Light Scattering and Baffle Configuration for LIGO
(Report Prepared for LIGO Baffle Review, 6&7 January 1995)

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We have carried out analytical calculations of what we believe to be the dominant scattered-light noise sources in the LIGO beam tube, for interferometers whose arms are Fabry Perot cavities. These calculations update and correct the 1988–89 analysis by one of us (Kip).

This report describes the results of our calculations and discusses their implications for the LIGO baffle design. The dominant noise sources are backscatter off baffles (and possibly also off the bare wall preceding the first baffle and off objects at the far end of the beam tube), and diffraction off baffles.

It is argued that the baffle design goal should be to keep scattered-light noise below 1/10 the standard quantum limit for a one-ton test mass in the 10 to 100 Hz frequency band. The present baffle design is probably inadequate for this goal. Some modest changes in the baffles are proposed, to bring them within this goal.

I. INTRODUCTION

A. Scattering Noise

Scattering noise in the LIGO beam tube arises as follows: Light in the main beam of an interferometer arm scatters off one of the arm's cavity mirrors. The scattered light then scatters, diffracts, and/or reflects off the beam-tube wall, baffles, or other objects—which are vibrating due to seismic and acoustic excitations and which thereby put a phase modulation on the scattered light. The light then recombines with the main beam, where its phase modulation mimics a gravitational-wave signal.

Recombination of the scattered light with the main beam can occur in two ways: by scattering into the main beam at a cavity mirror (*cavity recombination*), or by passing through the corner cavity mirror and on to the photodiode where the main-beam light and scattered light superpose to produce a modulated photocurrent (photodiode recombination).

In this report we shall identify various types of scattering noise by the recombination process and/or by the process that puts the phase modulation onto the scattered light. For example "cavity recombination noise" is all noise in which the scattered light rejoins the main beam via cavity recombination; "baffle backscatter noise" is all noise in which the phase modulation is put onto the scattered light via backscatter off baffles; and "baffle diffraction noise" is all noise in which the phase modulation arises via diffraction of scattered light off baffle edges.

B. Previous Analyses of Scattering Noise

In late 1988 and early 1989 one of us (Kip) carried out the first detailed analysis of light scattering noise in LIGO, and on the basis of that analysis he offered a set of guidelines for the design of the LIGO baffles [1]. Since then, there have been a number of other analyses of scattering noise and related issues, including:

- "Optical Properties of the LIGO Beam Tubes" by Rai Weiss (January 1989), which estimates the attenuation of scattered light as it travels down the beam tube via a sequence of specular reflections, and estimates the range of angles $\theta \lesssim \theta_o$ of light rays to the tube axis that must be attenuated by baffles [2]

- A Monte Carlo calculation of scattered light intensity in LIGO by Alan W. Greynolds and Gary L. Peterson of Breault Research Organization (BRO) [3], augmented with an analysis, by Rai Weiss and Stan Whitcomb, of the phase modulation put onto the scattered light by beam-tube wall vibrations [4].

- Two Laurea theses on light scattering and baffle design by students at the University of Pisa attached to the VIRGO Project, Marina Cobal (May 1990) [5] and Stefano Braccini (May 1992) [6]. This work will be summarized at the LIGO Baffle Review by Jean-Yves Vinet.

- Detailed analyses of scattering noise in VIRGO by Violette Brisson and Jean-Yves Vinet (ca 1991–94). Brisson and Vinet have discussed this work and its comparison to LIGO analyses with Weiss and Thorne from time to time over the past several years, but we have no documentation on it. Vinet will describe it at the baffle review.

There have also been measurements of various quantities that enter into the scattering calculations, including:

- Measurements of mirror irregularities and analyses of their implications for the angular distribution of scattered light [7].

- Measurements of roughness and scattering cross sections (BRDF) for candidate materials for the LIGO beam tubes and baffles [8,9].

- Measurements of spatial irregularities in photodiode efficiencies [10]

Because of these new analyses and measurements, and because the chosen LIGO baffle design is somewhat different from that treated in Kip's original scattering analysis, Kip's analysis is now very much out of date. Accordingly, in spring 1994 we embarked on a detailed reanalysis of light scattering by the same analytic techniques as Kip originally used.

