Linear Time Invariant (LTI) systems

In the previous lectures we have introduced a number of basic system properties. Two of these, *linearity and time-invariance (these two properties are sometimes called superposition property)*, plays an important role in signals and systems analysis.

Any system possess these two properties is called *linear time- invariant (LTI)* system.

LTI have an important property that, if the input to a LTI system can be represented in terms of linear combination of a set of basic signals, by the same way the output of the system can be computed using the superposition property in terms of it's response to these basic signals.

The *unit impulse* is an important basic signal, because any general signal $x[n]$ can be represented as a linear combination of delayed impulses.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

The above two properties allow us to develop a complete characterization of LTI system in terms to it's impulse response using *convolution sum for discrete time system* and *convolution integral for continuous time system*.

Discrete Time LTI systems: the convolution sum

The convolution sum is the mathematical relationship that links the input and output signals in any linear time-invariant discrete-time system. Given an LTI system and an input signal x[n], the convolution sum will allow us to compute the corresponding output signal y[n] of the system.

Representation of Discrete-Time Signals in Terms of Impulses

A discrete-time signal x[n] can be viewed as a linear combination of time-shifted impulses:

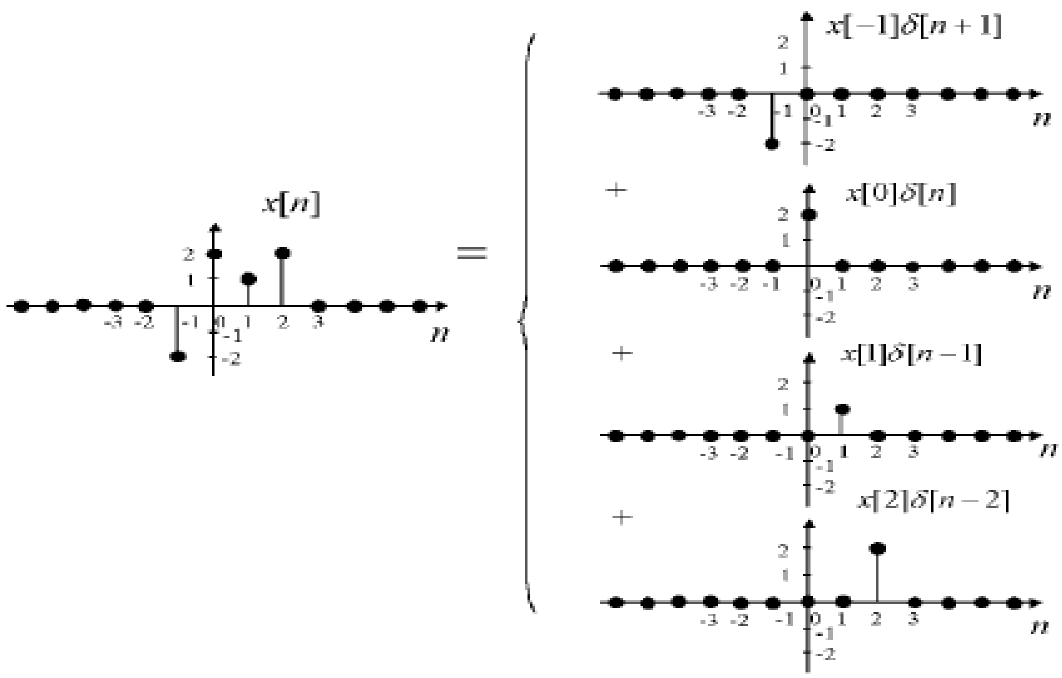
$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

Example 1:

The following signal can be written as:

$$x[n] = -2\,\delta[n+1] + 2\delta[n] + \delta[n-1] + 2\,\delta[n-2]$$

The finite-support signal $x[n]$ shown in Figure is the sum of four impulses.



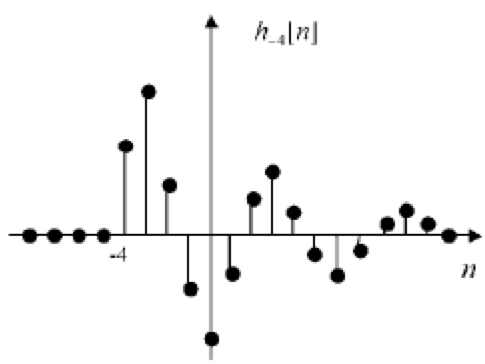
Decomposition of a discrete-time signal as a sum of shifted impulses.

Response of LTI as a linear combination of impulse response

The Principle of Superposition applies to the class of linear discrete-time systems. By the Principle of Superposition, the response y[n] of a discrete-time linear system is the sum of the responses to the individual shifted impulses making up the input signal x[n].

Let $h_k[n]$ be the response of the LTI system to the shifted impulse $\delta[n-k]$.

Example 2: For k=-4, the response $h_{-4}[n]$ to $\delta[n+4]$ might look like the one in the following figure.



Impulse response for an impulse occurring at time −4.

Example 3: For the input signal in Example 1

$$x[n] = -2\,\delta[n+1] + 2\delta[n] + \delta[n-1] + 2\,\delta[n-2]$$

, the response of a linear system would be $y[n] = -2\ h_{-1}[n] + 2h_0[n] + h_1[n] + 2\ h_2[n]$

Thus, the response to the input x[n] can be written as an infinite sum of all the impulse responses:

$$y[n] = \sum_{k=-\infty}^{\infty} x[n]\ h_k[n]$$

If we knew the response of the system to each shifted impulse $\delta[n-1]$, we would be able to calculate the response to any input signal x[n] using the above equation.

It gets better than this: for a linear time-invariant system (the time-invariance property is important here), the impulse responses $h_k[n]$ are just shifted versions of the same impulse response for k = 0.

$$h_k[n] = h_0[n-k]$$

Therefore, the impulse response of an LTI system $h[n] = h_0[n]$ characterizes it completely. This is not the case for a linear time-varying system: one has to specify all the impulse responses $h_k[n]$ (an infinite number) to characterize the system.

