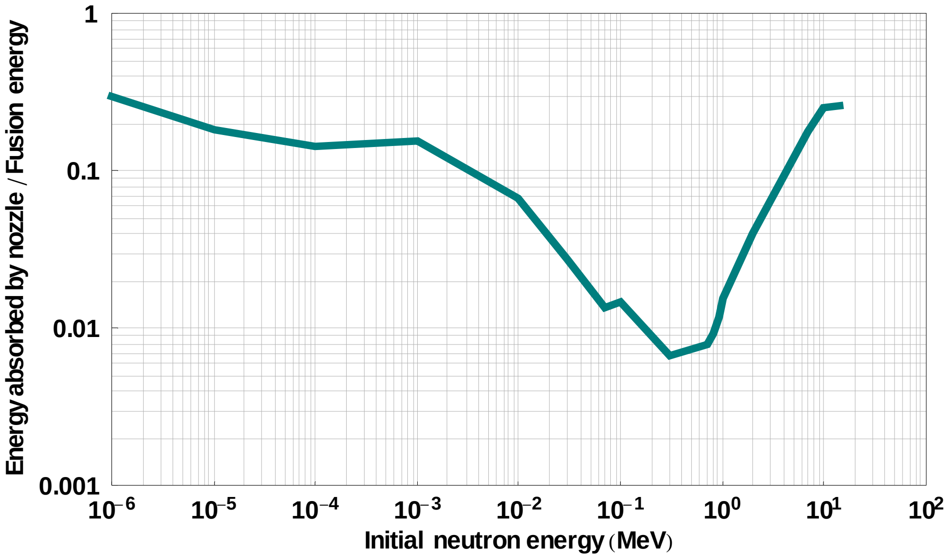


Figure 13. Energy absorbed by 5 cm of 316 stainless steel as a fraction of the total fusion energy for a given initial neutron energy.

3 Summary

This research supported NASA's Revolutionary Aerospace Systems Concepts / Human Outer Planet Exploration (RASC/HOPE) project by providing expertise on fusion propulsion and evaluating technical aspects of plasma-jet magnetized-target fusion (MTF). The 1-D radiation hydrodynamics of the plasma-jet induced implosion/explosion of the target plasma was modeled using the University of Wisconsin's BUCKY computer code [4]. The calculations qualitatively reproduce the anticipated behavior of the plasma regions, but the desired reference case [1,2] remains elusive. Further exploration of operating parameter space may reproduce this case, but it appears more likely that the discrepancy results from differences in the assumptions used. In particular, the BUCKY code does not include magnetic fields, and thereby neglects the localization of alpha-particle heating and the relationship between thermal conductivity and magnetic field, which will likely be very important factors. Also, the analytic model of Refs. 1 and 2 uses ideal-gas equations of state, whereas the BUCKY calculations use table lookup for the full deuterium and tritium equations of state and opacities. This is not, however, expected to be an important difference. The BUCKY computer code also includes independent ion and electron temperatures. This research also numerically assessed the heating of the structure and magnets of the magnetic nozzle by neutrons from the fusion burn. The resulting nozzle heating could be kept below 1% of the total fusion power by using a plasma liner to buffer the neutron energy into the 200-855 keV range. This assumes that the total thickness of the magnet's superconductor, stabilizer, structure, and shielding would be less than ~0.05 m, a value consistent with presently achieved superconducting magnet winding pack current densities.

4 Recommendations for Future Work

Several extensions to this research would improve the accuracy of plasma-jet MTF modeling and optimize design points for this concept:

1. Implement a magnetic-field dependent thermal conductivity and alpha-particle energy deposition in BUCKY and use that code to
    a. reproduce the published plasma-jet MTF analytic cases and
    b. optimize the burn dynamics of plasma-jet MTF.
2. Implement and test FRC equilibrium and stochastic magnetic field options in a 2-D particle-in-cell or discrete simulation Monte Carlo code. Use the chosen code to analyze alpha-particle transport in plasma-jet MTF.
3. Find optimized plasma-jet MTF cases for a range of experiments along the development path to a space-qualified fusion propulsion system.
4. Calculate the energy produced in the magnetic nozzle by neutrons and gamma rays for the full spectrum of neutron energies produced by the MTF fusion burn.

5 Acknowledgments

NASA funding for this research through Grant NAG8-1719 is gratefully acknowledged. Valuable discussions with Dr. Francis Thio have been held throughout this work.

6 Bibliography

1 Y.C.F. Thio, E. Panarella, R.C. Kirkpatrick, C.E. Knapp, F. Wysocki, P. Parks, and G. Schmidt, "Magnetized Target Fusion in a Spheroidal Geometry with Standoff Drivers," *Current Trends in International Fusion Research—Proceedings of the Second International Symposium* (National Research Council of Canada, Ottawa, Canada, 1999).

2 Y.C.F. Thio, B. Freeze, R.C. Kirkpatrick, B. Landrum, H. Gerrish, *et al.*, "High-Energy Space Propulsion based on Magnetized Target Fusion," *35th AIAA/ASMA/SAE/ASEE Joint Propulsion Conference,* paper AIAA 99-2703 (Los Angeles, California, 1999).

3 S. Wolfram, *The Mathematica Book, 3rd ed.* (Cambridge University Press, Cambridge, 1996).

4 J.J. MacFarlane, G.A. Moses, and R.R. Peterson, *BUCKY-1—A 1-D Radiation Hydrodynamics Code for Simulating Inertial Confinement Fusion High Energy Density Plasmas*, University of Wisconsin Fusion Technology Institute Report UWFDM-984 (Madison, Wisconsin, 1995).

5 R.R. Peterson, J.J. MacFarlane, J.F. Santarius, P. Wang, and G.A. Moses, "The BUCKY and ZEUS-2D Computer Codes for Simulating High Energy Density ICF Plasmas," *Fusion Technology* **30,** 783 (1996).

6 R. Hyde, L. Wood, and J. Nuckolls, "Prospects for Rocket Propulsion with Laser-Induced Fusion Microexplosions," *AIAA/SAE 8th Joint Propulsion Specialist Conference,* paper AIAA-72-1064 (New Orleans, LA, 1972).

7 A. Bond, A.R. Martin, R.A. Buckland, T.J. Grant, A.T. Lawton, *et al.*, "Project Daedalus," *Journal of the British Interplanetary Society* **31,** Supplement (1978).

8 R.A. Hyde, *A Laser Fusion Rocket for Interplanetary Propulsion*, Lawrence Livermore National Laboratory Report UCRL-88857 (1983).

9 G.P. Lasche, "Inertial Confinement Fusion with Direct Electric Generation," *Nuclear Technology/Fusion* **4,** 866 (1983).

10 C.D. Orth, G. Klein, J. Sercel, N. Hoffman, K. Murray, *et al.*, "The VISTA Spacecraft--Advantages of ICF for Interplanetary Fusion Propulsion Applications," *IEEE 12th Symp. on Fusion Engineering* (IEEE, Piscataway, NJ, 1987).

11 C.D. Orth, G. Klein, J. Sercel, N. Hoffman, K. Murray, *et al.*, "Transport Vehicle for Manned Mars Missions Powered by Inertial Confinement Fusion," *AIAA/SAE/ASME/ASEE 23rd Joint Propulsion Conference,* paper AIAA-87-1904 (San Diego, CA, 1987).

12 K. Mima, K. Yoshikawa, O. Morimiya, H. Takase, H. Takabe, *et al.*, "Preliminary Studies of Direct Energy Conversion in a D-3He Inertial Confinement Fusion Reactor," *Fusion Technology* **22,** 57 (1992).

13 Y. Nagamine and H. Nakashima, "Analysis of Plasma Behavior in a Magnetic Thrust Chamber of a Laser Fusion Rocket," *Fusion Technology* **35,** 62 (1999).

14 Y.P. Zakharov, A.G. Ponomarenko, A.V. Melekhov, V.G. Posukh, I.F. Shaikhislamov, *et al.*, “Plasma Confinement and Direct Energy Conversion of ICF-Microexplosion in Open Magnetic System,” *Transactions of Fusion Technology* **35,** 283 (1999).

15 R.J. Litchford, G.A. Robertson, C.W. Hawk, M.W. Turner, and S. Koelfgen, *Magnetic Flux Compression Reactor Concepts for Spacecraft Propulsion and Power*, paper AIAA-2000-2669 (Denver, 2000).

16 I.G. Mikellides, P.G. Mikellides, P.J. Turchi, and T.M. York, “Numerical Simulation of Magnetically-Guided Plasma Flows for the Design of a Fusion Propulsion System,” *Space Technology and Applications International Forum-2001*, p. 900 (AIP Press, Albuquerque, New Mexico, 2001).

17 R.E. Alcouffe et al., *DANTSYS 3.0, A Diffusion Accelerated Neutral Particle Transport Code System*, Los Alamos National Laboratory Report LA-12969-M (Albuquerque, 1995).

18 M. Herman and H. Wienke, *FENDL/MG-2.0 and FENDL/MC-2.0, The Processed Cross-Section Libraries For Neutron-Photon Transport Calculations*, International Atomic Energy Agency Report IAEA-NDS-176 (Vienna,1997).

19 O. Motojima, N. Yanagi, S. Imagawa, K. Takahara, S. Yamada, *et al.*, *Superconducting Magnet Design and Construction of Large Helical Device*, in *Fusion Energy 1996* (International Atomic Energy Agency, Vienna, 1997), p. 467.

Appendix: Final Report
Magnetized-Target Fusion for Space Propulsion

Mathematica **Notebook**
FinalReport_PlasmaMTF_03_Appendix.nb

John F. Santarius
University of Wisconsin--Madison
santarius@engr.wisc.edu
latest update: June 16, 2003

Introduction

This notebook contains calculations related to plasma-jet magnetized-target fusion (MTF), in which a magnetized plasmoid is compressed by plasma jets.

Units are SI, except that energies and temperatures are usually in eV or keV.

Notes and notation

■ Notes

★ *Note: spherical geometry used for BUCKY 1-D radiation hydrodynamics calculations.*

★ *Note: the densities in some plasma regimes are so high and the temperatures are so low that the Coulomb logarithms are very small (<5). Care must be exercised in interpreting results related to collisions.*

★ *Note: presently assuming fully ionized deuterium-tritium for target and jets.*

■ Neglecting magnetic field

The fastest initial interaction as the jets meet the target will be between the jet and target electrons. This will tend to equilibrate the electron temperatures, perhaps through a two-stream instability. Because the jet electrons have a drifted Maxwellian distribution with its peak at $\varepsilon_{\mathrm{ej}} < 1\ \mathrm{eV} \ll \frac{3}{2}\, T_e$, this should be a small effect. Any resulting electrostatic fluctuations will be transmitted to the ions on the ion-electron collisional time scale.

The target and undrifted jets, with T~2 eV and $n \sim 10^{22} - 10^{24}\, m^{-3}$, have internal mean free paths of 3-15 μm. This should mean that the internal dynamics can be separated from the interaction dynamics to a large extent, so the first calculations will neglect the internal scales.

The main momentum transfer should occur through slowing down of jet ions on target electrons and the resulting electrostatic pull on target ions. Plasma oscillations and possible instabilities may then be a factor. Shock waves will be induced. The key length scales should be the ion-electron slowing down mean free path and the ion-ion angle scattering mean free path.

Running BUCKY MTF cases on *ZEEP* PC computer cluster

Run X Windows (xwin32).
Log onto *ZEEP* using SecureCRT.
Start Emacs.
Modify input case: e.g., mtf_020807_01.inp
Start a shell within Emacs; run the command, e.g.,

```
qsub-v job=020807_01 bucky.sh
```

Results will be in the zeep directory bucky/cases/020708_01/ .
File transfer can be done using sftp.

Post-processing BUCKY/BPLOT files

The following BPLOT files are presently post-processed in this Mma notebook:
BPLOT.special (processed into plot.pfus.r.dat)
PLOT.mass.vel
PLOT.NeutronsYield.vs.t
PLOT.regn.dens
PLOT.regn.KEs
PLOT.regn.temp
PLOT.rho
PLOT.tempn
PLOT.vel
PLOT.zones.1
PLOT.zones.2

The first step is to change fortran exponential notation to Mma *^ notation. This can be done using sed:

```
...
sed 's/E+/*^/g' PLOT.vel |sed 's/E-/*^-/g' > plot.vel.dat
cat PLOT.zones.1 PLOT.zones.2 > PLOT.zones
sed 's/E+/*^/g' PLOT.zones |sed 's/E-/*^-/g' > plot.zones.dat
```

```
rm PLOT.zones
```

For the files listed in the previous paragraph, these commands are contained in the file ExpToMma.sed. The command "sh ExpToMma.sed" in a Cygwin window processes the files. Commands to process only the files from bptec4 generated at discrete times are in the file ExpToMmaTimes.sed.

Notation

Subscripts

b ≡ buffer (neutron-moderating layer of liner)
c ≡ compressed target
e ≡ electron
i ≡ ion
j ≡ jet (inner layer of liner)
t ≡ target
w ≡ wall

General parameter definitions

Symbol	***I/O Unit***	***Working Unit***	***Definition***
A_j	m^2	m^2	*Total* jet inner area at target impact
B_t	T	T	Target magnetic field
Δ_j	m	m	Jet pulse thickness
ε_{eb}	eV	eV	Buffer electron average energy
ε_{ib}	eV	eV	Buffer ion average energy
ε_{ec}	eV	eV	Compressed target electron average energy
ε_{ic}	eV	eV	Compressed target ion average energy
ε_{ej}	eV	eV	Jet electron average energy
ε_{ij}	eV	eV	Jet ion average energy
ε_{et}	eV	eV	Target electron average energy
ε_{it}	eV	eV	Target ion average energy
$\mathcal{E}_{\text{ej}}$	eV	eV	Jet electron directed energy
$\mathcal{E}_{\text{ij}}$	eV	eV	Jet ion directed energy
f_{rep}	Hz	Hz	Repetition rate
Γ_j	m^{-2}s	m^{-2}s	Jet ion flux
γ_t			Target gamma
$\mathcal{M}_j$	mg	kg	*Per jet* mass
m_b	g	kg	Buffer mass
m_j	mg	kg	Total jet mass
m_t	mg	kg	Target mass
$\mathfrak{n}_j$			Number of jets
$\mathcal{N}_b$			Number of zones in buffer
$\mathcal{N}_j$			Number of zones in jet
$\mathcal{N}_t$			Number of zones in target
μ_b	amu	amu	Buffer atomic mass
μ_j	amu	amu	Jet atomic mass

μ_t	amu	amu	Target atomic mass
n_b	m^{-3}	m^{-3}	Buffer number density
n_j	m^{-3}	m^{-3}	Jet number density
n_t	m^{-3}	m^{-3}	Target number density
Q_b	J	J	Buffer initial energy
Q_j	J	J	Jet initial energy
ρ_b	g/cm^3	kg/m^3	Buffer mass density
ρ_j	g/cm^3	kg/m^3	Jet mass density
ρ_t	g/cm^3	kg/m^3	Target mass density

Symbol	***I/O Unit***	***Working Unit***	***Definition***
r_b	m	m	Radius of outside edge of buffer
r_j	m	m	Radius of outside edge of jets
r_t	m	m	Radius of outside edge of target
r_w	m	m	Radius of inner edge of wall (distance to closest B-field coil)
T_{eb}	eV	eV	Buffer electron average temperature
T_{ib}	eV	eV	Buffer ion average temperature
T_{ec}	eV	eV	Compressed target electron average temperature
T_{ic}	eV	eV	Compressed target ion average temperature
T_{ej}	eV	eV	Jet electron average temperature
T_{ij}	eV	eV	Jet ion average temperature
T_{et}	eV	eV	Target electron average temperature
T_{it}	eV	eV	Target ion average temperature
τ_b	s	s	Buffer layer travel time to origin
τ_j	s	s	Jet layer travel time to origin
τ_{sb}	s	s	Buffer ion acoustic wave travel time across region
τ_{sj}	s	s	Jet ion acoustic wave travel time across region
τ_{st}	s	s	Target ion acoustic wave travel time across region

v_j	km/s	m/s	Jet velocity
v_{st}	m/s	m/s	Sound (ion-acoustic) velocity in target
V_j	m^{-3}	m^{-3}	*Total* jet volume
ζ_b			Buffer mass zone increment
ζ_j			Jet mass zone increment
ζ_t			Target mass zone increment
z_b			Buffer ion charge state
z_c			Compressed target ion charge state
z_j			Jet ion charge state
z_t			Target ion charge state

BUCKY-specific definitions

Parameter	***Input Unit***	***Bucky Unit***	***Mma calc Unit***	***Definition***
Δ_{region}	m	cm	m	Widths of regions
δ_{zone}	m	cm	m	Widths of individual zones
$\mathcal{N}_r$				Total number of regions
$\mathcal{N}_z$				Total number of zones
r_{region}	m	cm	m	Outer radius of regions

Package *plasma.m* functions

Note: run the Preliminaries section before running these commands.

```
?plasma`*
```

```
AlfvenVelocity          Ecrit                    ionThermalVelocity     nuieEnergy
alphasq                 electronGyroFrequency    lsParams               nuiePar
beta                    electronGyroRadius       lsParamsB0             nuiePerp
ChildLangmuirIspherical electronPlasmaFrequency  lsParamUnits           nuieSlow
CoulogEE                electronThermalVelocity  lsParamUnitsB0         nuiiPerp
CoulogEI                ionAcousticVelocity      lsParamUnitsB0Hz       plasma
CoulogIE                ionGyroFrequency         lsParamUnitsHz         plasmaParams
CoulogII                ionGyroRadius            nueePerp               plasmaParamsHz
DebyeLength             ionPlasmaFrequency       nueeSlow
```

```
?plasmaParams
```

```
plasmaParams[amu,B,Ee,Ei,gamma,ne,ni,Te, Ti,zi]
 generates a list of plasma parameters.
 plasmaParams[amu,B,Ee,Ei,gamma,ni,Te, Ti,zi]
 calculates ne=ni*zi and generates a list of plasma parameters.
```

Preliminaries

Mathematica setup

C:\Program Files\Common Files\Mathematica\4.0\FrontEnd*init.m* (*Mathematica* initializations) and F:\Mathematica\Configuration\Kernel*init.m* (my initializations) are run when Mma starts. My *init.m* adds package paths, turns off some error messages, sets the default font and table spacing, and invokes the *CleanSlate* package.

```
Off[System`Verbose::shdw];
```

Graphics

```
SetOptions[{Plot, ListPlot}, Frame → True,
  GridLines → {{Automatic, GrayLevel[0.7]}, {Automatic, GrayLevel[0.7]}}];
```

```
optionsListPlot = Options[ListPlot];
```

Convenience definition:

```
plotAll = PlotRange → All;
```

```
frameticksLog = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -30, 30}];
```

```
frameticks10 = Table[{10^i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -30, 30}];
```

FrameTicks usage: FrameTicks→{Automatic,frameticks10,None,None}

Input graphics package needed for many plots

```
Needs["Graphics`Graphics`"];
Needs["ExtendGraphics`Ticks`"];
Needs["Graphics`Legend`"];
```

```
SetOptions[{LogPlot, LogLogPlot, LogListPlot, LogLogListPlot},
  Frame → True, FrameTicks → {Automatic, Automatic, None, None},
  GridLines → {{Automatic, GrayLevel[0.7]},
              {Automatic, GrayLevel[0.7]}}];
```

```
SetOptions[LogLogListPlot, PlotJoined → True];
```

```
optionsLogListPlot = Options[LogListPlot];
```

PlotStyles

★ *Note: the order of the colors in colorStyle is changed in this notebook compared to most others.*

Define *lineStyle* depending on whether color or dashing is desired. Use PlotStyle->lineStyle[[1]] or dheStyle, etc. kLineStyle="color" or "dashing".

```
kLineStyle = "color";
```

```
thickness = Thickness[0.01];
thicknessOrig = thickness;
```

```
ptSize = PointSize[0.005];
ptSizeOrig = ptSize;
```

```
blue = RGBColor[0, 0, 1];
red = RGBColor[1, 0, 0];
green = RGBColor[0, 0.7, 0];
rose = RGBColor[0.667, 0, 0.333];
orange = RGBColor[0.75, 0.25, 0];
teal = RGBColor[0, 0.5, 0.5];
slategreen = RGBColor[0.2, 0.4, 0.4];
lightblue = RGBColor[0, 0.75, 1];
slateblue = RGBColor[0.167, 0.333, 0.5];
plum = RGBColor[0.5, 0, 0.5];
purple = RGBColor[0.333, 0, 0.667];
beige = RGBColor[0.8, 0.6, 0.6];
brown = RGBColor[0.5, 0.25, 0.25];
pinegreen = RGBColor[0.2, 0.7, 0.2];
gold = RGBColor[1, 155 / 255, 0];
fucshia = RGBColor[1, 0.2, 0.8];
```

```
solid = GrayLevel[0];
dot = Dashing[{0.005, 0.015}];
dash = Dashing[{0.03, 0.03}];
dashLong = Dashing[{0.05, 0.02, 0.05, 0.02}];
dashDot = Dashing[{0.05, 0.015, 0.01, 0.015}];
dashDotDot = Dashing[{0.05, 0.015, 0.01, 0.015, 0.01, 0.015}];
dashDashDot = Dashing[{0.05, 0.015, 0.05, 0.015, 0.015, 0.015}];
dashDashDotDot = Dashing[{0.05, 0.015, 0.05, 0.015, 0.015, 0.015, 0.015, 0.015}];
```

```
dashStyle = {solid, dash, dot, dashDot, dashDotDot, dashDashDot,
   dashDashDotDot, solid, dashLong, dashDashDot, dot, dashDotDot, dashDot};
dashStyleOrig = dashStyle;
```

```
colorStyle := {red, purple, teal, blue, orange, plum, green,
   rose, slategreen, lightblue, beige, slateblue, brown, pinegreen, gold};
colorStyleOrig = colorStyle;
```

Choose line style.

```
lineStyle := If[kLineStyle == "dashing",
   Map[{#, thickness} &, dashStyle],
   Map[{#, thickness} &, colorStyle] ];
```

```
 SetOptions[{Plot, LogPlot, LogLogPlot}, PlotStyle :> Table[lineStyle⟦i, 1⟧, {i, 1, 13}] ];
```

Point style

```
 pointStyle := Map[{#, ptSize} &, colorStyle]
```

Set styles specific to various fusion reactions

```
dtStyle := lineStyle[[3]];
dheStyle := lineStyle[[1]];
dhep5Style := lineStyle[[5]];
dhe3t1Style := lineStyle[[4]];
ddStyle := lineStyle[[2]];
ddptStyle := lineStyle[[4]];
ddn3Style := lineStyle[[5]];
heheStyle := lineStyle[[4]];
pbStyle := lineStyle[[5]];
pb2Style := lineStyle[[10]];
pb3Style := lineStyle[[4]];
pb4Style := lineStyle[[6]];
```

■ Fusion power and plasma packages

The fusion.m package imports plasma.m.

```
Needs["fusion`"]
```

■ Constants and units

```
Needs["Miscellaneous`PhysicalConstants`"];
Needs["Miscellaneous`Units`"]
```

```
ϵ0 = VacuumPermittivity
```

$$\frac{8.85419\times10^{-12}\ \text{Ampere Second}}{\text{Meter Volt}}$$

```
μ0 = VacuumPermeability // N
```

$$\frac{1.25664\times10^{-6}\ \text{Second Volt}}{\text{Ampere Meter}}$$

```
kB = 1.6*^-19 Joule / ElectronVolt
```

$$\frac{1.6\times10^{-19}\ \text{Joule}}{\text{ElectronVolt}}$$

```
me = ElectronMass
```

$$9.10939\times10^{-31}\ \text{Kilogram}$$

```
mp = ProtonMass
```

$$1.67262\times10^{-27}\ \text{Kilogram}$$

■ Rules

```
rlJoule = {Joule → Kilogram Meter^2 / Second^2};

rlToJoule = {Joule → Kilogram Meter^2 / Second^2};
```

■ Functions

```
CheckZoneBoundaryMasses[m1_, m2_] :=
 If[0.9 < m1/m2 < 1.1,
  Null,
  Print[StyleForm["*** Zone boundary mass disconnect ***",
    FontColor → red]]]
```

Parameters based on Thio, et al, Current Trends in International Fusion Research 2nd Symposium and Thio, et al, AIAA-99-2703

Note: see reference section (Thio 1999a and Thio 1999b).

Using BUCKY routine zoner3 with varying zone masses and thicknesses within a given region.

Miscellaneous parameters

$\mathbf{f_{rep} = 40\,Hertz;}$

$\mathbf{r_w = 1.4\,Meter;}$

Target before impact

Plasma parameters

$\mathbf{B_t = 1.0\,Tesla;}$
$\mathbf{\mu_t = 2.5;}$
$\mathbf{\gamma_t = 5/3;}$
$\mathbf{n_{it} = 2.0 \times 10^{24}\,Meter^{-3};}$
$\mathbf{r_t = 0.05\,Meter;}$
$\mathbf{T_{et} = 2\,ElectronVolt;}$
$\mathbf{z_t = 1;}$

Delayed definitions

$\mathbf{\varepsilon_{et} := 1.5\,T_{et};}$
$\mathbf{\varepsilon_{it} := 1.5\,T_{it};}$
$\mathbf{n_{et} := z_t\,n_{it};}$
$\mathbf{T_{it} := T_{et};}$

$$\mathbf{m_t = \mu_t\,m_p\,n_{it}\,\frac{4}{3}\,\pi\,r_t^3;}$$

Total, D, and T masses

$$\mathbf{\{1,\ 0.4,\ 0.6\}\,m_t\,\frac{10^6\,mg}{Kilogram}}$$

```
{4.37892 mg, 1.75157 mg, 2.62735 mg}
```

$\mathbf{plasmaParams}[\mu_t,\ \mathbf{B}_t[\![1]\!],\ 10^{-3}\ \varepsilon_{et}[\![1]\!],\ 10^{-3}\ \varepsilon_{it}[\![1]\!],\ \gamma_t,\ \mathbf{n}_{it}[\![1]\!],\ 10^{-3}\ \mathbf{T}_{et}[\![1]\!],\ 10^{-3}\ \mathbf{T}_{it}[\![1]\!],\ \mathbf{z}_t]$

```
Electron density          2.×10^24          m^-3
AlfvenVelocity            9749.26           m/s
beta                      3.2
CoulogEE                  2.70049
CoulogIE                  2.70049
CoulogII                  2.35392
DebyeLength               7.43135×10^-9     m
electronGyroFrequency     1.76×10^11        s^-1
electronGyroRadius        4.75835×10^-6     m
electronPlasmaFrequency   7.97616×10^13     s^-1
electronThermalVelocity   840762.           m/s
ionAcousticVelocity       11319.6           m/s
ionGyroFrequency          3.832×10^7        s^-1
ionGyroRadius             0.000322441       m
ionPlasmaFrequency        1.18064×10^12     s^-1
ionThermalVelocity        12388.5           m/s
nueePerp                  1.26808×10^13     s^-1
nueeSlow                  7.24618×10^12     s^-1
nuieEnergy                3.22053×10^7      s^-1
nuieSlow                  1.23185×10^9      s^-1
nuiiPerp                  1.46474×10^11     s^-1
```

The next few statements define quantities listed in the table but also required for calculations.

