Development of Hubbert's Peak Oil Theory and Analysis of its Continued Validity for U.S. Crude Oil Production

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ABSTRACT

In this paper we review the development of M. King Hubbert's basic petroleum resource depletion model, the Hubbert curve, that became the foundation for the variety of curve fitting techniques that are still widely used today. We then draw on current literature to assess the strengths and weaknesses of such models, before assessing their validity for continued use in the United States and analyzing whether or not U.S. crude oil production still follows the curve that Hubbert gained infamy for. Specifically, we focus on the impact that advances in petroleum production technology such as hydraulic fracturing and Enhanced Oil Recovery (EOR) techniques have had on the U.S. production cycle and test derivations of Hubbert's original model that might better approximate the impacts these advances have had. Our results demonstrate that while curve-fitting techniques remain useful, the original Hubbert curve no longer approximates the U.S. production cycle well.

M. KING HUBBERT & PEAK OIL THEORY (1956)

A Texas native, Marion King Hubbert obtained his B.S. from the University of Chicago in geology and physics with a mathematics minor in 1926 (Narvaez, 1989). He would continue his graduate work in the University's Department of Geology, at the time renowned as one of the nation's best, earning his M.S. in 1928, working on a theoretical study of thermodynamic processes contributing to geologic faults (Narvaez, 1989). Hubbert garnered applied geophysics experience working as an assistant geologist for the Amerada Petroleum Corporation (Narvaez, 1989). After briefly returning to the University of Chicago to pursue his Ph. D. and work as a teaching assistant, Hubbert went on to accept a faculty position at Columbia University in 1931 where he taught geophysics and completed his Ph. D. in 1937. Although colloquially best known for his theorization on peak oil, it was during this period that Hubbert published his seminal paper *Theory of Ground Water Motion*, demonstrating that groundwater flow is determined by both gravity and fluid pressure. The paper provided a physical interpretation of Darcy's Law using a new derivation of the Navier-Stokes equation. Previously thought to remain static in their reservoirs, the publication drastically influenced thinking on how gaseous and liquid hydrocarbons were transported through porous media (Priest, 2014). During World War II, after leaving his position at Columbia in 1941, Hubbert served as a senior analyst for the United States Board of Economic Warfare (Narvaez, 1989). As chief of the Board's Mineral Resources unit, Hubbert oversaw analysis of the Allied war effort's global natural resources, one the most critical of being petroleum (Priest, 2014).

World War I was vital to establishing, advancing and standardizing the science of estimating petroleum reserves and resources. The advent of motorized warfare shifted the view of petroleum as a commodity supplied by a small group of entrepreneurial prospectors to a national resource fundamental to victory in modern war (for perspective; when Britain entered World War I in 1914 their military possessed only 800 motor vehicles, four years later at the war's end, they had 56,000 trucks and 36,000 cars (ELC, 2015). In 1916 the United States Geological Survey (USGS) established their Oil & Gas Section. The Section applied methods such as depletion curves to track production and reserves remaining in known fields, and used their data to aid the U.S. military in planning the war effort and also advise the U.S. government on applying taxes to petroleum production (Priest, 2014). Throughout this time period and through the mid 1940s, the United States accounted for roughly 65% of world petroleum production (ELC, 2015).

Necessarily, the first attempts at quantifying the nation's reserves, resources and production rates arose from these analyses. In 1919 the USGS Oil & Gas Section estimated domestic petroleum reserves to be 6.74 billion barrels (Priest, 2014). Chief geologist and head of the Oil & Gas Section David White estimated the supply would last the country 17-18 years extrapolating from current consumption rates (Priest, 2014). A revised estimate of U.S. petroleum resources, this time including new production from fields in California, was issued in tandem with the American Association of Petroleum Geologists (AAPG) in 1921 (Priest, 2014). Without seismic imaging technology that arose in the mid 1920s, accurate assessment of the nation's oil fields was near impossible, and these early estimates of domestic resources and peak production proved wildly low.

Following his work for the Board of Economic Warfare, Hubbert was employed as a research geophysicist by Shell Petroleum Company from 1943 until 1964, and it was during this time period that he presented his early work on peak oil (Narvaez, 1989). In 1949 Hubbert presented his *Energy from Fossil Fuels* as part of the Symposium on Sources of Energy in Washington D.C., examining annual world production over time for both coal and petroleum. It is in this preliminary paper that we observe several aspects of what would lead to the infamous Hubbert curve and his predictions regarding peak oil. Hubbert's 1949 work focused on the rate of increase of production of both coal and petroleum (neglecting natural gas for want of world production statistics), noting that annual production of coal since 1913 had grown geometrically at a rate of 4% per year since 1913 (implying that annual production doubled every 17 years), while from 1860 to 1929 world crude oil production had grown geometrically at a rate of 9% a year (implying that annual production doubled every 7.5 years)(Hubbert, 1949). Pairing these observations with what he deemed "one of the most disturbing ecological influences of recent millennia" that was "the human species proclivity for the capture of energy, resulting in a progressive increase in human population." (Hubbert, 1949). Hubbert surmised that that the amount of any fossil fuel consumed at any given time would be proportional to the area underneath a curve similar to those which he had presented, concluding:

> Thus we may announce with certainty that the production curve of any given species of fossil fuel will rise, pass through one or several maxima, and then decline asymptotically to zero. Hence, whole there is an infinity of different shapes that such a curve may have, they all have this in common: that the area under each must be equal to or less than the amount initially present. (Hubbert, 1949)

Hubbert would wait another 7 years to speculate on what this curve might look like, although he would continue the paper by ominously discussing hydropower's ability to meet current energy demand and human existence on an absolute time scale.

In 1956, M. King Hubbert presented *Nuclear Energy and the Fossil Fuels* before the Spring Meeting of the Southern District Division of Production of the American Petroleum Institute (API). Using petroleum production data from the world, the continental United States (U.S. L48) and Texas, Hubbert created graphs for each in the same manner as he had done in his 1949 publication, plotting rate of production over time. While initial rates of production initially increased quite rapidly, as had observed for both coal and crude oil in 1949, he noted that this production growth was quite obviously unsustainable in that physical limits prevented production of a finite resource from behaving as such so over any prolonged period (Hubbert, 1956). To more effectively extrapolate growth curves for his observed data sets, Hubbert concluded that petroleum depletion could be modeled as function of cumulative production under the premises that:

> (1) For any production curve of a finite resource of fixed amount, two pointa on the curve are known at the outset, namely that at t = 0 and again at t = ∞. The production rate will be zero when the reference time is zero, and the rate will again be zero when the resource is exhausted.
>
> (2) The second consideration arises from the fundamental theorem of the integral calculus; namely, if there exists a single valued function y = f(x), then
>
> $$\int_0^{x_1} y dx = A$$
>
> where A is the area between the curve y = f(x) and the x-axis from the origin out to the distance is x_1.
>
> In the case of the production curve plotted against the time on an arithmetical scale, we have as the ordinate
>
> $$P = dQ/dt$$
>
> where dQ is the quantity of the resource produced in time dt. Likewise, from equation (1) the area under the curve up to any time t is given by
>
> $$A = \int_0^t P dt = \int_0^t \left(\frac{dQ}{dt}\right) dt = Q$$

where Q is the cumulative production up to the time t. Likewise, the ultimate production will be given by

$$Q_{max} = \int_0^{\infty} P dt$$

and will be represented on the graph of production-versus-time as the total area beneath the curve. (Hubbert, 1956)

The graphical representation of these basic relationships manifested itself in the form of a bell-shaped curve that Hubbert did not define mathematically in his 1956 publication. It is speculated that he drew the curve by hand, calculating the area beneath it to arrive at his original ultimate reserve estimates (Fig. 1).

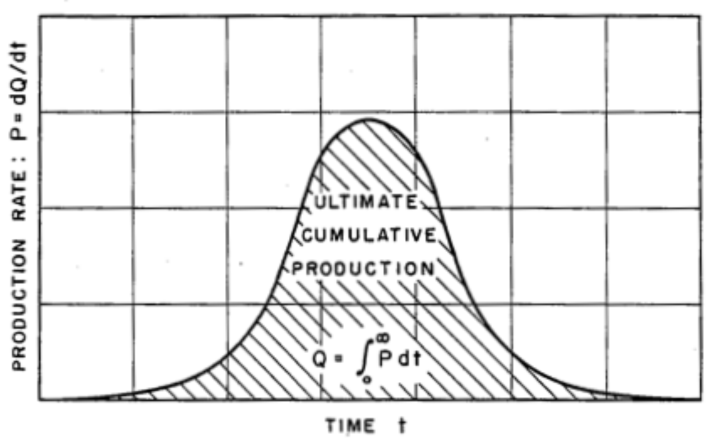


Fig. 1 Hubbert's proposed mathematical relationship encompassing the complete production cycle of a finite resource (Hubbert, 1956)

Hubbert's curve approach was beautifully simple in that it required only one known variable, Q_{max}, to fit his curve to a data set and extrapolate rates of production over time. The variable Q_{max}, representing ultimate production, is synonymous with Ultimately Recoverable Resources (URR), the approximated cumulative quantity of hydrocarbon that can be economically extracted from a reservoir over its producing lifespan. In his 1956 calculations, Hubbert used a URR of 1250 billion barrels to fit his world production curve (Fig. 2). Hubbert fit domestic production curves for both a conservative estimate of 150 billion barrel

EUR as well as a slightly more optimistic 200 billion barrel EUR (Fig. 3), using an EUR of 60 billion barrels for Texas (Fig. 4).