Initially in this reanalysis, we focussed on noise due to "backscatter" (backscatter off the baffles, off the unbaffled section of beam tube near each instrument chamber, and off objects at the far end of the beam tube), since such backscatter appeared to be the dominant scattering noise source and we were worried that the chosen baffle configuration might not control it adequately. We reported the results of that analysis last summer in a document that we shall refer to as "Backscatter" [11]. Since then we have reexamined all other scattering noise sources, and this report describes our conclusions.

C. Principal Changes Since Kip's Original Analysis

Our reanalysis and its results differ in major ways from Kip's original analysis and results [1]. These differences arise from the following items:

1. Vinet and Brisson found a serious error in Kip's calculation of how each cavity mirror in the interferometer's arms scatters scattered light back into the main beam (*cavity recombination*): Kip overlooked

the subsequent resonant buildup of the scattered light in each arm cavity and thereby missed a factor B^2 in Eq. (3.5) of [1]. This makes noise from cavity recombination much more serious than Kip had estimated.

2. Brian Lantz's measurements of the spectrum of spatial nonuniformities in photodiodes [10] showed that at the relevant wavenumbers the amplitudes of the nonuniformities are a factor 10^2 to 10^4 smaller than Kip originally assumed. This makes noise from *photodiode recombination* (recombination of scattered light with the main beam at the photodiode) far less serious than Kip estimated—and, in fact, when combined with item 1, it makes photodiode recombination always negligible compared to cavity recombination. Therefore, in this report we shall confine attention to cavity recombination.

3. Vinet and Brisson found a serious error in Kip's calculation of the phase modulation put onto light when it specularly reflects off the beam tube wall: Contrary to Kip's original calculation, the fluctuating tilt of the tube wall is *not* a significant source of phase modulation (the terms proportional to $\delta\mu$ and σ_{μ_o} should be deleted from Eqs. (4.17) of [1]). This reduction of *wall tilt noise* produces a corresponding reduction in *diffraction-aided reflection* (DAR) noise: DAR is no longer significant; it is replaced by *baffle diffraction* as the dominant non-backscatter source of noise.

4. The steel chosen for the LIGO beam tube was measured to have a somewhat larger rms surface height fluctuation than expected, $\sigma \sim 5\mu$m [9]. When combined with calculations by Weiss [2], this led to a much relaxed constraint on which scattered-light rays must be intercepted by baffles: All rays with angles $\theta < \theta_o$ to the beam-tube axis must be intercepted; Kip had assumed $\theta_o = 0.1$ rad; the new number is estimated [2,9] to be $\theta_o = 0.01$.

D. Outline of Report

The remainder of this report is organized as follows: Section II describes the present LIGO baffle confuration (Sec. II.A) and suggests a number of changes in it to bring it in line with our proposed scattered-light goal (Sec. II.B). Section III presents and discusses the results of our light scattering calculations. The standard quantum limit and our proposed scattered-light goal are discussed and are compared with the results of our noise calculations in Sec. III.A. Backscatter noise is discussed in Sec. III.B, specular reflection noise in Sec. III.C, and baffle diffraction noise in Sec. III.D. The diffraction discussion is rather long and complex. Subsection 1 presents a general phase-coherent formula for the diffraction noise, and that formula is then specialized to various situations in the subsequent subsections: mirrors offset from the beam-tube center in Subsec. 2, centered mirrors with unserrated baffles in Subsec. 3, centered mirrors with regularly serrated baffles in Subsec. 4, and centered mirrors with randomly serrated baffles in Subsec. 5.

II. PRESENT LIGO BAFFLE CONFIGURATION AND SUGGESTIONS FOR CHANGES

A. Present Configuration

The present configuration for the LIGO beam tube near each mirror is shown in Figure 1. Each test mass is ~ 10 or 30 meters from the end of a bare-walled 1.8 meter diameter tube; at the gate valve, the tube narrows to 1.2 meters and its wall remains bare for an additional 100 meters, where the first baffle is encountered. From there onward, baffles hide the tube wall from the view of the test mass. The baffles are made from the same oxidized steel as the tube. It is specified to have an rms roughness (peak to valley height variation) of $\sigma = 2.5\mu$m, but BRO measurements suggest an acutal roughness of $\sigma \simeq 5\mu$m. Its backscatter probability, for light incident at large angles to the beam tube axis (e.g. the 35^o relevant for light from a cavity mirror) is $dP/d\Omega_{\rm bs} \equiv \beta \simeq 0.01$ [9].