$\Lambda_{iit} = \mathbf{CoulogII}[\mu_t,\ \mathbf{n}_{it}[\![1]\!],\ 10^{-3}\ \mathbf{T}_{it}[\![1]\!],\ \mathbf{z}_t]$

```
2.35392
```

$\Lambda_{eet} = \mathbf{CoulogEE}[\mathbf{n}_{et}[\![1]\!],\ 10^{-3}\ \mathbf{T}_{et}[\![1]\!]]$

```
2.70049
```

$\mathbf{v}_{st} = \mathbf{ionAcousticVelocity}[\mu_t,\ \gamma_t,\ 10^{-3}\ \mathbf{T}_{et}[\![1]\!],\ \mathbf{z}_t]\ \mathbf{Meter / Second;}$

$$\tau_{st} = \frac{\mathbf{r}_t}{\mathbf{v}_{st}}$$

```
4.41712×10^-6 Second
```

$\tau_t = \mathbf{0.0\ Second;}$

$\mathbf{Q}_t = \mathbf{0.0\ Joule;}$

☐ Mean free paths

$$\lambda_{iiperp} = \frac{\mathbf{ionThermalVelocity}[\mu_t,\ 10^{-3}\ \mathbf{T}_{it}[\![1]\!]]}{\mathbf{nuiiPerp}[\Lambda_{iit},\ \mu_t,\ \mathbf{n}_{it}[\![1]\!],\ 10^{-3}\ \mathbf{T}_{it}[\![1]\!],\ \mathbf{z}_t]}$$

```
8.45785×10^-8
```

$$\lambda_{eeperp} = \frac{\mathbf{electronThermalVelocity}[10^{-3}\ \mathbf{T}_{et}[\![1]\!]]}{\mathbf{nueePerp}[\Lambda_{eet},\ \mathbf{n}_{et}[\![1]\!],\ 10^{-3}\ \mathbf{T}_{et}[\![1]\!]]}$$

```
6.63018×10^-8
```

■ BUCKY input

$\mathcal{N}_t = \mathbf{25;}$
$\xi_t = \mathbf{0.05;}$

$\Lambda_t = \mathbf{NestList}[((1.0 + \xi_t) * \#)\ \&,\ 1.0,\ \mathcal{N}_t - 1];$

Zone masses (kg)

$$\mathit{m}_\mathtt{t} = \frac{\Lambda_\mathtt{t}\ \mathtt{m}_\mathtt{t}[\![1]\!]}{\mathtt{Plus\ @@\ }\Lambda_\mathtt{t}};$$

```
{𝓂t⟦1⟧, 𝓂t⟦𝒩t⟧}
```

$\{9.17491\times10^{-8},\ 2.959\times10^{-7}\}$

■ Plasma jet before impact

Plasma parameters

```
Bj = 0.0 Tesla;
γj = 5 / 3;
μj = 2.5;
𝔫j = 60;
𝒩j = 100;
Tej = 26.7 ElectronVolt;
zj = 1;
```

Delayed definitions

```
εej := 1.5 Tej;
εij := 1.5 Tij;
nej := zj nij;
Tij := Tej;
```

```
mj = Convert[0.2 Gram, Kilogram]
```

```
0.0002 Kilogram
```

```
𝓜j = mj / 𝔫j
```

3.33333×10^{-6} Kilogram

$$\xi_\mathtt{j} = \mathtt{x - 1.0\ /.\ FindRoot}\left[\frac{\mathtt{m}_\mathtt{j}[\![1]\!]\ /\ \mathit{m}_\mathtt{t}[\![\mathcal{N}_\mathtt{t}]\!]}{\mathtt{Plus\ @@\ NestList[\,(x * \#)\ \&,\ 1.0,\ }\mathcal{N}_\mathtt{j}\mathtt{ - 1]}} \mathtt{== 1,\ \{x,\ 1\}}\right]$$

```
0.0315417
```

```
Λj = NestList[((1 + ξj) * #) &, 1.0, 𝒩j - 1];
```

Zone masses (kg)

$$\mathit{m}_\mathtt{j} = \frac{\Lambda_\mathtt{j}\ \mathtt{m}_\mathtt{j}[\![1]\!]}{\mathtt{Plus\ @@\ }\Lambda_\mathtt{j}};$$

```
{𝓂j⟦1⟧, 𝓂j⟦𝒩j⟧} // ScientificForm
```

$\{2.959\times10^{-7},\ 6.40229\times10^{-6}\}$

```
CheckZoneBoundaryMasses[𝓂t⟦𝒩t⟧, 𝓂j⟦1⟧]
```

```
rj = rt + 0.024 Meter;
```

$$\mathtt{n}_\mathtt{ij} = \nu\ \mathtt{/.\ Solve}\left[\mathtt{m}_\mathtt{j} == \mu_\mathtt{j}\ \mathtt{m}_\mathtt{p}\ \nu\ \frac{4\pi}{3}\ (\mathtt{r}_\mathtt{j}^3 - \mathtt{r}_\mathtt{t}^3),\ \nu\right][\![1]\!]$$

$$\frac{4.07472\times10^{25}}{\mathrm{Meter}^3}$$

$$\mathtt{v}_\mathtt{sj} = \mathtt{ionAcousticVelocity}[\mu_\mathtt{j},\ \gamma_\mathtt{j},\ 10^{-3}\ \mathtt{T}_\mathtt{ej}[\![1]\!],\ \mathtt{z}_\mathtt{j}]\ \mathtt{Meter / Second}$$

$$\frac{41359.2\ \mathrm{Meter}}{\mathrm{Second}}$$

Perfect acoustic match condition:

```
r1u1 = Solve[equ1 /. {ρ10 → nij, ρ0 → nit, c10 → vsj}, u1] // Flatten
```

$$\left\{\mathrm{u}_1 \to \frac{125231.\ \mathrm{Meter}}{\mathrm{Second}}\right\}$$

$$\mathbf{v_j = u_1 \;/.\; rlul}$$

$$\frac{125231.\,\text{Meter}}{\text{Second}}$$

$$\{\tau_j,\ \tau_{sj}\} = \frac{r_j}{\{v_j,\ v_{sj}\}}$$

$$\{5.90908\times10^{-7}\ \text{Second},\ 1.7892\times10^{-6}\ \text{Second}\}$$

$$Q_j = \mathbf{Convert}\left[\frac{m_j}{2}\, v_j^2,\ \mathbf{Joule}\right]$$

$$1.56828\times10^{6}\ \text{Joule}$$

Energy per jet

$$Q_j \,/\, \mathfrak{n}_j$$

```
26138. Joule
```

$$\mathbf{plasmaParams}[\mu_j,\ B_j[\![1]\!],\ 10^{-3}\,\varepsilon_{ej}[\![1]\!],\ 10^{-3}\,\varepsilon_{ij}[\![1]\!],\ \gamma_j,\ n_{ij}[\![1]\!],\ 10^{-3}\,T_{ej}[\![1]\!],\ 10^{-3}\,T_{ij}[\![1]\!],\ z_j]$$

```
Electron density          4.07472×10^25   m^-3
CoulogEE                  4.89219
CoulogIE                  4.89219
CoulogII                  4.73407
DebyeLength               6.01554×10^-9   m
electronPlasmaFrequency   3.60021×10^14   s^-1
electronThermalVelocity   3.07195×10^6    m/s
ionAcousticVelocity       41359.2         m/s
ionPlasmaFrequency        5.32909×10^12   s^-1
ionThermalVelocity        45264.7         m/s
nueePerp                  9.5952×10^12    s^-1
nueeSlow                  5.48297×10^12   s^-1
nuieEnergy                2.43688×10^7    s^-1
nuieSlow                  9.32105×10^8    s^-1
nuiiPerp                  1.23041×10^11   s^-1
```

The next two statements define quantities listed in the table but also required for calculations.

$$\Lambda_{iij} = \mathbf{CoulogII}[\mu_j,\ n_{ij}[\![1]\!],\ 10^{-3}\,T_{ij}[\![1]\!],\ z_j]$$

```
4.73407
```

$$\Lambda_{eej} = \mathbf{CoulogEE}[n_{ej}[\![1]\!],\ 10^{-3}\,T_{ej}[\![1]\!]]$$

```
4.89219
```

Mean free paths

$$\lambda_{\text{iiperp}} = \frac{\text{ionThermalVelocity}[\mu_j,\ 10^{-3}\ T_{ij}[\![1]\!]]}{\text{nuiiPerp}[\Lambda_{iij},\ \mu_j,\ n_{ij}[\![1]\!],\ 10^{-3}\ T_{ij}[\![1]\!],\ z_j]}$$

3.67884×10^{-7}

$$\lambda_{\text{eeperp}} = \frac{\text{electronThermalVelocity}[10^{-3}\ T_{ej}[\![1]\!]]}{\text{nueePerp}[\Lambda_{eej},\ n_{ej}[\![1]\!],\ 10^{-3}\ T_{ej}[\![1]\!]]}$$

3.20154×10^{-7}

BUCKY input

Area of jets at interface between jets and target:

$$A_j = 4\,\pi\, r_t^2$$

$0.0314159\ \text{Meter}^2$

Directed energy per ion or electron

$$\mathcal{E}_{ej} = \text{Convert}\left[\frac{m_e}{2}\, v_j^2,\ \text{ElectronVolt}\right]$$

$0.0445833\ \text{ElectronVolt}$

$$\mathcal{E}_{ij} = \text{Convert}\left[\frac{z_j\, m_p}{2}\, v_j^2,\ \text{ElectronVolt}\right]$$

$81.8618\ \text{ElectronVolt}$

$$Q_j = \text{Convert}\left[\frac{m_j}{2}\, v_j^2,\ \text{Joule}\right]$$

$1.56828 \times 10^{6}\ \text{Joule}$

$$\Gamma_j = n_{ij}\, v_j$$

$$\frac{5.10282 \times 10^{30}}{\text{Meter}^2\ \text{Second}}$$

Mass and density of jet specified; finding outer jet radius

$$r_j = \text{Module}\Big[\{s\},\ s\ /.\ \text{Solve}\left[\frac{2\, m_p}{m_j}\, n_{ij} * \frac{4\,\pi}{3}\,(s^3\ \text{Meter}^3 - r_t^3) == 1.0,\ s\right]\Big][\![3]\!]\ \text{Meter}$$

$0.0780399\ \text{Meter}$

$$\tau_j = \frac{r_j}{v_j}$$

$6.23167 \times 10^{-7}\ \text{Second}$

$$\Delta_j = r_j - r_t$$

$0.0280399\ \text{Meter}$

Buffer

Buffer before impact

$B_b = 0.0\ \text{Tesla};$
$\mu_b = 2.5;$
$\gamma_b = 5/3;$
$n_{ib} = 4.5 \times 10^{24}\ \text{Meter}^{-3};$
$\mathcal{N}_b = 24;$
$T_{eb} = T_{ej};$
$z_b = 1;$
$m_b = \text{Convert}[2.0\ \text{Gram},\ \text{Kilogram}];$

Delayed definitions

$$\varepsilon_{\mathrm{eb}} := 1.5\ \mathrm{T}_{\mathrm{eb}};$$
$$\varepsilon_{\mathrm{ib}} := 1.5\ \mathrm{T}_{\mathrm{ib}};$$
$$\mathrm{n}_{\mathrm{eb}} := \mathrm{z}_{\mathrm{b}}\ \mathrm{n}_{\mathrm{ib}};$$
$$\mathrm{T}_{\mathrm{ib}} := \mathrm{T}_{\mathrm{eb}};$$

$$\xi_{\mathrm{b}} = x - 1.0\ /.\ \mathtt{FindRoot}\left[\frac{\mathrm{m}_{\mathrm{b}}[\![1]\!]\ /\ m_{\mathrm{j}}[\![\mathcal{N}_{\mathrm{j}}]\!]}{\mathtt{Plus\ @@\ NestList}[(x * \#)\ \&,\ 1.0,\ \mathcal{N}_{\mathrm{b}} - 1]} == 1,\ \{x,\ 1\}\right]$$

0.185012

$$\Lambda_{\mathrm{b}} = \mathtt{NestList}[((1 + \xi_{\mathrm{b}}) * \#)\ \&,\ 1.0,\ \mathcal{N}_{\mathrm{b}} - 1];$$

Zone masses (kg)

$$m_{\mathrm{b}} = \frac{\Lambda_{\mathrm{b}}\ \mathrm{m}_{\mathrm{b}}[\![1]\!]}{\mathtt{Plus\ @@}\ \Lambda_{\mathrm{b}}};$$

$$\{m_{\mathrm{b}}[\![1]\!],\ m_{\mathrm{b}}[\![\mathcal{N}_{\mathrm{b}}]\!]\}\ //\ \mathtt{ScientificForm}$$

$$\{6.40229 \times 10^{-6},\ 3.17656 \times 10^{-4}\}$$

$$\mathtt{CheckZoneBoundaryMasses}[m_{\mathrm{j}}[\![\mathcal{N}_{\mathrm{j}}]\!],\ m_{\mathrm{b}}[\![1]\!]]$$

$$\rho_{\mathrm{b}} = \mu_{\mathrm{b}}\ \mathrm{m}_{\mathrm{p}}\ \mathrm{n}_{\mathrm{ib}}$$

$$\frac{0.018817\ \mathrm{Kilogram}}{\mathrm{Meter}^3}$$

$$\mathrm{r}_{\mathrm{b}} = \mathrm{s}\ /.\ \mathtt{Solve}\left[\mathrm{m}_{\mathrm{b}} == \frac{4\,\pi}{3}\ (\mathrm{s}^3 - \mathrm{r}_{\mathrm{j}}^3)\ \rho_{\mathrm{b}},\ \mathrm{s}\right][\![3]\!]$$

0.295676 Meter

$$\mathrm{v}_{\mathrm{sb}} = \mathtt{ionAcousticVelocity}[\mu_{\mathrm{b}},\ \gamma_{\mathrm{b}},\ 10^{-3}\ \mathrm{T}_{\mathrm{eb}}[\![1]\!],\ \mathrm{z}_{\mathrm{b}}]\ \mathrm{Meter}\ /\ \mathrm{Second}$$

$$\frac{41359.2\ \mathrm{Meter}}{\mathrm{Second}}$$

$$\mathrm{v}_{\mathrm{b}} = \mathrm{v}_{\mathrm{j}}$$

$$\frac{125231.\ \mathrm{Meter}}{\mathrm{Second}}$$

$$\{\tau_{\mathrm{b}},\ \tau_{\mathrm{sb}}\} = \frac{\mathrm{r}_{\mathrm{b}}}{\{\mathrm{v}_{\mathrm{b}},\ \mathrm{v}_{\mathrm{sb}}\}};\ \mathtt{ScientificForm}[\{\tau_{\mathrm{b}},\ \tau_{\mathrm{sb}}\}]$$

$$\{(2.36105 \times 10^{-6})\ \mathrm{Second},\ (7.149 \times 10^{-6})\ \mathrm{Second}\}$$

Flux of ions

$$\Gamma_{\mathrm{b}} = \mathrm{n}_{\mathrm{ib}}\ \mathrm{v}_{\mathrm{b}}\ //\ \mathrm{N}$$

$$\frac{5.6354 \times 10^{29}}{\mathrm{Meter}^2\ \mathrm{Second}}$$

Buffer kinetic energy

$$\mathrm{Q}_{\mathrm{b}} = \mathtt{Convert}\left[\frac{\mathrm{m}_{\mathrm{b}}}{2}\ \mathrm{v}_{\mathrm{b}}^2,\ \mathrm{Joule}\right]$$

$$1.56828 \times 10^{7}\ \mathrm{Joule}$$

$$\{\mu_{\mathrm{b}},\ \mathrm{B}_{\mathrm{b}}[\![1]\!],\ 10^{-3}\ \varepsilon_{\mathrm{eb}}[\![1]\!],\ 10^{-3}\ \varepsilon_{\mathrm{ib}}[\![1]\!],\ \gamma_{\mathrm{b}},\ \mathrm{n}_{\mathrm{ib}}[\![1]\!],\ 10^{-3}\ \mathrm{T}_{\mathrm{eb}}[\![1]\!],\ 10^{-3}\ \mathrm{T}_{\mathrm{ib}}[\![1]\!],\ \mathrm{z}_{\mathrm{b}}\}$$

$$\left\{2.5,\ 0.,\ 0.04005,\ 0.04005,\ \frac{5}{3},\ 4.5 \times 10^{24},\ 0.0267,\ 0.0267,\ 1\right\}$$

$\mathbf{plasmaParams}[\mu_b,\ B_b[\![1]\!],\ 10^{-3}\,\varepsilon_{eb}[\![1]\!],\ 10^{-3}\,\varepsilon_{ib}[\![1]\!],\ \gamma_b,\ n_{ib}[\![1]\!],\ 10^{-3}\,T_{eb}[\![1]\!],\ 10^{-3}\,T_{ib}[\![1]\!],\ z_b]$

```
Electron density          4.5×10^24        m^-3
CoulogEE                  5.99385
CoulogIE                  5.99385
CoulogII                  5.83573
DebyeLength               1.81016×10^-8    m
electronPlasmaFrequency   1.19642×10^14    s^-1
electronThermalVelocity   3.07195×10^6     m/s
ionAcousticVelocity       41359.2          m/s
ionPlasmaFrequency        1.77097×10^12    s^-1
ionThermalVelocity        45264.7          m/s
nueePerp                  1.29829×10^12    s^-1
nueeSlow                  7.41878×10^11    s^-1
nuieEnergy                3.29724×10^6     s^-1
nuieSlow                  1.26119×10^8     s^-1
nuiiPerp                  1.67503×10^10    s^-1
```

Jet-target interface at impact

Basic parameters

Note: ion parameters are for the jets, while electron parameters are for the target, except for the electron-electron slowing down frequency, because the dominant physics should be the ion-electron slowing-down interaction.

★ *Note: exercise caution if using this approach!*

$\mathbf{plasmaParams}[\mu_j,\ B_t[\![1]\!],\ 10^{-3}\,\mathcal{E}_{ej}[\![1]\!],\ 10^{-3}\,\mathcal{E}_{ij}[\![1]\!],\ \gamma_j,\ n_{et}[\![1]\!],\ n_{ij}[\![1]\!],\ 10^{-3}\,T_{et}[\![1]\!],\ 10^{-3}\,T_{ij}[\![1]\!],\ z_j]$

```
Electron density          2.×10^24         m^-3
AlfvenVelocity            2159.92          m/s
beta                      467.778
CoulogEE                  2.70049
CoulogIE                  2.70049
CoulogII                  4.73407
DebyeLength               7.43135×10^-9    m
electronGyroFrequency     1.76×10^11       s^-1
electronGyroRadius        4.75835×10^-6    m
electronPlasmaFrequency   7.97616×10^13    s^-1
electronThermalVelocity   840762.          m/s
ionAcousticVelocity       11319.6          m/s
ionGyroFrequency          3.832×10^7       s^-1
ionGyroRadius             0.00117812       m
ionPlasmaFrequency        5.32909×10^12    s^-1
ionThermalVelocity        45264.7          m/s
nueePerp                  1.26808×10^13    s^-1
nueeSlow                  4.87594×10^14    s^-1
nuieEnergy                2.3746×10^9      s^-1
nuieSlow                  1.23185×10^9     s^-1
nuiiPerp                  1.23041×10^11    s^-1
```

Coulomb logarithms

$\mathbf{\Lambda_{eeI} = CoulogEE}[n_{et}[\![1]\!],\ 10^{-3}\,T_{et}[\![1]\!]]$

```
2.70049
```

$$\Lambda_{\mathrm{ieI}} = \mathbf{CoulogIE}[\mu_j,\, n_{et}[\![1]\!],\, n_{ij}[\![1]\!],\, 10^{-3}\, T_{et}[\![1]\!],\, 10^{-3}\, \mathcal{E}_{ij}[\![1]\!],\, z_j]$$

2.70049

$$\Lambda_{\mathrm{iiI}} = \mathbf{CoulogII}[\mu_j,\, \mu_t,\, n_{ij}[\![1]\!],\, n_{it}[\![1]\!],\, 10^{-3}\, \mathcal{E}_{ij}[\![1]\!],\, 10^{-3}\, T_{it}[\![1]\!],\, z_j,\, z_t]$$

5.54139

Mean free paths

Note: the relevant mean free paths for this part of the problem are for the interaction of the jet ions, with energy $\mathcal{E}_i$, impacting the target ions or electrons, with temperatures T_i and T_e.

$$\lambda_{\mathrm{iiperp}} = \frac{\mathbf{ionThermalVelocity}[\mu_j,\, 10^{-3}\, \mathcal{E}_{ij}[\![1]\!]]}{\mathbf{nuiiPerp}[\Lambda_{\mathrm{iiI}},\, \mu_j,\, n_{ij}[\![1]\!],\, 10^{-3}\, \mathcal{E}_{ij}[\![1]\!],\, z_j]}$$

2.95439×10^{-6}

$$\lambda_{\mathrm{ieS}} = \frac{\mathbf{ionThermalVelocity}[\mu_j,\, 10^{-3}\, \mathcal{E}_{ij}[\![1]\!]]}{\mathbf{nuieSlow}[10^{-3}\, \mathcal{E}_{ij}[\![1]\!],\, \Lambda_{\mathrm{ieI}},\, \mu_j,\, n_{et}[\![1]\!],\, 10^{-3}\, T_{et}[\![1]\!],\, z_j]}$$

0.0000643408

$$\lambda_{\mathrm{ieE}} = \frac{\mathbf{ionThermalVelocity}[\mu_j,\, 10^{-3}\, \mathcal{E}_{ij}[\![1]\!]]}{\mathbf{nuieEnergy}[10^{-3}\, \mathcal{E}_{ij}[\![1]\!],\, \Lambda_{\mathrm{ieI}},\, \mu_j,\, n_{et}[\![1]\!],\, 10^{-3}\, T_{et}[\![1]\!],\, z_j]}$$

0.0000333776

$$\lambda_{\mathrm{eeperp}} = \frac{\mathbf{electronThermalVelocity}[10^{-3}\, T_{et}[\![1]\!]]}{\mathbf{nueePerp}[\Lambda_{\mathrm{eeI}},\, n_{et}[\![1]\!],\, 10^{-3}\, T_{et}[\![1]\!]]}$$

6.63018×10^{-8}

Collapse time scale

Based on jet velocity

$$\tau_{\mathrm{collapseJ}} = \frac{r_t}{v_j}$$

3.99262×10^{-7} Second

Based on ion-acoustic velocity

$$\tau_{\mathrm{collapseS}} = \frac{r_t}{v_{st}}$$

4.41712×10^{-6} Second

$$\tau_{\mathrm{collapse}} = \mathbf{Min}[\tau_{\mathrm{collapseJ}}[\![1]\!],\, \tau_{\mathrm{collapseS}}[\![1]\!]]\ \mathbf{Second}$$

3.99262×10^{-7} Second

Ion-electron interaction time scales

Slowing down

$$\tau_{\mathrm{ieS}} = \mathbf{Second} / \mathbf{nuieSlow}[10^{-3}\, \mathcal{E}_{ij}[\![1]\!],\, \Lambda_{\mathrm{ieI}},\, \mu_j,\, n_{et}[\![1]\!],\, 10^{-3}\, T_{et}[\![1]\!],\, z_j]$$

8.11786×10^{-10} Second

Energy transfer

$$\tau_{\mathrm{ieE}} = \mathbf{Second} / \mathbf{nuieEnergy}[10^{-3}\, \mathcal{E}_{ij}[\![1]\!],\, \Lambda_{\mathrm{ieI}},\, \mu_j,\, n_{et}[\![1]\!],\, 10^{-3}\, T_{et}[\![1]\!],\, z_j]$$

4.21124×10^{-10} Second

$$\tau_{\mathrm{ie}} = \mathbf{Min}[\tau_{\mathrm{ieS}}[\![1]\!],\, \tau_{\mathrm{ieE}}[\![1]\!]]\ \mathbf{Second}$$

4.21124×10^{-10} Second

Critical energy for ion slowing down

Jet ions into target

$$\mathbf{Ecrit[1,\ \{\mu_j,\ \mu_t\},\ \{n_{ij}[\![1]\!],\ n_{it}[\![1]\!]\},\ 10^{-3}\ T_{et}[\![1]\!],\ \{10^{-3}\ \mathcal{E}_{ij}[\![1]\!],\ 10^{-3}\ T_{it}[\![1]\!]\},\ \{z_j,\ z_t\}]\ 10^{3}\ ElectronVolt}$$

```
113.339 ElectronVolt
```

Pressure

Jet pressure at initial jet-target impact

$$P_{j0} = \mathrm{Convert}\left[\frac{\mathcal{M}_j\, v_j}{4\,\pi\, r_t^2\, \tau_{ie}},\ \mathrm{Pascal}\right]$$

3.15523×10^{10} Pascal

B-field pressure (Pa) at initial liner-target impact

$$P_{B0} = \mathrm{Convert}\left[\frac{B_t^2}{2\,\mu_0},\ \mathrm{Pascal}\right]$$

```
397887. Pascal
```

$$\frac{P_{j0}}{P_{B0}}$$

```
79299.5
```

Velocity

Perfect acoustic match condition:

$$\mathbf{rlul = Solve[equl\ /.\ \{\rho_{10}\to n_{ij},\ \rho_0\to n_{it},\ c_{10}\to v_{sj}\},\ u_1]\ //\ Flatten}$$

$$\left\{u_1\to\frac{125231.\ \mathrm{Meter}}{\mathrm{Second}}\right\}$$

Compressed target plasma

Basic parameters

B_c = 10.0 Tesla;
γ_c = 5 / 3;
$\mu_c = \mu_t$;
$n_{ic} = 1.0\times10^{25}$ Meter^{-3};
r_c = 0.0025 Meter;
T_{ec} = Convert[5 Kilo ElectronVolt, ElectronVolt];
z_c = 1;

Delayed definitions

ε_{ec} := 1.5 T_{ec};
ε_{ic} := 1.5 T_{ic};
n_{ec} := $z_c\, n_{ic}$;
T_{ic} := T_{ec};

$\texttt{plasmaParams}[\mu_c, B_c[\![1]\!], 10^{-3}\,\varepsilon_{ec}[\![1]\!], 10^{-3}\,\varepsilon_{ic}[\![1]\!], \gamma_c, n_{ic}[\![1]\!], 10^{-3}\,T_{ec}[\![1]\!], 10^{-3}\,T_{ic}[\![1]\!], z_c]$

```
Electron density          1.×10^25        m^-3
AlfvenVelocity            43600.          m/s
beta                      400.
CoulogEE                  10.8271
CoulogIE                  10.8271
CoulogII                  13.2853
DebyeLength               1.6617×10^-7    m
electronGyroFrequency     1.76×10^12      s^-1
electronGyroRadius        0.0000237918    m
electronPlasmaFrequency   1.78352×10^14   s^-1
electronThermalVelocity   4.20381×10^7    m/s
ionAcousticVelocity       565980.         m/s
ionGyroFrequency          3.832×10^8      s^-1
ionGyroRadius             0.0016122       m
ionPlasmaFrequency        2.64×10^12      s^-1
ionThermalVelocity        619426.         m/s
nueePerp                  2.03366×10^9    s^-1
nueeSlow                  1.16209×10^9    s^-1
nuieEnergy                5164.84         s^-1
nuieSlow                  197555.         s^-1
nuiiPerp                  3.30672×10^7    s^-1
```

■ Mean free paths

$\Lambda_{iic} = \texttt{CoulogII}[\mu_c, n_{ic}[\![1]\!], 10^{-3}\,T_{ic}[\![1]\!], z_c]$

13.2853

$$\lambda_{iiperp} = \frac{\texttt{ionThermalVelocity}[\mu_c, 10^{-3}\,T_{ic}[\![1]\!]]}{\texttt{nuiiPerp}[\Lambda_{iic}, \mu_c, n_{ic}[\![1]\!], 10^{-3}\,T_{ic}[\![1]\!], z_c]}$$

0.0187323

$\Lambda_{eec} = \texttt{CoulogEE}[n_{ec}[\![1]\!], 10^{-3}\,T_{ec}[\![1]\!]]$

10.8271

$$\lambda_{eeperp} = \frac{\texttt{electronThermalVelocity}[10^{-3}\,T_{ec}[\![1]\!]]}{\texttt{nueePerp}[\Lambda_{eec}, n_{ec}[\![1]\!], 10^{-3}\,T_{ec}[\![1]\!]]}$$

0.0206712

■ Critical energies for fusion-product slowing down

D-D alphas

$\texttt{Ecrit}[1, \{4, \mu_c\}, \{10^{-4}\,n_{ic}[\![1]\!], n_{ic}[\![1]\!]\}, 10^{-3}\,T_{ec}[\![1]\!], \{10^{-3}\,T_{ic}[\![1]\!], 10^{-3}\,T_{ic}[\![1]\!]\}, \{2, z_c\}]\ \texttt{keV}$

180.961 keV

D-D protons

$\texttt{Ecrit}[1, \{1, \mu_c\}, \{10^{-4}\,n_{ic}[\![1]\!], n_{ic}[\![1]\!]\}, 10^{-3}\,T_{ec}[\![1]\!], \{10^{-3}\,T_{ic}[\![1]\!], 10^{-3}\,T_{ic}[\![1]\!]\}, \{1, z_c\}]\ \texttt{keV}$

46.8453 keV

D-D tritons

$\texttt{Ecrit}[1, \{3, \mu_c\}, \{10^{-4}\,n_{ic}[\![1]\!], n_{ic}[\![1]\!]\}, 10^{-3}\,T_{ec}[\![1]\!], \{10^{-3}\,T_{ic}[\![1]\!], 10^{-3}\,T_{ic}[\![1]\!]\}, \{1, z_c\}]\ \texttt{keV}$

140.52 keV

D-^{3}He alphas

$\mathbf{Ecrit[1, \{4, \mu_c\}, \{10^{-4}\, n_{ic}[\![1]\!], n_{ic}[\![1]\!]\}, 10^{-3}\, T_{ec}[\![1]\!], \{10^{-3}\, T_{ic}[\![1]\!], 10^{-3}\, T_{ic}[\![1]\!]\}, \{2, z_c\}]\ keV}$

```
180.961 keV
```

Energy change

Increase in target energy

$$\Delta\varepsilon_t = 2\,\pi\, k_B\, ((n_{ic} + n_{ec})\, T_{ic}\, r_c^3 - (n_{it} + n_{et})\, T_{it}\, r_t^3)$$

```
565.487 Joule
```

Maximum fraction of jet energy remaining at full compression

$$f_{jc} = \frac{Q_j - \Delta\varepsilon_t}{Q_j}$$

```
0.999639
```

Pressure

Maximum jet pressure (Pa) remaining at full compression (jet momentum scales as square root of jet energy)

$$P_{jc} = \sqrt{f_{jc}}\, P_{j0}$$

3.15466×10^{10} Pascal

Target pressure (Pa) at full compression

$$P_{tc} = \mathrm{Convert}[n_{ic}\, T_{ic} + n_{ec}\, T_{ec}, \mathrm{Pascal}]$$

1.60218×10^{10} Pascal

B-field pressure (Pa) at full compression

$$P_{Bc} = \mathrm{Convert}\left[\frac{B_c^2}{2\,\mu_0}, \mathrm{Pascal}\right]$$

3.97887×10^{7} Pascal

Pressure ratios

```
Pressure ratios
```

$$\frac{P_{jc}}{\{P_{Bc}, P_{tc}\}}$$

```
{792.852, 1.96898}
```

Summary

Setup print lists

$\mathcal{N}_Z = \{\mathcal{N}_t, \mathcal{N}_j, \mathcal{N}_b\};$

$m_{region} = \{m_t, m_j, m_b\};$

$n_{region} = \{n_{it}, n_{ij}, n_{ib}\};$

$Q_{region} = \{Q_t, Q_j, Q_b\};$

$\tau_{region} = \{\tau_t, \tau_j, \tau_b\};$

$\tau_{sregion} = \{\tau_{st}, \tau_{sj}, \tau_{sb}\};$

$T_{region} = \{T_{et}, T_{ej}, T_{eb}\};$

$v_{region} = \{\{0\}, v_j, v_b\};$

$\zeta_{region} = \{\zeta_t, \zeta_j, \zeta_b\};$

Total number of zones

```
Plus @@ 𝒩_Z
149
```

Zone widths required for specified number of zones

```
r_region = {r_t, r_j, r_b};

Δ_region = -Apply[Subtract, Partition[Prepend[r_region, 0.0 Meter], 2, 1], 1];
```

Summary table

```
lsSummary = {𝒩_Z dummyunit, ζ_region dummyunit, Δ_region, r_region,
   10^3 m_region, n_region, T_region, 10^-3 v_region, 10^6 τ_region, 10^6 τ_sregion, 10^-6 Q_region};

TableForm[Map[#[[1]] &, lsSummary, {2}],
 TableHeadings → {{"zones", "zonfac", "Δ_region(m)", "r(m)", "m(g)", "n(m^-3)",
   "T_e(eV)", "v(km/s)", "τ(μs)", "τ_s(μs)", "KE(MJ)"}, {"Target", "Jets", "Buffer"}}]
```

```
             Target       Jets           Buffer
zones        25           100            24
zonfac       0.05         0.0315417      0.185012
Δ_region(m)  0.05         0.0280399      0.217637
r(m)         0.05         0.0780399      0.295676
m(g)         0.00437892   0.2            2.
n(m^-3)      2.×10^24     4.07472×10^25  4.5×10^24
T_e(eV)      2            26.7           26.7
v(km/s)      0            125.231        125.231
τ(μs)        0.           0.623167       2.36105
τ_s(μs)      4.41712      1.7892         7.149
KE(MJ)       0.           1.56828        15.6828
```

Parameters based roughly on Francis Thio's 3/23/01 output case

The values in this section are based on notes from a discussion with Francis Thio on 3/23/01, and an output case from Francis' MATLAB program.

Case 020718.01 uses these parameters.

