

Growth and Productivity

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Outline

- In previous lectures we have looked at **how to measure** economic success
- Today we want to:
 - 1 Identify which factors determine economic success
 - 2 Try to explain why some countries are so poor while others are rich

Production function

- Production function: mathematical function that relates inputs to output:

$$Y = AF(L, K, H, N)$$

- Y is output
- A is Technology
- L is Labor
- K is Capital
- H is Human Capital
- N is Natural Resources

Returns to Scale

- Qualitative measure of how output changes when you change inputs
 - Constant Returns to Scale
 - Increasing Returns to Scale
 - Decreasing Returns to Scale

- Constant Returns to Scale: Doubling inputs exactly doubles output

$$tF(K, L) = F(tK, tL) \quad \forall t \geq 1$$

- Decreasing Returns to Scale: Doubling inputs more than doubles output

$$tF(K, L) > F(tK, tL) \quad \forall t \geq 1$$

- Increasing Returns to Scale: Doubling inputs less than doubles output

$$tF(K, L) < F(tK, tL) \quad \forall t \geq 1$$

Examples: Returns to scale

Do the following functions exhibit constant returns to scale?

① $F(L, K, H, N) = K + L + H + N$

- Double Inputs: $F(tL, tK, tH, tN) = tK + tL + tH + tN$
- $\Leftrightarrow F(tL, tK, tH, tN) = t(K + L + H + N) = tF(L, K, H, N)$
- \Rightarrow Constant returns to scale

② $F(L, K, H, N) = K \times L \times H \times N$

- Double Inputs: $F(tL, tK, tH, tN) = tK \times tL \times tH \times tN$
- $\Leftrightarrow F(tL, tK, tH, tN) = t^4(K \times L \times H \times N) = t^4F(L, K, H, N)$
- \Rightarrow Increasing returns to scale

You Try

Does the production function, $F(K, L) = K + \sqrt{L}$, have increasing, decreasing, or constant returns to scale?

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Does the production function, $F(K, L) = K + \sqrt{L}$, have increasing, decreasing, or constant returns to scale?

$$F(tK, tL) = (tK) + \sqrt{tL} \qquad tF(K, L) = t(K + \sqrt{L})$$

$$\Leftrightarrow F(tK, tL) = tK + t^{0.5}\sqrt{L} \qquad \Leftrightarrow tF(K, L) = tK + t\sqrt{L}$$

$$\Rightarrow tF(K, L) > F(tK, tL)$$

- This example illustrates why it's important to only consider $t \geq 1$

Productivity

- The standard of living in an economy depends on the **economy's ability to produce** goods and services
 - Often measured as GDP per capita
 - Why not just GDP?
- Productivity can be defined as the quantity of goods and services produced from each unit of labor (hour worked)
 - $productivity = \frac{Y}{L}$
- Productivity is the main driver of increases in the standard of living
 - Kydland & Prescott show things like technology are key (not required)
 - Can think of labor augmenting technology, TFP, etc.

$$AF(K, L) \text{ v.s. } F(K, AL)$$

Factors that determine productivity

- 1 Physical capital per worker (sometimes just called capital)
 - More capital per worker makes workers more productive
 - Example: If I have 2 stove-tops, I can cook more than if I only have 1

$$prod = \frac{F(K, L)}{L} \Rightarrow \frac{\partial prod}{\partial K} > 0$$

- 2 Human capital per worker
 - Knowledge and skills that workers acquire through education, training, and experience
 - Romer-Jones model endogenizes productivity growth with sector for ideas (not required)
 - Example: Great calculus teacher can increase human capital and improve your productivity in economics classes

Factors that determine productivity

3 Natural resources per worker

- Natural resources (i.e. land, oil, etc.) are important inputs into production of goods and services
- Example: More land you have the more crops you can grow

4 Technological knowledge

- Understanding of the best way to produce goods and services
- Example: Cars were initially built one by one, but now companies use an assembly line which significantly increases productivity
- Kydland & Prescott's claim to fame!!!

$$Y = F(K, AL)$$

$$Y = AF(K, L)$$

- Where A could be subject to random shocks, growth trend, etc.

Example: Productivity

In-N-Out produces 1,000 hamburgers a day. Each employee works 5 hours a day. In-N-Out's productivity is 20 hamburgers per hour of labor. How many workers does In-N-Out employ?

- Each employee can produce: $5 \times 20 = 100$ hamburgers per day
- $\Rightarrow \frac{1,000}{100} = 10$ workers are needed

Example: Diminishing returns to capital

Bob the baker can make 240 cupcakes a day using 1 oven in his store. If Bob were to install a 2nd oven in his store, what would be the most realistic expectation for Bob's new per day cupcake production level?