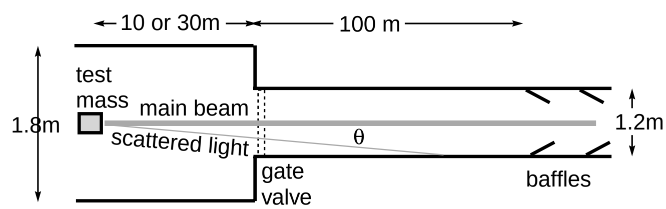

FIG. 1. LIGO beam tube and baffle configuration near a test-mass chamber.

All baffles are inclined at a 55^o angle to the vertical, leaning away from the nearest light-reflecting test-mass mirror, and have vertical heights $h = 6$cm. They are triangularly serrated with peak-to-valley serration heights of 3.5mm in the plane of the baffle, or $\Delta H = 2$mm in the vertical plane, and with serration widths (valley to valley) of 3.5mm.

There are two types of baffle families: the *end families* and the *central families*.

- Each end family begins 100m from the gate valve (Fig. 1) and extends for a total length of 150m with a uniform interbaffle spacing of $\Delta l = 7$m. Every third baffle in this family is at a support point of the beam tube; the intervening two baffles are not at support points.

- Each end family is followed by a central family, which thus begins 250m from the gate valve and extends onward with a uniform spacing of $\Delta l = 21$m.

All central-family baffles are at support points of the beam tube.

At Livingston, each arm has two end families (one near the corner station and one near the end station) with 23 ($\simeq 150/7$) baffles per family, and one central family with 173 ($\simeq 3500/21$) baffles; the total number of baffles in the arm is $N_b = 173 + 2 \times 23 = 219$.

At Hanford, each arm has four end families (one near the corner station, one near the end station, and one on each side of the mid station) with 23 baffles per family, and two central families (one on either side of the mid station) with 73 ($\simeq 1500/21$) baffles each; the total number of baffles in the arm is $N_b = 2 \times 73 + 4 \times 23 = 238$.

B. Suggestions for Changes

The results of our analyses (this report and our Backscatter report [11]) suggest that the following changes in the LIGO baffle design should be considered. These suggestions will be discussed at the LIGO Baffle Review:

Suggestion 1: Change the baffle design (scattered-light) goal from the standard quantum limit for a 1 ton test mass ($\tilde{h}_{\rm SQL}$) in the band from 10—100Hz, to $0.1\tilde{h}_{\rm SQL}$. See Sec. III.A for discussion.

Suggestion 2: Change the baffle material from the same oxidized steel as the walls are made of, to Martin Black or some other material with a comparably low backscatter probability. This would lower the backscatter noise by a factor $\simeq 3$. See Backscatter [11] for detailed justification.

Suggestion 3: If the beam-tube-wall backscatter at small incidence angles θ turns out to be $dP/d\Omega_{\rm bs} \sim \beta \simeq 0.01$ rather than $\sim \beta\theta$ (it is currently being measured by Weiss), then baffle the first 100 meters of beam tube wall with Martin-black baffles instead of leaving it bare. See Backscatter [11] for detailed justification.

Suggestion 4: Many of the baffles in the central family are not needed and should be removed so as to reduce single-diffraction noise. More specifically, at distances $l = 250\mathcal{N}$m to $250(\mathcal{N}+1)$m from the nearest light-reflecting mirror (where $\mathcal{N}$ is an integer), the baffle spacing should be $21\mathcal{N}$m. In other words, at $l = 250$ to 500m, the spacing should be 21m; at $l = 500$ to 750m, the spacing should be 42m; at $l = 750$ to 100m, the spacing should be 63m; etc. (This is a discrete version of the configuration originally proposed in Sec. II.1 of [1].) This will reduce the total number of baffles at Livingston from 211 to about 100, and at Hanford from 222 to about 120, which could produce a useful saving of money if the baffle material is changed to something expensive (Martin Black?), and it will reduce diffraction noise by about a factor $\sqrt{2}$ when the interferometer main beam is significantly offset from the center of the beam tube, and by about a factor 2.4 when the beam is centered. See Secs. III.D.2,4,5 for detailed discussion.