Input parameters

Miscellaneous parameters

```
f_rep = 40 Hertz;

r_w = 1.4 Meter;
```

Target before impact

```
μ_t = 2;
B_t = 1.0 Tesla;
γ_t = 5 / 3;
m_t = 3.56 × 10^-8 Kilogram;
r_t = 0.024 Meter;
T_et = 5.0 ElectronVolt;
z_t = 1;
```

$$n_{it} = \frac{m_t}{\mu_t\, m_p\, \frac{4}{3}\, \pi\, r_t^3}$$

$$\frac{1.83781\times 10^{23}}{\text{Meter}^3}$$

Delayed definitions

$\varepsilon_{et} := 1.5\ T_{et};$
$\varepsilon_{it} := 1.5\ T_{it};$
$n_{et} := z_t\ n_{it};$
$T_{it} := T_{et};$

Plasma jets before impact

B_j = 0.0 Tesla;
$\gamma_j = 5/3;$
$\mu_j = 2;$
$\mathcal{N}_j = 60;$
$\mathcal{M}_j = 3.21 \times 10^{-7}$ Kilogram;
$n_{ij} = 6.24 \times 10^{23}$ Meter^{-3};
T_{ej} = 2 ElectronVolt;
v_j = Convert[327 Kilo Meter / Second, Meter / Second];
$z_j = 1;$

Total jet mass

$m_j = \mathcal{N}_j\ \mathcal{M}_j$; Convert[$m_j$, Gram]

0.01926 Gram

Delayed definitions

$\varepsilon_{ej} := 1.5\ T_{ej};$
$\varepsilon_{ij} := 1.5\ T_{ij};$
$n_{ej} := z_j\ n_{ij};$
$T_{ij} := T_{ej};$

Buffer before impact

m_b = Convert[2.2 Gram, Kilogram];
$\mu_b = 2;$
$n_{ib} = 2.4 \times 10^{24}$ Meter^{-3};
v_b = Convert[125 Kilo Meter / Second, Meter / Second];

Compressed plasma

B_c = 10.0 Tesla;
$\gamma_c = 5/3;$
$\mu_c = \mu_t;$
$n_{ic} = 1.0 \times 10^{25}$ Meter^{-3};
r_c = 0.008 Meter;
T_{ec} = Convert[10 Kilo ElectronVolt, ElectronVolt];
$z_c = 1;$

Delayed definitions

$\varepsilon_{ec} := 1.5\ T_{ec};$
$\varepsilon_{ic} := 1.5\ T_{ic};$
$n_{ec} := z_c\ n_{ic};$
$T_{ic} := T_{ec};$

■ Target before impact

$$\mathbf{plasmaParams}[\mu_t,\ B_t[\![1]\!],\ 10^{-3}\,\varepsilon_{et}[\![1]\!],\ 10^{-3}\,\varepsilon_{it}[\![1]\!],\ \gamma_t,\ n_{it}[\![1]\!],\ 10^{-3}\,T_{et}[\![1]\!],\ 10^{-3}\,T_{it}[\![1]\!],\ z_t]$$

```
Electron density          1.83781×10^23  m^-3
AlfvenVelocity            35957.7        m/s
beta                      0.735122
CoulogEE                  5.26851
CoulogIE                  5.26851
CoulogII                  4.92194
DebyeLength               3.87617×10^-8  m
electronGyroFrequency     1.76×10^11     s^-1
electronGyroRadius        7.52362×10^-6  m
electronPlasmaFrequency   2.41785×10^13  s^-1
electronThermalVelocity   1.32936×10^6   m/s
ionAcousticVelocity       20010.4        m/s
ionGyroFrequency          4.79×10^7      s^-1
ionGyroRadius             0.000456       m
ionPlasmaFrequency        4.00137×10^11  s^-1
ionThermalVelocity        21900.         m/s
nueePerp                  5.75111×10^11  s^-1
nueeSlow                  3.28635×10^11  s^-1
nuieEnergy                1.82575×10^6   s^-1
nuieSlow                  6.98349×10^7   s^-1
nuiiPerp                  7.96009×10^9   s^-1
```

$$v_{st} = \mathbf{ionAcousticVelocity}[\mu_t,\ \gamma_t,\ 10^{-3}\,T_{et}[\![1]\!],\ z_t]\ \mathbf{Meter} / \mathbf{Second}$$

$$\frac{20010.4\ \text{Meter}}{\text{Second}}$$

□ Coulomb logarithms

$$\Lambda_{iit} = \mathbf{CoulogII}[\mu_t,\ n_{it}[\![1]\!],\ 10^{-3}\,T_{it}[\![1]\!],\ z_t]$$

4.92194

$$\Lambda_{eet} = \mathbf{CoulogEE}[n_{et}[\![1]\!],\ 10^{-3}\,T_{et}[\![1]\!]]$$

5.26851

■ Mean free paths

$$\lambda_{iiperp} = \frac{\mathbf{ionThermalVelocity}[\mu_t,\ 10^{-3}\,T_{it}[\![1]\!]]}{\mathbf{nuiiPerp}[\Lambda_{iit},\ \mu_t,\ n_{it}[\![1]\!],\ 10^{-3}\,T_{it}[\![1]\!],\ z_t]}$$

2.75122×10^{-6}

$$\lambda_{eeperp} = \frac{\mathbf{electronThermalVelocity}[10^{-3}\,T_{et}[\![1]\!]]}{\mathbf{nueePerp}[\Lambda_{eet},\ n_{et}[\![1]\!],\ 10^{-3}\,T_{et}[\![1]\!]]}$$

2.31149×10^{-6}

Plasma jet before impact

Basic parameters

plasmaParams[μ_j, B_j[[1]], 10^{-3} ε_{ej}[[1]], 10^{-3} ε_{ij}[[1]], γ_j, n_{ij}[[1]], 10^{-3} T_{ej}[[1]], 10^{-3} T_{ij}[[1]], z_j]

```
Electron density         6.24×10^23      m^-3
CoulogEE                 3.28287
CoulogIE                 3.28287
CoulogII                 2.9363
DebyeLength              1.33043×10^-8   m
electronPlasmaFrequency  4.45524×10^13   s^-1
electronThermalVelocity  840762.         m/s
ionAcousticVelocity      12655.7         m/s
ionPlasmaFrequency       7.37312×10^11   s^-1
ionThermalVelocity       13850.8         m/s
nueePerp                 4.80964×10^12   s^-1
nueeSlow                 2.74837×10^12   s^-1
nuieEnergy               1.52687×10^7    s^-1
nuieSlow                 5.84028×10^8    s^-1
nuiiPerp                 6.37349×10^10   s^-1
```

Coulomb logarithms

Λ_{iij} = **CoulogII**[μ_j, n_{ij}[[1]], 10^{-3} T_{ij}[[1]], z_j]

```
2.9363
```

Λ_{eej} = **CoulogEE**[n_{ej}[[1]], 10^{-3} T_{ej}[[1]]]

```
3.28287
```

Mean free paths

$$\lambda_{iiperp} = \frac{\mathbf{ionThermalVelocity}[\mu_j,\ 10^{-3}\ T_{ij}[[1]]]}{\mathbf{nuiiPerp}[\Lambda_{iij},\ \mu_j,\ n_{ij}[[1]],\ 10^{-3}\ T_{ij}[[1]],\ z_j]}$$

```
2.17319×10^-7
```

$$\lambda_{eeperp} = \frac{\mathbf{electronThermalVelocity}[10^{-3}\ T_{ej}[[1]]]}{\mathbf{nueePerp}[\Lambda_{eej},\ n_{ej}[[1]],\ 10^{-3}\ T_{ej}[[1]]]}$$

```
1.74808×10^-7
```

Jet parameters

Area of jets at interface between jets and target:

$A_j = 4\,\pi\,r_t^2$

0.00723823 Meter^2

Directed energy per ion or electron

$$\mathcal{E}_{ej} = \mathbf{Convert}\left[\frac{m_e}{2}\,v_j^2,\ \mathbf{ElectronVolt}\right]$$

0.303979 ElectronVolt

$$\mathcal{E}_{ij} = \mathbf{Convert}\left[\frac{z_j\,m_p}{2}\,v_j^2,\ \mathbf{ElectronVolt}\right]$$

558.153 ElectronVolt

$$Q_j = \mathbf{Convert}\left[\frac{m_j}{2}\,v_j^2,\ \mathbf{Joule}\right]$$

1.02973×10^{6} Joule

$\Gamma_j = n_{ij}\, v_j$

$$\frac{2.04048\times 10^{29}}{\text{Meter}^2\ \text{Second}}$$

$r_j = \text{Module}\left[\{s\},\ s\ /.\ \text{Solve}\left[\frac{2\, m_p}{m_j}\, n_{ij} * \frac{4\,\pi}{3}\,(s^3\ \text{Meter}^3 - r_t^3) == 1.0,\ s\right]\right][\![3]\!]\ \text{Meter}$

0.130384 Meter

$\tau_j = \frac{r_j}{v_j}$

3.98728×10^{-7} Second

$\Delta_j = r_j - r_t$

0.106384 Meter

Buffer

$\rho_b = \mu_b\, m_p\, n_{ib}$

$$\frac{0.00802859\ \text{Kilogram}}{\text{Meter}^3}$$

$\text{r1Rb} = \text{Module}\left[\{s\},\ s\ /.\ \text{Solve}\left[\rho_b == \frac{m_b}{\frac{4\,\pi}{3}\,(s^3 - r_j^3)},\ s\right]\right][\![3]\!]$

0.407432 Meter

Jet-target interface at impact

Basic parameters

Note: ion parameters are for the jets, while electron parameters are for the target, except for the electron-electron slowing down frequency, because the dominant physics should be the ion-electron slowing-down interaction.

★ *Note: exercise caution if using this approach!*

$\mathbf{plasmaParams}[\mu_j,\ B_t[\![1]\!],\ 10^{-3}\ \mathcal{E}_{ej}[\![1]\!],\ 10^{-3}\ \mathcal{E}_{ij}[\![1]\!],\ \gamma_j,\ n_{et}[\![1]\!],\ n_{ij}[\![1]\!],\ 10^{-3}\ T_{et}[\![1]\!],\ 10^{-3}\ T_{ij}[\![1]\!],\ z_j]$

```
Electron density          1.83781×10^23   m^-3
AlfvenVelocity            19514.1         m/s
beta                      1.7472
CoulogEE                  5.26851
CoulogIE                  5.26851
CoulogII                  2.9363
DebyeLength               3.87617×10^-8   m
electronGyroFrequency     1.76×10^11      s^-1
electronGyroRadius        7.52362×10^-6   m
electronPlasmaFrequency   2.41785×10^13   s^-1
electronThermalVelocity   1.32936×10^6    m/s
ionAcousticVelocity       20010.4         m/s
ionGyroFrequency          4.79×10^7       s^-1
ionGyroRadius             0.0002884       m
ionPlasmaFrequency        7.37312×10^11   s^-1
ionThermalVelocity        13850.8         m/s
nueePerp                  5.75111×10^11   s^-1
nueeSlow                  8.10832×10^12   s^-1
nuieEnergy                1.37818×10^8    s^-1
nuieSlow                  6.98349×10^7    s^-1
nuiiPerp                  6.37349×10^10   s^-1
```

Coulomb logarithms

$\Lambda_{eeI} = \mathbf{CoulogEE}[n_{et}[\![1]\!],\ 10^{-3}\ T_{et}[\![1]\!]]$

```
5.26851
```

$\Lambda_{ieI} = \mathbf{CoulogIE}[\mu_j,\ n_{et}[\![1]\!],\ n_{ij}[\![1]\!],\ 10^{-3}\ T_{et}[\![1]\!],\ 10^{-3}\ \mathcal{E}_{ij}[\![1]\!],\ z_j]$

```
5.26851
```

$\Lambda_{iiI} = \mathbf{CoulogII}[\mu_j,\ \mu_t,\ n_{ij}[\![1]\!],\ n_{it}[\![1]\!],\ 10^{-3}\ \mathcal{E}_{ij}[\![1]\!],\ 10^{-3}\ T_{it}[\![1]\!],\ z_j,\ z_t]$

```
9.28449
```

Mean free paths

Note: the relevant mean free paths for this part of the problem are for the interaction of the jet ions, with energy $\mathcal{E}_i$, impacting the target ions or electrons, with temperatures T_i and T_e.

$$\lambda_{iiperp} = \frac{\mathbf{ionThermalVelocity}[\mu_j,\ 10^{-3}\ \mathcal{E}_{ij}[\![1]\!]]}{\mathbf{nuiiPerp}[\Lambda_{iiI},\ \mu_j,\ n_{ij}[\![1]\!],\ 10^{-3}\ \mathcal{E}_{ij}[\![1]\!],\ z_j]}$$

```
0.00535284
```

$$\lambda_{ieS} = \frac{\mathbf{ionThermalVelocity}[\mu_j,\ 10^{-3}\ \mathcal{E}_{ij}[\![1]\!]]}{\mathbf{nuieSlow}[10^{-3}\ \mathcal{E}_{ij}[\![1]\!],\ \Lambda_{ieI},\ \mu_j,\ n_{et}[\![1]\!],\ 10^{-3}\ T_{et}[\![1]\!],\ z_j]}$$

```
0.00331332
```

$$\lambda_{ieE} = \frac{\mathbf{ionThermalVelocity}[\mu_j,\ 10^{-3}\ \mathcal{E}_{ij}[\![1]\!]]}{\mathbf{nuieEnergy}[10^{-3}\ \mathcal{E}_{ij}[\![1]\!],\ \Lambda_{ieI},\ \mu_j,\ n_{et}[\![1]\!],\ 10^{-3}\ T_{et}[\![1]\!],\ z_j]}$$

```
0.00167892
```

$$\lambda_{eeperp} = \frac{\mathbf{electronThermalVelocity}[10^{-3}\ T_{et}[\![1]\!]]}{\mathbf{nueePerp}[\Lambda_{eeI},\ n_{et}[\![1]\!],\ 10^{-3}\ T_{et}[\![1]\!]]}$$

```
2.31149×10^-6
```

Collapse time scale

Based on jet velocity

$$\tau_{\mathrm{collapseJ}} = \frac{r_t}{v_j}$$

7.33945×10^{-8} Second

Based on ion-acoustic velocity

$$\tau_{\mathrm{collapseS}} = \frac{r_t}{v_{st}}$$

1.19938×10^{-6} Second

$\tau_{\mathrm{collapse}} = \mathrm{Min}[\tau_{\mathrm{collapseJ}}[[1]], \tau_{\mathrm{collapseS}}[[1]]]$ Second

7.33945×10^{-8} Second

Ion-electron interaction time scales

Slowing down

$\tau_{\mathrm{ieS}} = \mathrm{Second} / \mathrm{nuieSlow}[10^{-3}\, \mathcal{E}_{ij}[[1]], \Lambda_{\mathrm{ieI}}, \mu_j, n_{et}[[1]], 10^{-3}\, T_{et}[[1]], z_j]$

1.43195×10^{-8} Second

Energy transfer

$\tau_{\mathrm{ieE}} = \mathrm{Second} / \mathrm{nuieEnergy}[10^{-3}\, \mathcal{E}_{ij}[[1]], \Lambda_{\mathrm{ieI}}, \mu_j, n_{et}[[1]], 10^{-3}\, T_{et}[[1]], z_j]$

7.25597×10^{-9} Second

$\tau_{\mathrm{ie}} = \mathrm{Min}[\tau_{\mathrm{ieS}}[[1]], \tau_{\mathrm{ieE}}[[1]]]$ Second

7.25597×10^{-9} Second

Critical energy for ion slowing down

Jet ions into target

$\mathrm{Ecrit}[1, \{\mu_j, \mu_t\}, \{n_{ij}[[1]], n_{it}[[1]]\}, 10^{-3}\, T_{et}[[1]], \{10^{-3}\, \mathcal{E}_{ij}[[1]], 10^{-3}\, T_{it}[[1]]\}, \{z_j, z_t\}]\ 10^3$ ElectronVolt

150.474 ElectronVolt

Pressure

Jet pressure at initial jet-target impact

$$P_{j0} = \mathrm{Convert}\left[\frac{\mathcal{M}_j\, v_j}{4\, \pi\, r_t^2\, \tau_{ie}}, \mathrm{Pascal}\right]$$

1.9986×10^9 Pascal

B-field pressure (Pa) at initial liner-target impact

$$P_{B0} = \mathrm{Convert}\left[\frac{B_t^2}{2\, \mu_0}, \mathrm{Pascal}\right]$$

397887. Pascal

$$\frac{P_{j0}}{P_{B0}}$$

5023.02

Compressed target plasma

Basic parameters

$\mathbf{plasmaParams}[\mu_c,\ B_c[\![1]\!],\ 10^{-3}\,\varepsilon_{ec}[\![1]\!],\ 10^{-3}\,\varepsilon_{ic}[\![1]\!],\ \gamma_c,\ n_{ic}[\![1]\!],\ 10^{-3}\,T_{ec}[\![1]\!],\ 10^{-3}\,T_{ic}[\![1]\!],\ z_c]$

```
Electron density          1. × 10^25        m^-3
AlfvenVelocity            48746.3           m/s
beta                      800.
CoulogEE                  11.5203
CoulogIE                  11.5203
CoulogII                  14.325
DebyeLength               2.35 × 10^-7      m
electronGyroFrequency     1.76 × 10^12      s^-1
electronGyroRadius        0.0000336466      m
electronPlasmaFrequency   1.78352 × 10^14   s^-1
electronThermalVelocity   5.94508 × 10^7    m/s
ionAcousticVelocity       894893.           m/s
ionGyroFrequency          4.79 × 10^8       s^-1
ionGyroRadius             0.00203929        m
ionPlasmaFrequency        2.95161 × 10^12   s^-1
ionThermalVelocity        979398.           m/s
nueePerp                  7.65036 × 10^8    s^-1
nueeSlow                  4.37164 × 10^8    s^-1
nuieEnergy                2428.69           s^-1
nuieSlow                  92897.3           s^-1
nuiiPerp                  1.40939 × 10^7    s^-1
```

Mean free paths

$\Lambda_{iic} = \mathbf{CoulogII}[\mu_c,\ n_{ic}[\![1]\!],\ 10^{-3}\,T_{ic}[\![1]\!],\ z_c]$

14.325

$$\lambda_{iiperp} = \frac{\mathbf{ionThermalVelocity}[\mu_c,\ 10^{-3}\,T_{ic}[\![1]\!]]}{\mathbf{nuiiPerp}[\Lambda_{iic},\ \mu_c,\ n_{ic}[\![1]\!],\ 10^{-3}\,T_{ic}[\![1]\!],\ z_c]}$$

0.0694908

$\Lambda_{eec} = \mathbf{CoulogEE}[n_{ec}[\![1]\!],\ 10^{-3}\,T_{ec}[\![1]\!]]$

11.5203

$$\lambda_{eeperp} = \frac{\mathbf{electronThermalVelocity}[10^{-3}\,T_{ec}[\![1]\!]]}{\mathbf{nueePerp}[\Lambda_{eec},\ n_{ec}[\![1]\!],\ 10^{-3}\,T_{ec}[\![1]\!]]}$$

0.0777098

Critical energies for fusion-product slowing down

D-D alphas

$\mathbf{Ecrit}[1,\ \{4,\ \mu_c\},\ \{10^{-4}\,n_{ic}[\![1]\!],\ n_{ic}[\![1]\!]\},\ 10^{-3}\,T_{ec}[\![1]\!],\ \{10^{-3}\,T_{ic}[\![1]\!],\ 10^{-3}\,T_{ic}[\![1]\!]\},\ \{2,\ z_c\}]\ \mathbf{keV}$

424.251 keV

D-D protons

$\mathbf{Ecrit}[1,\ \{1,\ \mu_c\},\ \{10^{-4}\,n_{ic}[\![1]\!],\ n_{ic}[\![1]\!]\},\ 10^{-3}\,T_{ec}[\![1]\!],\ \{10^{-3}\,T_{ic}[\![1]\!],\ 10^{-3}\,T_{ic}[\![1]\!]\},\ \{1,\ z_c\}]\ \mathbf{keV}$

109.549 keV

D-D tritons

$$\mathrm{Ecrit}[1,\ \{3,\ \mu_c\},\ \{10^{-4}\, n_{ic}[\![1]\!],\ n_{ic}[\![1]\!]\},\ 10^{-3}\, T_{ec}[\![1]\!],\ \{10^{-3}\, T_{ic}[\![1]\!],\ 10^{-3}\, T_{ic}[\![1]\!]\},\ \{1,\ z_c\}]\ \mathrm{keV}$$

```
328.617 keV
```

Energy change

Increase in target energy

$$\Delta\varepsilon_t = 2\,\pi\, k_B\, ((n_{ic} + n_{ec})\, T_{ic}\, r_c^3 - (n_{it} + n_{et})\, T_{it}\, r_t^3)$$

```
102918. Joule
```

Maximum fraction of jet energy remaining at full compression

$$f_{jc} = \frac{Q_j - \Delta\varepsilon_t}{Q_j}$$

```
0.900053
```

Pressure

Maximum jet pressure (Pa) remaining at full compression (jet momentum scales as square root of jet energy)

$$P_{jc} = \sqrt{f_{jc}}\, P_{j0}$$

1.89609×10^9 Pascal

Target pressure (Pa) at full compression

$$P_{tc} = \mathrm{Convert}[n_{ic}\, T_{ic} + n_{ec}\, T_{ec},\ \mathrm{Pascal}]$$

3.20435×10^{10} Pascal

B-field pressure (Pa) at full compression

$$P_{Bc} = \mathrm{Convert}\left[\frac{B_c^2}{2\,\mu_0},\ \mathrm{Pascal}\right]$$

3.97887×10^7 Pascal

Pressure ratios

```
Pressure ratios
```

$$\frac{P_{jc}}{\{P_{Bc},\ P_{tc}\}}$$

```
{47.6539, 0.0591723}
```

BUCKY input parameters for selected cases

■ Case 030201.03

Based on Thio, et al, *Current Trends (1999). Using BUCKY routine* zoner3 *with varying zone masses and thicknesses within a given region.*

■ Target

$\mu_t = 2.5;$
$\gamma_t = 5/3;$
$n_{it} = 1.87 \times 10^{24}\ \mathrm{Meter}^{-3};$
$T_{et} = 2\ \mathrm{ElectronVolt};$
$z_t = 1;$
$\zeta_t = 0.05;$

$r_t = 0.05\ \mathrm{Meter}$

$0.05\ \mathrm{Meter}$

$$m_t = \mu_t\, m_p\, n_{it}\, \frac{4\pi}{3}\, r_t^3$$

$4.09429 \times 10^{-6}\ \mathrm{Kilogram}$

Total, D, and T masses

`Convert[m_t, Milli Gram] {1, 0.4, 0.6}`

`{4.09429 Gram Milli, 1.63771 Gram Milli, 2.45657 Gram Milli}`

$\mathcal{N}_t = 25;$

$\xi_t = 1.0 + \zeta_t$

1.05

$\Lambda_t = \mathrm{NestList}[(\xi_t * \#)\ \&,\ 1.0,\ \mathcal{N}_t - 1]$

```
{1., 1.05, 1.1025, 1.15763, 1.21551, 1.27628, 1.3401, 1.4071,
 1.47746, 1.55133, 1.62889, 1.71034, 1.79586, 1.88565, 1.97993, 2.07893,
 2.18287, 2.29202, 2.40662, 2.52695, 2.6533, 2.78596, 2.92526, 3.07152, 3.2251}
```

Zone masses (kg)

$$m_t = \frac{\Lambda_t\, \mathrm{m}_t[\![1]\!]}{\mathrm{Plus}\ @@\ \Lambda_t}$$

$\{8.57854 \times 10^{-8}, 9.00747 \times 10^{-8}, 9.45784 \times 10^{-8}, 9.93073 \times 10^{-8}, 1.04273 \times 10^{-7}, 1.09486 \times 10^{-7}, 1.14961 \times 10^{-7},$
$1.20709 \times 10^{-7}, 1.26744 \times 10^{-7}, 1.33081 \times 10^{-7}, 1.39735 \times 10^{-7}, 1.46722 \times 10^{-7}, 1.54058 \times 10^{-7},$
$1.61761 \times 10^{-7}, 1.69849 \times 10^{-7}, 1.78342 \times 10^{-7}, 1.87259 \times 10^{-7}, 1.96622 \times 10^{-7}, 2.06453 \times 10^{-7},$
$2.16775 \times 10^{-7}, 2.27614 \times 10^{-7}, 2.38995 \times 10^{-7}, 2.50945 \times 10^{-7}, 2.63492 \times 10^{-7}, 2.76666 \times 10^{-7}\}$

$v_{st} = \mathrm{ionAcousticVelocity}[\mu_t,\ \gamma_t,\ 10^{-3}\ T_{et}[\![1]\!],\ z_t]\ \mathrm{Meter}/\mathrm{Second}$

$$\frac{11319.6\ \mathrm{Meter}}{\mathrm{Second}}$$

$$\frac{r_t}{v_{st}}$$

$4.41712 \times 10^{-6}\ \mathrm{Second}$

■ Jet

$\gamma_j = 5/3;$
$\mu_t = 2.5;$
$T_{ej} = 12\ \mathrm{ElectronVolt};$
$z_j = 1;$

$$\mathfrak{n} = \mathbf{60;}$$

$$\mathbf{m_j = Convert[0.4\,Gram,\ Kilogram]}$$

$$0.0004\ \mathrm{Kilogram}$$

$$\mathcal{M}_j = \mathbf{m_j} / \mathfrak{n}_j$$

$$\frac{0.0004\ \mathrm{Kilogram}}{60_j}$$

$$\mathcal{N}_j = \mathbf{100;}$$

$$\xi_j = x\ \mathbf{/.\ FindRoot}\left[\frac{\mathbf{m_j[\![1]\!]} / m_t[\![\mathcal{N}_t]\!]}{\mathbf{Plus\ @@\ NestList[(}x * \#\mathbf{)\ \&,\ 1.0,\ }\mathcal{N}_j - \mathbf{1]}} == \mathbf{1},\ \{x,\ \mathbf{1}\}\right]$$

$$1.04212$$

$$\zeta_j = \xi_j - \mathbf{1.0}$$

$$0.0421166$$

$$\Lambda_j = \mathbf{NestList[(}\xi_j * \#\mathbf{)\ \&,\ 1.0,\ }\mathcal{N}_j - \mathbf{1];}$$

Zone masses (kg)

$$m_j = \frac{\Lambda_j\ \mathbf{m_j[\![1]\!]}}{\mathbf{Plus\ @@}\ \Lambda_j};$$

$$\{m_j[\![1]\!],\ m_j[\![\mathcal{N}_j]\!]\}\ \mathbf{//\ ScientificForm}$$

$$\{2.76666 \times 10^{-7},\ 1.64313 \times 10^{-5}\}$$

$$\mathbf{r_j = r_t + 0.024\ Meter;}$$

$$\mathbf{n_{ij}} = \nu\ \mathbf{/.\ Solve}\left[\mathbf{m_j == 2.5\ m_p}\ \nu\ \frac{4\pi}{3}\ (\mathbf{r_j^3 - r_t^3}),\ \nu\right][\![1]\!]$$

$$\frac{8.14944 \times 10^{25}}{\mathrm{Meter}^3}$$

$$\mathbf{v_{sj} = ionAcousticVelocity[}\mu_j,\ \gamma_j,\ \mathbf{10^{-3}\ T_{ej}[\![1]\!],\ z_j]\ Meter / Second}$$

$$\frac{31000.\ \mathrm{Meter}}{\mathrm{Second}}$$

Perfect acoustic match condition:

$$\mathbf{rlul = Solve[equl\ /.\ \{}\rho_{10} \to \mathbf{n_{ij}},\ \rho_0 \to \mathbf{n_{it},\ c_{10} \to v_{sj}\},\ u_1]\ //\ Flatten}$$

$$\left\{u_1 \to \frac{137281.\ \mathrm{Meter}}{\mathrm{Second}}\right\}$$

$$\mathbf{v_j = u_1\ /.\ rlul}$$

$$\frac{137281.\ \mathrm{Meter}}{\mathrm{Second}}$$

$$\frac{\mathbf{r_j}}{\mathbf{\{v_j,\ v_{sj}\}}}$$

$$\{5.3904 \times 10^{-7}\ \mathrm{Second},\ 2.3871 \times 10^{-6}\ \mathrm{Second}\}$$

$$\mathbf{Q_j = Convert}\left[\frac{\mathbf{m_j}}{\mathbf{2}}\ \mathbf{v_j^2},\ \mathbf{Joule}\right]$$

$$3.76923 \times 10^{6}\ \mathrm{Joule}$$

Energy per jet

$$\mathbf{Q_j} / \mathfrak{n}_j$$

$$\frac{3.76923 \times 10^{6}\ \mathrm{Joule}}{60_j}$$

Buffer

$\mu_b = 2.5;$
$\gamma_b = 5/3;$
$T_{eb} = T_{ej};$
$z_b = 1;$

$n_{ib} = 4.5\times 10^{24}\ \mathrm{Meter}^{-3};$

$m_b = \mathrm{Convert}[1.8\ \mathrm{Gram},\ \mathrm{Kilogram}];$

$\mathcal{N}_b = 24;$

$$\xi_b = x\ /.\ \mathrm{FindRoot}\left[\frac{m_b[[1]]\ /\ m_j[[\mathcal{N}_j]]}{\mathrm{Plus} @@ \mathrm{NestList}[(x * \#)\ \&,\ 1.0,\ \mathcal{N}_b - 1]} == 1,\ \{x,\ 1\}\right]$$

1.11481

$\zeta_b = \xi_b - 1.0$

0.114808

$\Lambda_b = \mathrm{NestList}[(\xi_b * \#)\ \&,\ 1.0,\ \mathcal{N}_b - 1];$

Zone masses (kg)

$$m_b = \frac{\Lambda_b\, m_b[[1]]}{\mathrm{Plus} @@ \Lambda_b};$$

$\{m_b[[1]],\ m_b[[\mathcal{N}_b]]\}\ //\ \mathrm{ScientificForm}$

$\{1.64313\times 10^{-5},\ 2.00111\times 10^{-4}\}$

$\rho_b = 2.5\ m_p\ n_{ib}$

$$\frac{0.018817\ \mathrm{Kilogram}}{\mathrm{Meter}^3}$$

$$r_b = s\ /.\ \mathrm{Solve}\left[m_b == \frac{4\pi}{3}\,(s^3 - r_j^3)\,\rho_b,\ s\right][[3]]$$

0.28538 Meter

$v_{sb} = \mathrm{ionAcousticVelocity}[\mu_b,\ \gamma_b,\ 10^{-3}\ T_{eb}[[1]],\ z_b]\ \mathrm{Meter}\ /\ \mathrm{Second}$

$$\frac{27727.2\ \mathrm{Meter}}{\mathrm{Second}}$$

$v_b = \mathrm{Convert}[100\ \mathrm{Kilo}\ \mathrm{Meter}\ /\ \mathrm{Second},\ \mathrm{Meter}\ /\ \mathrm{Second}];$

$$\frac{r_b}{\{v_b,\ v_{sb}\}}\ //\ \mathrm{ScientificForm}$$

$\{(2.8538\times 10^{-6})\ \mathrm{Second},\ (1.02924\times 10^{-5})\ \mathrm{Second}\}$

Flux of ions

$\Gamma_b = n_{ib}\ v_b\ //\ N$

$$\frac{4.5\times 10^{29}}{\mathrm{Meter}^2\ \mathrm{Second}}$$

Buffer kinetic energy

$$Q_b = \mathrm{Convert}\left[\frac{m_b}{2}\,v_b^2,\ \mathrm{Joule}\right]$$

$9.\times 10^{6}\ \mathrm{Joule}$

Zone widths required for specified number of zones

$\mathcal{N}_z = \{\mathcal{N}_t,\ \mathcal{N}_j,\ \mathcal{N}_b\}$

$\{25,\ 100,\ 24\}$

m_{region} = {m_t, m_j, m_b}; Map[Convert[#, Gram] &, m_{region}]

{0.00409429 Gram, 0.4 Gram, 1.8 Gram}

r_{region} = {r_t, r_j, r_b}

{0.05 Meter, 0.074 Meter, 0.28538 Meter}

Δ_{region} = -Apply[Subtract, Partition[Prepend[r_{region}, 0], 2, 1], 1]

{0.05 Meter, 0.024 Meter, 0.21138 Meter}

Total number of zones

Plus @@ $\mathcal{N}_Z$

149

Summary

**lsSummary = {{ζ_t, ζ_j, ζ_b} dummyunit, Δ_{region},
r_{region}, 10^3 {m_t, m_j, m_b}, {n_{it}, n_{ij}, n_{ib}}, {T_{et}, T_{ej}, T_{eb}}, 10^{-3} {{0}, v_j, v_b}};**

**TableForm[Map[#[[1]] &, lsSummary, {2}],
TableHeadings → {{"zonfac", "Δ_{region}(m)", "r(m)", "m(g)", "n(m^{-3})", "T_e(eV)", "v(km/s)"},
{"Target", "Jets", "Buffer"}}]**

```
             Target        Jets            Buffer
zonfac       0.05          0.0421166       0.114808
Δ_region(m)  0.05          0.024           0.21138
r(m)         0.05          0.074           0.28538
m(g)         0.00409429    0.4             1.8
n(m^-3)      1.87×10^24    8.14944×10^25   4.5×10^24
T_e(eV)      2             12              12
v(km/s)      0             137.281         100
```

Case 030313.01 (also 030201.05)

Based on Thio, et al*, Current Trends (1999) with* v_{jet}=398 km/s for "perfect acoustic matching."
Using BUCKY routine zoner3 *with varying zone masses and thicknesses within a given region.*