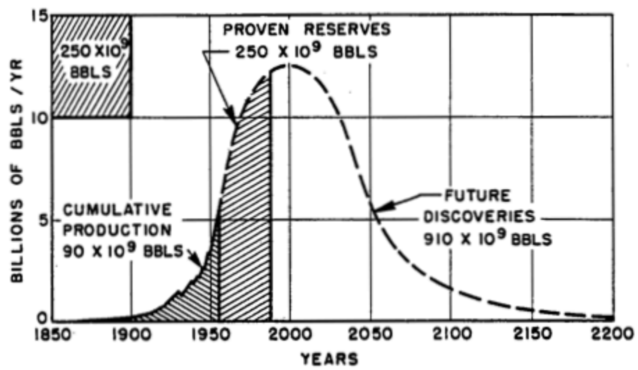


Fig. 2 Hubbert's projection for ultimate world crude oil production, assuming a 1250 billion barrel EUR (Hubbert, 1956).

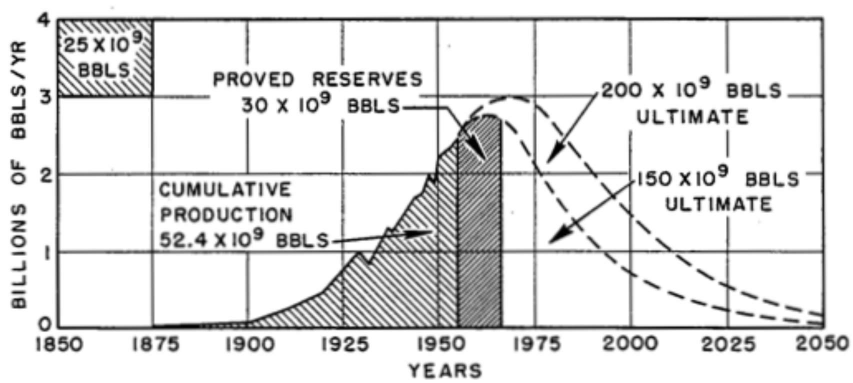


Fig. 3 Hubbert's projection for ultimate crude oil production from the Lower 48 United States, the bottom curve assuming a 150 billion barrel URR and the upper curve assuming a 200 billion barrel EUR (Hubbert, 1956).

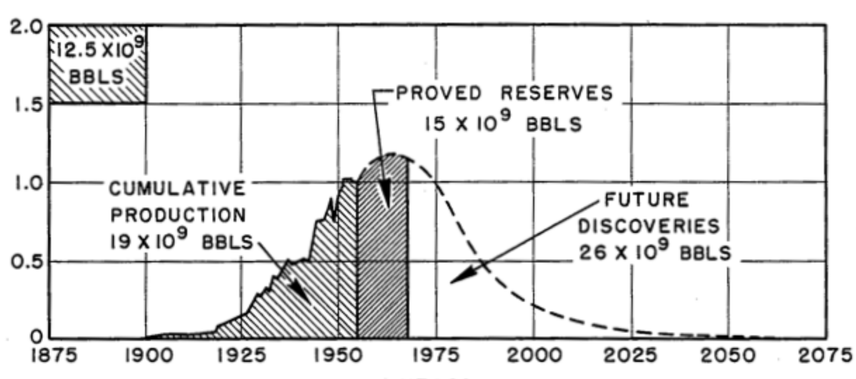


Fig. 4 Hubbert's projection for ultimate crude oil production from the state of Texas, assuming a 60 billion barrel EUR (Hubbert, 1956).

Hubbert’s concept of a moment of peak production implicit to the single curve he had applied to his data sets was not one that had been considered in previous petroleum resource assessments (Priest, 2014). This would develop into what became known as the theory of peak oil, which by nature of Hubbert’s bell-shaped symmetric model occurred when exactly half of the ultimate reserve had been produced. His 1956 paper calculated that world peak production of crude oil would occur in the year 2000, with a maximum rate of production of roughly 12.5 billion barrels annually (Hubbert, 1956). For U.S. Lower 48 EURs of 150 billion barrels and 200 billion barrels, Hubbert predicted peaks in 1965 and 1970, respectively, with maximum peak production rates of roughly 2.7 billion barrels annually and 3 billion barrels annually (Hubbert, 1956). Hubbert was intrigued by the fact that even when domestic EUR was increased by 33% from 150 billion barrels to 200 billion barrels delayed the occurrence of peak oil by only 5 years (Hubbert, 1956). Noting that the versatility and ease of extraction of liquid and gaseous hydrocarbon products had resulted in a rate of production disparate to their magnitude, Hubbert rather sensationally predicted that should the world continue to use fossil fuels as their primary energy source, that “On the basis of the present estimates of the ultimate reserves of petroleum and natural gas, it appears that the culmination of world production of these products should occur within about half a century, while the culmination for petroleum and natural gas in both the United States and the state of Texas should occur within the next few decades.” (Hubbert, 1956).

Hubbert considered peak production to be a critical event in the lifecycle of an exhaustible resource and believed that this point would have significant implications for the manners in which they would be produced and consumed, exacerbated by the increasing demand for energy would come with a growing population (Priest, 2014). He would touch briefly on the consequences a large energy imbalance held for both domestic industry and national defense purposes before going on to assess alternative energy options as he had done in his 1949 paper, this time focusing on energy from nuclear sources (Hubbert, 1956). As Tyler

Priest comments in his 2014 paper *Hubbert's Peak: the Great Debate Over the End of Oil*, "What was so shocking about Hubbert's projections was that it offered a unique and intuitive interpretation of widely published data that overturned conventional wisdom. That wisdom held that petroleum resources were plentiful, not poised for decline." (Priest, 2014) Hubbert was well a well-respected petroleum geologist and geophysicist, his work held significant weight in the oil industry and the gravity even his preliminary work held independently established peak oil theory as a well known, if contentious, topic that would be the subject of debate in the field of petroleum resource assessment and beyond for decades to come.

HUBBERT'S CONTINUED WORK

Following his initial 1956 work, Hubbert would continue to refine his methods of analysis. Most importantly, in his *Techniques of Prediction With Application to the Petroleum Industry* presented at the 44th Annual Meeting of the American Association of Petroleum Geologists (AAPG), he assigned mathematical function to the bell curve he had fit to historical data in his earlier work. Hubbert asserted that cumulative production over time could be best approximated by the logistic growth function, with the first derivative of this function, representing annual rate of production, taking the same form as the curve he had drawn in 1956 (Hubbert, 1959).

In the same paper, Hubbert proposed that magnitude of ultimate reserves could be reached independent of industry or academic expert speculation by examining the relationship between cumulative discoveries, production and proved reserves. At the time of his 1956 publication, Hubbert had declined to make his own ultimate reserve calculations, instead using previously calculated industry estimates to make his preliminary assertions. In the 1950s, best practice in estimating ultimate reserves (used by both the petroleum industry and the USGS) was a method known as volumetric yield analysis (Priest, 2014). Volumetric yield analysis entailed averaging oil yield produced from a unit volume of sediment in producing basins and assuming that as of yet undrilled, geologically similar basins would produce the same amount per unit volume of sediment to extrapolate ultimate reserves. This form of analysis both assumed comparable yield per unit volume of sediment without scientific justification for doing so, and hinged on subjective measures of geologic similarity (Priest, 2014). Hubbert realized that his model's dependence on an inherently unreliable variable was a significant flaw, and devised a mathematical means by which to ascertain what he viewed as a more reliable estimate indirectly.