The Convolution Sum

we obtain the *convolution sum*
that gives the response of a discrete-time LTI system to an arbitrary input.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Remark: In general, for each time n, the summation for the single value $y[n]$ runs over all values (an infinite number) of the input signal $x[n]$ and of the impulse response $h[n]$.

The Convolution Operation

More generally, the *convolution* of two discrete-time signals $v[n]$ and $w[n]$, denoted as $v[n]*w[n]$ (or sometimes $(v*w)[n]$), is defined as follows:

$$v[n]*w[n] = \sum_{k=-\infty}^{\infty} v[k]w[n-k].$$

The convolution operation has the following properties. It is

- Commutative:

$$v[n]*w[n] = \sum_{k=-\infty}^{\infty} v[k]w[n-k] = \sum_{m=\infty}^{-\infty} v[n-m]w[m] = \sum_{m=-\infty}^{+\infty} w[m]v[n-m] = w[n]*v[n]$$

(after the change of variables $m = n-k$)

- Associative:

$$v[n]*(w[n]*y[n]) = v[n]*(y[n]*w[n])$$

- Distributive:

$$x[n]*(v[n]+w[n]) = \sum_{k=-\infty}^{+\infty} x[k](v[n-k]+w[n-k])$$
$$= \sum_{k=-\infty}^{+\infty} x[k]v[n-k] + \sum_{k=-\infty}^{+\infty} x[k]w[n-k] = x[n]*v[n]+x[n]*w[n]$$

- Commutative with respect to multiplication by a scalar:

$$a(v[n]*w[n]) = (av[n])*w[n] = v[n]*(aw[n])$$

- Time-shifted when one of the two signals is time-shifted:

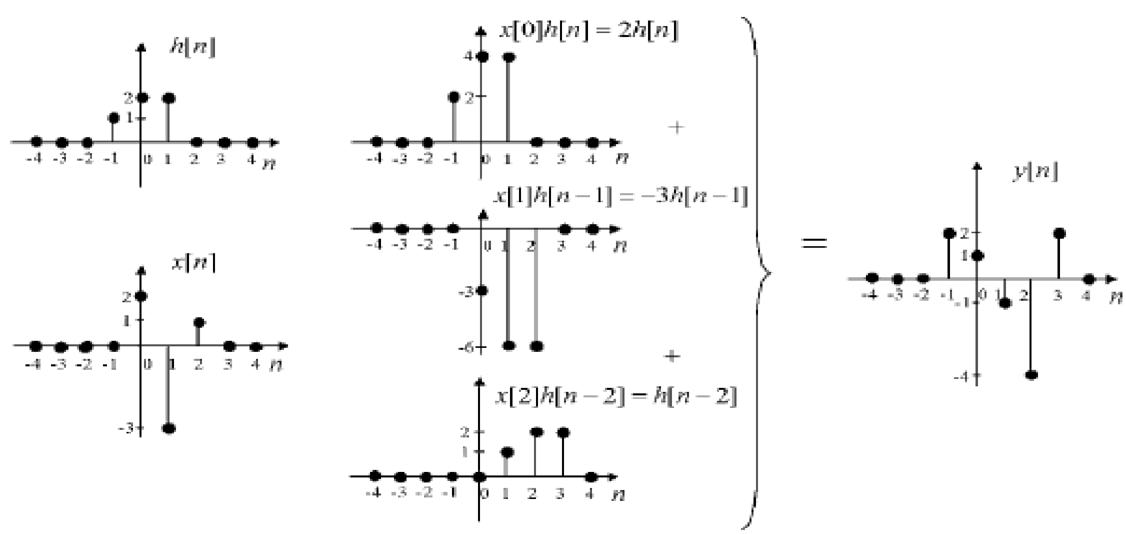
$$v[n]*w[n-N] = \sum_{k=-\infty}^{\infty} v[k]w[n-N-k] = (v*w)[n-N]$$

Finally, the convolution of a signal with a unit impulse leaves the signal unchanged (this is just Equation $x[n] = \sum_{k=-\infty}^{\infty\infty} x[k]\,\delta[n-k]$), and therefore the LTI system defined by the impulse response $h[n] = \delta[n]$ is the identity system.

Graphical Computation of a Convolution

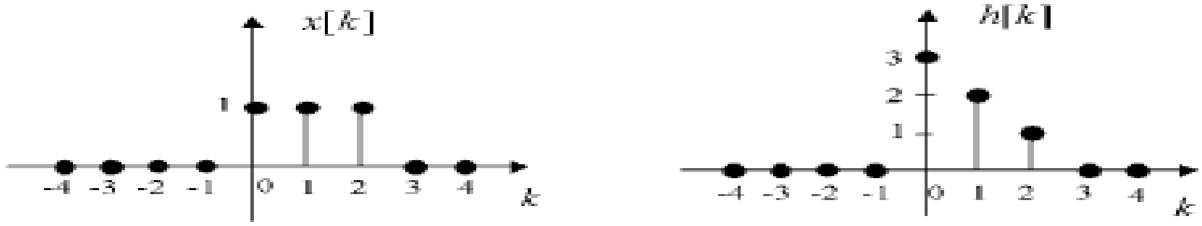
One way to visualize the convolution sum for simple examples is to draw the weighted and shifted impulse responses one above the other and to add them up.

Example : Let us compute $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=0}^{2} x[k]h[n-k]$ for the impulse response and input signal shown in Figure



Graphical computation of a convolution.

***Example*: Let us compute y[0] and y[1] for the input signal and impulse response of an LTI system shown in the following Figure.**



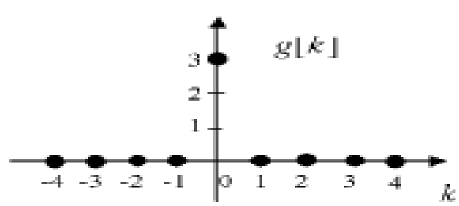
Convolution of an input signal with an impulse response.

Case $n = 0$:

Step 1: Sketch $x[k]$ and $h[0-k] = h[-k]$ as in Figure

Impulse response flipped around the vertical axis.