Suggestion 5: Make the peaks or the valleys of the baffle serrations random in height by ≥ 1mm in the vertical plane (≥ 1.7mm in the plane of the baffle), or make the peaks and the valleys both random by ≥ 0.5mm, so (in the latter case) in the vertical plane the valleys vary randomly from 5.7cm to 5.75cm above the tube wall, and the peaks vary randomly from 5.95 to 6.0cm above the wall. The coherence length, along the baffle, for these random variations should be $\lesssim \sqrt{\lambda L/4} \simeq 2.2$cm. This randomization is needed as a safety measure to control coherent effects in the scattering noise. The present, regular serrations are less adequate. The reduction in the baffle height safety factor that accompanies this change is acceptable because DAR is no longer a serious noise source.

Suggestion 6: The current baffle design (with or without the above changes) is adequate only to control light that scatters at angles $\theta < 11.4{\rm cm}/700{\rm cm} = 0.016$ to the beam tube axis. We should make sure, by specular reflectivity measurements on the actual beam-tube material rather than solely by theory, that rays with $\theta > 0.016$ are sufficiently attenuated that the baffles do not need to intercept them.

Suggestion 7: For the current baffle design, it is possible for light rays to depart from the scattering mirror at a scattering angle $\theta < 0.01$ in some small range (e.g., some but by no means all rays with θ between $(60{\rm cm} + 6{\rm cm})/(110 \text{ or } 130{\rm m}) \simeq 0.006$ and $2 \times (60{\rm cm} - 6{\rm cm})/150{\rm m} = 0.0072$ if the mirrors are at the center of the beam tube), reflect once on the bare wall before the first end family of baffles, pass over the first end family, then reflect specularly between baffles of the central family, skip over the second end family, bounce once on the bare end wall, and recombine at the far mirror. To stop this (admittedly small) set of dangerous, specularly reflecting rays, we suggest that one or more baffles be added at appropriate locations.

III. RESULTS OF SCATTERING CALCULATIONS

A. Standard Quantum Limit and Baffling Goal

It has become conventional to express the goal of the LIGO baffle design in terms of the standard quantum limit for a 1 ton test mass, [1,12]

$$\tilde{h}_{\rm SQL} = \left(\frac{8\hbar}{m(2\pi f L)^2}\right)^{1/2} = \frac{4 \times 10^{-24}}{\sqrt{\rm Hz}} \frac{10{\rm Hz}}{f} . \qquad (1)$$

Here $m = 1$ton is the mass of the test mass. In his original scattered light analysis [1], Kip proposed a goal that scattered light noise should not exceed $\tilde{h}_{\rm SQL}$ in the band 10Hz to 100Hz; and he proposed a factor 10 margin of safety to allow for inaccuracies in the scattering calculations, so the calculated noise should be below $0.1\tilde{h}_{\rm SQL}$.

We recommend (*Suggestion 1* in Sec. II.B) that the goal for the actual scattering noise be tightened by a factor 10, to $0.1\tilde{h}_{\rm SQL}$, and correspondingly, if inaccuracies are still thought to be as large as a factor 10, the calculated noise should be kept below $0.01\tilde{h}_{\rm SQL}$. Our reasoning is simple: The LIGO facilities are likely to be in operation for several decades, during which the interferometer sensitivities will improve substantially. It is not unreasonable to hope that, 15 to 30 years into LIGO operations some LIGO interferometers will incorporate a factor ~ 10 of quantum nondemolition, bringing them (even with test masses of $\sim$ 100kg rather than 1 ton) near $0.1\tilde{h}_{\rm SQL}$.

Figure 2 compares this goal with our best estimates of the two dominant noise sources [baffle backscatter, and baffle diffraction with off-centered mirrors, Eqs. (2) and (7) below] for the current baffle design, *assuming no amplification of ground motion by normal-mode vibrations of the beam tube.* The diffraction noise satisfies our suggested $0.1\tilde{h}_{\rm SQL}$ goal, with nearly a factor 10 safety margin for vibrational amplification plus calculational inaccuracies (if, as we assume in Fig. 2, coherence effects are adequately controlled for centered mirrors by making the baffle serrations random; our *Suggestion 5*). However, we would feel a little more secure if unnecessary baffles were removed, thereby reducing the off-center diffraction noise by $\sqrt{2}$. The backscatter noise leaves no margin at 10 Hz for vibrational amplification or calculational inaccuracies. Thus, we regard as quite important our *Suggestion 2*, to change the baffle material to something like Martin Black, thereby reducing the backscatter noise by about a factor $\sqrt{10}$.