Under the premise that cumulative discoveries (Q_D) for any time t is the sum of oil that has already been produced (Q_P) and oil that has been discovered but not yet produced, (i.e. reserves, represented by Q_R), or:

$$Q_D = Q_P + Q_R$$

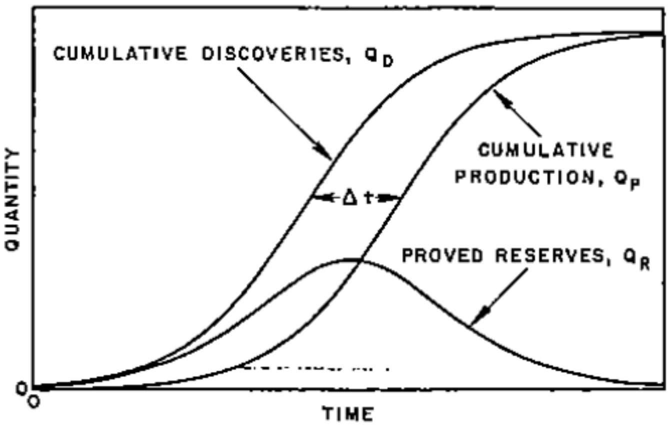
Hubbert also ascertained that the curve of cumulative production and the curve of cumulative discovery would be of very nearly identical shape, and have the same asymptote, with cumulative production trailing cumulative discovery with a lag in time that he represented with Δt (7). With the derivative of this equation with respect to time (t):

$$\frac{dQ_D}{dt} = \frac{dQ_P}{dt} + \frac{dQ_R}{dt}$$

Noting that when proved reserves reach their maximum, their rate of increase would be equal to zero, setting the rate of discovery equal to the rate of production at this time and implying that the ascending rate of production curve would intersect the rate of discovery curve which would have begun to decrease:

$$for\ \frac{dQ_R}{dt} = 0, \qquad \frac{dQ_D}{dt} = \frac{dQ_P}{dt}$$

With these mathematical foundations in place (graphical interpretations below in Fig. 5, 6), Hubbert had established what would come to be known as the basic tools of modern Hubbert analysis (Hubbert, 1959, Brandt, 2006).



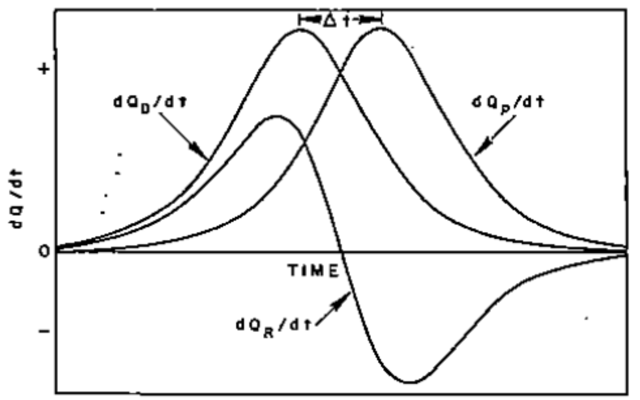


Fig. 5 (LEFT) Hubbert's theoretical single-cycle growth curves for cumulative discoveries, production, and proved reserves (Hubbert, 1959).
Fig. 6 (RIGHT) Hubbert's theoretical rate of discovery and production, rate of proved reserve increase (Hubbert, 1959).

Hubbert would go on to apply the methods described in his 1959 publication to substantiate his claims in a report submitted to the Committee of Natural Resources in 1962 while serving as chair of the subcommittee on energy. Using crude oil production data from 1860 onwards and estimates of U.S. Lower 48 proved crude oil reserves since from 1937 onward published annually by the API, Hubbert calculated a time lag between U.S. cumulative production and discovery of 10-11 years (best fit for Δt was 10.5), asserting that discovery rate had reached its peak in 1956 while projecting that peak production rate would peak between 1966 and 1967 with ultimate cumulative production of 175 billion barrels (Hubbert, 1962). Considering a contingency allowance of an additional 50 billion barrels, bringing the total to 225 billion barrels, Hubbert claimed this addition would only push peak production to the early 1970s. Hubbert's contentious predictions for peak crude oil production of the U.S. Lower 48 were validated as production rates peaked in 1970 and declined every year subsequently. However, while Hubbert had successfully predicted the timing of U.S. peak oil, his 1962 curve indicating a peak production rate of roughly 2.7 billion barrels annually fell materially short of the actual 1970 production rate of 3.5 billion barrels per year (Hubbert, 1962, EIA).

ESTABLISHING THE PARAMETERS OF CURVE FITTING TECHNIQUES AND THE UNCERTAINTY THEY INTRODUCE

While it is beyond the scope of this paper to rigorously mathematically justify the use of curve fitting techniques versus other models that have been developed for the analysis of petroleum resource production and consumption, it is important to both acknowledge their limitations as well as establish sound parameters for when their application is both valid and useful.

Ultimately Recoverable Resource (URR)

Ultimately Recoverable Resource (URR) is defined by Sorrell et al. (2010) as the sum of cumulative discoveries, future reserve growth at known fields and the volume of oil estimated to be economically recoverable from fields that have not yet been discovered (Sorrell et al., 2010). Inherently, Hubbert's model relies on accurate estimation of URR. Because ultimate production (area under the rate of production curve) delineates projected production over time, it has a larger effect on predictions the model makes than any other variable (Brandt, 2006). URR estimates can be either exogenous or endogenous.

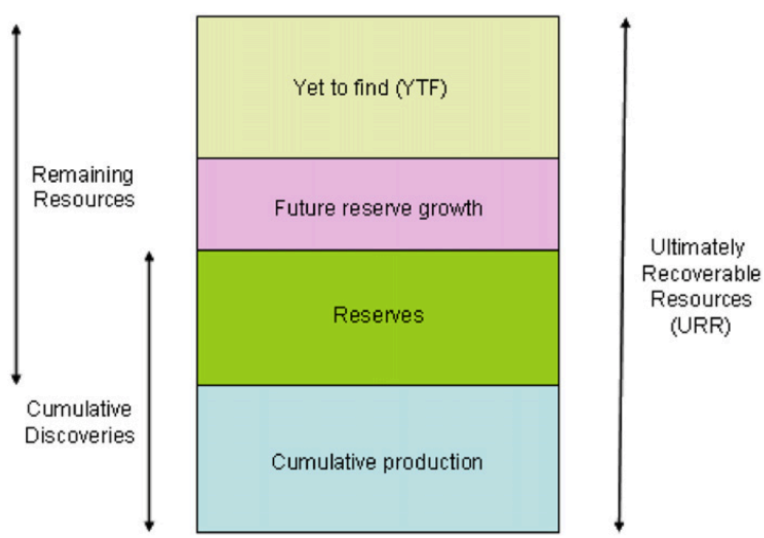


Fig. 7 Breakdown of Ultimately Recoverable Resources (Sorrell et al., 2010)

Endogenous Estimates of URR

As mentioned earlier, Hubbert devised his own method of calculating URR so that his models could be independent of unreliable (and in some cases politically motivated) estimates. While at the time this was a reasonable and necessary assumption to make, as methods of geologic estimation have become more certain with improving technology endogenous estimates are increasingly less relied upon. Hubbert's method, which extrapolates peak production rate and time of peak production from the inflection point of the cumulative production curve has been shown to be unreliable. As Laherrere (2000) demonstrates by comparing Gaussian and a Hubbert curves, the Gaussian inflection point (which occurs earlier compared to that of the Hubbert curve, resulted in a peak rate 10% higher and a URR 33% larger than that of a Hubbert curve modeling the same data set (Laherrere, 2000). Utilization of the inflection point in calculating URR is also problematic in that it cannot be applied to a cumulative production data set that has yet to reach an inflection point, limiting the predictive power of these models early in the production cycle.

Exogenous Estimates of URR and Reserve Additions

Exogenous URR estimates are more commonly used in studies employing curve-fitting techniques and act as a limiting external variable independent of the actual curve fitting process (Wang, Feng, 2016). These estimates are generally sourced from national or global geologic authorities such as the USGS or EIA. These estimates present their own aspects of uncertainty to curve-fitting techniques in that historically they have underestimated future additions due to new discoveries and reserve growth (Brandt, 2009). Reserve growth refers to the tendency of estimates for individual fields to increase over time without the addition of new discoveries, and contributes to the majority of reserve additions in most regions of the world, and reserve growth in the Unites States contributed to 89% of proved reserve additions between 1978 and 1990 (Sorrell et al., 2012). This effect is attributable to a number of different factors, including improving geologic understanding of an area, improving technology and definitional factors (Sorrell et al., 2012). While technological improvements in seismic imaging technology can contribute to reserve growth in older fields where they were not used in original delineation of the play, in the future they will likely mitigate reserve growth attributable to changing geologic understanding of a play as new discoveries will be better understood in the first place.

Improving technology has played a roll in reserve increases due to both other factors as it has improved the accuracy of seismic survey techniques as well as cheapened the extraction process, influencing the definition of what can be referred to as economically extractable. URR estimates are also prone to increase over time as technology improves methods of finding and extracting hydrocarbon, and fluctuate as oil price dictates plays that are economically extractable. While this is potentially problematic, factors that have only long term impacts such as improvements in technology will act more significantly to slow rate of decline after peak production is reached and will have negligible impacts on timing of the peak (Sorrell et al., 2010).