Step 2: Multiply $x[k]$ and $h[-k]$ to get $g[k]$ shown in Figure



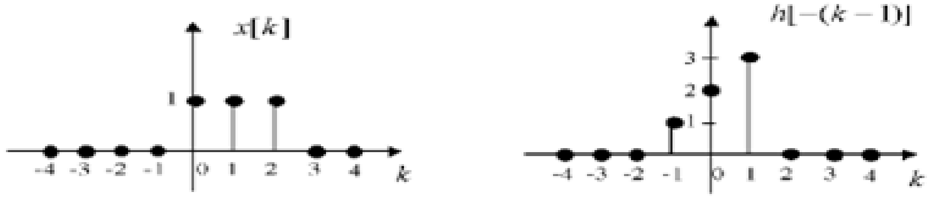
Product of flipped impulse response with input signal for $n = 0$.

Step 3: Sum all values of $g[k]$ from $k = -\infty$ to $+\infty$ to get $y[0]$:

$$y[0] = 3$$

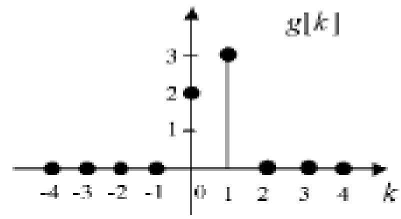
Case $n = 1$:

Step 1: Sketch $x[k]$ and $h[1-k] = h[-(k-1)]$ (i.e., the signal $h[-k]$ delayed by 1) as in Figure



Time-reversed and shifted impulse response for $n = 1$.

Step 2: Multiply $x[k]$ and $h[1-k]$ to get $g[k]$ shown in Figure



Product of flipped and shifted impulse response with input signal for $n = 1$.

Step 3: Sum all values of $g[k]$ from $k = -\infty$ to $+\infty$ to get $y[1]$:

$$y[1] = 2 + 3 = 5$$

Solved Problems

1. **Let $x[n] = \delta[n] + 2\,\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$**

   Compute and plot each of the following convolutions:

   a. $y[n] = x[n] * h[n]$

(a) We know that

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \qquad (S2.1-1)$$

The signals $x[n]$ and $h[n]$ are as shown in Figure S2.1.

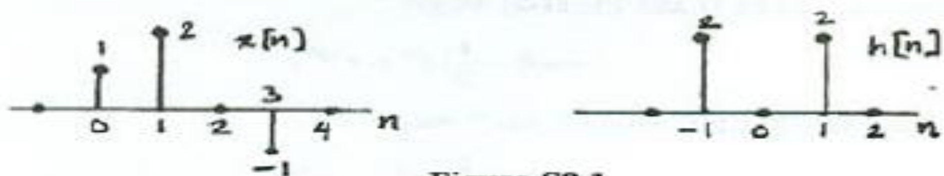


Figure S2.1

From this figure, we can easily see that the above convolution sum reduces to

$$\begin{aligned} y_1[n] &= h[-1]x[n+1] + h[1]x[n-1] \\ &= 2x[n+1] + 2x[n-1] \end{aligned}$$

This gives

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

   b. $y[n] = x[n+2] * h[n]$

(b) We know that

$$y_2[n] = x[n+2] * h[n] = \sum_{k=-\infty}^{\infty} h[k]x[n+2-k]$$

Comparing with eq. (S2.1-1), we see that

$$y_2[n] = y_1[n+2]$$

   c. $y[n] = x[n] * h[n+2]$

(c) We may rewrite eq. (S2.1-1) as

$$y_1[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Similarly, we may write

$$y_3[n] = x[n] * h[n+2] = \sum_{k=-\infty}^{\infty} x[k]h[n+2-k]$$

Comparing this with eq. (S2.1), we see that

$$y_3[n] = y_1[n+2]$$

2. Compute and plot y[n] = x[n] ∗ h[n] where

$$x[n] = \begin{cases} 1 & 3 \le n \le 8 \\ 0 & otherwise \end{cases}, \quad h[n] = \begin{cases} 1 & 4 \le n \le 15 \\ 0 & otherwise \end{cases}$$

We know that

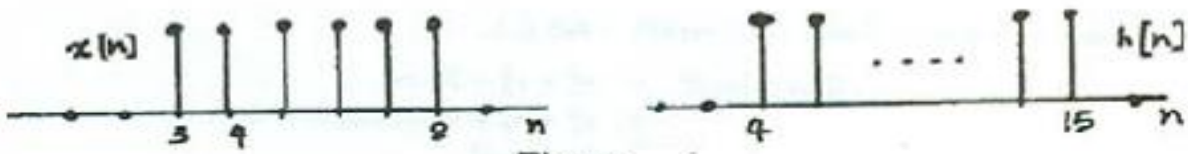
$$y[n] = x[n] * h[n] = \sum_{k=\infty}^{\infty} x[k]h[n-k]$$

The signals x[n] and y[n] are as shown in Figure From this figure, we see that the above summation reduces to

$$y[n] = x[3]h[n-3] + x[4]h[n-4] + x[5]h[n-5] + x[6]h[n-6] + x[7]h[n-7] + x[8]h[n-8]$$

This gives

$$y[n] = \begin{cases} n-6, & 7 \le n \le 11 \\ 6, & 12 \le n \le 18 \\ 24-n, & 19 \le n \le 23 \\ 0, & \text{otherwise} \end{cases}$$



Figure

3. **A linear system has the relationship**

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$$

Between its input x[n] and its output y[n] where g[n]=u[n]-u[n-4].

a. Determine y[n] when $x[n] = \delta[n-1]$

b. Determine y[n] when $x[n] = \delta[n-2]$

c. Is the system is LTI?

d. Determine y[n] when $x[n] = u[n]$.

(a) Given that

$$x[n] = \delta[n-1],$$

we see that

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k] = g[n-2] = u[n-2] - u[n-6]$$

(b) Given that $x[n] = \delta[n-2]$,

we see that $$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k] = g[n-4] = u[n-4] - u[n-8]$$

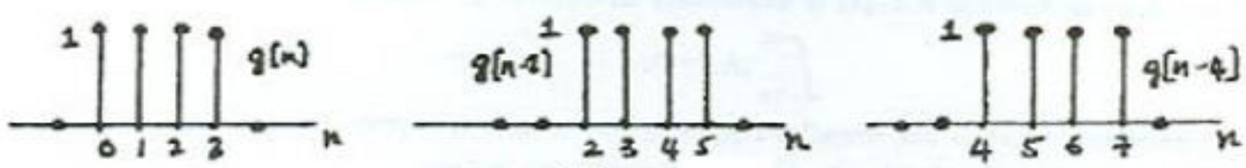
(c) The input to the system in part (b) is the same as the input in part (a) shifted by 1 to the right. If S is time invariant then the system output obtained in part (b) has to the be the same as the system output obtained in part (a) shifted by 1 to the right. Clealry, this is not the case. Therefore, the system is not LTI.