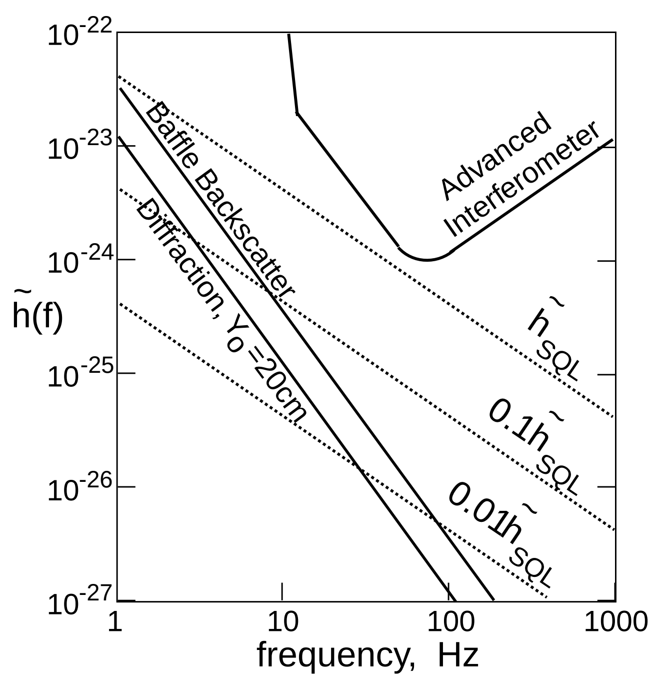

FIG. 2. Comparison of the standard quantum limit for a 1 ton test mass with our best estimates of the dominant scterred light noise, *assuming no amplification of ground noise by normal-mode vibrations of the beam tube, and assuming the site-selection specification for the ground noise.* The baffle backscatter noise is given by Eq. (2) and associated discussion; the diffraction noise, with the interferometer's main beam a distance 20cm from the tops of the baffles, is given by Eq. (7) and associated discussion. The "Advanced Interferometer" curve is from Figure 7 of the LIGO *Science* article [13].

B. Backscatter Noise

Baffle backscatter noise arises when light scatters off a cavity mirror, then backscatters from a vibrating baffle (acquiring thereby a phase modulation), then returns to the mirror and there recombines with the main beam. (Recall that we are ignoring photodiode recombination.) We have calculated the spectrum of baffle backscatter noise in our previous report (Backscatter [11]). For comparison with other noise sources and to remind the reader of our notation, we here quote our result:

$$
\begin{aligned}
\tilde{h}_{\rm baf\ bksct}(f) &= \left[4\pi\alpha^2\beta\ln\left(\frac{l_1}{l_2}\right)\, J_0(\rho)\right]^{1/2} \bar{A}(f)\frac{\lambda}{R}\frac{\tilde{\xi}_s(f)}{L} \\
&= \frac{3\times10^{-25}}{{\rm Hz}^{1/2}}\left(\frac{10{\rm Hz}}{f}\right)^2 \bar{A}(f)\frac{\sqrt{J_0(\rho)}}{2}\,\frac{\alpha}{10^{-6}}\left(\frac{\beta}{0.01}\right)^{1/2} \\
&\times\left(\frac{\ln(l_1/l_2)}{\ln(4{\rm km}/120{\rm m})}\right)^{1/2}\frac{\tilde{\xi}_{\rm s}(f)}{10^{-7}{\rm cmHz}^{-1/2}(f/10{\rm Hz})^{-2}}\ .
\end{aligned}
\tag{2}
$$