Reserve growth functions have been employed as a means of predicting further reserve growth and rely on the use of measured growth of sample fields to make their forecasts. Typical U.S. 1P data sets exhibit rapid growth immediately after discovery with growth slowing over time, and old fields still record growth after 80 year time periods (Sorrell et al., 2012). Like Hubbert's model, these functions neglect economic variables, but the reason for their growth varies over the production cycle of the play: growth recently after first production is heavily influenced by additions to Original Oil In Place (OOIP), with later additions more commonly attributed to changing recovery factors (Sorrell et al., 2012). Unfortunately, reserve growth functions have been found to be unreliable when projecting potential of a wide range of areas (McGlade, 2012). Calculated rates of reserve growth remain contentious, but it is clear that this effect introduces significant uncertainty to exogenous estimates utilized by curve fitting techniques.

Assumptions of the Hubbert Model and their relevancy to U.S. L48

In theory, the United States present a nearly perfect sample with which to test the continued viability of Hubbert's proposed model. As Sorrell & Speirs (2010) note, extrapolation techniques are most justifiably applicable to geologically homogenous areas where exploration has continued relatively unimpeded over time: the U.S. L48 fits this criteria well. Additionally, the U.S. has well documented petroleum production and discovery data sets through 1859 that are publicly available through the U.S. Energy Information Administration. We maintain that curve fitting techniques' utilization of publicly available aggregate data, often the most granular available, remains a huge benefit to their continued use**.** Until very recently, Hubbert's model fit U.S. data incredibly well. While the United States discovery curve imitates the symmetric nature that Hubbert describes best of any major petroleum producing country, Sorrell & Speirs (2010) attribute this fit to the limited early drilling capabilities and exploration and first production occurring in relatively minor resource plays, with major basins being discovered slightly later. They also note that while Hubbbert's assumption that the resource discovery cycle trails production cycle symmetrically is not mathematically well supported, U.S. data set mimics this behavior well (Sorrell & Speirs, 2010).

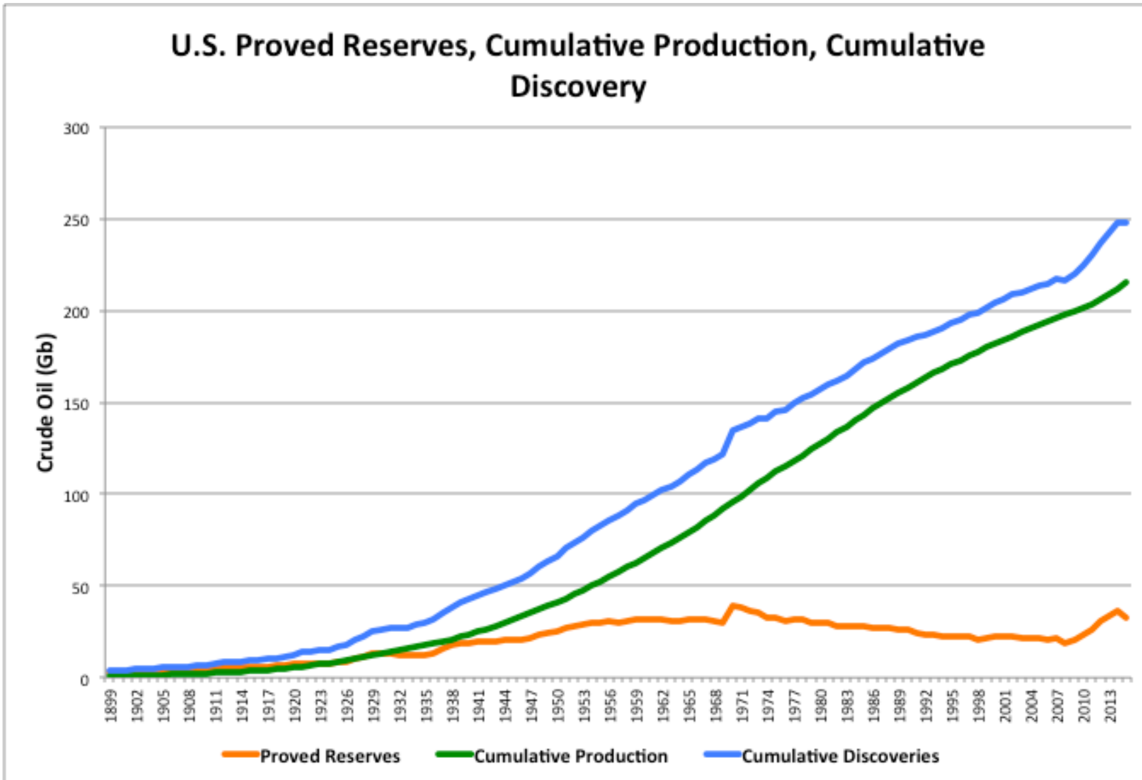


Fig. 8 U.S. proved reserves, cumulative discovery and production mimics Hubbert's predictions well (EIA, 2017).

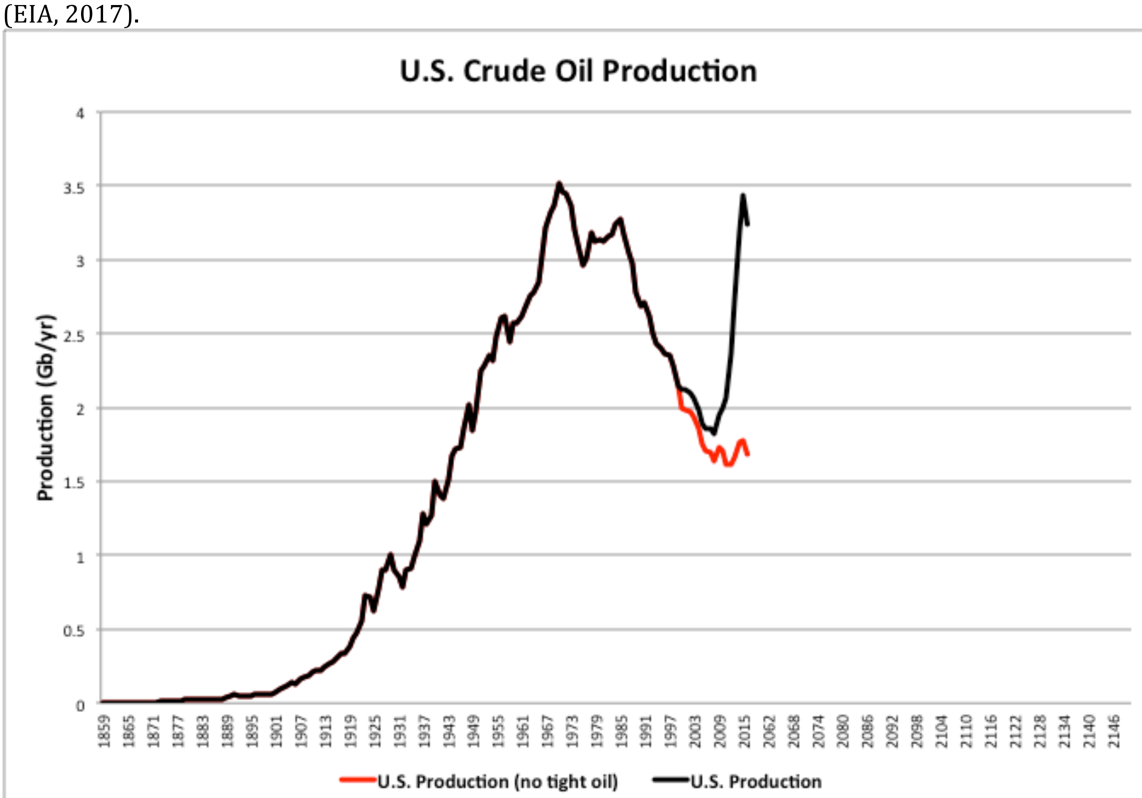


Fig. 9 U.S. crude oil production over time. Production without tight oil resembles Hubbert's curve well, but exploitation of tight oil resources has resulted in a significant deviation from this model.

MODERN HUBBERT ANALYSIS

A wide variety of techniques M. King Hubbert directly or indirectly contributed to the development of are still applied today in various forms, with the general framework he created for resource assessment broadly referred to as Hubbert analysis. These methods are often characterized by their production of a single-value estimate obtained by extrapolating one or multiple curves fit to historic production or discovery data from country-level regions where only aggregate data is available (Sorrell & Speirs, 2010). We first generate our own single Hubbert Curve estimate before assessing two other classes of models that have evolved from it. Here we examine two aspects of Hubbert's model: 1.) Increase and decrease in production rates approaching and following peak production are roughly symmetrical and well approximated using a bell shaped curve. 2.) the monocyclic nature of resource production.