(d) If $x[n] = u[n]$, then

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k] = \sum_{k=0}^{\infty} g[n-2k]$$

The signal $g[n-2k]$ is plotted for $k = 0, 1, 2$ in Figure . From this figure it is clear that

$$y[n] = \begin{cases} 1, & n = 0, 1 \\ 2, & n > 1 \\ 0, & \text{otherwise} \end{cases} = 2u[n] - \delta[n] - \delta[n-1]$$



Figure

4. Find the impulse response for each of the following discrete-time systems:

a. $y[n] + 0.2\,y[n-1] = x[n] - x[n-1]$.

$$y[n] + 0.2\,y[n-1] = x[n] - x[n-1].\ \ let\ y[n] = h[n] \text{ then } x[n] = \delta[n]$$

$$h[n] + 0.2\,h[n-1] = \delta[n] - \delta[n-1]$$

$$h[n] = -0.2\,h[n-1] + \delta[n] - \delta[n-1]$$

$at\ n = 0$ ➔ $h[0] = -0.2\,h[0-1] + \delta[0] - \delta[0-1]$

$$= -0.2\,h[-1] + \delta[0] - \delta[-1] = -0.2 * 0 + 1 - 0 = 1$$ ➔(1)

$at\ n = 1$ ➔ $h[1] = -0.2\,h[1-1] + \delta[1] - \delta[1-1]$

$= -0.2\,h[0] + \delta[1] - \delta[0] = -0.2 * 1 + 0 - 1 = -1.2$ ➔ (2)

$at\ n = 2$ ➔ $h[2] = -0.2\,h[1] + \delta[2] - \delta[1] = -0.2 * -1.2 + 0 - 0 = 0.24$ ➔ (3)

$at\ n = 3$ ➔ $h[3] = -0.2\,h[2] + \delta[3] - \delta[2] = -0.2 * 0.24 + 0 - 0 = -0.48$ ➔ (4)

From (1) , (2) , (3) , (4) $h[n] = (-0.2)^{n+1}(-1.2) \quad for\ n \geq 1$

---

b. $y[n] + 1.2\,y[n-1] = 2\,x[n-1]$.

$y[n] = -1.2\,y[n-1] + 2\,x[n-1]$. Put y[n]=h[n] and x[n]=$\delta[n]$

$h[n] = -1.2\,h[n-1] + 2\,\delta[n-1]$

$h[0] = -1.2\,h[-1] + 2\,\delta[-1] = -1.2 * 0 + 2 * 0 = 0$ ➔ Put n=0

$h[1] = -1.2\,h[0] + 2\,\delta[0] = -1.2 * 0 + 2 * 1 = 2$ ➔ put n=1

$h[2] = -1.2\,h[1] + 2\,\delta[1] = -1.2 * 2 + 2 * 0 = -1.2 * (2)$ ➔ put n=2

$h[3] = -1.2\,h[2] + 2\,\delta[2] = -1.2 * -1.2 * (2) + 2 * 0 = (-1.2)^2 * (2)$➔put n=3

$from\ the\ above\ equations$

$\mathrm{h[n]} = (-1.2)^{\mathrm{n}-1} * 2$ and $\mathrm{n} \geq 0$

---

c. $y[n] = 0.24\,\{ x[n] + x[n-1] + x[n-2] + x[n-3] \}$.

Let x[n] = $\delta[n]$ then y[n] will be h[n]

h[n] = 0.24 { $\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$ }.

$$h[n] = f(x) = \begin{cases} 0.24, & 0 \leq n \leq 3 \\ o, & Otherwise \end{cases}$$

---

d. $y[n] = x[n] + 0.5x[n-1] + x[n-2]$

Let x[n] = $\delta[n]$ then y[n] will be h[n]

h[n] = $\delta[n] + 0.5\delta[n-1] + \delta[n-2]$

h[n]=[1 0.5 1], h[n]=0 for all other n.

---

5. Perform the following convolutions, x[n]*v[n]

a. $x[n] = u[n] - u[n-4], v[n] = 0.5^n u[n]$

$$x[n] * v[n] = \sum_{k=-\infty}^{\infty} x[k]\, v[n-k]$$

$$= \sum_{k=-\infty}^{\infty} (u[k] - u[k-4])\, 0.5^{n-k} u[n-k]$$

if $0 \le n \le 4$

$$= \sum_{k=0}^{n} 0.5^{n-k} = 0.5^{n} \frac{1-2^{n+1}}{1-2} = -(0.5^{n} - 2)$$

if $4 < n$

$$= \sum_{k=0}^{4} 0.5^{n-k} = 0.5^{n} \frac{1-2^{5}}{1-2} = -0.5^{n} + 0.5^{n-5}$$

---

b. $x[n] = [1\ 4\ 8\ 2]; v[n] = [0\ 1\ 2\ 3\ 4]$ ($the\ sequences\ starts\ at\ n = 0$).