Here the various symbols have the following meanings: The probability for a photon in the main beam to scatter off a mirror and into a direction making an angle θ to the main beam, per unit solid angle, is assumed (in accord with measurements and analyses [7,1,3]) to be $dP/d\Omega_{\rm ms} = \alpha/\theta^2$, with $\alpha \simeq 10^{-6}$. The probability for a photon arriving at a baffle from the scattering mirror to backscatter back toward that same mirror, per unit solid angle, is assumed, in accord with measurements by the Breault Research Organization, BRO [9], to be $dP/d\Omega_{\rm bs} = \beta \simeq 10^{-2}$; and this is true whether the photon hits the back face of a distant baffle or the front face of a more nearby baffle. The most distant baffle is at a distance $l_1 \simeq$ 4km from the mirror, and the nearest baffle is at a distance $l_2 \simeq$ 120m. The mirror is offset from the center of the beam tube's cross section by a distance ρR, where $R =$ 60cm is the beam-tube radius; and the function $J_0(\rho)$ (*not* a Bessel function), which deals with this offset, is plotted in Backscatter [11]; it varies from $\sqrt{J_0(0)} = 1$ for a centered mirror to $\sqrt{J_0(2/3)} \simeq 2$ for a mirror 20cm from the tube wall. The spectrum of the baffles' horizontal vibrational displacement is $A(f)\tilde{\xi}_{\rm s}(f)$, where $\tilde{\xi}_{\rm s}(f)$ is the horizontal seismic noise spectrum and $A(f)$ accounts for amplification due to excitation of beam-tube normal modes; the quantity $\bar{A}(f)$ is an average of $A(f)$ over all regions of all baffles, with the average weighted by the backscattered light amplitude that is returned to the main beam from each region of a baffle. We choose as our fiducial seismic noise spectrum the upper-limit specification for LIGO sites in the 10 to 100 Hz band, $\tilde{\xi}_{\rm s}(f) = 10^{-7}{\rm cm/Hz}^{-1/2}(f/10{\rm Hz})^{-2}$. Finally, $L = 4$km and $\lambda = 0.5\mu$m are the beam-tube length and the wavelength of the main-beam light.

For the parameter values shown in Eq. (2), this baffle backscatter noise has the values plotted in Fig. 2, and is $\tilde{h}_{\rm baf\ bkst} = 0.08(10{\rm Hz}/f)\tilde{h}_{\rm SQL}$. By changing the baffle material from beam-tube steel to Martin black, β would be reduced by a factor 10 and $\tilde{h}_{\rm baf\ bkst}$ would be reduced by a factor $\sqrt{10} \simeq 3$.

It may well turn out that backscatter off the $\simeq 120$m of bare beam tube wall that precedes the first baffle and backscatter off objects at the far end of the beam tube both produce noise comparable to that from backscatter off baffles; see Secs. I.B and I.C of Backscatter [11] for details, implications, and recommendations.

C. Specular Reflection Noise

Specular reflection noise arises when light rays scatter off one of the cavity mirrors, then travel down the beam tube from one end to the other via specular reflections off the tube walls (which phase modulate the light), avoiding all baffles as they go, then recombine with the main beam at the other cavity mirror.

For the present LIGO design, such specular reflection paths are possible only if the light rays' angles to the tube axis are $\theta \gtrsim 0.114{\rm m}/7{\rm m} = 0.016$rad. Presumably (we need to be sure of this!) all scattered light in this range gets strongly attenuated by scattering off the walls. If not, then the resulting noise from rays in a small range of angles θ to $\theta + \Delta\theta$ will be

$$\begin{aligned}\tilde{h}_{\rm refl}(f) &= \alpha\sqrt{2\mathcal{A}(\theta)\mathcal{M}(\theta)}\frac{\lambda}{\sqrt{LR}}\frac{\bar{A}(f)\tilde{\xi}_s(f)}{R}\sqrt{\Delta\theta} \\ &= \frac{2.4\times10^{-24}}{{\rm Hz}^{1/2}}\left(\frac{10{\rm Hz}}{f}\right)^2\sqrt{\mathcal{A}(\theta)\mathcal{M}(\theta)}\bar{A}(f) \\ &\quad\times\left(\frac{\Delta\theta}{0.01}\right)^{1/2}\frac{\tilde{\xi}_{\rm s}(f)}{10^{-7}(f/10{\rm Hz})^{-2}}\ . \qquad (3)\end{aligned}$$

Here the notation is as in Eq. (2), except for the following changes or new symbols: $\mathcal{A}(\theta)$ is the attenuation factor for photons that travel with angle θ from one end of the beam to the other (i.e., the fraction of the photons that survive the trip); $\mathcal{M}(\theta)$ is a magnification factor due to phase coherence between light rays that travel along different paths separated by more than one Fresnel zone width ($\mathcal{M} = 1$ corresponds to phase incoherence of the light arriving at the recombining mirror); $A(f)\tilde{\xi}_s(f)$ is the spectrum of radial vibrations of the tube wall (in Eq. (2) it was the spectrum of horizontal baffle vibrations), at frequency f and averaged over the wall; and these wall vibrations are broken down into a driving seismic noise $\tilde{\xi}_s(f)$ and an amplification factor $\bar{A}(f)$ (at freqency f and averaged over the wall) due to the excitation of wall normal modes.