Target

μ_t = 2.5;
γ_t = 5 / 3;
n_{it} = 0.43 × 10^{24} Meter^{-3};
T_{et} = 2 ElectronVolt;
z_t = 1;
ζ_t = 0.05;

r_t = 0.05 Meter

0.05 Meter

$$m_t = \mu_t\, m_p\, n_{it}\, \frac{4\,\pi}{3}\, r_t^3$$

9.41467 × 10^{-7} Kilogram

Total, D, and T masses

Convert[m_t, Milli Gram] {1, 0.4, 0.6}

{0.941467 Gram Milli, 0.376587 Gram Milli, 0.56488 Gram Milli}

$\mathcal{N}_t$ = 25;

ξ_t = 1.0 + ζ_t

1.05

```
Λt = NestList[(ξt * #) &, 1.0, 𝒩t - 1]
```

```
{1., 1.05, 1.1025, 1.15763, 1.21551, 1.27628, 1.3401, 1.4071,
 1.47746, 1.55133, 1.62889, 1.71034, 1.79586, 1.88565, 1.97993, 2.07893,
 2.18287, 2.29202, 2.40662, 2.52695, 2.6533, 2.78596, 2.92526, 3.07152, 3.2251}
```

Zone masses (kg)

$$m_t = \frac{\Lambda_t\, \mathbf{m}_t[\![1]\!]}{\text{Plus @@ } \Lambda_t}$$

$$\{1.97261\times10^{-8},\ 2.07124\times10^{-8},\ 2.1748\times10^{-8},\ 2.28354\times10^{-8},\ 2.39771\times10^{-8},\ 2.5176\times10^{-8},\ 2.64348\times10^{-8},\ 2.77565\times10^{-8},\ 2.91444\times10^{-8},\ 3.06016\times10^{-8},\ 3.21317\times10^{-8},\ 3.37382\times10^{-8},\ 3.54252\times10^{-8},\ 3.71964\times10^{-8},\ 3.90562\times10^{-8},\ 4.1009\times10^{-8},\ 4.30595\times10^{-8},\ 4.52125\times10^{-8},\ 4.74731\times10^{-8},\ 4.98467\times10^{-8},\ 5.23391\times10^{-8},\ 5.4956\times10^{-8},\ 5.77038\times10^{-8},\ 6.0589\times10^{-8},\ 6.36185\times10^{-8}\}$$

$$\mathbf{v}_{st} = \text{ionAcousticVelocity}[\mu_t,\ \gamma_t,\ 10^{-3}\ \mathbf{T}_{et}[\![1]\!],\ \mathbf{z}_t]\ \text{Meter / Second}$$

$$\frac{11319.6\ \text{Meter}}{\text{Second}}$$

$$\tau_{st} = \frac{\mathbf{r}_t}{\mathbf{v}_{st}}$$

$$4.41712\times10^{-6}\ \text{Second}$$

```
τt = 0.0 Second;
```

```
Qt = 0.0 Joule;
```

Jet

```
γj = 5 / 3;
μj = 2.5;
Tej = 29 ElectronVolt;
zj = 1;
```

```
𝔫 = 60;
```

```
mj = Convert[0.4 Gram, Kilogram]
```

```
0.0004 Kilogram
```

```
𝓜j = mj / 𝔫j
```

$$\frac{0.0004\ \text{Kilogram}}{60_j}$$

```
𝒩j = 100;
```

$$\xi_j = x\ /.\ \text{FindRoot}\Big[\frac{\mathbf{m}_j[\![1]\!]\ /\ m_t[\![\mathcal{N}_t]\!]}{\text{Plus @@ NestList}[(x * \#)\ \&,\ 1.0,\ \mathcal{N}_j - 1]} == 1,\ \{x,\ 1\}\Big]$$

```
1.0614
```

$$\zeta_j = \xi_j - 1.0$$

```
0.0613959
```

```
Λj = NestList[(ξj * #) &, 1.0, 𝒩j - 1];
```

Zone masses (kg)

$$m_j = \frac{\Lambda_j\, \mathbf{m}_j[\![1]\!]}{\text{Plus @@ } \Lambda_j};$$

$$\{m_j[\![1]\!],\ m_j[\![\mathcal{N}_j]\!]\}\ \text{// ScientificForm}$$

$$\{6.36185\times10^{-8},\ 2.31977\times10^{-5}\}$$

```
rj = rt + 0.024 Meter;
```

$$\mathbf{n_{ij} = \nu \text{ /. Solve}\left[m_j == 2.5\, m_p\, \nu \frac{4\,\pi}{3}\,(r_j^3 - r_t^3),\ \nu\right][\![1]\!]}$$

$$\frac{8.14944\times 10^{25}}{\text{Meter}^3}$$

$$\mathbf{v_{sj} = \text{ionAcousticVelocity}[\mu_j,\ \gamma_j,\ 10^{-3}\ T_{ej}[\![1]\!],\ z_j]\ \text{Meter / Second}}$$

$$\frac{43103.8\ \text{Meter}}{\text{Second}}$$

Perfect acoustic match condition:

$$\mathbf{\text{rlul} = \text{Solve}[\text{equl /. }\{\rho_{10} \to n_{ij},\ \rho_0 \to n_{it},\ c_{10} \to v_{sj}\},\ u_1]\ \text{// Flatten}}$$

$$\left\{u_1 \to \frac{398062.\ \text{Meter}}{\text{Second}}\right\}$$

$$\mathbf{v_j = u_1 \text{ /. rlul}}$$

$$\frac{398062.\ \text{Meter}}{\text{Second}}$$

$$\mathbf{\{\tau_j,\ \tau_{sj}\} = \frac{r_j}{\{v_j,\ v_{sj}\}}}$$

$$\{1.85901\times 10^{-7}\ \text{Second},\ 1.71679\times 10^{-6}\ \text{Second}\}$$

$$\mathbf{Q_j = \text{Convert}\left[\frac{m_j}{2}\, v_j^2,\ \text{Joule}\right]}$$

$$3.16907\times 10^{7}\ \text{Joule}$$

Energy per jet

$$\mathbf{Q_j \,/\, n_j}$$

$$\frac{3.16907\times 10^{7}\ \text{Joule}}{60_j}$$

Buffer

$$\mathbf{\mu_b = 2.5;}$$
$$\mathbf{\gamma_b = 5/3;}$$
$$\mathbf{T_{eb} = T_{ej};}$$
$$\mathbf{z_b = 1;}$$

$$\mathbf{n_{ib} = 4.5\times 10^{24}\ \text{Meter}^{-3};}$$

$$\mathbf{m_b = \text{Convert}[1.8\ \text{Gram},\ \text{Kilogram}];}$$

$$\mathbf{\mathcal{N}_b = 24;}$$

$$\mathbf{\xi_b = x \text{ /. FindRoot}\left[\frac{m_b[\![1]\!] \,/\, \mathit{m}_j[\![\mathcal{N}_j]\!]}{\text{Plus @@ NestList}[(x * \#)\ \&,\ 1.0,\ \mathcal{N}_b - 1]} == 1,\ \{x,\ 1\}\right]}$$

$$1.09074$$

$$\mathbf{\zeta_b = \xi_b - 1.0}$$

$$0.0907387$$

$$\mathbf{\Lambda_b = \text{NestList}[(\xi_b * \#)\ \&,\ 1.0,\ \mathcal{N}_b - 1];}$$

Zone masses (kg)

$$\mathbf{\mathit{m}_b = \frac{\Lambda_b\, m_b[\![1]\!]}{\text{Plus @@ }\Lambda_b};}$$

$$\mathbf{\{\mathit{m}_b[\![1]\!],\ \mathit{m}_b[\![\mathcal{N}_b]\!]\}\ \text{// ScientificForm}}$$

$$\{2.31977\times 10^{-5},\ 1.7101\times 10^{-4}\}$$

$$\mathbf{\rho_b = 2.5\, m_p\, n_{ib}}$$

$$\frac{0.018817\ \text{Kilogram}}{\text{Meter}^3}$$

$r_b = s \text{ /. Solve}\left[m_b == \frac{4\,\pi}{3}\,(s^3 - r_j^3)\,\rho_b,\ s\right][[3]]$

$0.28538\ \text{Meter}$

$v_{sb} = \text{ionAcousticVelocity}[\mu_b,\ \gamma_b,\ 10^{-3}\ T_{eb}[[1]],\ z_b]\ \text{Meter / Second}$

$\frac{43103.8\ \text{Meter}}{\text{Second}}$

$v_b = \text{Convert[100 Kilo Meter / Second, Meter / Second];}$

$\{\tau_b,\ \tau_{sb}\} = \frac{r_b}{\{v_b,\ v_{sb}\}};\ \text{ScientificForm}[\{\tau_b,\ \tau_{sb}\}]$

$\{(2.8538\times10^{-6})\ \text{Second},\ (6.62078\times10^{-6})\ \text{Second}\}$

Flux of ions

$\Gamma_b = n_{ib}\, v_b\ \text{// N}$

$\frac{4.5\times10^{29}}{\text{Meter}^2\ \text{Second}}$

Buffer kinetic energy

$Q_b = \text{Convert}\left[\frac{m_b}{2}\, v_b^2,\ \text{Joule}\right]$

$9.\times10^{6}\ \text{Joule}$

Summary

Lists

$\mathcal{N}_Z = \{\mathcal{N}_t,\ \mathcal{N}_j,\ \mathcal{N}_b\}$

{25, 100, 24}

$m_{region} = \{m_t,\ m_j,\ m_b\};\ \text{Map[Convert[\#, Gram] \&, }m_{region}]$

$\{0.000941467\ \text{Gram},\ 0.4\ \text{Gram},\ 1.8\ \text{Gram}\}$

$n_{region} = \{n_{it},\ n_{ij},\ n_{ib}\}$

$\left\{\frac{4.3\times10^{23}}{\text{Meter}^3},\ \frac{8.14944\times10^{25}}{\text{Meter}^3},\ \frac{4.5\times10^{24}}{\text{Meter}^3}\right\}$

$Q_{region} = \{Q_t,\ Q_j,\ Q_b\}$

$\{0.\ \text{Joule},\ 3.16907\times10^{7}\ \text{Joule},\ 9.\times10^{6}\ \text{Joule}\}$

$\tau_{region} = \{\tau_t,\ \tau_j,\ \tau_b\}$

$\{0.\ \text{Second},\ 1.85901\times10^{-7}\ \text{Second},\ 2.8538\times10^{-6}\ \text{Second}\}$

$\tau_{sregion} = \{\tau_{st},\ \tau_{sj},\ \tau_{sb}\}$

$\{4.41712\times10^{-6}\ \text{Second},\ 1.71679\times10^{-6}\ \text{Second},\ 6.62078\times10^{-6}\ \text{Second}\}$

$T_{region} = \{T_{et},\ T_{ej},\ T_{eb}\}$

{2 ElectronVolt, 29 ElectronVolt, 29 ElectronVolt}

$v_{region} = \{\{0\},\ v_j,\ v_b\}$

$\left\{\{0\},\ \frac{398062.\ \text{Meter}}{\text{Second}},\ \frac{100000\ \text{Meter}}{\text{Second}}\right\}$

$\zeta_{region} = \{\zeta_t,\ \zeta_j,\ \zeta_b\}$

{0.05, 0.0613959, 0.0907387}

Total number of zones

$\text{Plus @@ }\mathcal{N}_Z$

149

☐ Zone widths required for specified number of zones

```
r_region = {r_t, r_j, r_b}
{0.05 Meter, 0.074 Meter, 0.28538 Meter}
```

```
Δ_region = -Apply[Subtract, Partition[Prepend[r_region, 0], 2, 1], 1]
{0.05 Meter, 0.024 Meter, 0.21138 Meter}
```

☐ Summary table

```
lsSummary = {𝒩_Z dummyunit, ζ_region dummyunit, Δ_region, r_region,
   10^3 m_region, n_region, T_region, 10^-3 v_region, 10^6 τ_region, 10^6 τ_sregion, 10^-6 Q_region};

TableForm[Map[#[[1]] &, lsSummary, {2}],
 TableHeadings → {{"zones", "zonfac", "Δ_region(m)", "r(m)", "m(g)", "n(m^-3)",
    "T_e(eV)", "v(km/s)", "τ(μs)", "τ_s(μs)", "KE(MJ)"}, {"Target", "Jets", "Buffer"}}]
```

```
            Target        Jets             Buffer
zones       25            100              24
zonfac      0.05          0.0613959        0.0907387
Δ_region(m) 0.05          0.024            0.21138
r(m)        0.05          0.074            0.28538
m(g)        0.000941467   0.4              1.8
n(m^-3)     4.3×10^23     8.14944×10^25    4.5×10^24
T_e(eV)     2             29               29
v(km/s)     0             398.062          100
τ(μs)       0.            0.185901         2.8538
τ_s(μs)     4.41712       1.71679          6.62078
KE(MJ)      0.            31.6907          9.
```

■ Case 030313.02

Based on Thio *(*1999a*). Using BUCKY routine* zoner3 *with varying zone masses and thicknesses within a given region. See Subsection "*Thio: Current Trends in International Fusion Research 2nd Symposium Paper*".*

■ Target

```
μ_t = 2.5;
γ_t = 5 / 3;
n_it = 2.0 × 10^24 Meter^-3;
𝒩_t = 25;
r_t = 0.05 Meter;
T_et = 2 ElectronVolt;
z_t = 1;
ζ_t = 0.05;
ξ_t = 1.0 + ζ_t
1.05
```

```
m_t = μ_t m_p n_it (4/3) π r_t^3;
```

Total, D, and T masses

```
{1, 0.4, 0.6} m_t (10^6 mg / Kilogram)
{4.37892 mg, 1.75157 mg, 2.62735 mg}
```

```
Λ_t = NestList[(ξ_t * #) &, 1.0, 𝒩_t - 1];
```

Zone masses (kg)

$$m_t = \frac{\Lambda_t\, m_t[\![1]\!]}{\text{Plus @@ }\Lambda_t};$$

$$\{m_t[\![1]\!],\ m_t[\![\mathcal{N}_t]\!]\}$$

$$\{9.17491\times10^{-8},\ 2.959\times10^{-7}\}$$

$$v_{st} = \text{ionAcousticVelocity}[\mu_t,\ \gamma_t,\ 10^{-3}\ T_{et}[\![1]\!],\ z_t]\ \text{Meter / Second}$$

$$\frac{11319.6\ \text{Meter}}{\text{Second}}$$

$$\tau_{st} = \frac{r_t}{v_{st}}$$

$$4.41712\times10^{-6}\ \text{Second}$$

$$\tau_t = 0.0\ \text{Second};$$

$$Q_t = 0.0\ \text{Joule};$$

Jet

$$\gamma_j = 5/3;$$
$$\mu_j = 2.5;$$
$$\mathfrak{n}_j = 60;$$
$$\mathcal{N}_j = 100;$$
$$T_{ej} = 26.7\ \text{ElectronVolt};$$
$$z_j = 1;$$

$$m_j = \text{Convert}[0.2\ \text{Gram, Kilogram}]$$

$$0.0002\ \text{Kilogram}$$

$$\mathcal{M}_j = m_j / \mathfrak{n}_j$$

$$3.33333\times10^{-6}\ \text{Kilogram}$$

$$\xi_j = x\ /.\ \text{FindRoot}\left[\frac{m_j[\![1]\!] / m_t[\![\mathcal{N}_t]\!]}{\text{Plus @@ NestList}[(x * \#)\ \&,\ 1.0,\ \mathcal{N}_j - 1]} == 1,\ \{x,\ 1\}\right]$$

$$1.03154$$

$$\zeta_j = \xi_j - 1.0$$

$$0.0315417$$

$$\Lambda_j = \text{NestList}[(\xi_j * \#)\ \&,\ 1.0,\ \mathcal{N}_j - 1];$$

Zone masses (kg)

$$m_j = \frac{\Lambda_j\, m_j[\![1]\!]}{\text{Plus @@ }\Lambda_j};$$

$$\{m_j[\![1]\!],\ m_j[\![\mathcal{N}_j]\!]\}\ //\ \text{ScientificForm}$$

$$\{2.959\times10^{-7},\ 6.40229\times10^{-6}\}$$

$$\text{CheckZoneBoundaryMasses}[m_t[\![\mathcal{N}_t]\!],\ m_j[\![1]\!]]$$

$$r_j = r_t + 0.024\ \text{Meter};$$

$$n_{ij} = \nu\ /.\ \text{Solve}\left[m_j == \mu_j\, m_p\, \nu\, \frac{4\,\pi}{3}\,(r_j^3 - r_t^3),\ \nu\right][\![1]\!]$$

$$\frac{4.07472\times10^{25}}{\text{Meter}^3}$$

$$v_{sj} = \text{ionAcousticVelocity}[\mu_j,\ \gamma_j,\ 10^{-3}\ T_{ej}[\![1]\!],\ z_j]\ \text{Meter / Second}$$

$$\frac{41359.2\ \text{Meter}}{\text{Second}}$$

Perfect acoustic match condition:

$$\mathrm{rlul} = \mathrm{Solve}[\mathrm{equl}\ /.\ \{\rho_{10} \to n_{ij},\ \rho_0 \to n_{it},\ c_{10} \to v_{sj}\},\ u_1]\ //\ \mathrm{Flatten}$$

$$\left\{u_1 \to \frac{125231.\ \mathrm{Meter}}{\mathrm{Second}}\right\}$$

$$v_j = u_1\ /.\ \mathrm{rlul}$$

$$\frac{125231.\ \mathrm{Meter}}{\mathrm{Second}}$$

$$\{\tau_j,\ \tau_{sj}\} = \frac{r_j}{\{v_j,\ v_{sj}\}}$$

$$\{5.90908 \times 10^{-7}\ \mathrm{Second},\ 1.7892 \times 10^{-6}\ \mathrm{Second}\}$$

$$Q_j = \mathrm{Convert}\left[\frac{m_j}{2}\ v_j^2,\ \mathrm{Joule}\right]$$

$$1.56828 \times 10^{6}\ \mathrm{Joule}$$

Energy per jet

$$Q_j\ /\ \mathfrak{n}_j$$

$$26138.\ \mathrm{Joule}$$

Buffer

$$\mu_b = 2.5;$$
$$\gamma_b = 5\ /\ 3;$$
$$n_{ib} = 4.5 \times 10^{24}\ \mathrm{Meter}^{-3};$$
$$\mathcal{N}_b = 24;$$
$$T_{eb} = T_{ej};$$
$$z_b = 1;$$
$$m_b = \mathrm{Convert}[2.0\ \mathrm{Gram},\ \mathrm{Kilogram}];$$

$$\xi_b = x\ /.\ \mathrm{FindRoot}\left[\frac{\mathrm{m}_b\llbracket 1\rrbracket\ /\ \mathit{m}_j\llbracket \mathcal{N}_j\rrbracket}{\mathrm{Plus} \mathbin{@@} \mathrm{NestList}[(x * \#)\ \&,\ 1.0,\ \mathcal{N}_b - 1]} == 1,\ \{x,\ 1\}\right]$$

$$1.18501$$

$$\zeta_b = \xi_b - 1.0$$

$$0.185012$$

$$\Lambda_b = \mathrm{NestList}[(\xi_b * \#)\ \&,\ 1.0,\ \mathcal{N}_b - 1];$$

Zone masses (kg)

$$\mathit{m}_b = \frac{\Lambda_b\ \mathrm{m}_b\llbracket 1\rrbracket}{\mathrm{Plus} \mathbin{@@} \Lambda_b};$$

$$\{\mathit{m}_b\llbracket 1\rrbracket,\ \mathit{m}_b\llbracket \mathcal{N}_b\rrbracket\}\ //\ \mathrm{ScientificForm}$$

$$\{6.40229 \times 10^{-6},\ 3.17656 \times 10^{-4}\}$$

$$\mathrm{CheckZoneBoundaryMasses}[\mathit{m}_j\llbracket \mathcal{N}_j\rrbracket,\ \mathit{m}_b\llbracket 1\rrbracket]$$

$$\rho_b = \mu_b\ m_p\ n_{ib}$$

$$\frac{0.018817\ \mathrm{Kilogram}}{\mathrm{Meter}^3}$$

$$r_b = s\ /.\ \mathrm{Solve}\left[m_b == \frac{4\,\pi}{3}\ (s^3 - r_j^3)\ \rho_b,\ s\right]\llbracket 3\rrbracket$$

$$0.295409\ \mathrm{Meter}$$

$$v_{sb} = \mathrm{ionAcousticVelocity}[\mu_b,\ \gamma_b,\ 10^{-3}\ T_{eb}\llbracket 1\rrbracket,\ z_b]\ \mathrm{Meter}\ /\ \mathrm{Second}$$

$$\frac{41359.2\ \mathrm{Meter}}{\mathrm{Second}}$$

$$v_b = v_j$$

$$\frac{125231.\ \text{Meter}}{\text{Second}}$$

$$\{\tau_b,\ \tau_{sb}\} = \frac{r_b}{\{v_b,\ v_{sb}\}};\ \text{ScientificForm}[\{\tau_b,\ \tau_{sb}\}]$$

$$\{(2.35891\times10^{-6})\ \text{Second},\ (7.14253\times10^{-6})\ \text{Second}\}$$

Flux of ions

$$\Gamma_b = n_{ib}\, v_b\ //\ \text{N}$$

$$\frac{5.6354\times10^{29}}{\text{Meter}^2\ \text{Second}}$$

Buffer kinetic energy

$$Q_b = \text{Convert}\left[\frac{m_b}{2}\, v_b^2,\ \text{Joule}\right]$$

$$1.56828\times10^{7}\ \text{Joule}$$

■ Summary

☐ Setup print lists

$\mathcal{N}_Z = \{\mathcal{N}_t,\ \mathcal{N}_j,\ \mathcal{N}_b\};$

$m_{region} = \{m_t,\ m_j,\ m_b\};$

$n_{region} = \{n_{it},\ n_{ij},\ n_{ib}\};$

$Q_{region} = \{Q_t,\ Q_j,\ Q_b\};$

$\tau_{region} = \{\tau_t,\ \tau_j,\ \tau_b\};$

$\tau_{sregion} = \{\tau_{st},\ \tau_{sj},\ \tau_{sb}\};$

$T_{region} = \{T_{et},\ T_{ej},\ T_{eb}\};$

$v_{region} = \{\{0\},\ v_j,\ v_b\};$

$\zeta_{region} = \{\zeta_t,\ \zeta_j,\ \zeta_b\};$

Total number of zones

$\text{Plus}\ @@\ \mathcal{N}_Z$

149

☐ Zone widths required for specified number of zones

$r_{region} = \{r_t,\ r_j,\ r_b\};$

$\Delta_{region} = -\text{Apply}[\text{Subtract},\ \text{Partition}[\text{Prepend}[r_{region},\ 0.0\ \text{Meter}],\ 2,\ 1],\ 1];$

☐ Summary table

$$\begin{aligned}&\text{lsSummary} = \{\mathcal{N}_Z\ \text{dummyunit},\ \zeta_{region}\ \text{dummyunit},\ \Delta_{region},\ r_{region},\\ &\quad 10^3\, m_{region},\ n_{region},\ T_{region},\ 10^{-3}\, v_{region},\ 10^6\, \tau_{region},\ 10^6\, \tau_{sregion},\ 10^{-6}\, Q_{region}\};\end{aligned}$$

```
TableForm[Map[#[[1]] &, lsSummary, {2}],
 TableHeadings → {{"zones", "zonfac", "Δregion(m)", "r(m)", "m(g)", "n(m^-3)",
    "Te(eV)", "v(km/s)", "τ(μs)", "τs(μs)", "KE(MJ)"}, {"Target", "Jets", "Buffer"}}]
```

```
            Target       Jets            Buffer
zones       25           100             24
zonfac      0.05         0.0315417       0.185012
Δregion(m)  0.05         0.024           0.221409
r(m)        0.05         0.074           0.295409
m(g)        0.00437892   0.2             2.
n(m^-3)     2.×10^24     4.07472×10^25   4.5×10^24
Te(eV)      2            26.7            26.7
v(km/s)     0            125.231         125.231
τ(μs)       0.           0.590908        2.35891
τs(μs)      4.41712      1.7892          7.14253
KE(MJ)      0.           1.56828         15.6828
```

Case 030313.03

Case 030313.02 with v_{jet}=250 km/s, maintaining "perfect acoustic matching."

Target

$\mu_t = 2.5;$

$\gamma_t = 5/3;$

$n_{it} = 2.0 \times 10^{24}\ \text{Meter}^{-3};$

$\mathcal{N}_t = 25;$

$r_t = 0.05\ \text{Meter};$

$T_{et} = 2\ \text{ElectronVolt};$

$z_t = 1;$

$\zeta_t = 0.05;$

$\xi_t = 1.0 + \zeta_t$

```
1.05
```

$$m_t = \mu_t\, m_p\, n_{it}\, \frac{4}{3}\, \pi\, r_t^3;$$

Total, D, and T masses

$$\{1,\ 0.4,\ 0.6\}\ m_t\ \frac{10^6\ \text{mg}}{\text{Kilogram}}$$

```
{4.37892 mg, 1.75157 mg, 2.62735 mg}
```

$\Lambda_t = \text{NestList}[(\xi_t * \#)\ \&,\ 1.0,\ \mathcal{N}_t - 1];$

Zone masses (kg)

$$m_t = \frac{\Lambda_t\, m_t[[1]]}{\text{Plus @@ }\Lambda_t};$$

```
{mt[[1]], mt[[Nt]]}
{9.17491×10^-8, 2.959×10^-7}
```

```
vst = ionAcousticVelocity[μt, γt, 10^-3 Tet[[1]], zt] Meter / Second
```

$$\frac{11319.6\ \text{Meter}}{\text{Second}}$$

$$\tau_{st} = \frac{r_t}{v_{st}}$$

```
4.41712×10^-6 Second
```

```
τt = 0.0 Second;
```

```
Qt = 0.0 Joule;
```

Jet

```
γj = 5 / 3;
μj = 2.5;
ɳj = 60;
Nj = 100;
Tej = 106 ElectronVolt;
zj = 1;
```

```
mj = Convert[0.2 Gram, Kilogram]
0.0002 Kilogram
```

```
Mj = mj / ɳj
3.33333×10^-6 Kilogram
```

$$\xi_j = x\ /.\ \text{FindRoot}\left[\frac{m_j[[1]]\ /\ m_t[[N_t]]}{\text{Plus}\ @@\ \text{NestList}[(x * \#)\ \&,\ 1.0,\ N_j - 1]} == 1,\ \{x, 1\}\right]$$

```
1.03154
```

```
ζj = ξj - 1.0
0.0315417
```

```
Λj = NestList[(ξj * #) &, 1.0, Nj - 1];
```

Zone masses (kg)

$$m_j = \frac{\Lambda_j\ m_j[[1]]}{\text{Plus}\ @@\ \Lambda_j};$$

```
{mj[[1]], mj[[Nj]]} // ScientificForm
{2.959×10^-7, 6.40229×10^-6}
```

```
CheckZoneBoundaryMasses[mt[[Nt]], mj[[1]]]
```

```
rj = rt + 0.024 Meter;
```

$$n_{ij} = \nu\ /.\ \text{Solve}\left[m_j == \mu_j\, m_p\, \nu\, \frac{4\pi}{3}\,(r_j^3 - r_t^3),\ \nu\right][[1]]$$

$$\frac{4.07472\times 10^{25}}{\text{Meter}^3}$$

```
vsj = ionAcousticVelocity[μj, γj, 10^-3 Tej[[1]], zj] Meter / Second
```

$$\frac{82407.9\ \text{Meter}}{\text{Second}}$$

Perfect acoustic match condition:

$$\mathbf{rlul = Solve[equl\ /.\ \{\rho_{10} \to n_{ij},\ \rho_0 \to n_{it},\ c_{10} \to v_{sj}\},\ u_1]\ //\ Flatten}$$

$$\left\{u_1 \to \frac{249522.\ \text{Meter}}{\text{Second}}\right\}$$

$$\mathbf{v_j = u_1\ /.\ rlul}$$

$$\frac{249522.\ \text{Meter}}{\text{Second}}$$

$$\mathbf{\{\tau_j,\ \tau_{sj}\} = \frac{r_j}{\{v_j,\ v_{sj}\}}}$$

$$\{2.96567 \times 10^{-7}\ \text{Second},\ 8.97972 \times 10^{-7}\ \text{Second}\}$$

$$\mathbf{Q_j = Convert\left[\frac{m_j}{2}\ v_j^2,\ Joule\right]}$$

$$6.22613 \times 10^{6}\ \text{Joule}$$

Energy per jet

$$\mathbf{Q_j\ /\ \mathfrak{n}_j}$$

$$103769.\ \text{Joule}$$

Buffer

$$\mathbf{\mu_b = 2.5;}$$
$$\mathbf{\gamma_b = 5\ /\ 3;}$$
$$\mathbf{n_{ib} = 4.5 \times 10^{24}\ Meter^{-3};}$$
$$\mathbf{\mathcal{N}_b = 24;}$$
$$\mathbf{T_{eb} = T_{ej};}$$
$$\mathbf{z_b = 1;}$$
$$\mathbf{m_b = Convert[2.0\ Gram,\ Kilogram];}$$

$$\mathbf{\xi_b = x\ /.\ FindRoot\left[\frac{m_b[[1]]\ /\ m_j[[\mathcal{N}_j]]}{Plus\ @@\ NestList[(x * \#)\ \&,\ 1.0,\ \mathcal{N}_b - 1]} == 1,\ \{x,\ 1\}\right]}$$

$$1.18501$$

$$\mathbf{\zeta_b = \xi_b - 1.0}$$

$$0.185012$$

$$\mathbf{\Lambda_b = NestList[(\xi_b * \#)\ \&,\ 1.0,\ \mathcal{N}_b - 1];}$$

Zone masses (kg)

$$\mathbf{m_b = \frac{\Lambda_b\ m_b[[1]]}{Plus\ @@\ \Lambda_b};}$$

$$\mathbf{\{m_b[[1]],\ m_b[[\mathcal{N}_b]]\}\ //\ ScientificForm}$$

$$\{6.40229 \times 10^{-6},\ 3.17656 \times 10^{-4}\}$$

$$\mathbf{CheckZoneBoundaryMasses[m_j[[\mathcal{N}_j]],\ m_b[[1]]]}$$

$$\mathbf{\rho_b = \mu_b\ m_p\ n_{ib}}$$

$$\frac{0.018817\ \text{Kilogram}}{\text{Meter}^3}$$

$$\mathbf{r_b = s\ /.\ Solve\left[m_b == \frac{4\,\pi}{3}\ (s^3 - r_j^3)\ \rho_b,\ s\right][[3]]}$$

$$0.295409\ \text{Meter}$$

$$\mathbf{v_{sb} = ionAcousticVelocity[\mu_b,\ \gamma_b,\ 10^{-3}\ T_{eb}[[1]],\ z_b]\ Meter\ /\ Second}$$

$$\frac{82407.9\ \text{Meter}}{\text{Second}}$$

$\mathbf{v_b = v_j}$

$$\frac{249522.\,\text{Meter}}{\text{Second}}$$

$$\{\tau_b,\ \tau_{sb}\} = \frac{r_b}{\{v_b,\ v_{sb}\}};\ \mathbf{ScientificForm}[\{\tau_b,\ \tau_{sb}\}]$$

$\{(1.1839 \times 10^{-6})\ \text{Second},\ (3.58472 \times 10^{-6})\ \text{Second}\}$