$x[n]$	1	4	8	2	0	0	0	0	0
$h[n]$	0	1	2	3	4	0	0	0	0
	0	1	2	3	4	0	0	0	0
		0	4	8	12	16	0	0	0
			0	8	16	24	32	0	0
				0	2	4	6	8	0
$h[n]$	0	1	6	19	34	44	38	8	0

---

c. $x[n] = u[n] \quad v[n] = 2 * (0.8)^{n} u[n]$

$$x[n] * v[n] = \sum_{k=-\infty}^{\infty} u[k]\, 2\, .8^{n-k} u[n-k]$$

$$= \sum_{k=0}^{n} 2\, (.8)^{n-k}$$

$$= 2\, (.8)^{n} \sum_{k=0}^{n} .8^{-k} = 2\, (.8)^{n} \frac{1-1.25^{n+1}}{1-1.25}$$

$$= -8\, [0.8^{n} - 1.25], \quad n \ge 0$$

$$= -8\, (.8)^{n} + 10, \quad n \ge 0$$

---

a. $x[n] = u[n-1] \quad v[n] = 2 * (0.5)^n u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} u[k-1]\, 2\, (.5)^{n-k} u[n-k]$$

$$= \sum_{k=1}^{n} 2\, (.5)^{n-k} \quad , \quad n \geq 1$$

$$= 2\, (.5)^n \sum_{k=1}^{n} 2^k$$

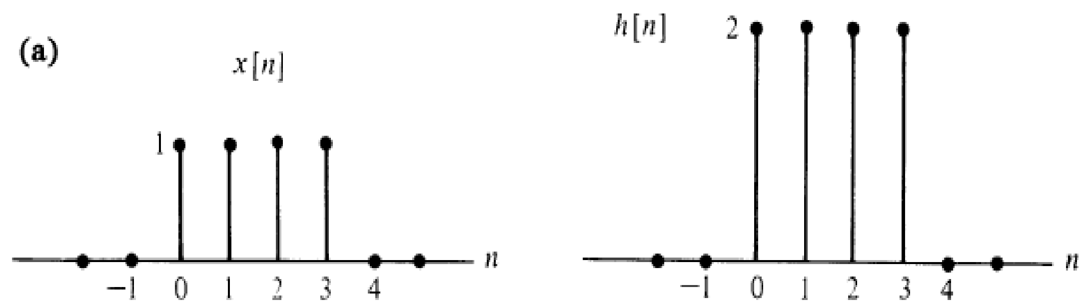
$$= 2\, (.5)^n \left(\sum_{k=0}^{n} 2^k - 1 \right)$$

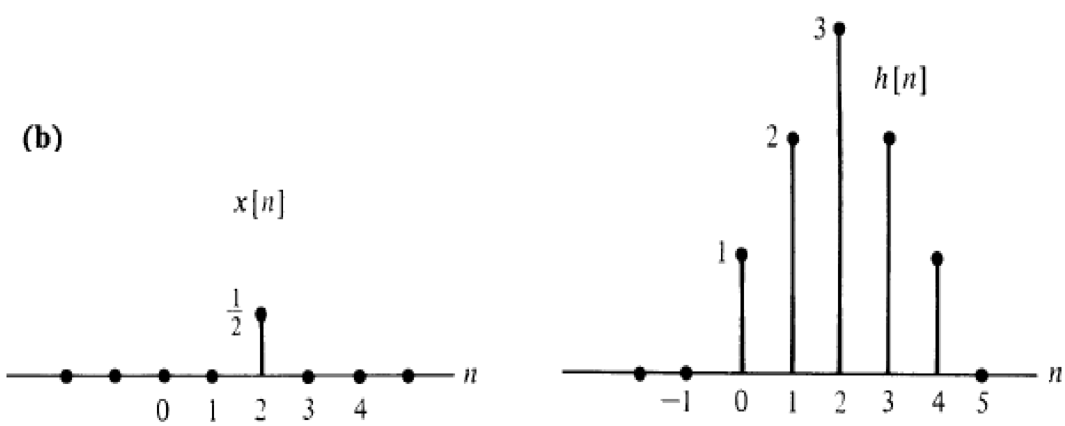
$$= 2\, (.5)^n \left[\frac{1 - 2^{n+1}}{1-2} - 1 \right]$$

$$= 2\, (.5)^n \left(-2 + 2^{n+1} \right)$$

$$= -(.5)^{n-2} + 4 \quad , \quad n \geq 1$$

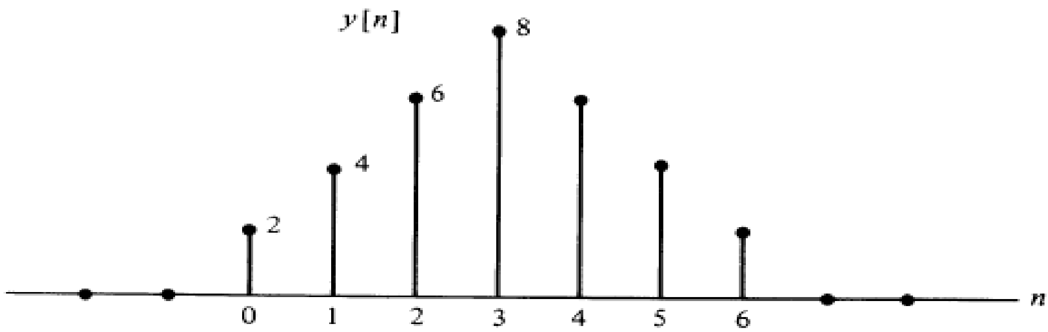
6. Determine the discrete-time convolution of $x[n]$ and $h[n]$ for the following two cases.



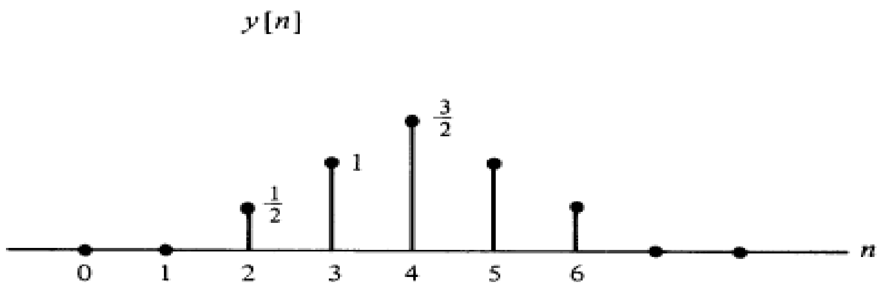


The required convolutions are most easily done graphically by reflecting $x[n]$ about the origin and shifting the reflected signal.

(a) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure



(b) By reflecting $x[n]$ about the origin, shifting, multiplying, and adding, we see that $y[n] = x[n] * h[n]$ is as shown in Figure



Notice that $y[n]$ is a shifted and scaled version of $h[n]$.