For the parameter values shown in Eq. (3) (including no reflective attenuation or coherent magnification or vibrational amplification), the noise is $\tilde{h}_{\rm refl} = 0.6\tilde{h}_{\rm SQL}(10{\rm Hz}/f)$. This value (6 times larger than our suggested goal with no margin for coherence or vibrational amplification) is a reminder of the importance of strong attenuation of reflecting rays by wall scatter or baffle encounters.

D. Diffraction Noise

1. General Phase-Coherent Formula

The dominant form of *baffle diffraction noise* arises when light scatters off a cavity mirror, then diffracts off the edge of a vibrating baffle, then recombines with the main beam at the other cavity mirror.

If the mirrors (and main beam) are near the beam tube's center, and if the baffles are round and not properly serrated, then the diffraction noise is in danger of being magnified significantly by phase coherence between light diffracted off different regions of the same baffle. (Such phase coherence is not a danger for backscatter noise; roughness of the baffles enforces backscatter incoherence). One can use coherent, paraxial wave propagation techniques to compute the baffle diffraction noise, obtaining the following formula in the time domain (cf. Eq. (4.68) of Kip's original report [1]):

$$\begin{aligned}h_{\rm diff}(t) = \sum_{\rm mir}\sum_{n=1}^{N_B}\frac{\lambda}{4\pi L}\Im\Bigg\{\int_0^{2\pi}&\left[\frac{\sqrt{\alpha}}{Y_n}f_{\rm sm}(\mathbf{Y}_n)e^{ikY_n^2/2l_n}\right] \\ \times&\left[\frac{\sqrt{\alpha}}{Y_n}f_{\rm rm}(\mathbf{Y}_n)e^{ikY_n^2/2l'_n}\right]RA_n\xi_s d\varphi\Bigg\}\ . \qquad (4)\end{aligned}$$

This formula can be understood as follows: The first sum is over the interferometer's four cavity mirrors, each of which can act as a "sending mirror" that creates scattered light, the second sum is over each baffle (labeled by the integer n) that the light can diffract from, and the integral is around the edge of baffle n. The quantity inside the curly brackets $\{...\}$ is the fractional contribution $\delta\psi_{\rm mb}/\psi_{\rm mb}$ of the scattered, diffracted light field to the main beam field after recombination at the "receiving mirror"; and the factor $(\lambda/4\pi L)\Im$ (where $\Im$ means "take the imaginary part") comes from the standard formula $\delta\psi_{\rm mb} = i(4\pi Lh/\lambda)\psi_{\rm mb}$ for the modulated main beam light field created by a gravitational wave h when it pushes the cavity mirrors back and forth (cf. Eq. (8) of

Backscatter [11]). The expression, inside the curly brackets, for the scattered light's fractional contribution to the main beam field, can be understood as follows:

- The term in the first square brackets [...] is the scattered light field (in units of the square root of the main beam intensity $\sqrt{I_{\rm mb}}$ and in paraxial optics formalism) arriving at the point $\mathcal{P}$ on the edge of baffle n at angle φ around the baffle; cf. Fig. 3; $\mathbf{Y}_n(\varphi)$ is the transverse vector (vector in the plane of the baffle) from the main beam axis to baffle point $\mathcal{P}$; $Y_n(\varphi) = |\mathbf{Y}_n(\varphi)|$ is the transverse distance from the main beam axis to baffle point $\mathcal{P}$; and l_n is the distance of the baffle plane from the sending mirror. The $\sqrt{\alpha}/Y_n$ part of the scattered field is $(1/r)\sqrt{\alpha/\theta^2}$, where α/θ^2 is the probability for a main beam photon to scatter into angle α and r is distance from the mirror to the baffle point $\mathcal{P}$. The exponential term (with $k = 2\pi/\lambda$) is the propagator's standard transverse phase factor, and $f_{\rm sm}(\mathbf{Y}_n)$ is a complex factor of order unity and with unit mean, which describes random variations in phase and amplitude (laser beam "speckles") due to the randomness of the surface of the scattering mirror (cf. Eqs. (4.5) of Kip's original report [1]). Elementary diffraction theory dictates that the phase of this $f_{\rm sm}$ varies significantly only on lengthscales $\gtrsim \sqrt{\lambda l_n}$.