Flux of ions

$\Gamma_b = n_{ib}\, v_b\ \mathbf{//\ N}$

$$\frac{1.12285 \times 10^{30}}{\text{Meter}^2\ \text{Second}}$$

Buffer kinetic energy

$$Q_b = \mathbf{Convert}\left[\frac{m_b}{2}\, v_b^2,\ \mathbf{Joule}\right]$$

6.22613×10^7 Joule

■ Summary

□ Setup print lists

$\mathcal{N}_Z = \{\mathcal{N}_t,\ \mathcal{N}_j,\ \mathcal{N}_b\};$

$m_{region} = \{m_t,\ m_j,\ m_b\};$

$n_{region} = \{n_{it},\ n_{ij},\ n_{ib}\};$

$Q_{region} = \{Q_t,\ Q_j,\ Q_b\};$

$\tau_{region} = \{\tau_t,\ \tau_j,\ \tau_b\};$

$\tau_{sregion} = \{\tau_{st},\ \tau_{sj},\ \tau_{sb}\};$

$T_{region} = \{T_{et},\ T_{ej},\ T_{eb}\};$

$v_{region} = \{\{0\},\ v_j,\ v_b\};$

$\zeta_{region} = \{\zeta_t,\ \zeta_j,\ \zeta_b\};$

Total number of zones

$\mathbf{Plus\ @@}\ \mathcal{N}_Z$

149

□ Zone widths required for specified number of zones

$r_{region} = \{r_t,\ r_j,\ r_b\};$

$\Delta_{region} = -\mathbf{Apply[Subtract,\ Partition[Prepend[}r_{region},\ 0.0\ \mathbf{Meter],\ 2,\ 1],\ 1]};$

□ Summary table

$$\mathbf{lsSummary} = \{\mathcal{N}_Z\ \mathbf{dummyunit},\ \zeta_{region}\ \mathbf{dummyunit},\ \Delta_{region},\ r_{region},\ 10^3\, m_{region},\ n_{region},\ T_{region},\ 10^{-3}\, v_{region},\ 10^6\, \tau_{region},\ 10^6\, \tau_{sregion},\ 10^{-6}\, Q_{region}\};$$

```
TableForm[Map[#⟦1⟧ &, lsSummary, {2}],
 TableHeadings → {{"zones", "zonfac", "Δ_region(m)", "r(m)", "m(g)", "n(m^-3)",
    "T_e(eV)", "v(km/s)", "τ(μs)", "τ_s(μs)", "KE(MJ)"}, {"Target", "Jets", "Buffer"}}]
```

```
           Target      Jets           Buffer
zones      25          100            24
zonfac     0.05        0.0315417      0.185012
Δ_region(m) 0.05       0.024          0.221409
r(m)       0.05        0.074          0.295409
m(g)       0.00437892  0.2            2.
n(m^-3)    2.×10^24    4.07472×10^25  4.5×10^24
T_e(eV)    2           106            106
v(km/s)    0           249.522        249.522
τ(μs)      0.          0.296567       1.1839
τ_s(μs)    4.41712     0.897972       3.58472
KE(MJ)     0.          6.22613        62.2613
```

BUCKY cases 030121.06&7 (D-T, perfect acoustic matching, v_{jet}=400 km/s)

- ★ *Note: the compression for this case is too good to be true, because the masses were not matched well at the interfaces, but this is the sort of behavior that is sought. The case is left here as a potential starting point for other calculations.*
- ★ *Note: the full set of commands to generate the plots is shown in the section "BUCKY case 030313.02" below. In this section only the plots and the outline of the structure appear.*

■ Setup

```
caseID = "030121_06";
```

```
params = Graphics[Text["Case " <> caseID <> "\n
m_t=0.1 mg; Δ_t=3.1 cm\n
m_j=0.4 g; Δ_j=3.6 cm\n
m_b=1.8 g; Δ_b=21 cm\n
v_j=398 km/s; v_b=100 km/s", {0, 0}, {0, 0}]];
```

■ 0-1 μs

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\0_1_us\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 1000;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";
```

```
thickness = Thickness[0.0002];
```

☐ **Read data**

☐ **Number of time steps**

☐ **Number and list of radial zones in file**

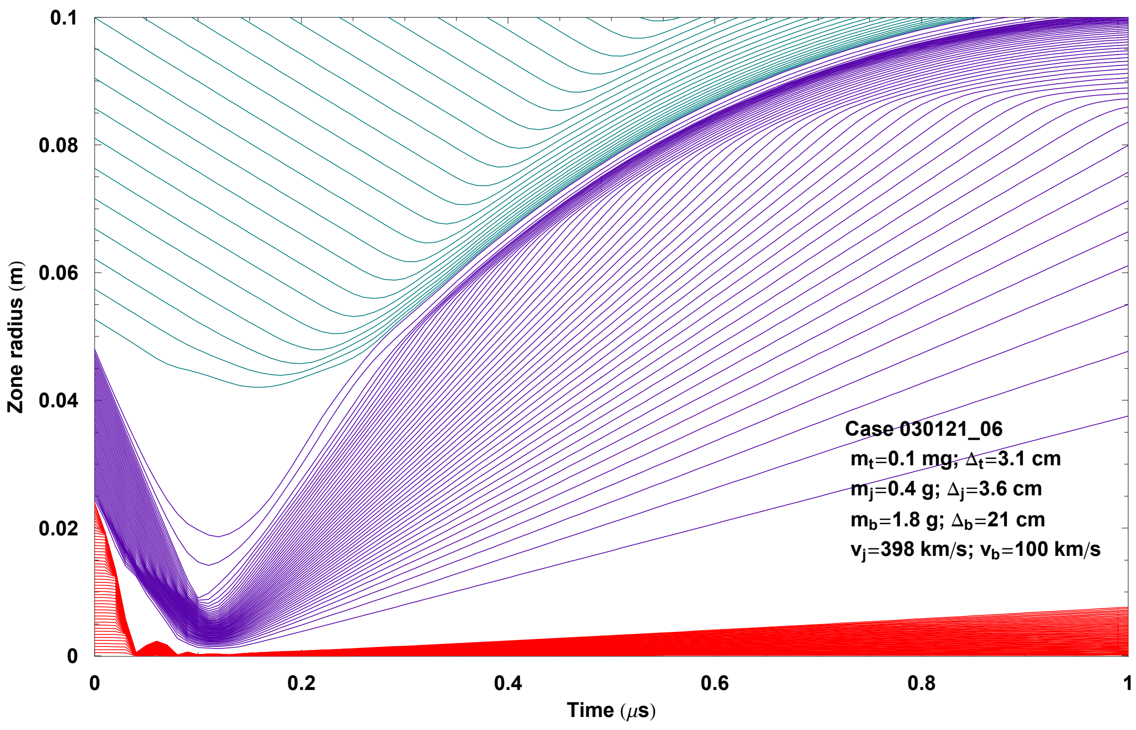
☐ **Convert data from {s, cm} to {μs, m}**

☐ **Plots**

- **r < 0.1 m**

```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.1}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10^-3 tEnd, 0.024}, {0.95 * 10^-3 tEnd, 0.028}, params],
  DisplayFunction → Identity];

lsPlotsTarget = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → red], {i, 1, 𝒩t}];

lsPlotsJets = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];

lsPlotsBuffer = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



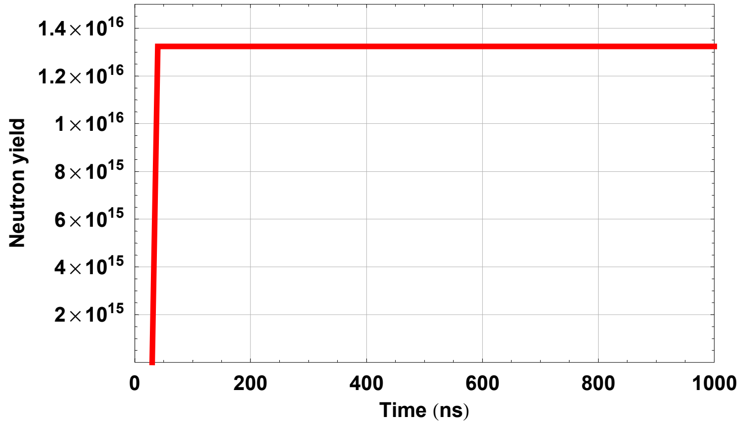
☐ **Undo options**

■ Neutron yield vs time (PLOT.NeutronsYield.vs.t)

☐ Set up problem and import data

☐ Plot

```
plotrange = PlotRange → {{-10^-3, tEnd}, {0.999 × 10^12, 1.5 × 10^16}};

plls = ListPlot[lsData[plotID], plotrange, PlotJoined → True];
```



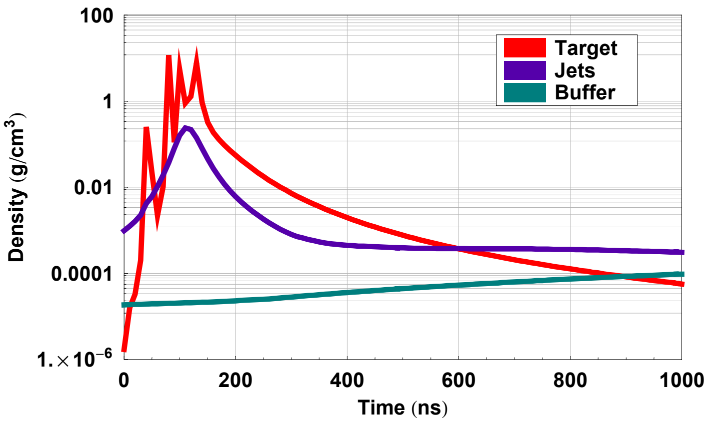
☐ Undo options

■ Density vs time in each region (PLOT.regn.dens)

☐ Set up problem and import data

☐ Plot

```
plotrange = PlotRange → {{-10^-3, tEnd}, {10^-6.001, 1 × 10^2.001}};

plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange, PlotJoined → True,
    PlotStyle → lineStyle⟦i, All⟧, DisplayFunction → Identity], {i, 1, 3}];

ShowLegend[Show[plls],
  {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
   LegendPosition → {0.4, 0.3}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



☐ Undo options

Ion temperature vs time in each region (PLOT.regn.temp)

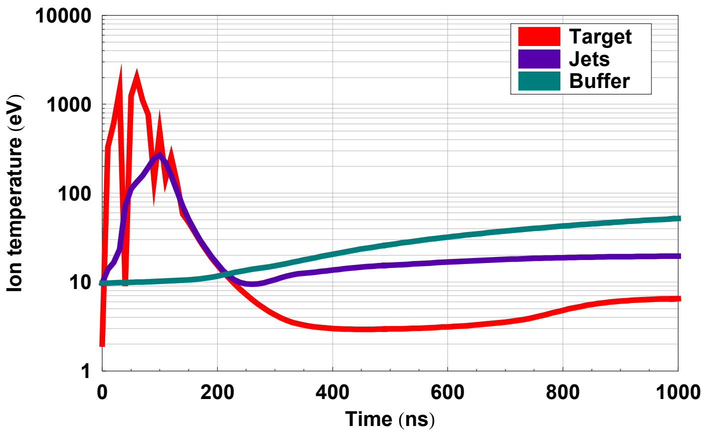
Set up problem and import data

Plot

```
plotrange = PlotRange → {{-10^-3, tEnd}, {0.999, 10^4}};
```

```
plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange,
    PlotJoined → True, PlotStyle → lineStyle⟦i⟧, DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls],
  {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
   LegendPosition → {0.45, 0.35}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



Undo options

Kinetic energy vs time in each region (PLOT.regn.KEs)

Set up problem and import data

Plot

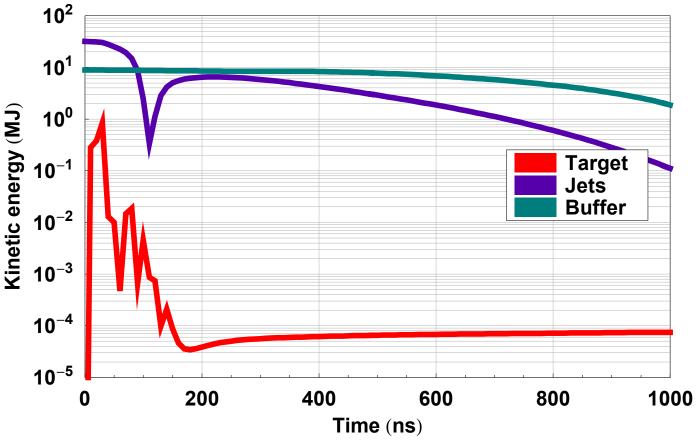
```
plotrange = PlotRange → {{-10^-3, tEnd}, {10^-5.0001, 100}};
```

```
gridlinesY = Map[{#, {GrayLevel[0.7]}} &, Flatten[Table[Log[10, j * 10^i], {i, -5, 2}, {j, 1, 9}]]];
```

```
frameticksY = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -5, 2}];
```

```
plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange,
    PlotJoined → True, PlotStyle → lineStyle⟦i⟧, DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls, GridLines → {{Automatic, GrayLevel[0.7]}, gridlinesY},
   FrameTicks → {Automatic, frameticksY, None, None}],
  {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
   LegendPosition → {0.45, 0}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



☐ **Undo options**

■ **Ion temperature vs radius (PLOT.tempn)**

■ **Mass density vs radius (PLOT.rho)**

■ **Fluid velocity vs radius (PLOT.vel)**

■ **Fusion power vs radius (BPLOT variables 30 vs 1)**

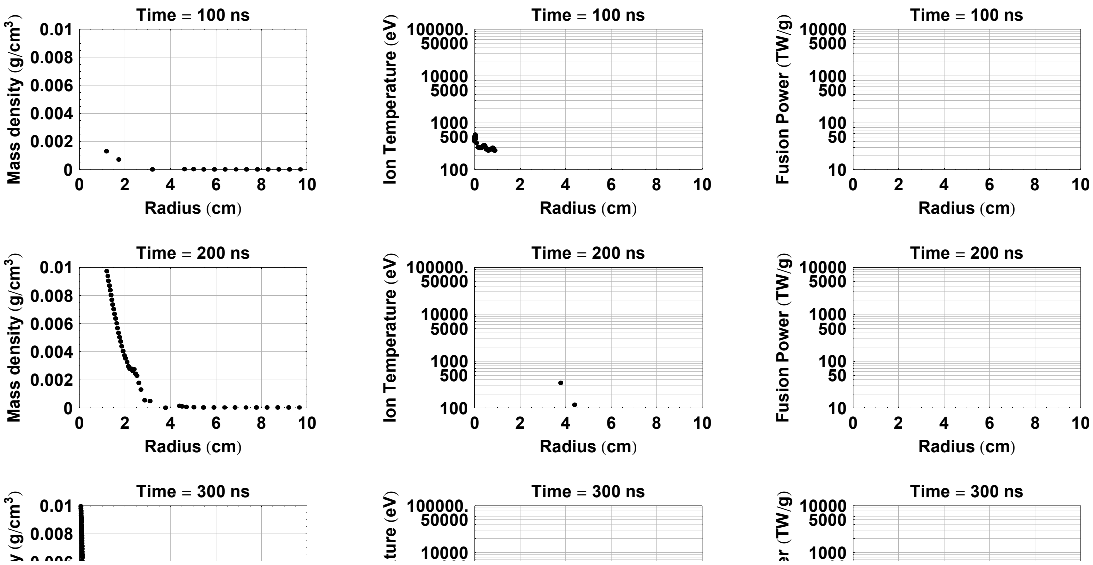
■ **Combined plots: mass density, ion temperature, and fusion power vs radius**

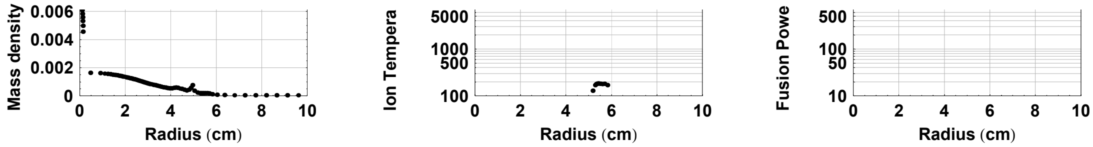
☐ **Setup**

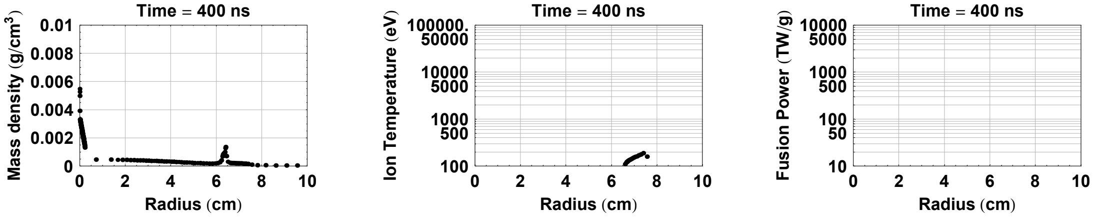
☐ **Table of plots**

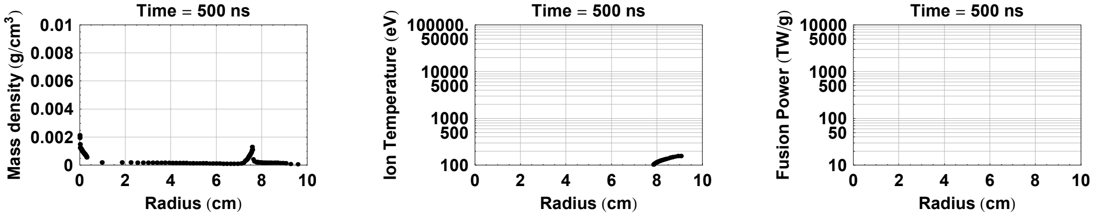
☐ **Array of plots**

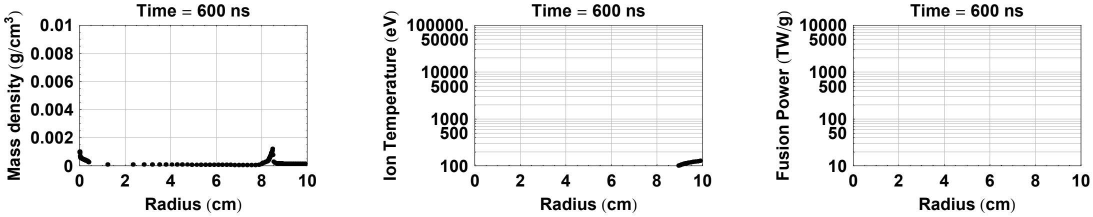
```
Show[GraphicsArray[Transpose[{tb1, tb2, tb3}]], DisplayFunction → $DisplayFunction];
```

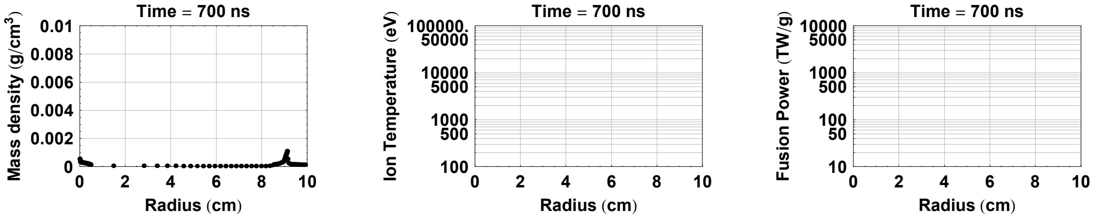


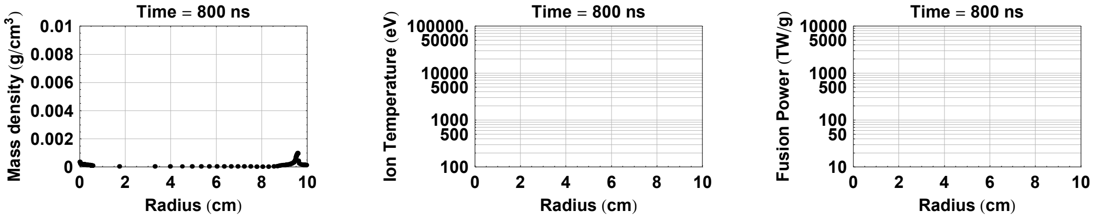

Mass density
0.006
0.004
0.002
0
0
2
4
6
8
10
Radius (cm)
Ion Tempera
5000
1000
500
100
Fusion Powe
500
100
50
10

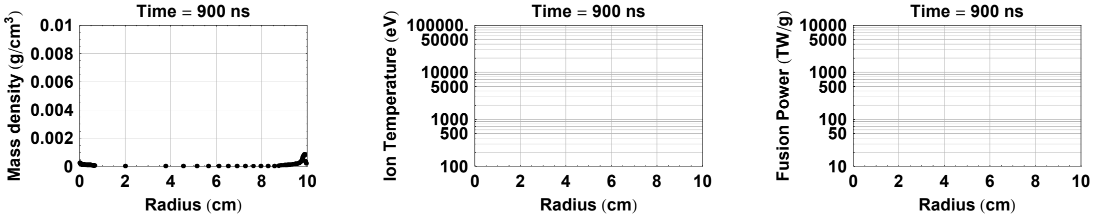

Time = 400 ns
Mass density (g/cm^3)
0.01
0.008
0.006
0.004
0.002
0
Radius (cm)
Ion Temperature (eV)
100000.
50000
10000
5000
1000
500
100
Fusion Power (TW/g)
10000
5000
1000
500
100
50
10
0
2
4
6
8
10

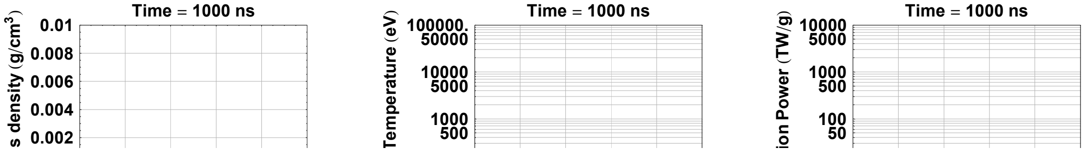

Time = 500 ns
Mass density (g/cm^3)
Ion Temperature (eV)
Fusion Power (TW/g)
Radius (cm)

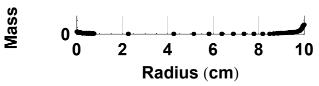

Time = 600 ns
Mass density (g/cm^3)
Ion Temperature (eV)
Fusion Power (TW/g)
Radius (cm)

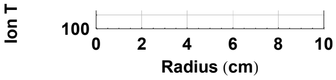

Time = 700 ns
Mass density (g/cm^3)
Ion Temperature (eV)
Fusion Power (TW/g)
Radius (cm)

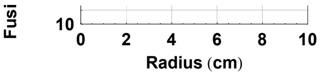

Time = 800 ns
Mass density (g/cm^3)
Ion Temperature (eV)
Fusion Power (TW/g)
Radius (cm)


Time = 900 ns
Mass density (g/cm^3)
Ion Temperature (eV)
Fusion Power (TW/g)
Radius (cm)


Time = 1000 ns
s density (g/cm^3)
Temperature (eV)
ion Power (TW/g)







☐ Undo options

■ 0-100 ns

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\0_100_ns\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 100;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";
```

```
thickness = Thickness[0.0002];
```

☐ Read data

☐ Number of time steps

☐ Number and list of radial zones in file

☐ Convert data from {s, cm} to {μs, m}

☐ Plots

- **r < 0.03 m**

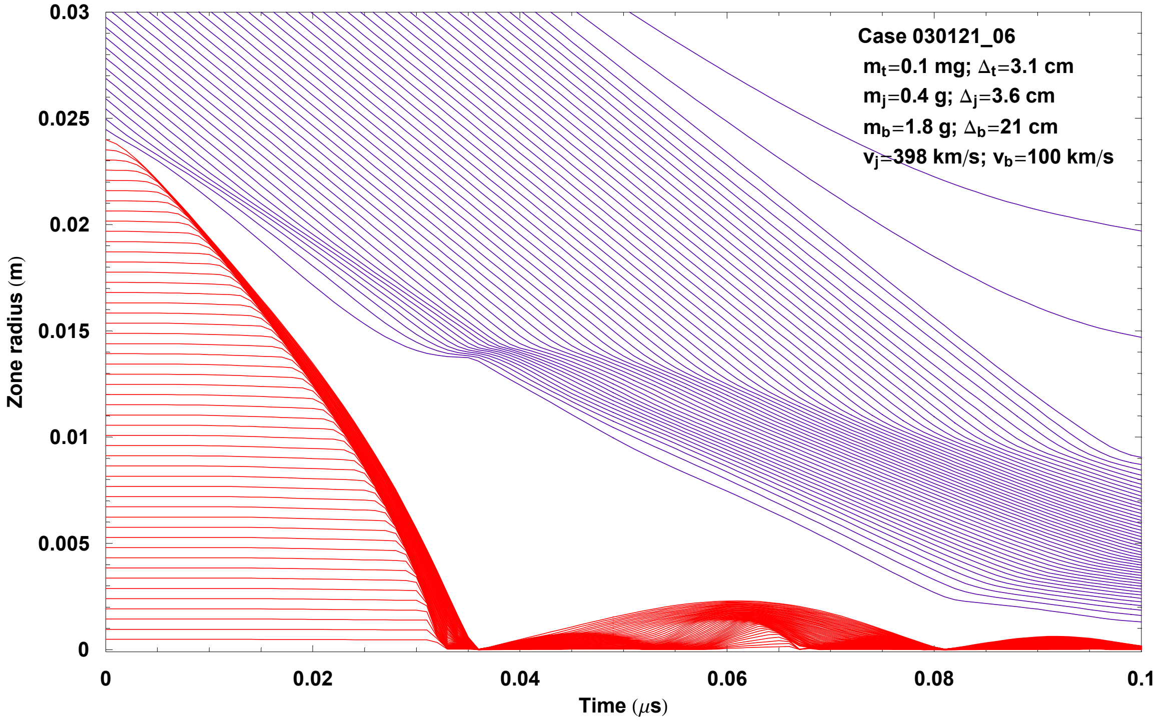
```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.03}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10^-3 tEnd, 0.024}, {0.95 * 10^-3 tEnd, 0.028}, params],
  DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, 𝒩t}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

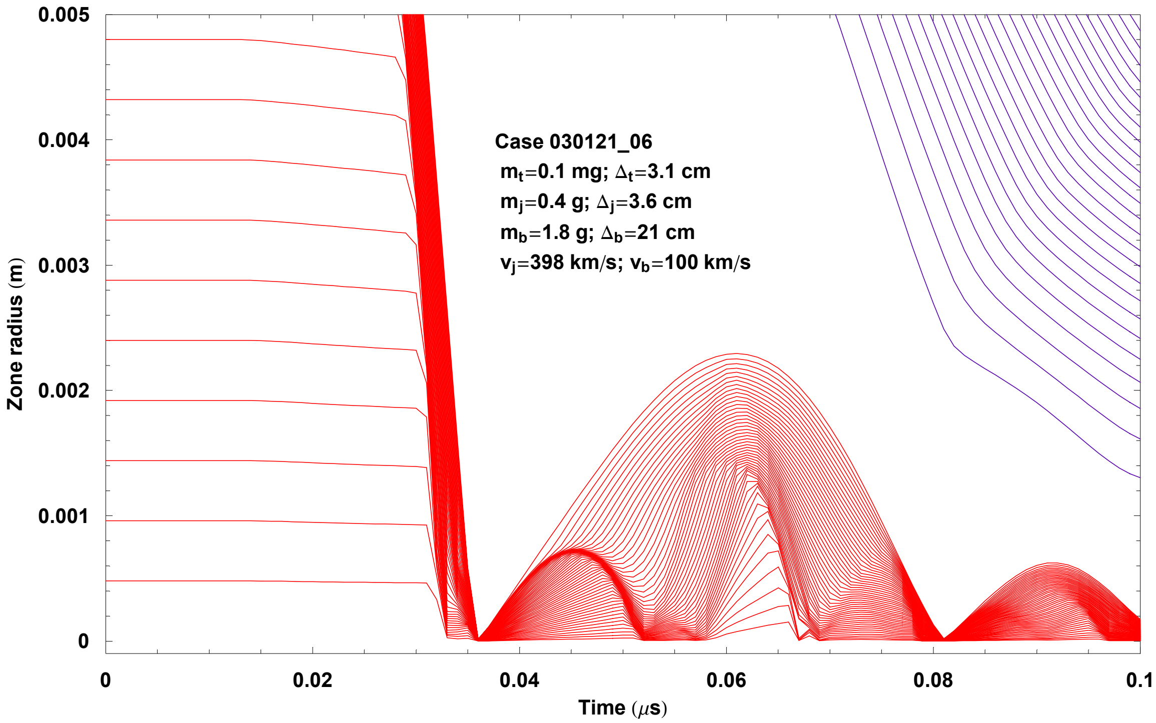
```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



- **r < 0.005 m**

```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.005}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.4 * 10^-3 tEnd, 0.003}, {0.6 * 10^-3 tEnd, 0.004}, params],
  DisplayFunction → Identity];

lsPlotsTarget = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → red], {i, 1, 𝒩t}];

lsPlotsJets = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];

lsPlotsBuffer = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



☐ **Undo options**

■ Neutron yield vs time (PLOT.NeutronsYield.vs.t)

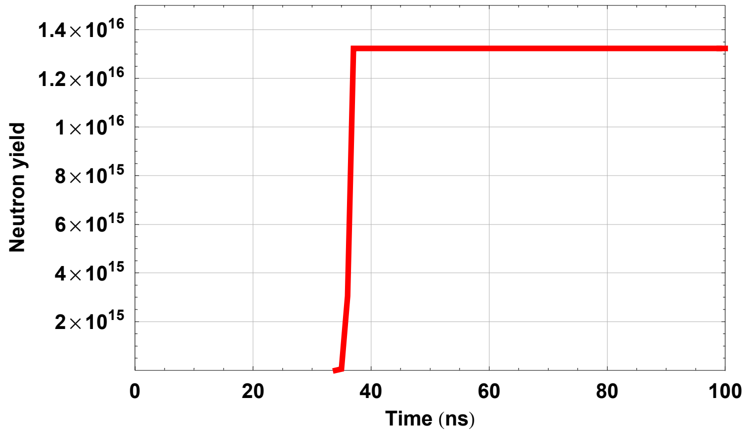
☐ Set up problem and import data

☐ Plot

```
SetOptions[{ListPlot, LogListPlot},
  PlotStyle → lineStyle⟦1, All⟧, FrameLabel → {"Time (ns)", "Neutron yield"}];
```

```
plotrange = PlotRange → {{-10^-3, tEnd}, {0.999 × 10^12, 1.5 × 10^16}};
```

```
plls = ListPlot[lsData[plotID], plotrange, PlotJoined → True];
```



☐ **Undo options**

Density vs time in each region (PLOT.regn.dens)

Set up problem and import data

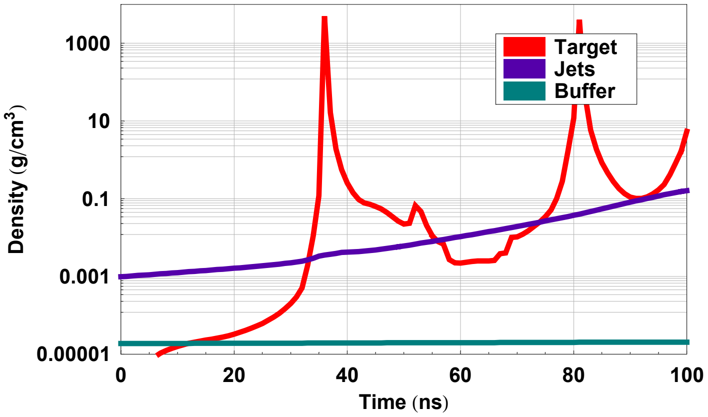
Plot