Application: The distributive property sometimes facilitates the evaluation of a convolution integral.

Example : Suppose we want to calculate the output of an LTI system with impulse response $h[n]=\left(\frac{1}{4}\right)^n u[n]+4^n u[-n]$ to the input signal $x[n]=u[n]$; it is easier to break down $h[n]$ as a sum of its two components, $h_1[n]=\left(\frac{1}{4}\right)^n u[n]$ and $h_2[n]=4^n u[-n]$, then calculate the two convolutions $y_1[n]=x[n]*h_1[n]$, $y_2[n]=x[n]*h_2[n]$ and sum them to obtain $y[n]$.

Exercise 1

Compute the convolutions $y[n] = x[n] * h[n]$:

(a) $x[n] = \alpha^n u[n],\ h[n] = \beta^n u[n],\quad \alpha \neq \beta$. Sketch the output signal $y[n]$ for the case $\alpha = 0.8,\ \beta = 0.9$.

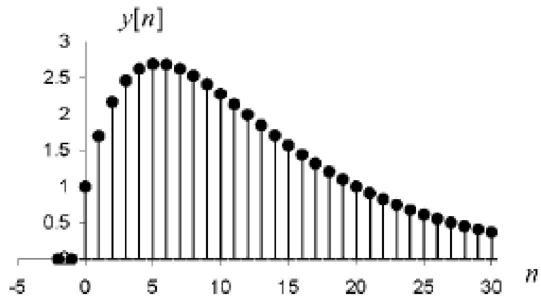
Answer: $$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=-\infty}^{+\infty} \alpha^k u[k]\beta^{n-k}u[n-k] = \beta^n \sum_{k=0}^{n}\left(\frac{\alpha}{\beta}\right)^k, \quad n \geq 0$$

$$= \beta^n \left(\frac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\left(\frac{\alpha}{\beta}\right)} \right) u[n] \quad = \left(\frac{\beta^{n+1} - \alpha^{n+1}}{\beta - \alpha} \right) u[n], \quad \alpha \neq \beta$$

For $\alpha = 0.8,\ \beta = 0.9$, we obtain

$$y[n] = \left(\frac{(0.9)^{n+1} - (0.8)^{n+1}}{0.1} \right) u[n] = \left[9(0.9)^n - 8(0.8)^n \right] u[n]$$

which is plotted in Figure

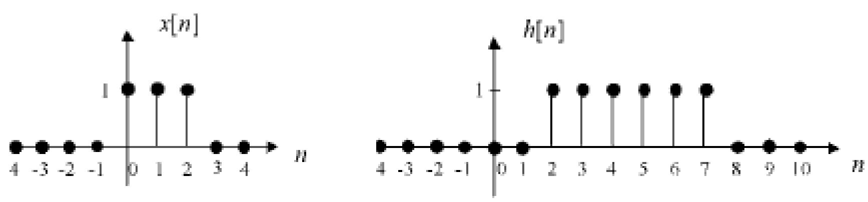


Output of discrete-time LTI system obtained by convolution in Exercise 1(a).

(b) $x[n] = \delta[n] - \delta[n-2],\ h[n] = u[n]$

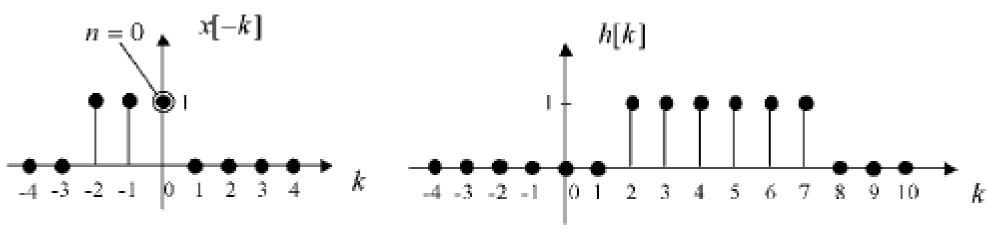
Answer: $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=-\infty}^{+\infty} \left(\delta[k] - \delta[k-2]\right) u[n-k] = u[n] - u[n-2].$

(c) The input signal and impulse response depicted in Figure Sketch the output signal $y[n]$.



Input signal and impulse response in Problem 2 (c).

Answer: Let us compute this one by time-reversing and shifting $x[k]$ (note that time-reversing and shifting $h[k]$ would lead to the same answer)



Time-reversing and shifting the input signal to compute the convolution in Exercise 2. (c).

Intervals:

$n < 2$ $\quad h[k]x[k-n] = 0, \forall k \quad y[n] = 0$

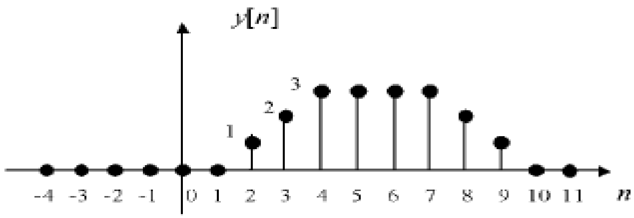
$2 \le n \le 4:$ $\quad h[k]x[k-n] = 1, 2 \le k \le n \quad y[n] = \sum_{k=2}^{n} 1 = n-(2)+1 = n-1$

$5 \le n \le 7:$ $\quad h[k]x[k-n] = 1, n-2 \le k \le n \quad y[n] = \sum_{k=n-2}^{n} 1 = 3$

$8 \le n \le 9:$ $\quad h[k]x[k-n] = 1, n-2 \le k \le 7 \quad y[n] = \sum_{k=n-2}^{7} 1 = 7-(n-2)+1 = 10-n$

$n \ge 10$ $\quad h[k]x[k-n] = 0, \forall k \quad y[n] = 0$

the Figure shows a plot of the output signal:



Output of discrete-time LTI system obtained by convolution in Exercise 2. (c).

(d) $x[n] = u[n],\ h[n] = u[n]$

Answer: $y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] = \sum_{k=-\infty}^{+\infty} u[k]u[n-k]$

$$= \sum_{k=-\infty}^{+\infty} u[k]u[-(k-n)] = \begin{cases} \sum_{k=0}^{n} 1 = n+1, & n \ge 0 \\ 0, & n < 0 \end{cases}$$

$= (n+1)u[n]$

Evaluate $y[n] = x[n] * h[n]$, where $x[n]$ and $h[n]$ are shown in Fig. 2-23, (a) by an analytical technique, and (b) by a graphical method.