- A. 600
- B. 550
- C. 500
- D. 450

Example: Diminishing returns to capital

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- A. 600
- B. 550
- C. 500
- D. 450

Answer: D

**The second oven can't be more productive than the first oven;
idea of diminishing marginal product**

$$MP_k = \frac{\partial F}{\partial K} \quad \frac{\partial MP_k}{\partial K} < 0$$

Growth

- Economic growth is a result of increased productivity
- Productivity depends on the 4 factors we discussed earlier (physical capital per worker, human capital per worker, etc.)
- Growth: How can a country increase productivity ?

Growth

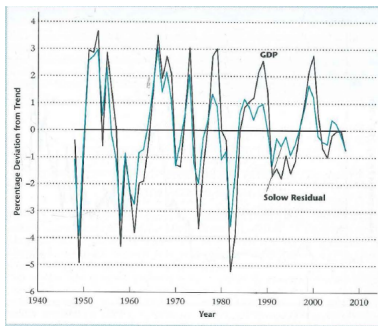
Growth: How can a country increase productivity ?

- Save and invest
 - Instead of consuming spend output to accumulate capital
 - Next period there will be more capital per worker, which will increase productivity
- Invest in improved education
- Health and nutrition
- Property rights and political stability
- Free trade and markets
- Research and Development

Solow Model

Everything from here on is just food for thought! (i.e. not required)

- Reduced form model to describe economic growth, capital accumulation, and dynamics of aggregates
 - All long-run growth is driven by technology
- Takes production function, savings rate, and technology progress as given



- Assume a Cobb-Douglas production function and standard evolution of capital

$$F(K, AL) = K^\alpha (AL)^{1-\alpha} \quad \frac{\partial K}{\partial t} = sF(K, AL) - \delta K$$

- pop. growth rate = n , savings rate = s , depreciation rate = δ , capital share = α , and productivity growth rate = g
- We can define the change of variables $k = \frac{K}{AL}$ and derive $\frac{\partial k}{\partial t}$

$$\frac{\partial k}{\partial t} = sk^\alpha - (n + g + \delta)k \Rightarrow k^* = \left(\frac{s}{n + g + \delta} \right)^{\frac{1}{1-\alpha}}$$

- From the steady state value of k^* , we can describe dynamics of any aggregate ratio
 - $\frac{Y}{L} = y^* A$
 - $\frac{K}{L} = k^* A$

- The dynamics of the k can be summarized in the following diagram
 - What's the effect of an increase in the aggregate savings rate?
 - Recall: $\frac{\partial k}{\partial t} = sk^\alpha - (n + g + \delta)k$

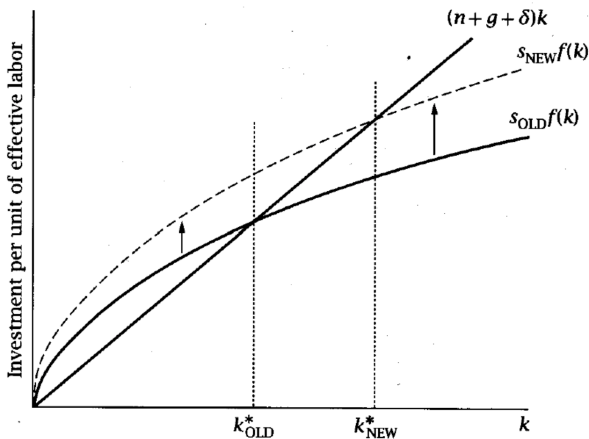


FIGURE 1.4 The effects of an increase in the saving rate on investment

- What's the effect of an increase in the aggregate savings rate?
 - Recall: $\frac{\partial k}{\partial t} = sk^\alpha - (n + g + \delta)k$

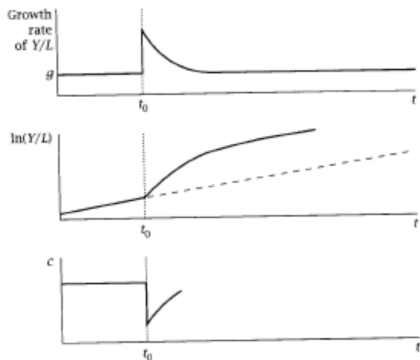
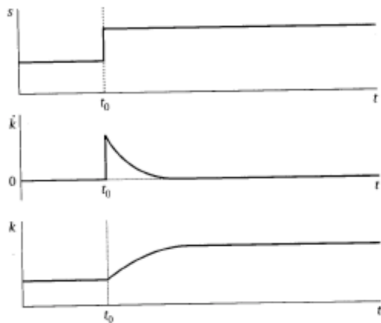


FIGURE 1.5 The effects of an increase in the saving rate