```
SetOptions[{ListPlot, LogListPlot}, FrameLabel → {"Time (ns)", "Density (g/cm^3)"}];
```

```
plotrange = PlotRange → {{-10^-3, tEnd}, {10^-5.001, 1 × 10^3.999}};
```

```
plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange, PlotJoined → True,
    PlotStyle → lineStyle⟦i, All⟧, DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls],
  {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
   LegendPosition → {0.4, 0.3}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



Undo options

Ion temperature vs time in each region (PLOT.regn.temp)

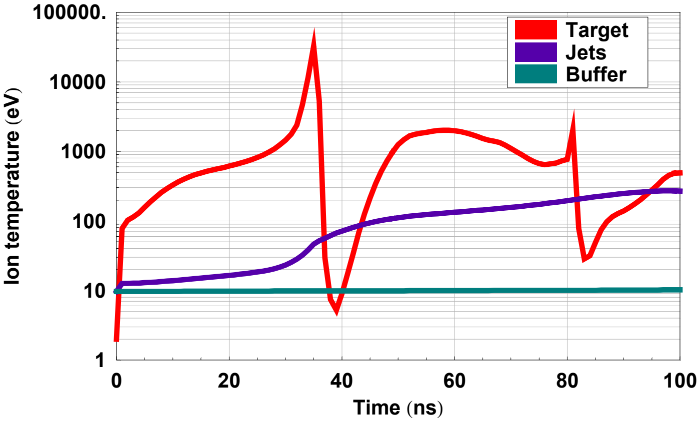
Set up problem and import data

Plot

```
plotrange = PlotRange → {{-10^-3, tEnd}, {0.999, 10^5}};
```

```
plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange,
    PlotJoined → True, PlotStyle → lineStyle⟦i⟧, DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls],
  {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
   LegendPosition → {0.45, 0.35}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



☐ Undo options

■ Kinetic energy vs time in each region (PLOT.regn.KEs)

☐ Set up problem and import data

☐ Plot

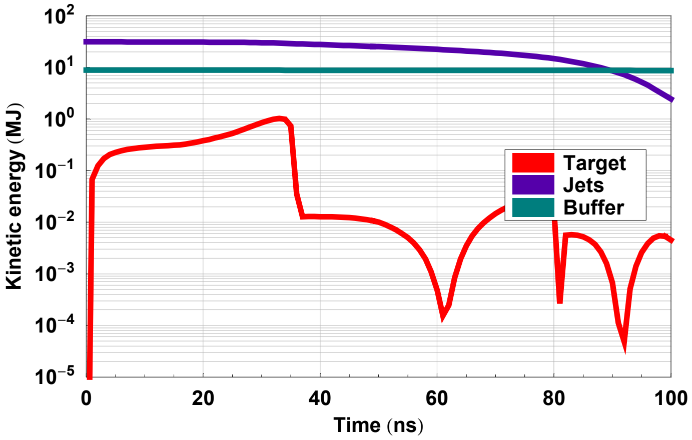
```
plotrange = PlotRange → {{-10^-3, tEnd}, {10^-5.0001, 100}};
```

```
gridlinesY = Map[{#, {GrayLevel[0.7]}} &, Flatten[Table[Log[10, j * 10^i], {i, -5, 2}, {j, 1, 9}]]];
```

```
frameticksY = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -5, 2}];
```

```
plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange,
    PlotJoined → True, PlotStyle → lineStyle⟦i⟧, DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls, GridLines → {{Automatic, GrayLevel[0.7]}, gridlinesY},
   FrameTicks → {Automatic, frameticksY, None, None}],
  {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
   LegendPosition → {0.45, 0}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



☐ Undo options

■ Ion temperature vs radius (PLOT.tempn)

■ Mass density vs radius (PLOT.rho)

■ Fluid velocity vs radius (PLOT.vel)

■ Fusion power vs radius (BPLOT variables 30 vs 1)

■ Combined plots: mass density, ion temperature, and fusion power vs radius

☐ Setup

☐ Table of plots

Partition length

```
partitionL = 3;
```

```
SetOptions[{ListPlot, LogListPlot},
  PlotStyle → PointSize[0.02], DefaultFont → {"Helvetica-Bold", 10}];
```

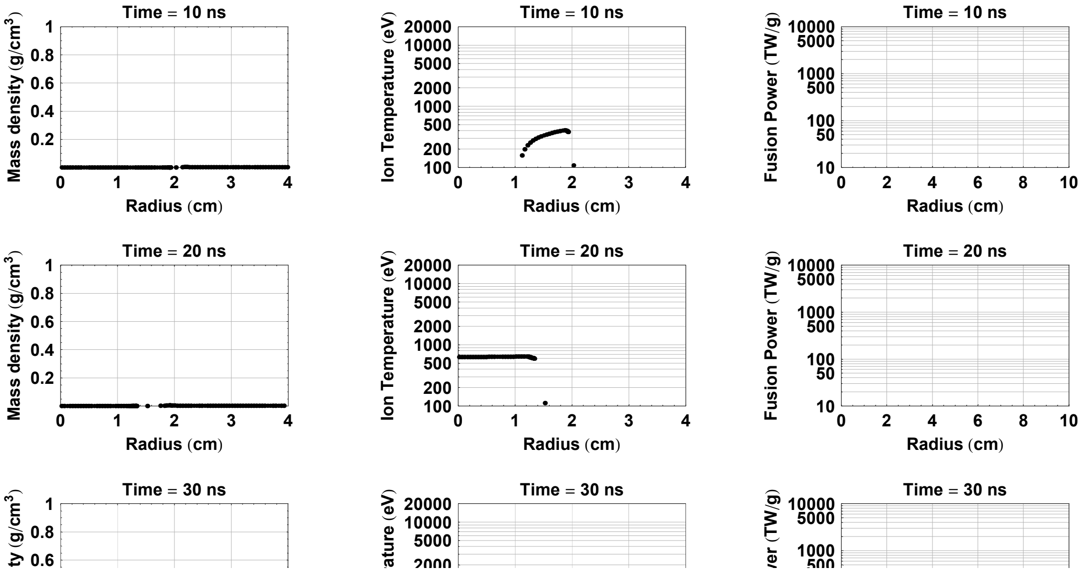
```
tb1 = Table[ListPlot[lsData[plotID1]〚i〛,
    plotrangeRho, PlotLabel → "Time = " <> ToString[Round[lsTimes〚i〛]] <> " ns",
    FrameLabel → {"Radius (cm)", "Mass density (g/cm^3)"},
    DisplayFunction → Identity], {i, 1, Length[lsData[plotID1]]}];
```

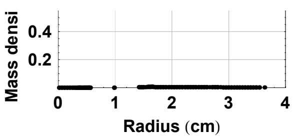
```
tb2 = Table[LogListPlot[lsData[plotID2]〚i〛,
    plotrangeTi, PlotLabel → "Time = " <> ToString[Round[lsTimes〚i〛]] <> " ns",
    FrameLabel → {"Radius (cm)", "Ion Temperature (eV)"},
    DisplayFunction → Identity], {i, 1, Length[lsData[plotID2]]}];
```

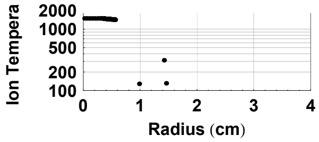
```
tb3 = Table[LogListPlot[lsData[plotID3]〚i〛,
    plotrangePfus, PlotLabel → "Time = " <> ToString[Round[lsTimes〚i〛]] <> " ns",
    FrameLabel → {"Radius (cm)", "Fusion Power (TW/g)"},
    DisplayFunction → Identity], {i, 1, Length[lsData[plotID3]]}];
```

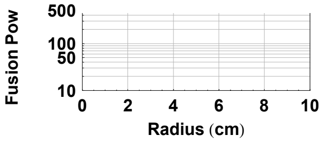
☐ Array of plots

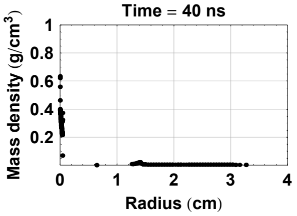
```
Show[GraphicsArray[Transpose[{tb1, tb2, tb3}]], DisplayFunction → $DisplayFunction];
```

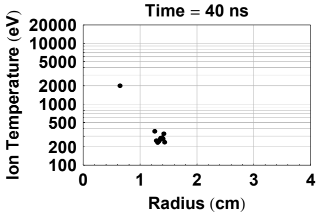


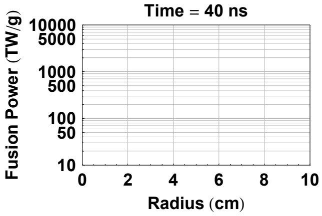


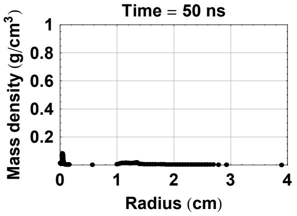


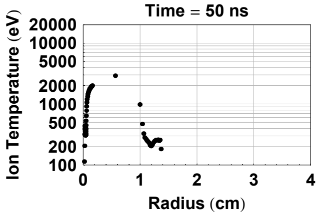


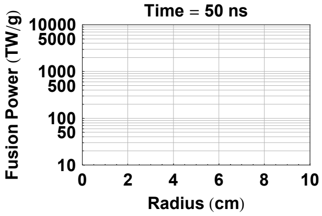

Time = 40 ns

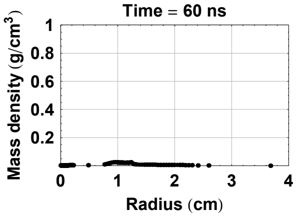

Time = 40 ns

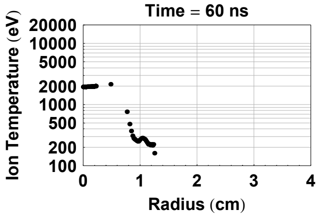

Time = 40 ns

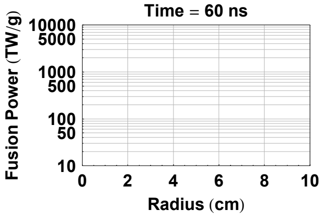

Time = 50 ns

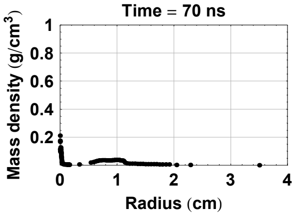

Time = 50 ns

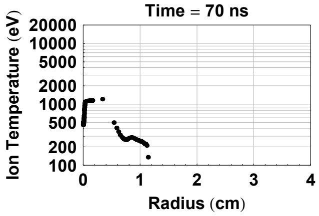

Time = 50 ns

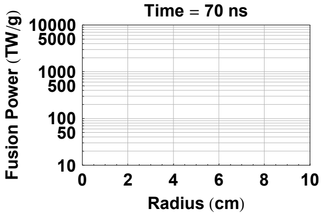

Time = 60 ns

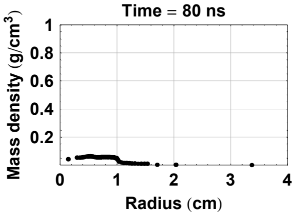

Time = 60 ns

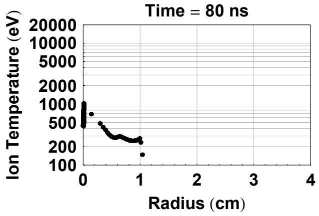

Time = 60 ns

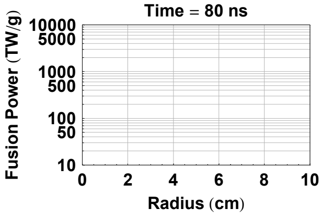

Time = 70 ns

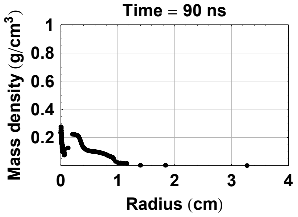

Time = 70 ns

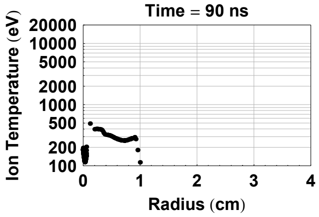

Time = 70 ns

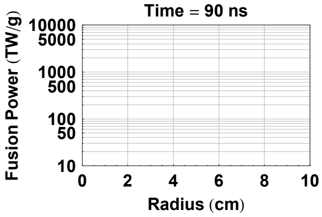

Time = 80 ns

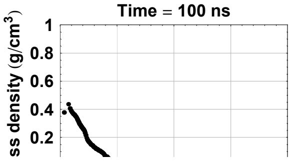

Time = 80 ns

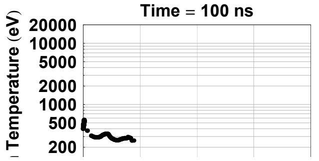

Time = 80 ns

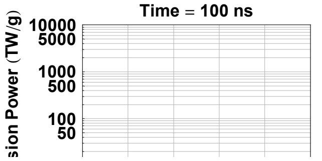

Time = 90 ns

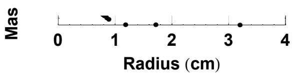

Time = 90 ns

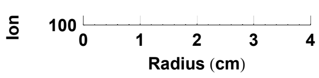

Time = 90 ns

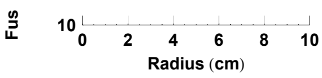

Time = 100 ns


Time = 100 ns


Time = 100 ns







☐ Undo options

BUCKY case 030201.05 (D-T, perfect acoustic matching, 149 zones, v_{jet}=398 km/s)

★ Note: the full set of commands to generate the plots is shown in the section "BUCKY case 030313.02" below. In this section only the plots and the outline of the structure appear.

■ Setup

```
caseID = "030201_05";
```

```
params = Graphics[Text["Case " <> caseID <> "\n
m_t=1.0 mg; Δ_t=5.0 cm\n
m_j=0.4 g; Δ_j=2.4 cm\n
m_b=1.8 g; Δ_b=21.1 cm\n
v_j=398 km/s; v_b=100 km/s", {0, 0}, {0, 0}]];
```

■ 0-200ns

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 200;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";
```

```
thickness = Thickness[0.0002];
```

☐ Read data

☐ Number of time steps

☐ Number and list of radial zones in file

☐ Convert data from {s, cm} to {μs, m}

☐ Plots

• r < 0.3 m

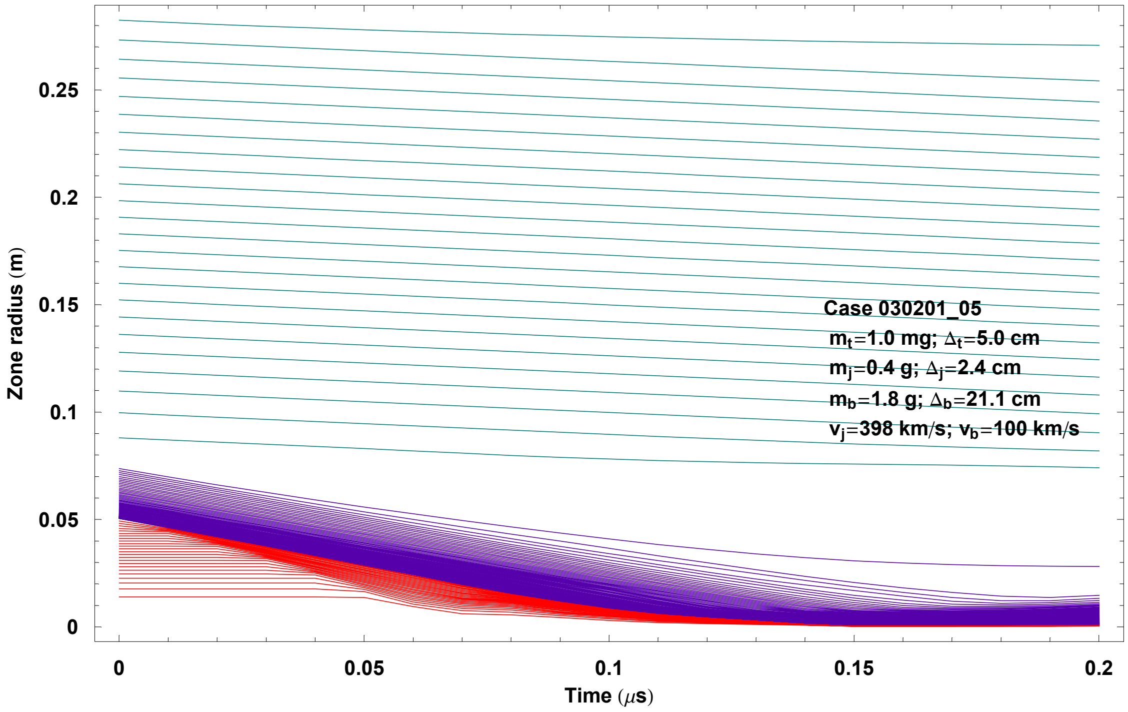
```
SetOptions[ListPlot, PlotRange → All, PlotJoined → True,
  GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"}, Epilog →
   Rectangle[{0.75 * 10^-3 tEnd, 0.07}, {0.95 * 10^-3 tEnd, 0.17}, params], DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → red], {i, 1, Nt}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → purple], {i, Nt + 1, Nt + Nj}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → teal], {i, Nt + Nj + 1, Nz}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



• r < 0.05 m

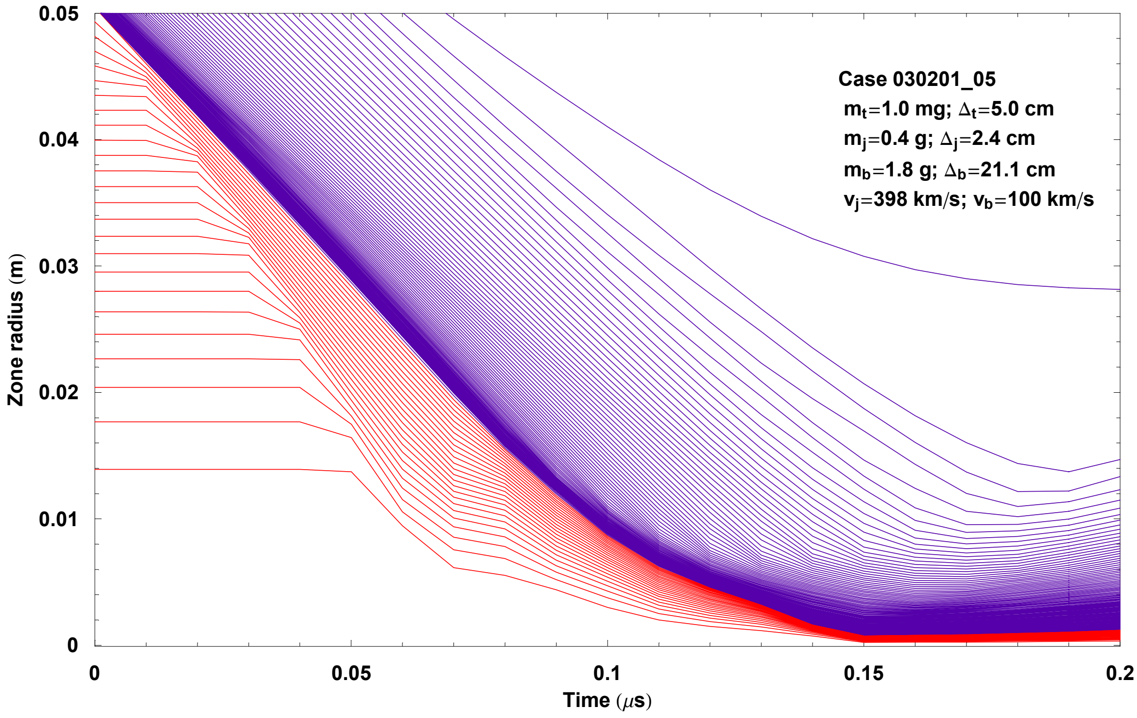
```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.05}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10^-3 tEnd, 0.035}, {0.95 * 10^-3 tEnd, 0.045}, params],
  DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → red], {i, 1, 𝒩t}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



• r < 0.02 m

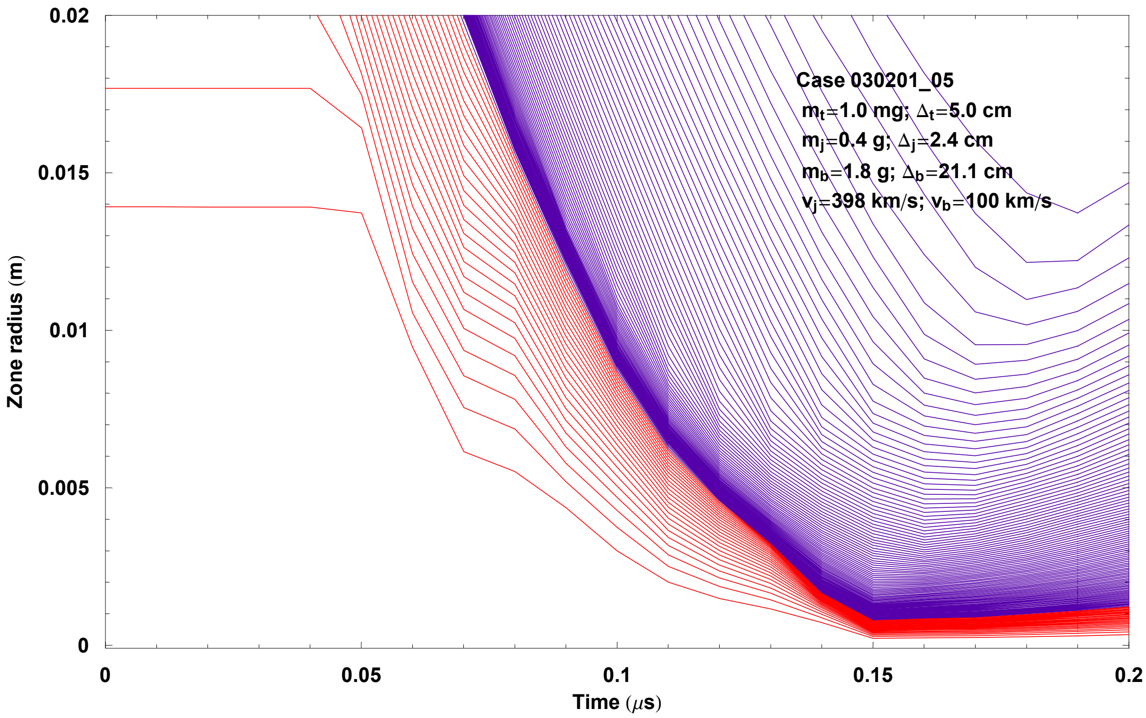
```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.02}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.7 * 10^-3 tEnd, 0.014}, {0.9 * 10^-3 tEnd, 0.018}, params],
  DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → red], {i, 1, 𝒩t}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



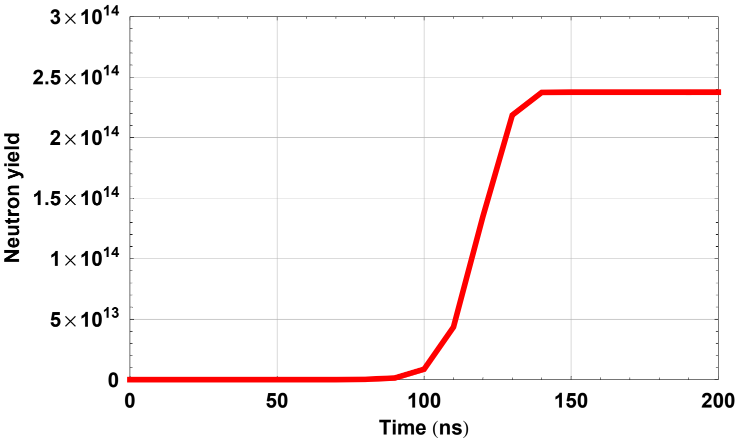
☐ Undo options

Neutron yield vs time (PLOT.NeutronsYield.vs.t)

Set up problem and import data

Plot

```
SetOptions[{ListPlot, LogListPlot},
  PlotStyle → lineStyle⟦1, All⟧, FrameLabel → {"Time (ns)", "Neutron yield"}];

plotrange = PlotRange → {{-10^-3, tEnd}, {-10^11, 3×10^14}};

plls = ListPlot[lsData[plotID], plotrange, PlotJoined → True];
```



Undo options

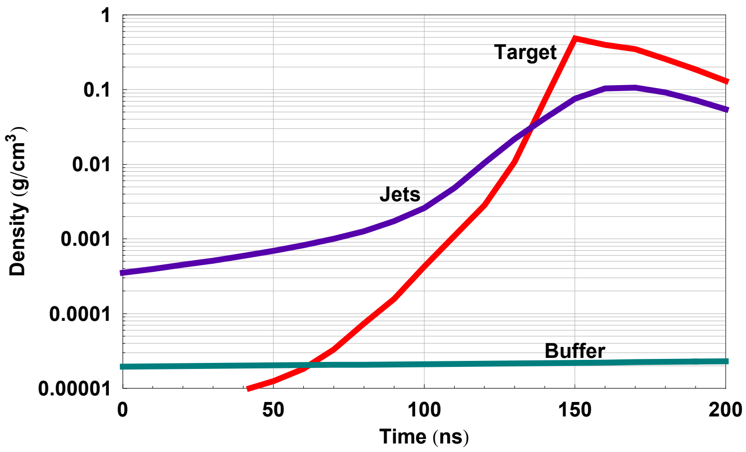
Density vs time in each region (PLOT.regn.dens)

Set up problem and import data

Plot

```
SetOptions[{ListPlot, LogListPlot}, FrameLabel → {"Time (ns)", "Density (g/cm^3)"}];

plotrange = PlotRange → {{-10^-3, tEnd}, {10^-5.001, 1}};

plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange, PlotJoined → True,
    PlotStyle → lineStyle⟦i, All⟧, DisplayFunction → Identity], {i, 1, 3}];

text = {Graphics[Text["Target", {145, -0.5}, {1, 0}]],
   Graphics[Text["Jets", {100, -2.4}, {1, 0}]],
   Graphics[Text["Buffer", {150, -4.5}, {0, 0}]]};
```

```
Show[plls, text,
  FrameLabel → {"Time (ns)", "Density (g/cm^3)"}, DisplayFunction → $DisplayFunction];
```



☐ Undo options

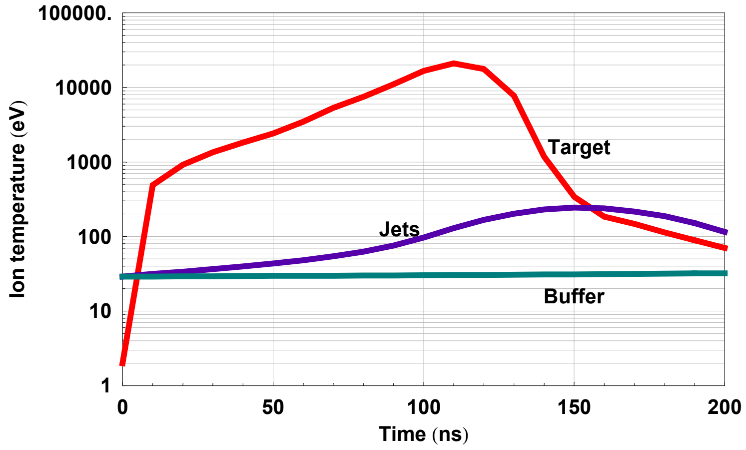
■ Ion temperature vs time in each region (PLOT.regn.temp)

☐ Set up problem and import data

☐ Plot

```
plotrange = PlotRange → {{-10^-3, tEnd}, {0.999, 10^5}};
```

```
plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange,
    PlotJoined → True, PlotStyle → lineStyle⟦i⟧, DisplayFunction → Identity], {i, 1, 3}];

text = {Graphics[Text["Target", {140, 3.2}, {-1, 0}]],
   Graphics[Text["Jets", {100, 2.1}, {1, 0}]],
   Graphics[Text["Buffer", {150, 1.2}, {0, 0}]]};

Show[plls, text, FrameLabel → {"Time (ns)", "Ion temperature (eV)"},
  DisplayFunction → $DisplayFunction];
```



☐ Undo options

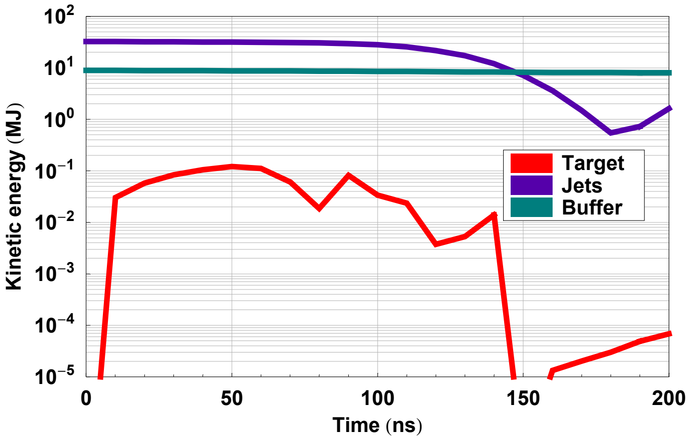
Kinetic energy vs time in each region (PLOT.regn.KEs)

Set up problem and import data

Plot

```
plotrange = PlotRange → {{-10^-3, tEnd}, {10^-5.0001, 100}};

gridlinesY = Map[{#, {GrayLevel[0.7]}} &, Flatten[Table[Log[10, j * 10^i], {i, -5, 2}, {j, 1, 9}]]];

frameticksY = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -5, 2}];

plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange,
    PlotJoined → True, PlotStyle → lineStyle⟦i⟧, DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls, GridLines → {{Automatic, GrayLevel[0.7]}, gridlinesY},
   FrameTicks → {Automatic, frameticksY, None, None}],
  {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
   LegendPosition → {0.45, 0}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



Undo options

Ion temperature vs radius (PLOT.tempn)

Mass density vs radius (PLOT.rho)

Fluid velocity vs radius (PLOT.vel)

Fusion power vs radius (BPLOT variables 30 vs 1)

Combined plots: mass density, ion temperature, and fusion power vs radius

Setup

Table of plots

Partition length

```
partitionL = 3;
```

```
SetOptions[{ListPlot, LogListPlot},
  PlotStyle → PointSize[0.02], DefaultFont → {"Helvetica-Bold", 10}];
```