Fig. 2-23

(a) Note that $x[n]$ and $h[n]$ can be expressed as

$$x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$
$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

Now, using Eqs. (2.38), (2.130), and (2.131), we have

$$\begin{aligned} x[n] * h[n] &= x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2]\} \\ &= x[n] * \delta[n] + x[n] * \delta[n-1] + x[n] * \delta[n-2]\} \\ &= x[n] + x[n-1] + x[n-2] \end{aligned}$$

Thus,
$$\begin{aligned} y[n] = \delta[n] &+ \delta[n-1] + \delta[n-2] + \delta[n-3] \\ &+ \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] \\ &+ \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5] \end{aligned}$$

or
$$y[n] = \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 3\delta[n-3] + 2\delta[n-4] + \delta[n-5]$$

or
$$y[n] = \{1, 2, 3, 3, 2, 1\}$$

(b) Sequences $h[k]$, $x[k]$ and $h[n-k]$, $x[k]h[n-k]$ for different values of n are sketched in Fig. 2-24. From Fig. 2-24 we see that $x[k]$ and $h[n-k]$ do not overlap for $n < 0$ and $n > 5$, and hence $y[n] = 0$ for $n < 0$ and $n > 5$. For $0 \le n \le 5$, $x[k]$ and $h[n-k]$ overlap. Thus, summing $x[k]h[n-k]$ for $0 \le n \le 5$, we obtain

$y[0] = 1$ $y[1] = 2$ $y[2] = 3$ $y[3] = 3$ $y[4] = 2$ $y[5] = 1$

or

$$y[n] = \{1, 2, 3, 3, 2, 1\}$$

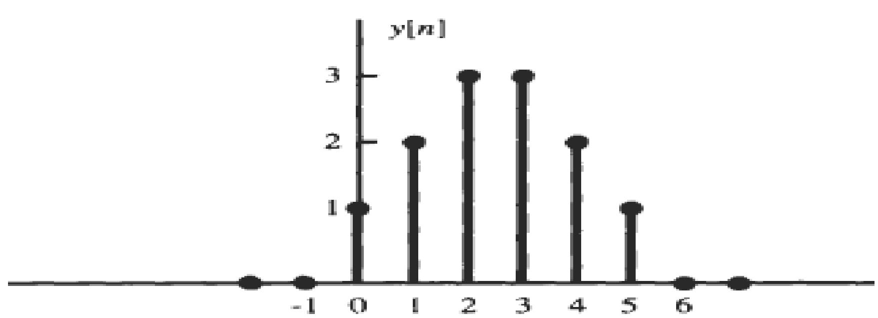


Fig. 2-25

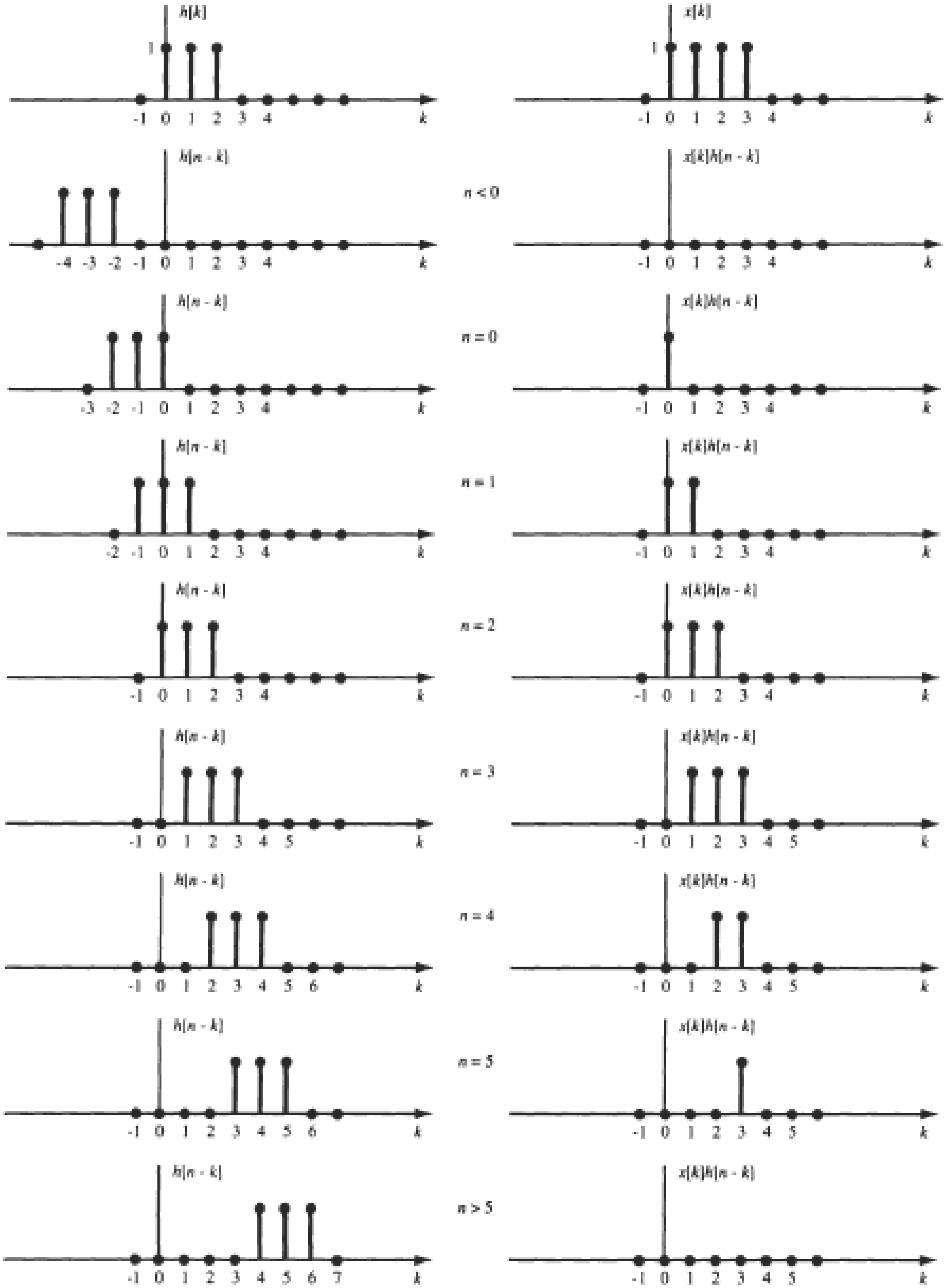


Fig. 2-24

2.34. The impulse response $h[n]$ of a discrete-time LTI system is shown in Fig. 2-26(a). Determine and sketch the output $y[n]$ of this system to the input $x[n]$ shown in Fig. 2-26(b) without using the convolution technique.