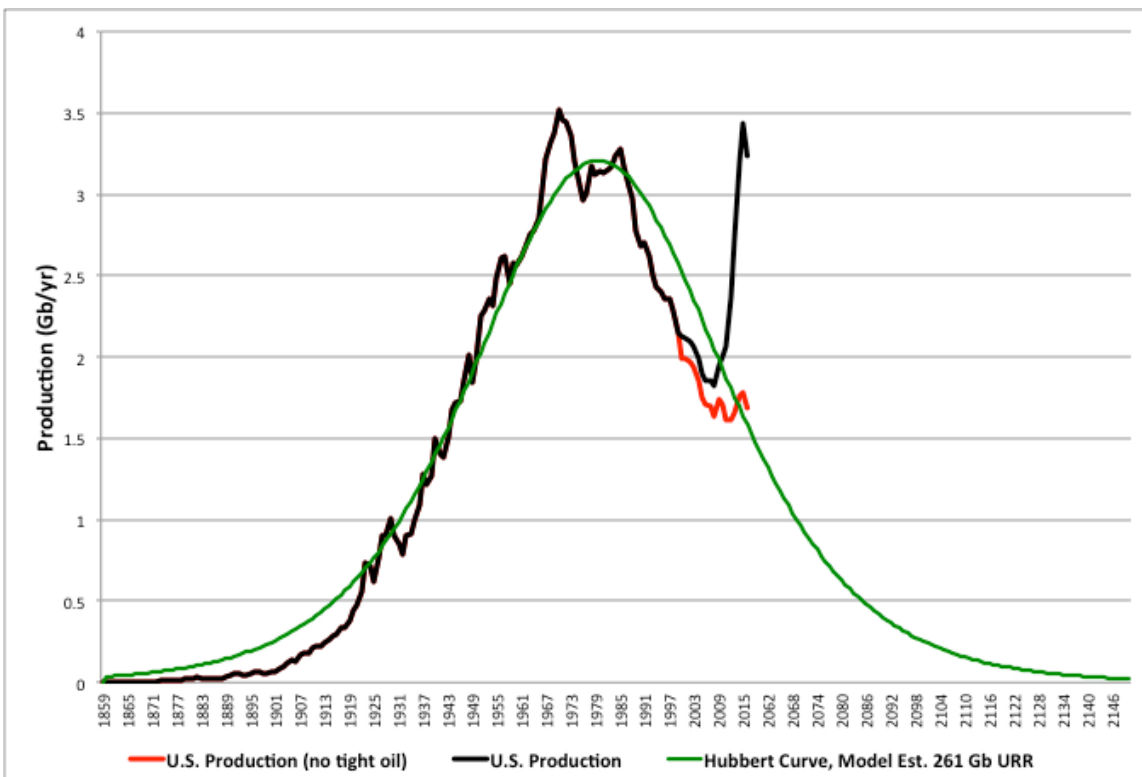


Fig. 10 U.S. production overlain with a 261Gb Hubbert curve, the URR estimate generated by our model.

Asymmetric Curves

Hubbert's assumption that production rate over the lifecycle of a resource is symmetric has been criticized for neglecting the interaction of geologic, technical, economic and social parameters and it has been posited that it is unreasonable to expect that this will scenario will be applicable to a wide range of production cycles (Wang et al., 2016). Asymmetry can be explained by the fact that operators are motivated to increase production while minimizing decline rates to maximize post-peak profit via methods such as Enhanced Oil Recovery (EOR) (Wang et al., 2016). EOR techniques are often applied in plays with heavy oil that exhibit poor permeability and include chemical flooding, gas injection, and thermal recovery methods that alter the makeup of the reservoir. While expensive to apply, they are increasingly popular and have the potential to more than double current average domestic oil recovery of 30% (DOE, 2017). Increasing application of EOR could sustain post-peak production of currently produced reserves and dampen rate of decrease.

Reviewing production data from 139 regions, Brandt (2006) found that production was significantly asymmetric in one direction: median rate of increase was 7.8% whole median rate of decline was found to be 2.6% (Brandt, 2006). However, Brandt (2006) also notes that these asymmetric models are especially difficult to fit to past production data when a peak in production is not yet evident: Hubbert's model ascertains that decline rate is most simply approximated when assumed to be equal to that of the rate of increase if there is no information to warrant the use of a more complex approach (Brandt, 2006). Here we test an asymmetric model based on a Gaussian curve, described by:

$$P(t) = P_{max} e^{-(t-T_{peak})^2/2(f(t)^2}$$

where *f(t)* is the sigmoid function that changes the standard deviation in the vicinity of t=T_{peak} described by:

$$f(t) = \sigma_{dec} - \frac{\sigma_{dec} - \sigma_{inc}}{1 + e^{k(t-T_{peak})}}$$

where P(t) is production in year t, P_{max} is maximum production, T_{peak} is the year of peak production, σ_{inc} is the standard deviation of the pre-peak production curve and σ_{dec} is the standard deviation of the post-peak production curve (Brandt, 2006).

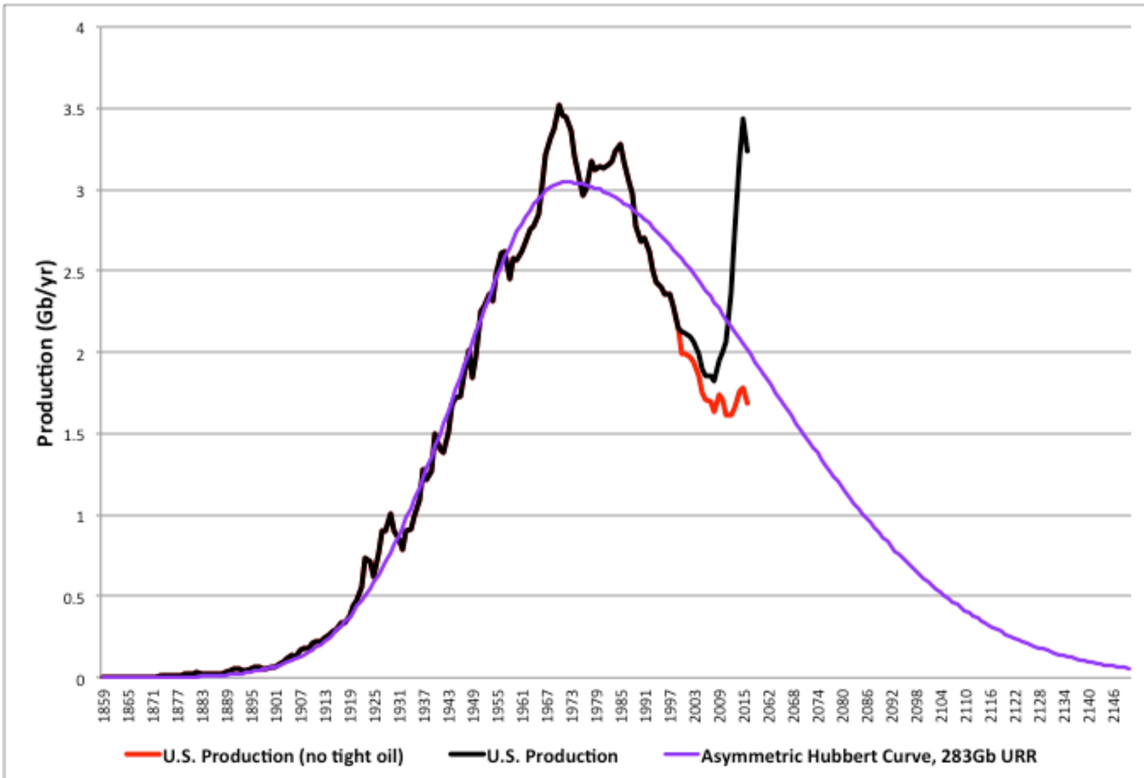


Fig. 11 Asymmetric Gaussian curve fit to U.S. L48 production data.