```
tb1 = Table[ListPlot[lsData[plotID1]⟦i⟧,
    plotrangeRho, PlotLabel → "Time = " <> ToString[Round[lsTimes⟦i⟧]] <> " ns",
    FrameLabel → {"Radius (cm)", "Mass density (g/cm^3)"},
    DisplayFunction → Identity], {i, 1, Length[lsData[plotID1]]}];
```

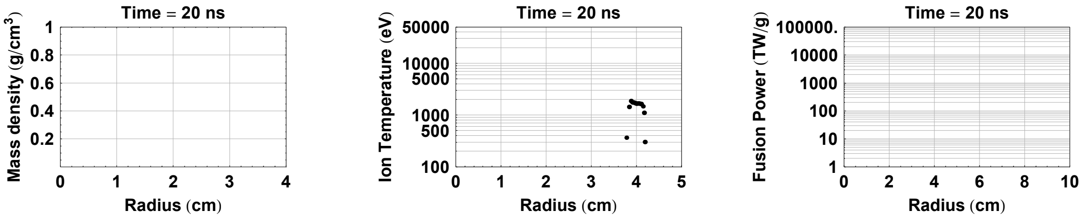
```
tb2 = Table[LogListPlot[lsData[plotID2]⟦i⟧,
    plotrangeTi, PlotLabel → "Time = " <> ToString[Round[lsTimes⟦i⟧]] <> " ns",
    FrameLabel → {"Radius (cm)", "Ion Temperature (eV)"},
    DisplayFunction → Identity], {i, 1, Length[lsData[plotID2]]}];
```

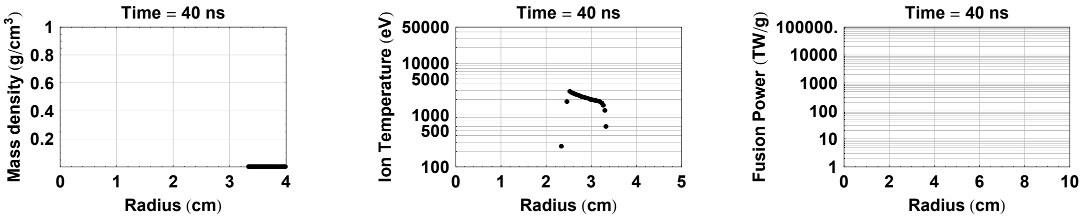
```
tb3 = Table[LogListPlot[lsData[plotID3]⟦i⟧,
    plotrangePfus, PlotLabel → "Time = " <> ToString[Round[lsTimes⟦i⟧]] <> " ns",
    FrameLabel → {"Radius (cm)", "Fusion Power (TW/g)"},
    DisplayFunction → Identity], {i, 1, Length[lsData[plotID3]]}];
```

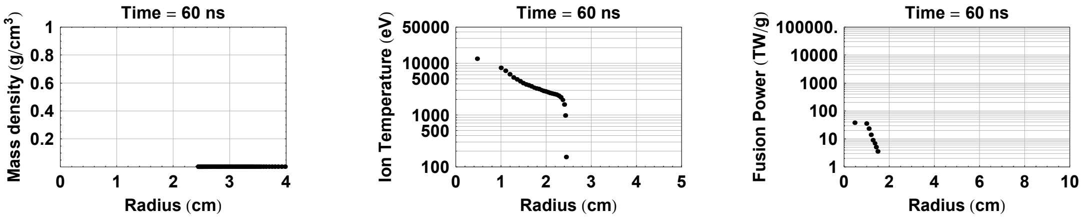
Array of plots

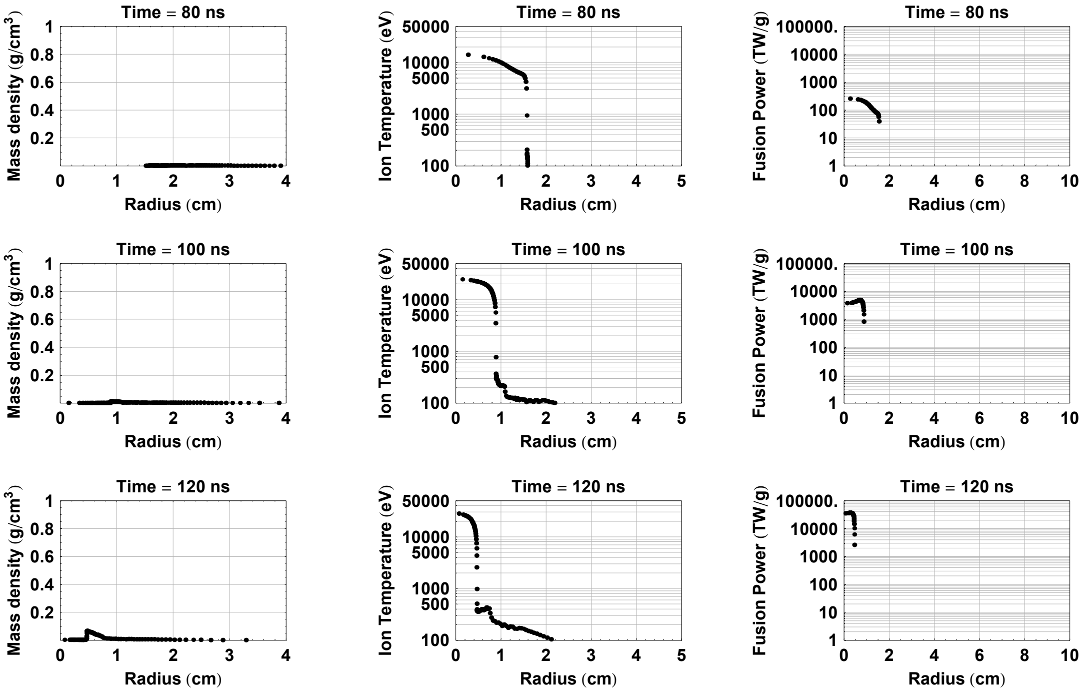
```
tbArray = Transpose[{tb1, tb2, tb3}];
```

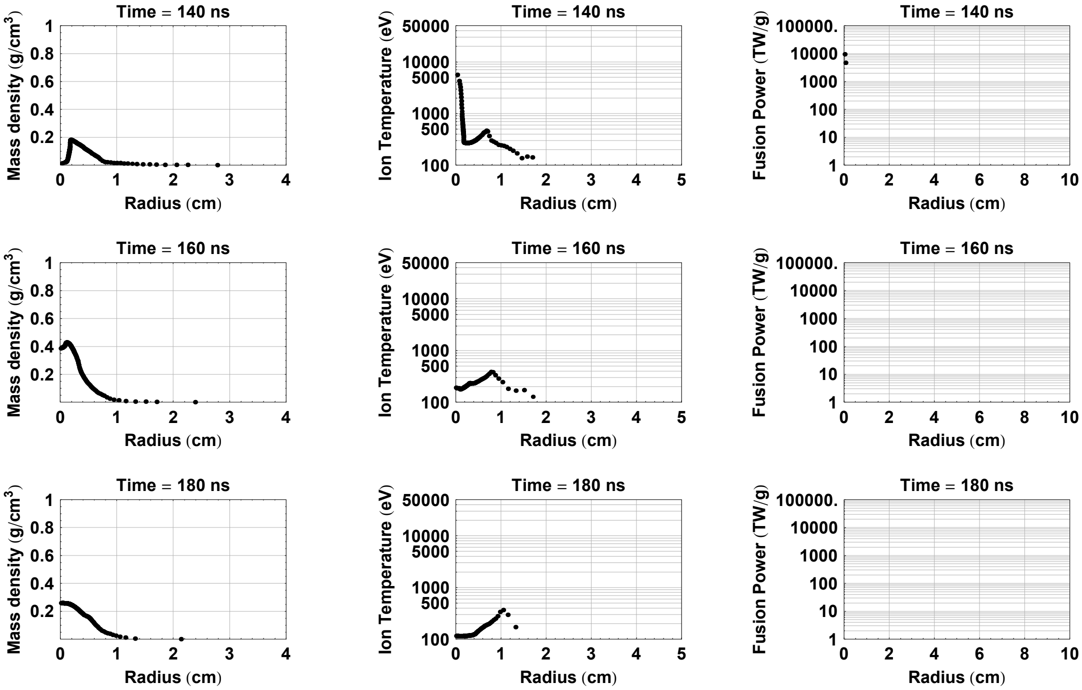
```
Table[Show[GraphicsArray[tbArray⟦{3 i + 1, 3 i + 2, 3 i + 3}⟧],
   DisplayFunction → $DisplayFunction], {i, 0, 2}];
```



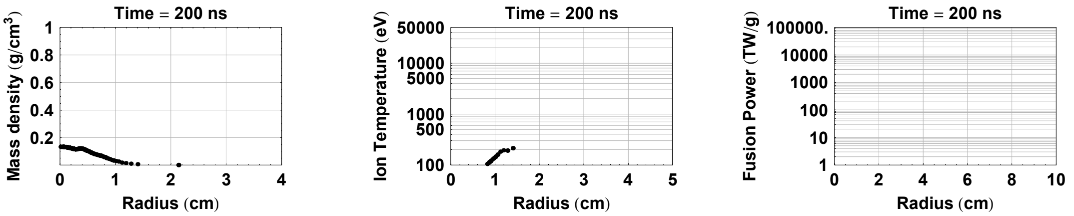





Time = 80 ns
Time = 100 ns
Time = 120 ns
Mass density (g/cm^3)
1
0.8
0.6
0.4
0.2
Ion Temperature (eV)
50000
10000
5000
1000
500
100
Fusion Power (TW/g)
100000.
10000
1000
100
10
1
Radius (cm)
0
1
2
3
4
5
6
8
10


Time = 140 ns
Time = 160 ns
Time = 180 ns
Mass density (g/cm^3)
1
0.8
0.6
0.4
0.2
Ion Temperature (eV)
50000
10000
5000
1000
500
100
Fusion Power (TW/g)
100000.
10000
1000
100
10
1
Radius (cm)
0
1
2
3
4
5
6
8
10

```
Show[GraphicsArray[tbArray[[10]]], DisplayFunction → $DisplayFunction];
```



☐ Undo options

BUCKY case 030313.02: Thio, Current Trends (1999) 149 zones (v_{jet}=125 km/s)

■ Setup

```
caseID = "030313_02";
```

```
params = Graphics[Text["Case " <> caseID <> "\n
m_t=4.38 mg; Δ_t=5.0 cm\n
m_j=0.2 g; Δ_j=2.4 cm\n
m_b=2.0 g; Δ_b=22.1 cm\n
v_j=125 km/s; v_b=125 km/s", {0, 0}, {0, 0}]];
```

■ 0-2 μs

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 2000;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";
```

```
thickness = Thickness[0.0002];
```

☐ Read data

☐ Number of time steps

☐ Number and list of radial zones in file

☐ Convert data from {s, cm} to {μs, m}

Plots

r < 0.3 m

```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.3}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10^-3 tEnd, 0.07}, {0.95 * 10^-3 tEnd, 0.17}, params],
  DisplayFunction → Identity];
```

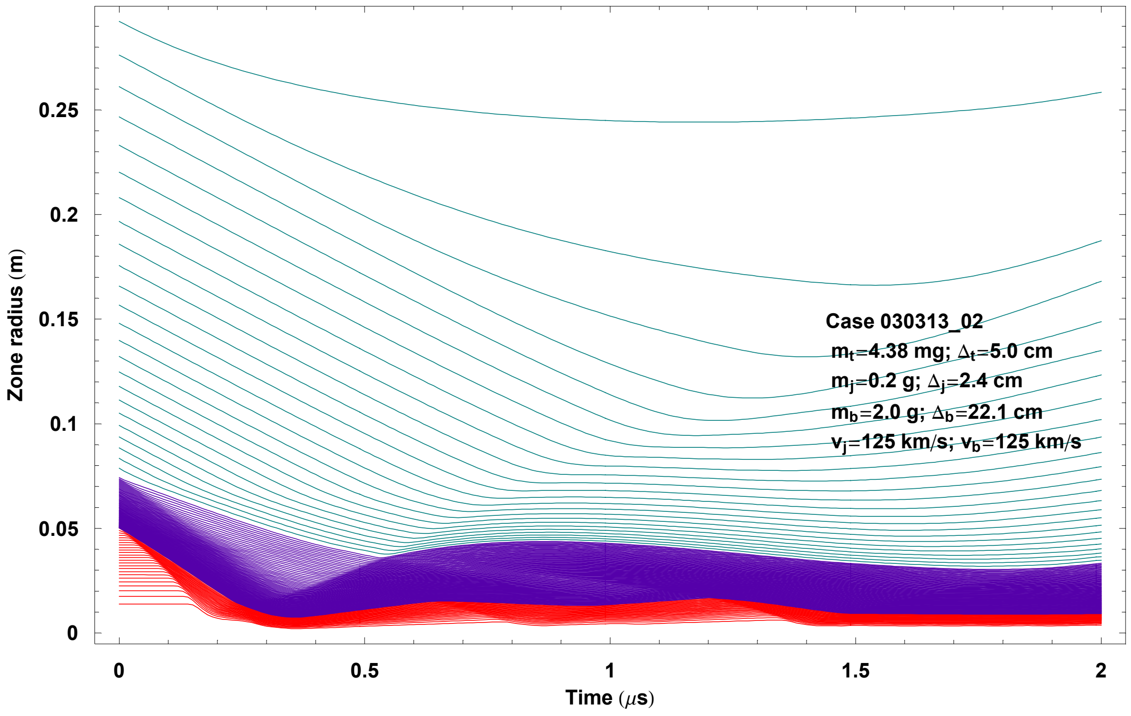
```
SetOptions[ListPlot, PlotRange → All, PlotJoined → True,
  GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"}, Epilog →
   Rectangle[{0.75 * 10^-3 tEnd, 0.07}, {0.95 * 10^-3 tEnd, 0.17}, params], DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → red], {i, 1, 𝒩t}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



- **r < 0.05 m**

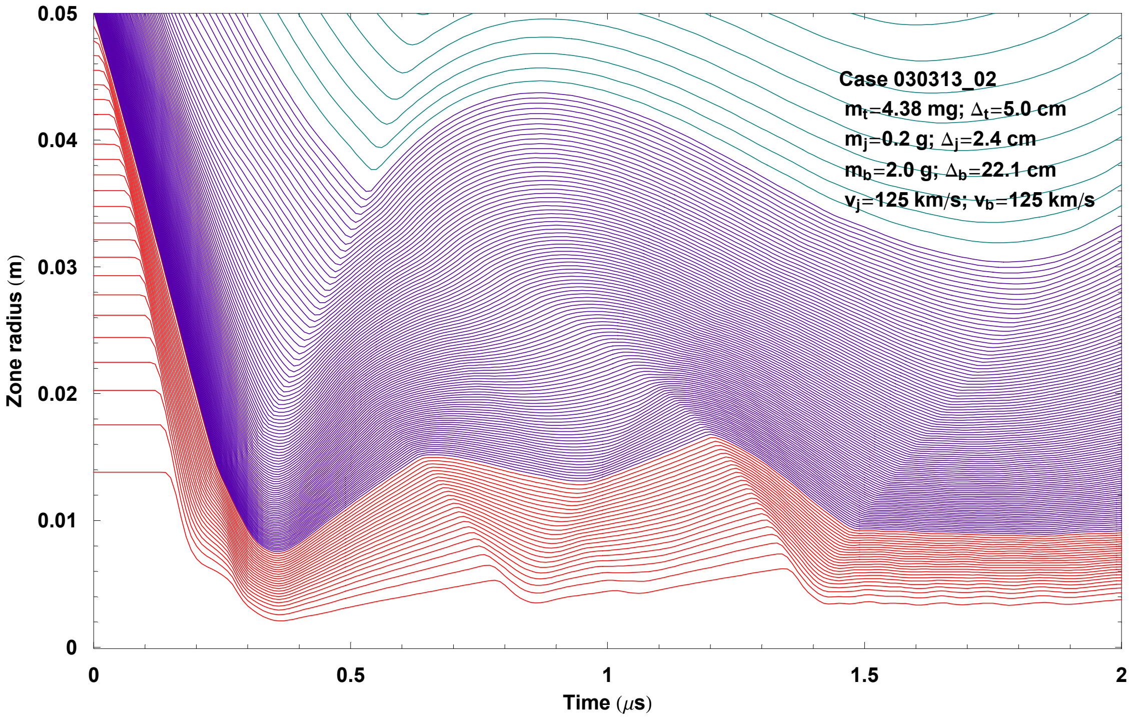
```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.05}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10^-3 tEnd, 0.035}, {0.95 * 10^-3 tEnd, 0.045}, params],
  DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID][[i]], PlotStyle → red], {i, 1, 𝒩t}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID][[i]], PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID][[i]], PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



- **r < 0.05 m**

```
tEndOrig = tEnd;
tEnd = 400;
```

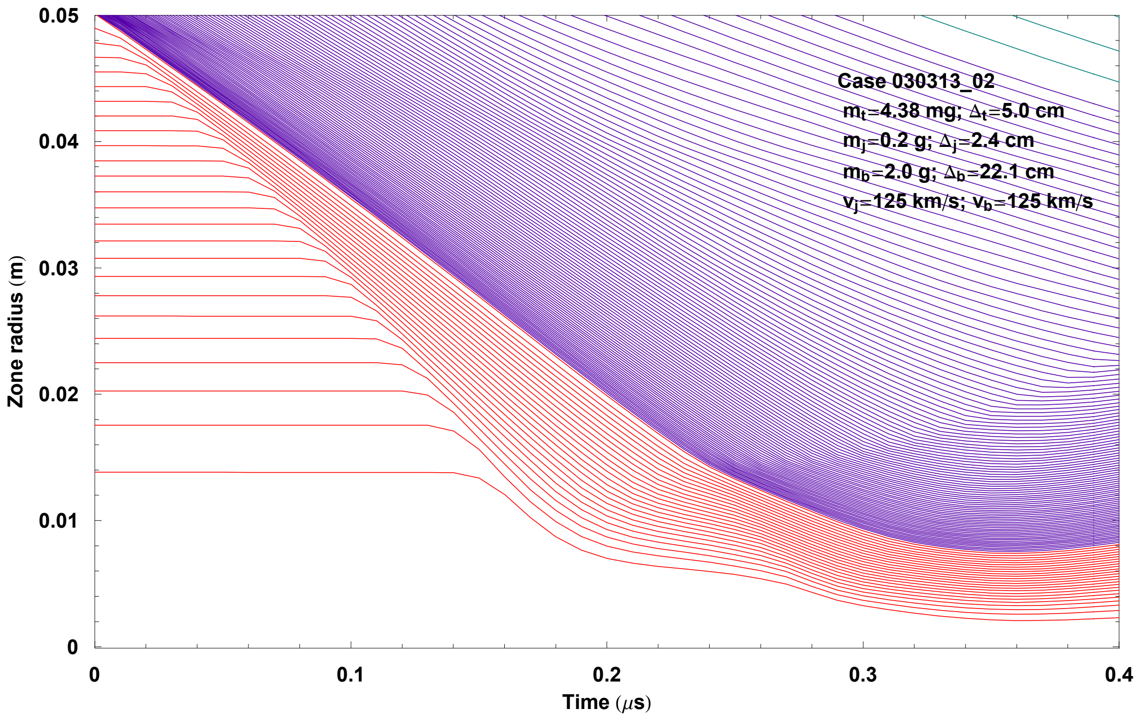
```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.05}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10^-3 tEnd, 0.035}, {0.95 * 10^-3 tEnd, 0.045}, params],
  DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → red], {i, 1, Nt}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → purple], {i, Nt + 1, Nt + Nj}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → teal], {i, Nt + Nj + 1, Nz}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



```
tEnd = tEndOrig;
```

- **r < 0.02 m**

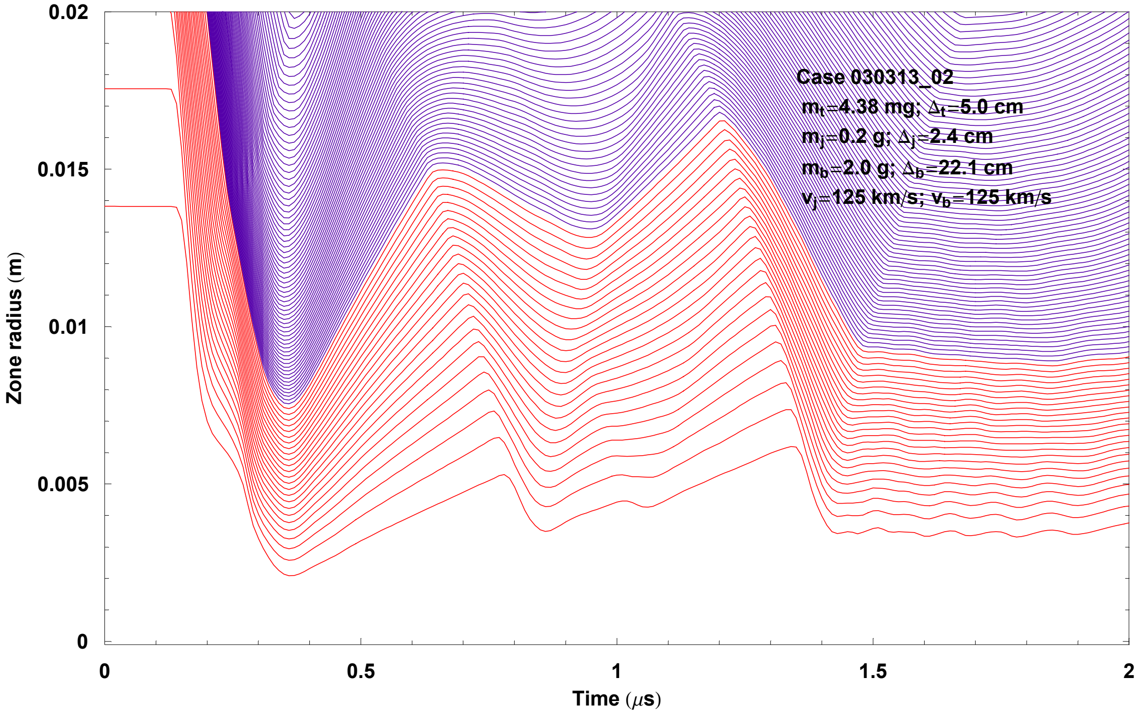
```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.02}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.7 * 10^-3 tEnd, 0.014}, {0.9 * 10^-3 tEnd, 0.018}, params],
  DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → red], {i, 1, Nt}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → purple], {i, Nt + 1, Nt + Nj}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID]〚i〛, PlotStyle → teal], {i, Nt + Nj + 1, Nz}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



☐ **Undo options**

■ Density vs time in each region (PLOT.regn.dens)

□ Set up problem and import data

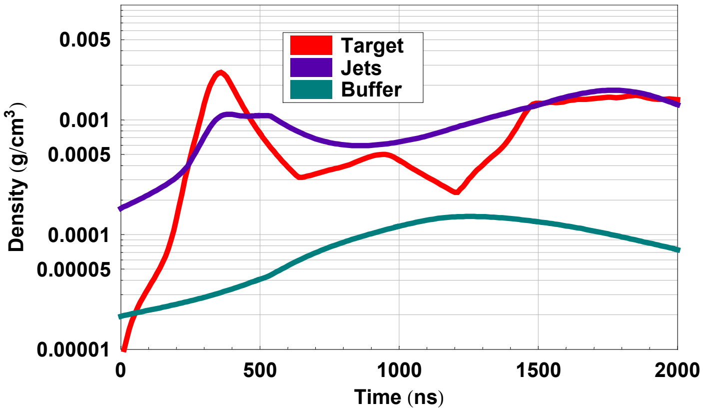
□ Plot

```
SetOptions[{ListPlot, LogListPlot}, FrameLabel → {"Time (ns)", "Density (g/cm^3)"}];
```

```
plotrange = PlotRange → {{-10^-3, tEnd}, {10^-5.001, 10^-2}};
```

```
plls = Table[LogListPlot[lsData[plotID][[All, i]], plotrange, PlotJoined → True,
    PlotStyle → lineStyle[[i, All]], DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls],
  {{{lineStyle[[1, 1]], "Target"}, {lineStyle[[2, 1]], "Jets"}, {lineStyle[[3, 1]], "Buffer"}},
   LegendPosition → {-0.2, 0.3}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



□ Undo options

■ Ion temperature vs time in each region (PLOT.regn.temp)

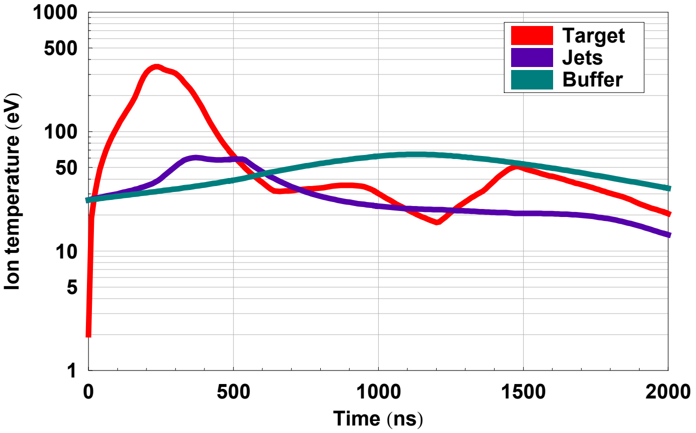
□ Set up problem and import data

□ Plot

```
plotrange = PlotRange → {{-10^-3, tEnd}, {0.999, 10^3}};
```

```
plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange,
     PlotJoined → True, PlotStyle → lineStyle⟦i⟧, DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls],
   {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
    LegendPosition → {0.45, 0.35}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



☐ **Undo options**

■ **Kinetic energy vs time in each region (PLOT.regn.KEs)**

☐ **Set up problem and import data**

☐ **Plot**

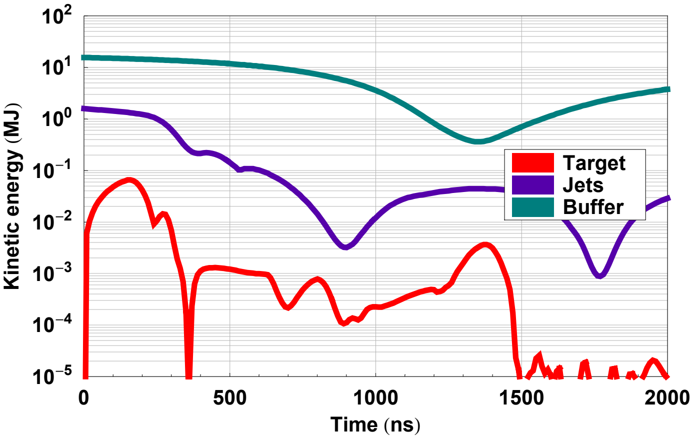
```
plotrange = PlotRange → {{-10^-3, tEnd}, {10^-5.0001, 100}};
```

```
gridlinesY = Map[{#, {GrayLevel[0.7]}} &, Flatten[Table[Log[10, j * 10^i], {i, -5, 2}, {j, 1, 9}]]];
```

```
frameticksY = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -5, 2}];
```

```
plls = Table[LogListPlot[lsData[plotID]⟦All, i⟧, plotrange,
     PlotJoined → True, PlotStyle → lineStyle⟦i⟧, DisplayFunction → Identity], {i, 1, 3}];
```

```
ShowLegend[Show[plls, GridLines → {{Automatic, GrayLevel[0.7]}, gridlinesY},
    FrameTicks → {Automatic, frameticksY, None, None}],
   {{{lineStyle⟦1, 1⟧, "Target"}, {lineStyle⟦2, 1⟧, "Jets"}, {lineStyle⟦3, 1⟧, "Buffer"}},
    LegendPosition → {0.45, 0}, LegendSize → {0.4, 0.2}, LegendShadow → {0, 0}}];
```



☐ **Undo options**

■ **Ion temperature vs radius (PLOT.tempn)**

■ **Mass density vs radius (PLOT.rho)**

■ **Fluid velocity vs radius (PLOT.vel)**

■ **Fusion power vs radius (BPLOT variables 30 vs 1)**

★ *Note: no fusion power was produced for this case.*

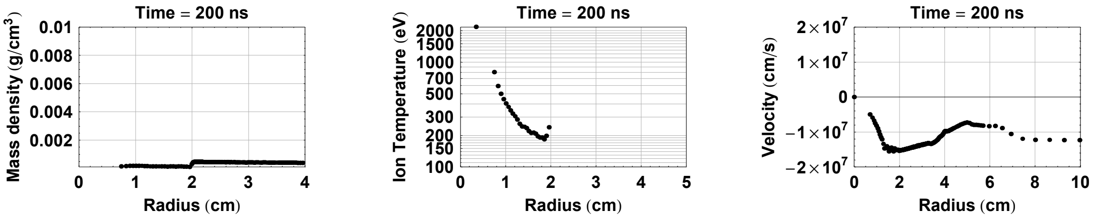
■ **Combined plots: mass density, ion temperature, and *fluid velocity* vs radius**

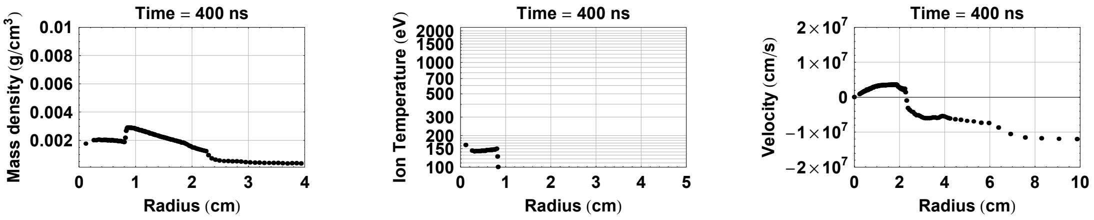
☐ **Setup**

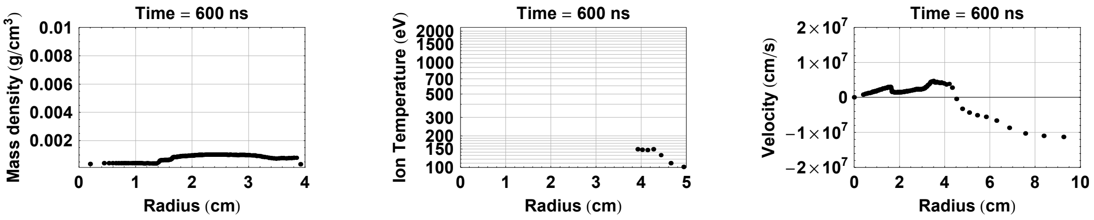
☐ **Table of plots**

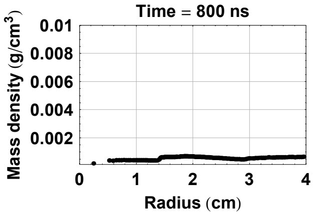
☐ **Array of plots**

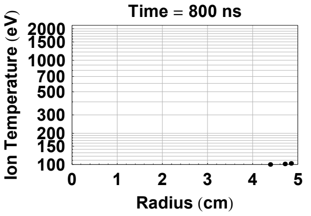
```
tbArray = Transpose[{tb1, tb2, tb3}];

Table[Show[GraphicsArray[tbArray⟦{3 i + 1, 3 i + 2, 3 i + 3}⟧],
   DisplayFunction → $DisplayFunction], {i, 0, 2}];
```

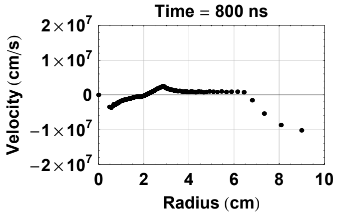


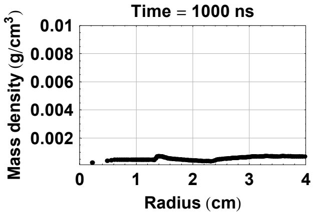


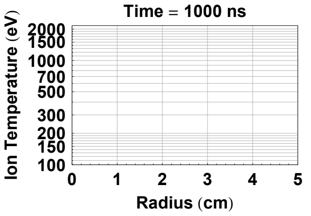


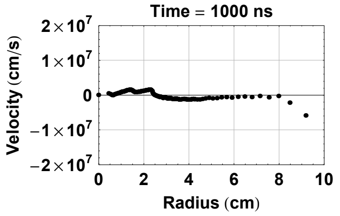

Time = 800 ns
Mass density (g/cm^3)
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
Radius (cm)

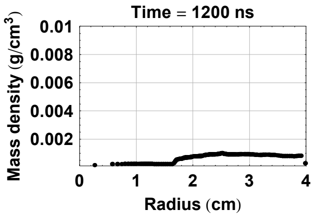

Time = 800 ns
Ion Temperature (eV)
2000
1500
1000
700
500
300
200
150
100
0
1
2
3
4
5
Radius (cm)