From Fig. 2-26(b) we can express $x[n]$ as

$$x[n] = \delta[n-2] - \delta[n-4]$$

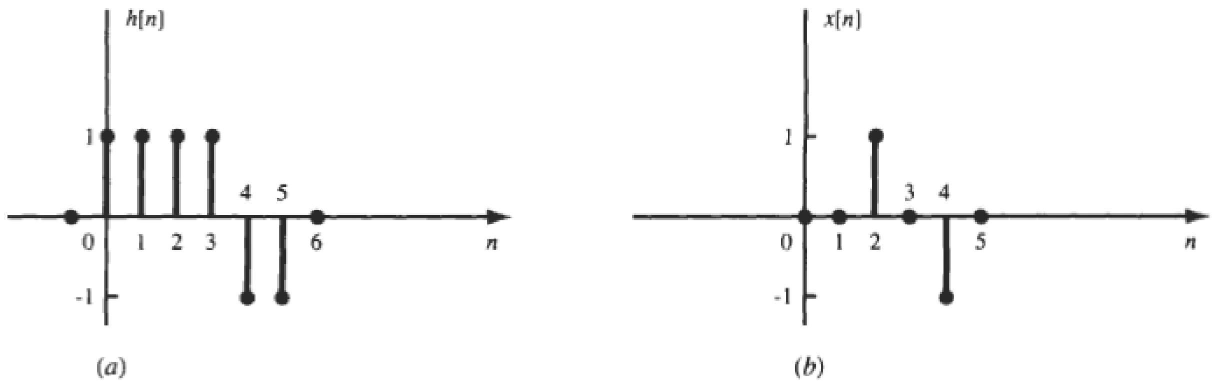


Fig. 2-26

Since the system is linear and time-invariant and by the definition of the impulse response, we see that the output $y[n]$ is given by

$$y[n] = h[n-2] - h[n-4]$$

which is sketched in Fig. 2-27.

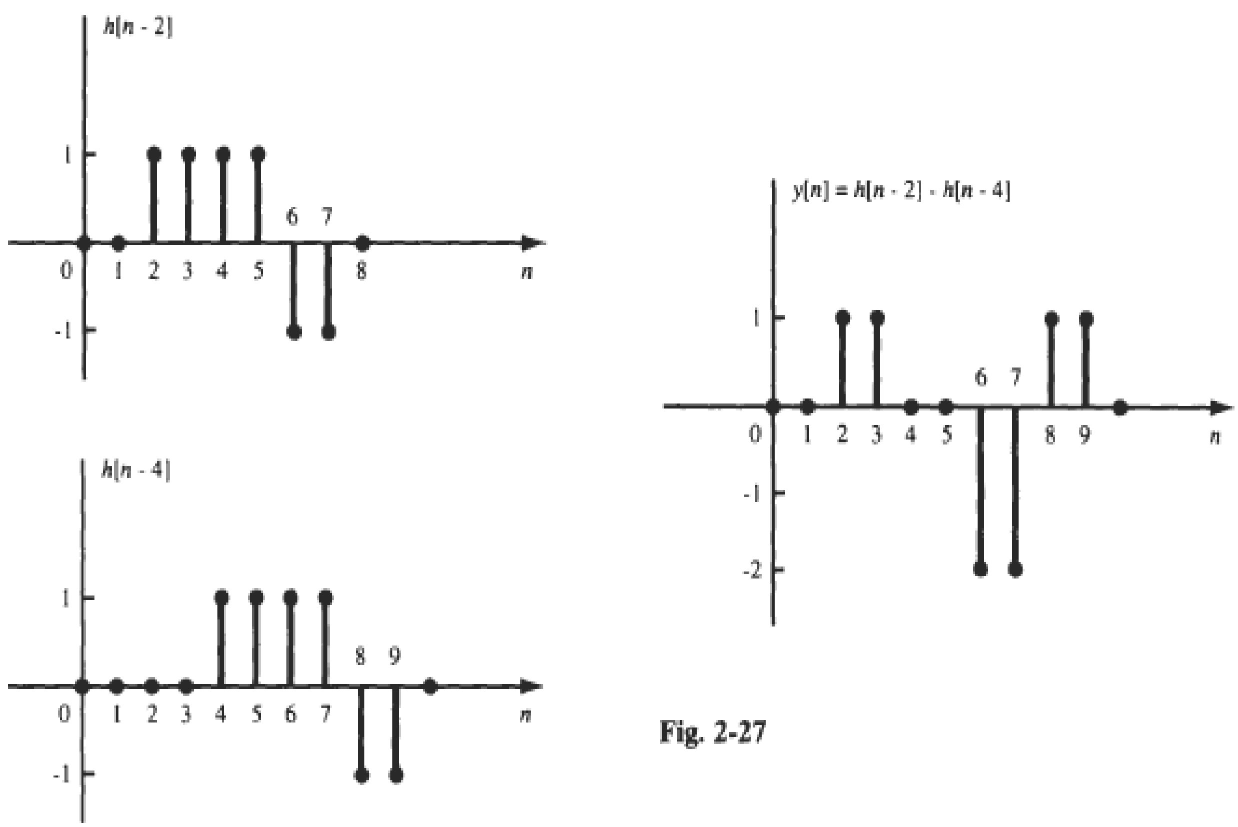


Fig. 2-27