Multi-Cyclic Models

Curve fitting techniques, especially monocyclic models such as the Hubbert's, rely on the assumption that future exploration cycles will not occur, or if they do they will have a negligible impact on aggregate resource production due to their relative size (Sorrell & Speirs, 2010). This is primarily an issue in countries where political instability or geographic inaccessibility has limited extent or continuity of exploration. For well-delineated regions where the exploration cycle is relatively well advanced (like the United States), this is assumption appeared valid. However, hydraulic fracturing has drastically impacted the U.S. L48 production cycle. Here we argue that the advent of hydraulic fracturing in the United States necessitates the use of a multi-cyclic model for production data. Since the EIA began tracking tight oil production in 2000, it has accounted for 25% of all crude oil produced in the U.S. L48 between 2000 and 2016, and accounted for 51% of crude oil produced in the U.S. L48 in 2016 (EIA, 2017). Examined as an entity separate from that of the purely conventional resources Hubbert applied his analysis to, cumulative tight oil production corresponds with the postulated logistic growth function extremely well (URR 48Gb assumed) (EIA, 2013) (Fig. 12). Tight oil in the United States is primarily sourced from the Bakken shale formation of North Dakota and the Eagle Ford shale formation of southern Texas, with Bakken production responsible for an increase of 2.2 million barrels of oil per day in 2013 (Murray & Henson, 2013).

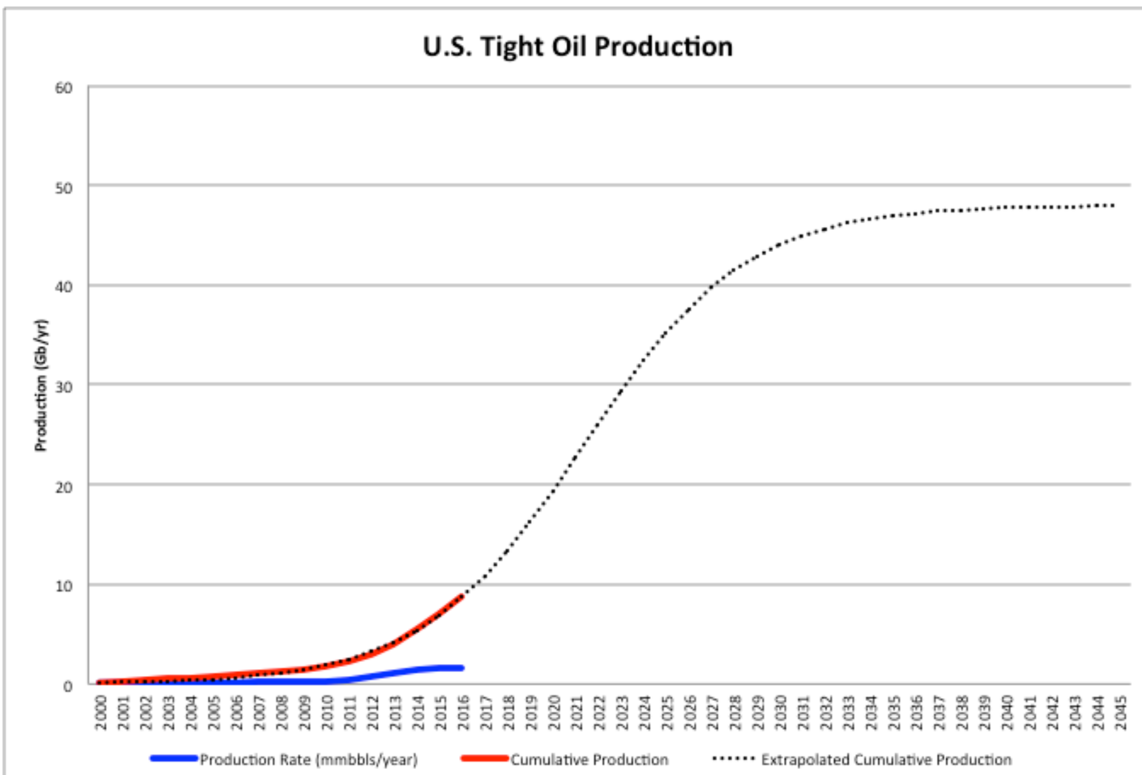


Fig. 12 U.S. tight oil production over time fits the logistic curve Hubbert postulated to approximate cumulative production over time quite well (EIA, 2017).

Laherrere (2000) introduced a variant of the Hubbert curve generally known as multi-cycle models, which fit a number of Hubbert curves to production data. In general, these models will provide a better fit to production data, however this can sometimes be a result of "over-fitting". Theoretically, each curve is representative of a new, well-defined resource that is easily distinguishable from the original play, but in practice these additional cycles can also result from improvement in exploration and production technology such as Enhanced Oil Recovery Techniques (EOR) or hydraulic fracturing (Brandt, 2009). Hubbert's single curve model fit U.S. L48 production relatively well until widespread use of hydraulic fracturing (fracking) began in the early 2000's. Production over time using Laherrere's multi- cycle model is typically described as:

$$Q(t) = \sum_{i=1}^{k} Q(t)_i = \sum_{i=1}^{k} \left\{ \frac{2Q_{max}}{1 + \cosh\,[b(t - t_m))} \right\}_i$$

where k represents the total number of logistic curves fit to production data, and Q_{max} and t_m represent peak production and time of peak production respectively for each cycle (Wang et al., 2011).

Laherrere's multi-cycle curves have previously been applied to regions such as France and Illionois whose production cycles follow significant bimodal discovery trend (Brandt, 2006, Laherrere, 2005). Here applying Laherrere's multi-cyclic model, we use two curves, one representing production of U.S. L48 conventional resources over time and representing U.S. L48 production of tight oil over time. Summing these curves, we arrive at a multi-cyclic model that corresponds with actual production quite well. We used the 219Gb estimate our model calculated for conventional URR and a tight oil URR of 48Gb estimate for tight oil (EIA, 2013).

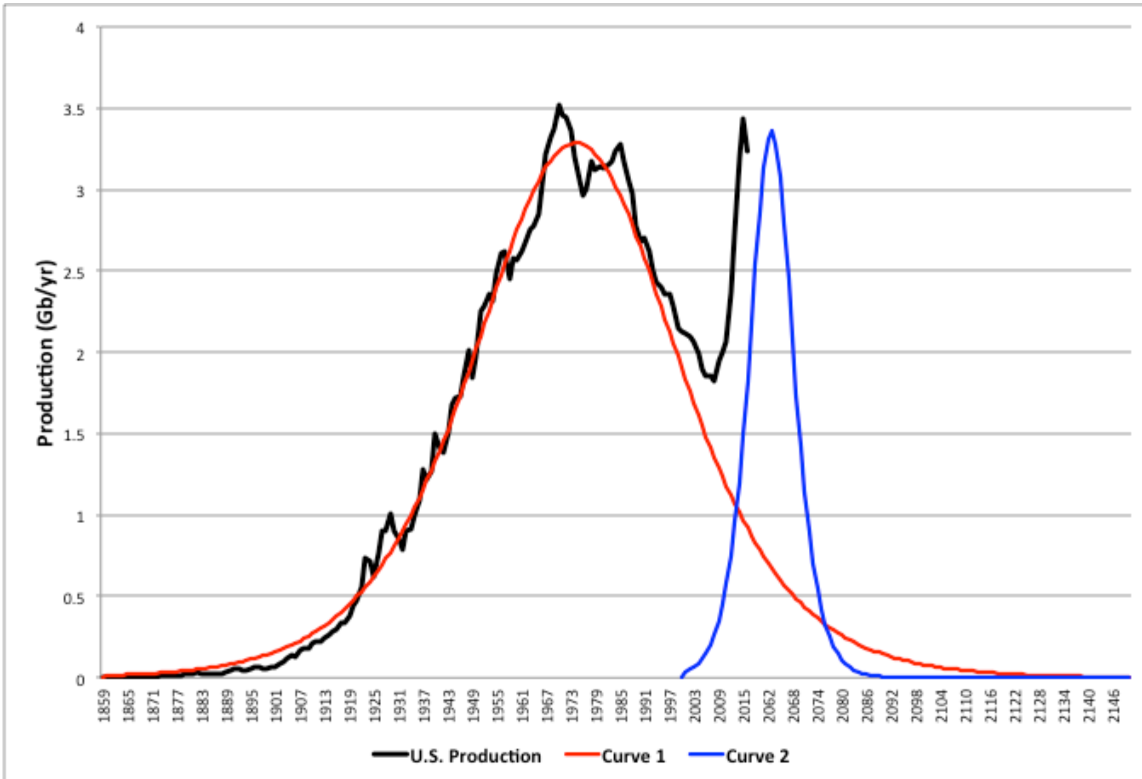


Fig. 13 U.S. production data fit with two curves, the first approximating conventional crude oil production and the second approximating tight oil production.