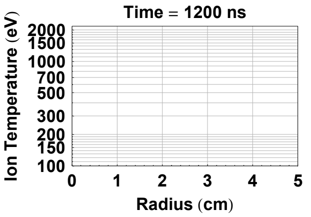

Time = 800 ns
Velocity (cm/s)
2×10^7
1×10^7
0
−1×10^7
−2×10^7
0
2
4
6
8
10
Radius (cm)

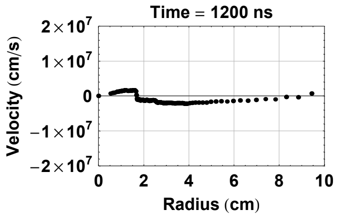

Time = 1000 ns
Mass density (g/cm^3)
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
Radius (cm)

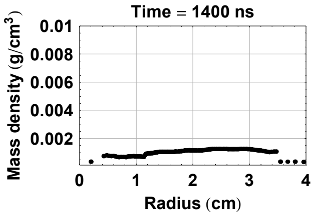

Time = 1000 ns
Ion Temperature (eV)
2000
1500
1000
700
500
300
200
150
100
0
1
2
3
4
5
Radius (cm)

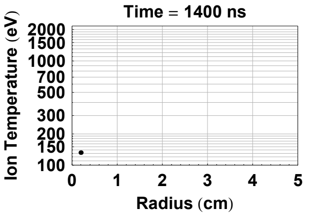

Time = 1000 ns
Velocity (cm/s)
2×10^7
1×10^7
0
−1×10^7
−2×10^7
0
2
4
6
8
10
Radius (cm)

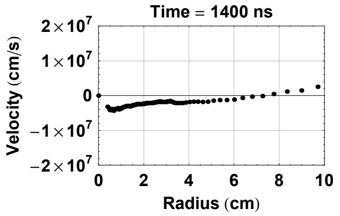

Time = 1200 ns
Mass density (g/cm^3)
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
Radius (cm)

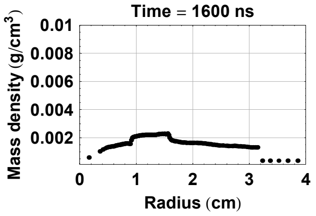

Time = 1200 ns
Ion Temperature (eV)
2000
1500
1000
700
500
300
200
150
100
0
1
2
3
4
5
Radius (cm)

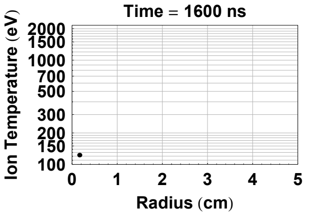

Time = 1200 ns
Velocity (cm/s)
2×10^7
1×10^7
0
−1×10^7
−2×10^7
0
2
4
6
8
10
Radius (cm)

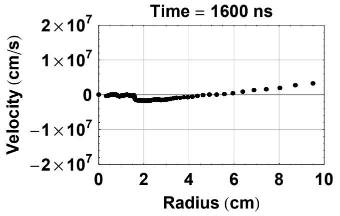

Time = 1400 ns
Mass density (g/cm^3)
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
Radius (cm)

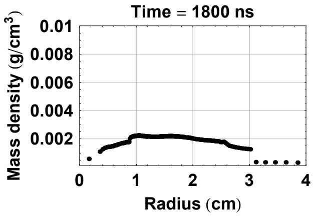

Time = 1400 ns
Ion Temperature (eV)
2000
1500
1000
700
500
300
200
150
100
0
1
2
3
4
5
Radius (cm)

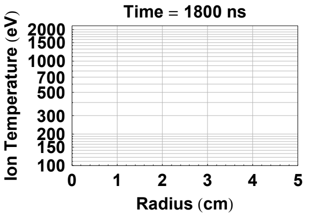

Time = 1400 ns
Velocity (cm/s)
2×10^7
1×10^7
0
−1×10^7
−2×10^7
0
2
4
6
8
10
Radius (cm)

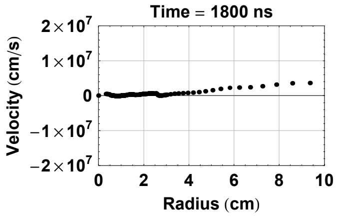

Time = 1600 ns
Mass density (g/cm^3)
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
Radius (cm)


Time = 1600 ns
Ion Temperature (eV)
2000
1500
1000
700
500
300
200
150
100
0
1
2
3
4
5
Radius (cm)

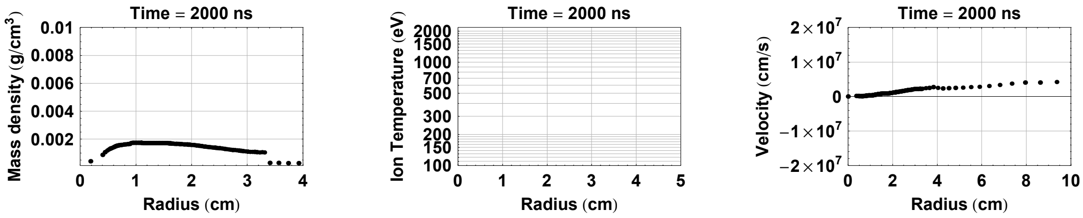

Time = 1600 ns
Velocity (cm/s)
2×10^7
1×10^7
0
−1×10^7
−2×10^7
0
2
4
6
8
10
Radius (cm)


Time = 1800 ns
Mass density (g/cm^3)
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
Radius (cm)


Time = 1800 ns
Ion Temperature (eV)
2000
1500
1000
700
500
300
200
150
100
0
1
2
3
4
5
Radius (cm)


Time = 1800 ns
Velocity (cm/s)
2×10^7
1×10^7
0
−1×10^7
−2×10^7
0
2
4
6
8
10
Radius (cm)

```
Show[GraphicsArray[tbArray[[10]], DisplayFunction → $DisplayFunction];
```



☐ Undo options

BUCKY case 030313.04 (v_{jet}=250 km/s)

★ *Note: the full set of commands to generate the plots is shown in the section "BUCKY case 030313.02" above. In this section only the plots and the outline of the structure appear.*

■ Setup

```
caseID = "030313_04";
```

```
params = Graphics[Text["Case " <> caseID <> "\n
m_t=4.38 mg; Δ_t=5.0 cm\n
m_j=0.2 g; Δ_j=2.4 cm\n
m_b=2.0 g; Δ_b=22.1 cm\n
T_ej=T_eb=106 eV\n
v_j=250 km/s; v_b=250 km/s", {0, 0}, {0, 0}]];
```

■ 0-500 ns

```
dataDir = "c:\\bucky\\cases\\" <> caseID <> "\\";
```

The variable tEnd (ns) is used for all of the time plots in this section.

```
tEnd = 500;
```

■ Zone radius vs time (PLOT.zones.1 & 2)

```
plotID = "zones";
```

```
thickness = Thickness[0.0002];
```

☐ Read data

☐ Number of time steps

☐ Number and list of radial zones in file

☐ Convert data from {s, cm} to {μs, m}

☐ Plots

• r < 0.3 m

```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.3}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10^-3 tEnd, 0.07}, {0.95 * 10^-3 tEnd, 0.17}, params],
  DisplayFunction → Identity];
```

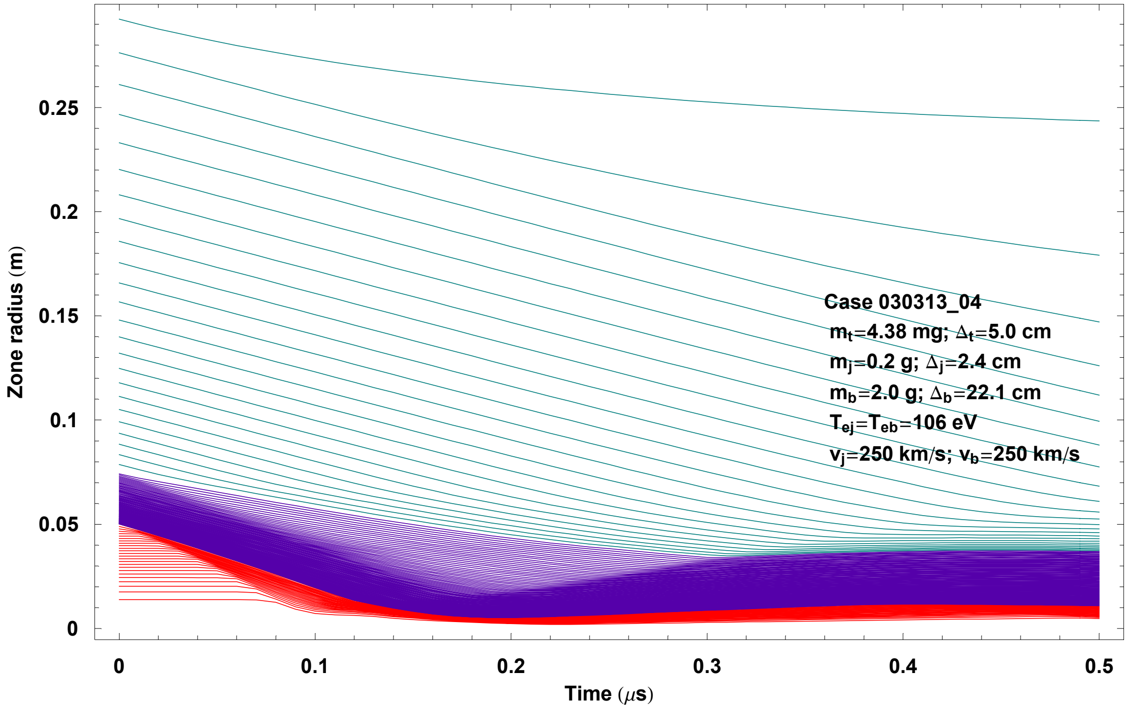
```
SetOptions[ListPlot, PlotRange → All, PlotJoined → True,
  GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"}, Epilog →
   Rectangle[{0.75 * 10^-3 tEnd, 0.07}, {0.95 * 10^-3 tEnd, 0.17}, params], DisplayFunction → Identity];
```

```
lsPlotsTarget = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → red], {i, 1, 𝒩t}];
```

```
lsPlotsJets = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];
```

```
lsPlotsBuffer = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

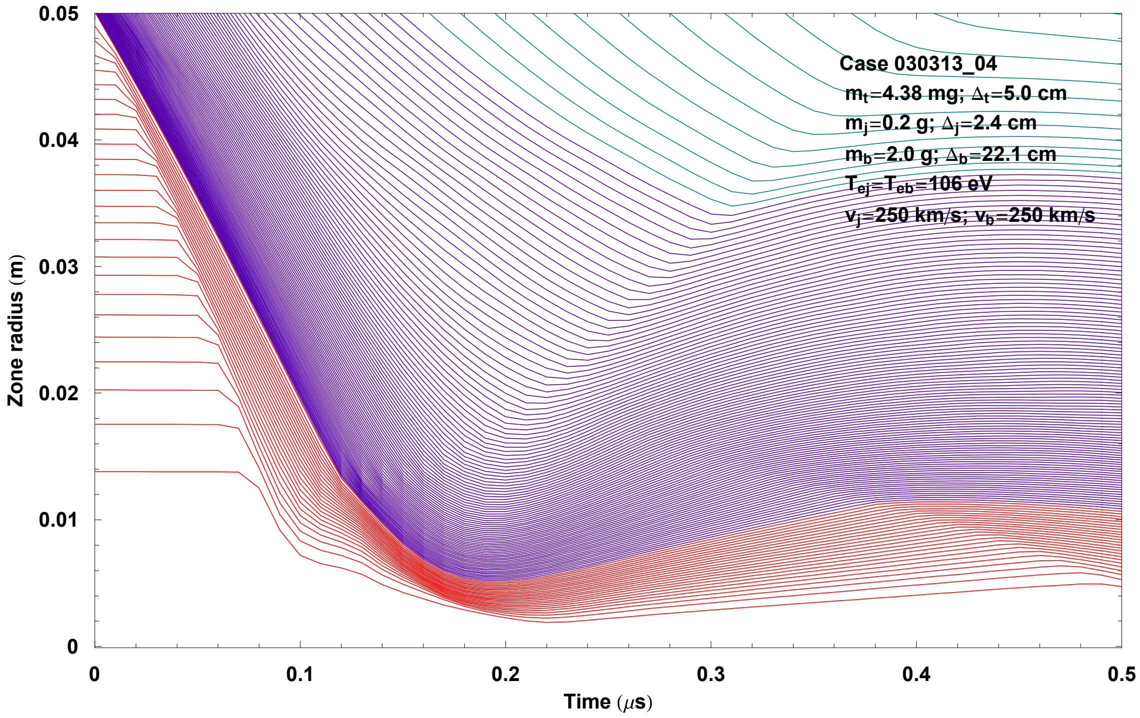
```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



• r < 0.05 m

```
SetOptions[ListPlot, PlotRange → {{-10^-6, 10^-3 tEnd}, {-10^-4, 0.05}},
  PlotJoined → True, GridLines → None, FrameLabel → {"Time (μs)", "Zone radius (m)"},
  Epilog → Rectangle[{0.75 * 10^-3 tEnd, 0.035}, {0.95 * 10^-3 tEnd, 0.045}, params],
  DisplayFunction → Identity];

lsPlotsTarget = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → red], {i, 1, 𝒩t}];

lsPlotsJets = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → purple], {i, 𝒩t + 1, 𝒩t + 𝒩j}];

lsPlotsBuffer = Table[ListPlot[lsData[plotID]⟦i⟧, PlotStyle → teal], {i, 𝒩t + 𝒩j + 1, 𝒩z}];
```

```
Show[lsPlotsTarget, lsPlotsJets, lsPlotsBuffer, DisplayFunction → $DisplayFunction];
```



• r < 0.02 m

☐ Undo options

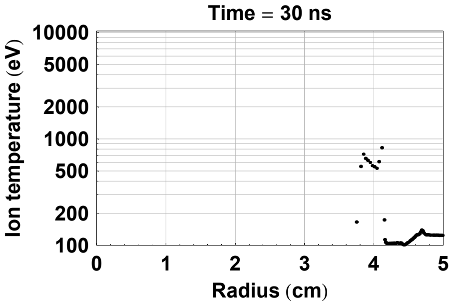
■ Ion temperature vs radius (PLOT.tempn)

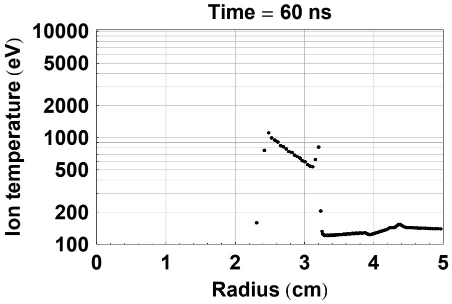
☐ Set up problem and import data

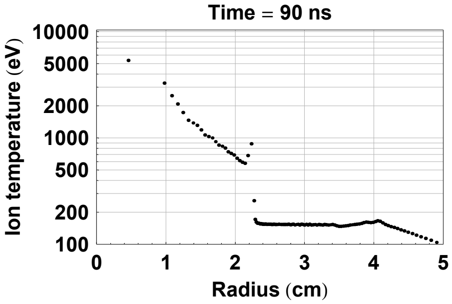
☐ Plots

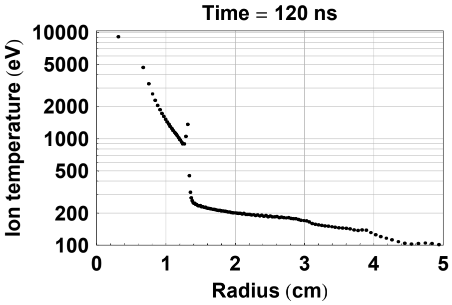
Partition length

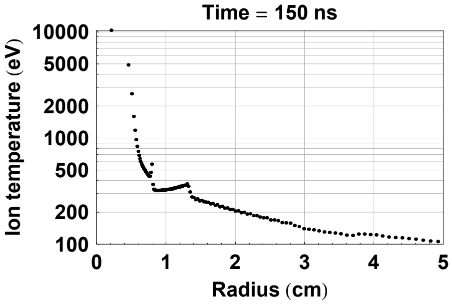
```
partitionL = 2;

plotrangeTi = PlotRange → {{-10^-3, 5}, {99.9, Max[lsData[plotID]⟦All, All, 2⟧]}};

tbData = Table[LogListPlot[lsData[plotID]⟦i⟧, plotrangeTi, PlotStyle → PointSize[0.01],
     PlotLabel → "Time = " <> ToString[Round[lsTimes⟦i⟧]] <> " ns", DisplayFunction → Identity],
    {i, 1, Length[lsTimes]}];

Map[Show[#, DisplayFunction → $DisplayFunction] &, tbData];
```

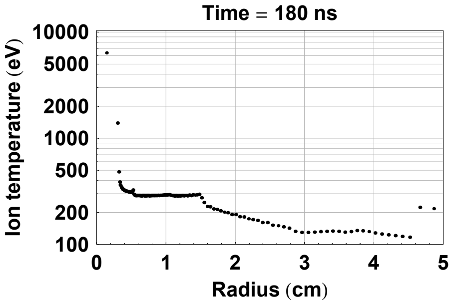


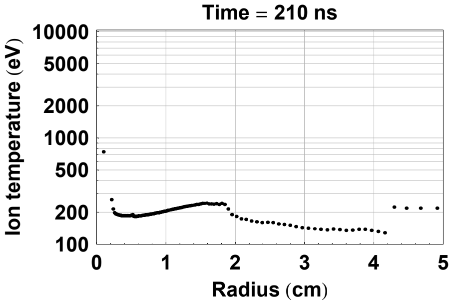


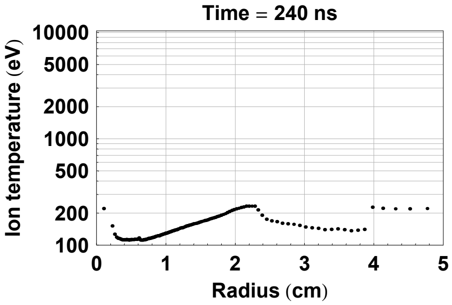


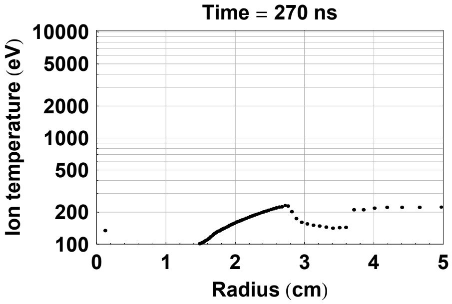

Time = 120 ns

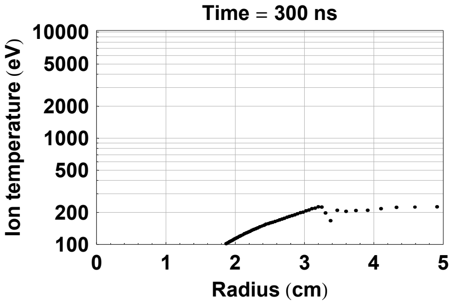

Time = 150 ns


Time = 180 ns


Time = 210 ns







☐ Undo options

Neutronics

MCNP cases

Notes

Mohamed Sawan ran MCNP cases that roughly simulate neutron heating of the magnetic nozzle. The conductor and structure were approximated by 0.05 m of 316 stainless steel. The cross sections for the actual magnet materials do not differ too greatly, so this approximation should be sufficient for a zero-order analysis.

The list *lsNHeating* contains sublists with $\{\mathcal{E}_0, \mathcal{E}_n, \mathcal{E}_\gamma, \mathcal{E}_{\text{tot}}\}$, all in MeV, defined as

$\mathcal{E}_0 \equiv$ Initial neutron energy
$\mathcal{E}_n \equiv$ Resulting neutron energy in magnet and structure
$\mathcal{E}_\gamma \equiv$ Resulting gamma energy in magnet and structure
$\mathcal{E}_{\text{tot}} \equiv$ Resulting total energy in magnet and structure

Data

```
lsNHeating = {{10^-6, 0.099, 5.09, 5.18}, {10^-5, 0.063, 3.18, 3.24},
    {10^-4, 0.027, 2.53, 2.56}, {10^-3, 0.071, 2.69, 2.76}, {10^-2, 0.029, 1.16, 1.19},
    {0.030, 0.018, 0.47, 0.49}, {0.070, 0.021, 0.219, 0.240}, {0.1, 0.033, 0.225, 0.257},
    {0.3, 0.039, 0.077, 0.117}, {0.7, 0.077, 0.061, 0.138}, {0.8, 0.102, 0.063, 0.165},
    {0.93, 0.090, 0.122, 0.210}, {1, 0.099, 0.175, 0.274}, {2, 0.154, 0.560, 0.712},
    {7, 0.640, 2.53, 3.18}, {10, 0.76, 3.66, 4.42}, {14.1, 1.25, 3.31, 4.56}} // N;
TableForm[lsNHeating, TableHeadings → {None, {ℰ0, ℰn, ℰγ, ℰtot}}]
```

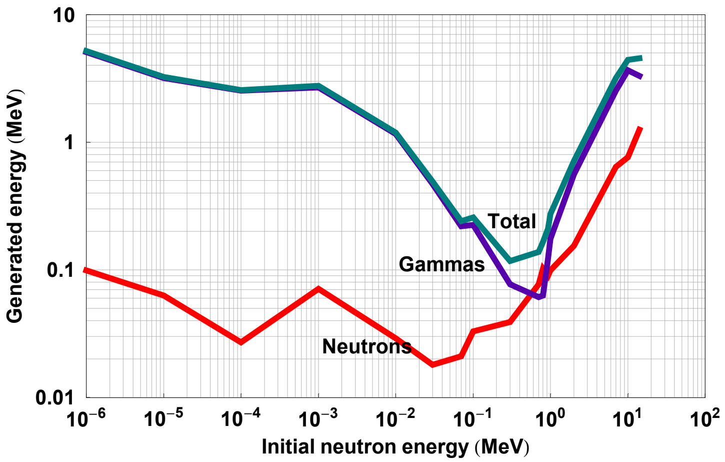
Check:

```
 lsNHeating⟦All, 2⟧ + lsNHeating⟦All, 3⟧
 ---------------------------------------
          lsNHeating⟦All, 4⟧
{1.00174, 1.00093, 0.998828, 1.00036, 0.99916, 0.995918,
 1., 1.00389, 0.991453, 1., 1., 1.00952, 1., 1.00281, 0.996855, 1., 1.}
```

Resulting energy vs n energy

```
gridlinesX = Map[{#, {GrayLevel[0.7]}} &, Flatten[Table[Log[10, j * 10^i], {i, -6, 2}, {j, 1, 9}]]];

gridlinesY = Map[{#, {GrayLevel[0.7]}} &, Flatten[Table[Log[10, j * 10^i], {i, -3, 2}, {j, 1, 9}]]];

frameticksX = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -6, 2}];

frameticksY = Table[{i, ToString[If[i < 0, N[10^i], 10^i]]}, {i, -3, 2}];

lllsPlots = Table[LogLogListPlot[Transpose[{lsNHeating⟦All, 1⟧, lsNHeating⟦All, i⟧}],
    PlotRange → All, PlotStyle → lineStyle⟦i - 1⟧,
    FrameLabel → {"Initial neutron energy (MeV)", "Generated energy (MeV)"},
    DisplayFunction → Identity], {i, 2, 4}];

text = {Graphics[Text["Neutrons", {-3.0, -1.6}, {-1, 0}]],
   Graphics[Text["Gammas", {-0.8, -0.85}, {1, 1}]],
   Graphics[Text["Total", {-0.5, -0.62}, {0, 0}]]};
```

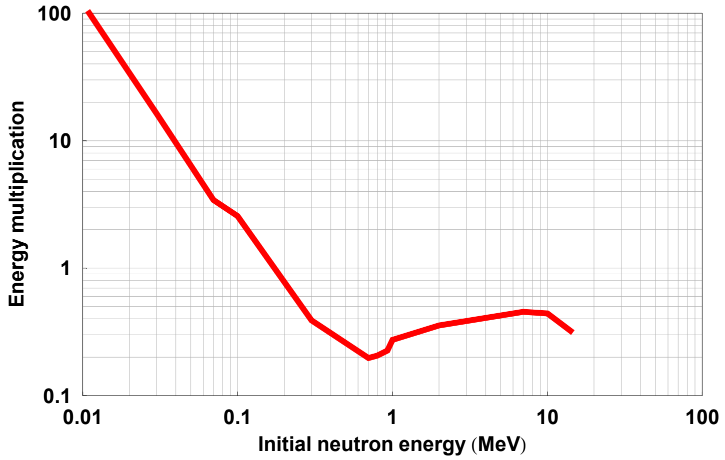
```
Show[lllsPlots, text, PlotRange → {{-6.001, 2}, {-2.001, 1}},
  GridLines → {gridlinesX, gridlinesY},
  FrameTicks → {frameticksX, frameticksY, None, None}, DisplayFunction → $DisplayFunction];
```



Energy multiplication vs n energy

```
lllsNMult = LogLogListPlot[Transpose[{lsNHeating⟦All, 1⟧, lsNHeating⟦All, 4⟧ / lsNHeating⟦All, 1⟧}],
   PlotRange → {{0.999 × 10^-2, 100}, {0.0999, 10^2}}, PlotStyle → lineStyle⟦1⟧, FrameLabel →
    {"Initial neutron energy (MeV)", "Energy multiplication"}, DisplayFunction → Identity];

frameticksX = Table[{i, ToString[If[i < 0, N[10^i], 10^i]]}, {i, -3, 2}];
```

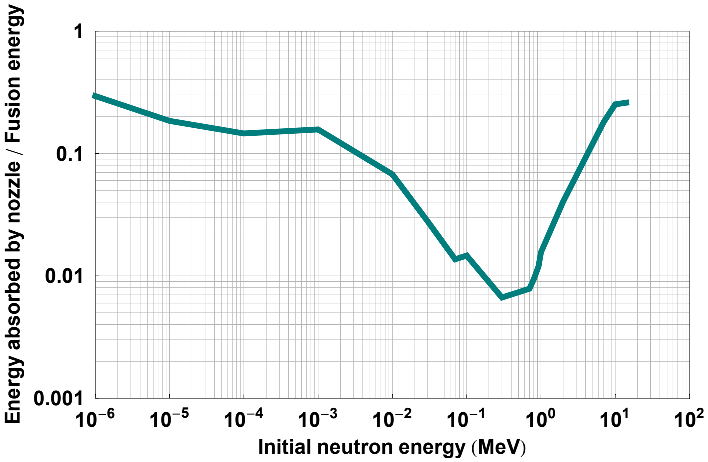
```
Show[lllsNMult, GridLines → {gridlinesX, gridlinesY},
  FrameTicks → {frameticksX, frameticksY, None, None}, DisplayFunction → $DisplayFunction];
```



Energy absorbed by nozzle vs n energy

```
lllsNPfus = LogLogListPlot[Transpose[{lsNHeating⟦All, 1⟧, lsNHeating⟦All, 4⟧ / 17.58}],
   PlotRange → {{0.999 × 10^-6, 100}, {0.0009999, 1}}, PlotStyle → lineStyle⟦3⟧,
   GridLines → {gridlinesX, gridlinesY}, FrameTicks → {frameticksX, frameticksY, None, None},
   FrameLabel → {"Initial neutron energy (MeV)", "Energy absorbed by nozzle / Fusion energy"},
   DisplayFunction → Identity];

frameticksX = Table[{i, "\!\(10\^" <> ToString[i] <> "\)"}, {i, -6, 2}];
```

```
Show[lllsNPfus, GridLines → {gridlinesX, gridlinesY},
  FrameTicks → {frameticksX, frameticksY, None, None}, DisplayFunction → $DisplayFunction];
```



□ Function

```
lsNPfus = Transpose[{Log[10, lsNHeating⟦All, 1⟧], lsNHeating⟦All, 4⟧ / 17.58}];
```

```
nHt = Interpolation[lsNPfus];
```

```
Map[FindRoot[nHt[Log[10, En]] == 0.01, {En, #}] &, {0.2, 1}]
```

```
{{En → 0.201552}, {En → 0.854865}}
```

```
FindMinimum[nHt[Log[10, En]], {En, 0.2, 0.5}]
```

```
{0.00620698, {En → 0.459338}}
```

■ Calculation of neutron slowing down in deuterium

■ Setup

□ Read data

```
dataDirN = "c:\\nuclearphysics\\";
```

```
lsRT0 = Import[dataDirN <> "sigmaTotalVsE_ENDF_H-2f.dat"];
```

```
lsRT = Partition[Flatten[Drop[lsRT0, 2]], 2];
```

□ Function

```
logSigmaND = Interpolation[lsRT /. {x_, y_} → {Log[10, x], y}]
```

```
InterpolatingFunction[{{-5., 8.17609}}, <>]
```

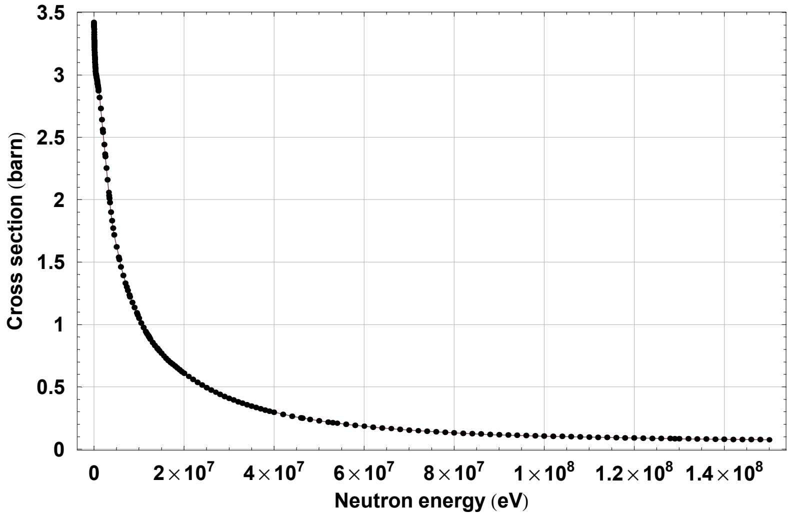
```
sigmaND[En_] := logSigmaND[Evaluate[Log[10, En]]]
```

□ Plots and check of function

```
pls1 = ListPlot[lsRT, FrameLabel → {"Neutron energy (eV)", "Cross section (barn)"},
   PlotRange → All, DisplayFunction → Identity];
```

```
pls2 = Plot[sigmaND[En], {En, 100, 1.5*^8}, PlotRange → All, DisplayFunction → Identity];
```

```
Show[pls2, pls1, PlotRange → All, FrameLabel → {"Neutron energy (eV)", "Cross section (barn)"},
  DisplayFunction → $DisplayFunction];
```



References

Thio 1999a: Y.C.F. Thio, E. Panarella, R.C. Kirkpatrick, C.E. Knapp, F. Wysocki, P. Parks, and G. Schmidt, "Magnetized Target Fusion in a Spheroidal Geometry with Standoff Drivers," Current Trends in International Fusion Research--Proceedings of the Second International Symposium (National Research Council of Canada, Ottawa, Canada, 1999).

Thio 1999b: Y.C.F. Thio, B. Freeze, R.C. Kirkpatrick, B. Landrum, H. Gerrish, and G.R. Schmidt, "High-Energy Space Propulsion Based on Magnetized Target Fusion," 35th AIAA/ASMA/SAE/ASEE Joint Propulsion Conference paper AIAA-99-2703 (1999).