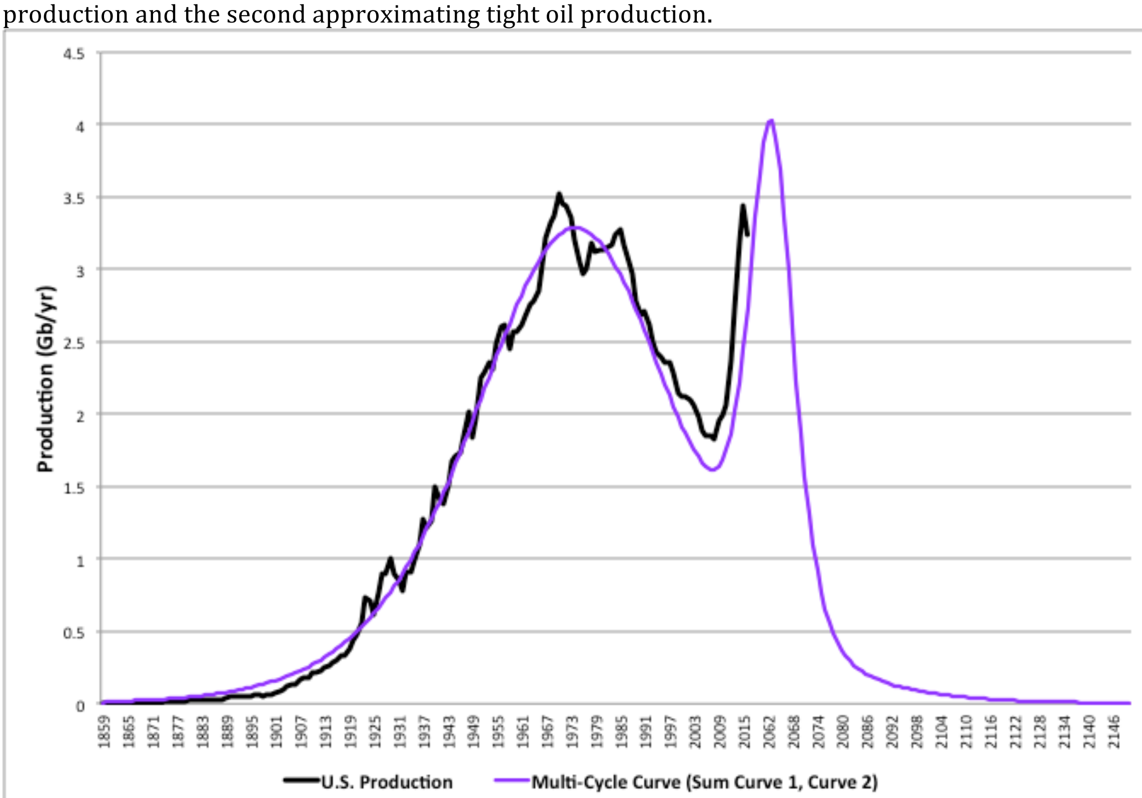


Fig. 14 U.S. production data fit with the curve summed from the two separate curves displayed in Fig. 13, this represents Laherrere's Multi-Cyclic model.

Lastly, we attempted to create a multi-cyclic model incorporating an asymmetric curve that might better approximate the U.S. production cycle when tight oil was modeled by a separate curve (Curve 2). This model necessarily incorporates more variables as it combines two different equations, the asymmetric curve described by Brandt (2006) and a symmetric Hubbert curve modeling tight oil production. While it is plausible and indeed likely that the tight oil production cycle may eventually be better fit by an asymmetric curve model rather than a symmetric, we cannot model this behavior as production has yet to peak.

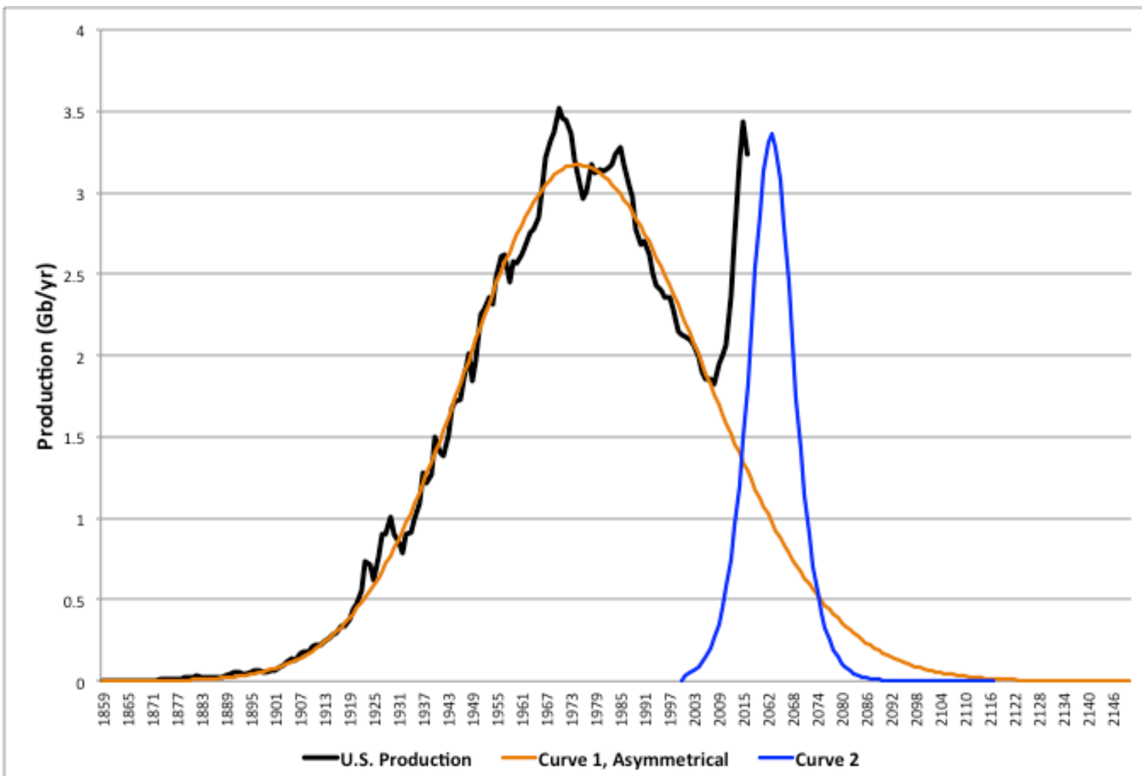


Fig. 15 U.S. production data fit with an asymmetric curve (Curve 1) approximating the production cycle of conventional resources, with Curve 2 fit to the production of tight oil.

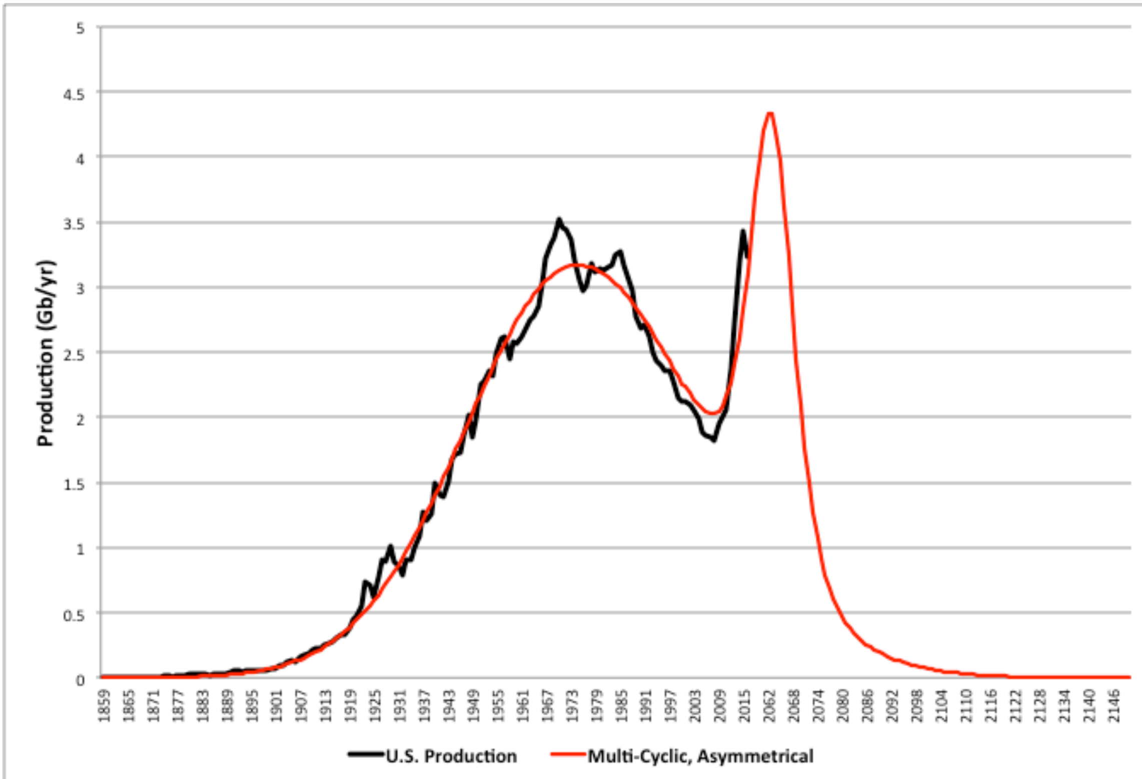


Fig. 16 Our Multi-Cyclic, Asymmetrical model (sum of Curve 1, Curve 2 in Fig. 15)

COMPARING MODELS

As Brandt (2006) notes, comparing models of differing complexity (i.e. different numbers of parameters) using the sum of squared errors of prediction (SSE) or the root mean square error (RMSE) is inadequate as more complex models inherently fit better when measured by SSE due to their increased flexibility (Brandt, 2006). We replicate Brandt's (2006) procedure, using the corrected Akaike's Information Criterion (AIC_C) score, allowing the comparison of models of differing complexity. The AIC_C formula is given by:

$$AIC_C = Nln\left(\frac{SSE}{N}\right) + 2K + \frac{2K(K+1)}{N-K-1}$$

where N is the number of data points in the series, SSE is the sum of squared errors and K is the number of model parameters (Motulsky & Christopolus, 2004, Brandt, 2006). While a low AIC_C score indicates a high likelihood of best-fit, a model cannot be directly accepted or rejected by comparison AIC_C as the score is based on information theory and not statistics. By calculating the difference in AIC_C score of two models in question (Δ AIC_C), we establish the probability that one model is 'more correct' than the other using:

$$\Delta AIC = AIC_C\ best\ fitting\ model - AIC_C\ second\ best\ fitting\ model$$

$$Probability = \frac{e^{-0.5\Delta AIC}}{1 + e^{-0.5\Delta AIC}}$$

where a probability >99% is considered strong evidence of best fit (Motulsky & Christopolus, 2004, Brandt, 2006).

We used this method to compare the fit of our single, multi and assymetric curve variants (for our set of annual production data ranging spanning 1859-2016, N=158):

Model	SSE	N	K	AIC
Single Hubbert	14.78	158	2	-370.32
Multi-Cyclic	5.59	158	5	-517.60
Asymmetric Hubbert	10.16	158	5	-423.20
Multi-Cyclic, Asymmetric	2.70	158	7	-627.95

All variants of the original Hubbert model scored better than the original, and all proved to be significantly better fits. Comparing them sequentially by magnitude of AIC score:

Single Hubbert with the Asymmetric Hubbert:

ΔAIC	Probability
-52.88	1.00

The Asymmetric Hubbert with the Multi-Cyclic:

ΔAIC	Probability
-94.3954858	1.00

And finally the Mutli-Cyclic with the Mutli-Cyclic model incorporating an asymmetric curve:

ΔAIC	Probability
-110.36	1.00

By an indirect comparison of the three models tested we find that the symmetric single cycle model Hubbert proposed provides the poorest fit for U.S. production data, while the Multi-Cyclic Asymmetric model provides the best fit even while accounting for the larger number of variables it incorporates.

DISCUSSION & CONCLUSION

As our results demonstrate, technological advances have significantly impacted the continued validity of Hubbert's single curve model for the U.S. production cycle. The advent of hydraulic fracturing and EOR techniques has introduced new production cycles and damped decline rates in manners that Hubbert had no way of accounting for. The best fit of both Multi-Cyclic models is unsurprising considering the large, well delineated resource hydraulic fracturing has made exploitable. The better fit of both Asymmetric models demonstrates the fallibility of Hubbert's assumed symmetry. While Hubbert's single cycle model should be applied with caution, it is important to highlight the useful properties all curve fitting techniques share in that they can make the most of regional level data that is often the most granular available. As Brandt (2006) notes, Hubbert-like models based on good URR estimates will not be erroneous on decade timescales due to the power of exponential growth, and as we prove with the application Multi Cyclic curves for U.S. production, modified curve fitting methods are useful when used with discretion (i.e. avoiding over fitting).

Model	P (2100), mmbbl	Model Estimated URR, Gb
Single Hubbert	251.36	261.46
Multi-Cyclic	80.88	267.52
Asymmetric Hubbert	610.59	283.10
Multi-Cyclic, Asymmetric	76.42	278.24

As Laherrere (2005) states “what goes up must come down… what is born will die… constant growth has no future in a limited world”. The Multi-Cyclic, Asymmetric model that fit production cycle data best estimated a URR of 283.10Gb. Current cumulative U.S. production amounts to more than 75% of that estimate. While our models provided a wide range of estimates for production in the year 2100, the most optimistic (Asymmetric Hubbert) was still less than 20% of U.S. production in 2016. Future production cycles spurred by exploitation of other harder to access or unconventional resources will come at the cost of increasingly diminishing EROI, and it is important to consider the economic and political implications this holds for an increasingly energy dependent nation.

WORKS CITED

1. U.S. Field Production of Crude Oil. 31 Mar. 2017. Raw data. U.S. Energy Information Administration (EIA), Washington D.C.
2. U.S. Tight Oil Production . 31 Mar. 2017. Raw data. U.S. Energy Information Administration (EIA), Washington D.C.
3. Brandt, Adam R. "Testing Hubbert." *Energy Policy* 35 (2006): 3074-088.
4. Brandt, Adam R. "Review of mathematical models of future oil supply: Historical overview and synthesizing critique." Energy 35 (2009): 3958-974.
5. "ENHANCED OIL RECOVERY." *Energy.gov*. U.S. Department of Energy (DOE), 2017. Web. 23 Apr. 2017.
6. Hubbert, M. King. "Energy from Fossil Fuels." *American Association for the Advancement of Science* 109.2823 (1949): 103-09. Web.
7. Hubbert, M. King. "Nuclear Energy and the Fossil Fuels." *Shell Development Company Exploration and Production Division* 59 (1956).
8. Hubbert, M. King. "Techniques of Prediction With Application to the Petroleum Industry." *American Association of Petroleum Geologists* 43.7 (1959): 1767-768.
9. Hubbert, M. King. "Energy Resources, A Report to the Committee on Natural Resources of the National Academy of Sciences-National Research Council." *National Academy of Sciences-National Research Council* 1000-D (1962).
10. Laherrere, J. H. "THE HUBBERT CURVE : ITS STRENGTHS AND WEAKNESSES." *Oil and Gas Journal* 98.16 (2000): 63-76.
11. Laherrere, J.H. "Peak oil and other peaks: Presentation at the CERN meeting on 3 Oct. 2005." (2005).
12. McGlade, C.E. "A review of the uncertainties in estimates of global oil resources." *Energy* 47 (2012): 262-70.
13. Motulskey, Harvey, and Arthur Christopolous. *Fitting Models to Biological Data Using Linear and Non-linear Regression: A Practical Guide to Curve Fitting.* New York: Oxford U Press, 2004.
14. Murray, James W., and Richard Henson. "Peak Oil and Energy Independence: Myth and Reality, ." *Eos* 24.28 (2013): 245-52..
15. Narvaez, Alfonso A. "M. King Hubbert, 86, Geologist; Research Changed Oil Production." *The New York Times* 17 Oct. 1989.
16. "Petroleum History ." *Enviroliteracy*. The Environmental Literacy Council (ELC), 2015.
17. Priest, Tyler. "Hubbert's Peak: the Great Debate Over the End of Oil." *HIST STUD NAT SCI* 44.1 (2014).
18. Sorrell, Steve, and Jamie Speirs. "Hubbert's Legacy: A Review of Curve-Fitting Methods to Estimate Ultimately Recoverable Resources." *Natural Resources Research* 19.3 (2010): 209-30.
19. Sorrell, Steve, Richard Miller, Roger Bentley Bentley, and Jamie Speirs.

"Oil futures: A comparison of global supply forecasts." *Energy Policy* 38 (2010): 4990-5003.

20. Sorrell, Steve, Jamie Speirs, Roger Bentley, Richard Miller, and Erica Thompson. "Shaping the global oil peak: A review of the evidence on field sizes, reserve growth, decline rates and depletion rates." *Energy* 37 (2012): 709-24.
21. *Technically Recoverable Shale Oil and Shale Gas Resources: An Assessment of 137 Shale Formations in 41 Countries Outside the United States*. Rep. Washington, D.C.: U.S. Energy Information Administration (EIA), 2013.
22. Wang, Jianliang, and Lianyong Feng. "Curve-fitting models for fossil fuel production forecasting: Key influence factors." *Journal of Natural Gas Science and Engineering* 32 (2016): 136-49.
23. Wang, Jianliang, Lianyong Feng, Lin Zhao, Simon Snowden, and Xu Wang. "A comparison of two typical multicyclic models used to forecast the world's conventional oil production." *Energy Policy* 39 (2011): 7616-621.