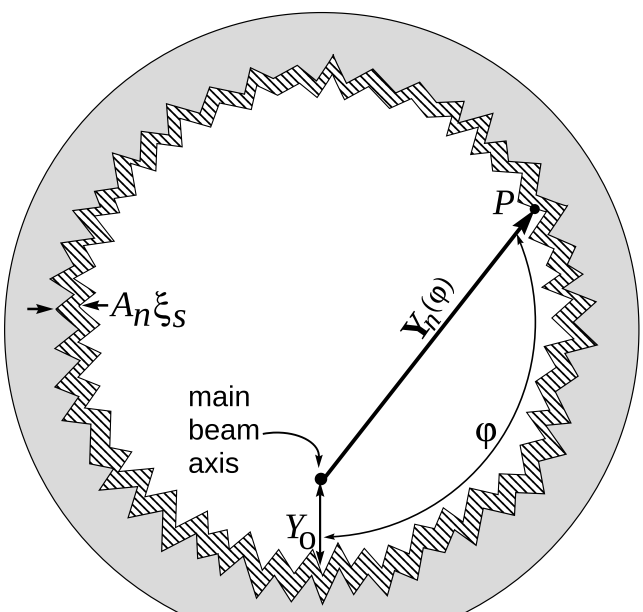

FIG. 3. Diagram, in the plane of baffle n, depicting the geometry used in a phase-coherent computation of diffraction noise.

- According to the paraxial formalism for diffraction, the modulated part of the scattered light consists of all light that passes through the cross-hatched region in the baffle plane of Fig. 3, which has been opened up by the baffle's radial vibrational displacement $A_n(\varphi,t)\xi_s(t)$. (Here A_n is the factor by which the seismic noise ξ_s is amplified at location φ and time t on baffle n.) This explains the integration element $RA_n\xi_s d\varphi$, which represents the area of the cross-hatched region between φ and $\varphi+d\varphi$. As the baffle vibrates, the cross-hatched region changes size and shape, thereby changing the scattered light field $\delta\psi_{\rm mb}$ being added to the main beam; this is the origin of the phase modulation that mimics the gravitational wave.

- After passing through point $\mathcal{P}$ in the cross-hatched region, the scattered light propagates onward to the receiving mirror, where it then scatters back into the main beam. There is a reciprocity relation for scattering into and out of the main beam (cf. the appendix of our Backscatter document [11]), which guarantees that this propagation from baffle to receiving mirror and recombination is described by a factor (second square bracket [...]) with identically the same form as that for scattering out of the main beam and propagating to the baffle (first square bracket). In the propagation and recombination term (second square bracket), $l'_n = L - l_n$ is distance from the baffle plane to the receiving mirror, and $f_{\rm rm}(\mathbf{Y}_n)$ is a complex factor of order unity and with unit mean, which describes random variations (on lengthscales $\gtrsim \sqrt{\lambda l'_n}$) of phase and amplitude due to the randomness of the surface of the receiving mirror.

2. Off-Centered Mirrors

In expression (4) for baffle diffraction noise, the two transverse phase factors can be combined to give

$$\exp\left(\frac{ikY_n^2}{2l_n}\right)\exp\left(\frac{ikY_n^2}{2l'_n}\right) = \exp\left(\frac{ikY_n^2}{2l_{nR}}\right) , \qquad (5)$$

where

$$l_{nR} = \frac{l_n l'_n}{(l_n + l'_n)} = \frac{l_n(L-l_n)}{L} \qquad (6)$$

is the baffle's "reduced distance" from the mirrors. This phase factor describes a Fresnel zone pattern surrounding the main beam axis, with the central zone having a radius $\sqrt{\lambda l_{nR}} \le \sqrt{\lambda L/4} = 2.2$cm; see Fig. 4.

If the mirrors are displaced a distance $\gg \sqrt{\lambda L} = 4.5$cm from the beam tube center (as in Figs. 3 and 4), then the Fresnel zone pattern will be substantially offset from the averaged (smoothed) beam tube edge. There then will be near cancellations of the diffraction-noise contributions from adjacent Fresnel zones, and these near cancellations—even in the absence of baffle serrations—will make the scattering noise essentially incoherent. A quantitative analysis based on Eq. (4) shows in this case that the baffle serration neither helps nor hurts; the noise is essentially independent of whether the baffles are serrated and whether the serrations are regular or random; see Sec. IV.A.5 of Kip's original scattering report [1] for details. By performing the integral and sums in (4) for the case of no baffle serrations, we obtain the following expression for the diffraction noise:

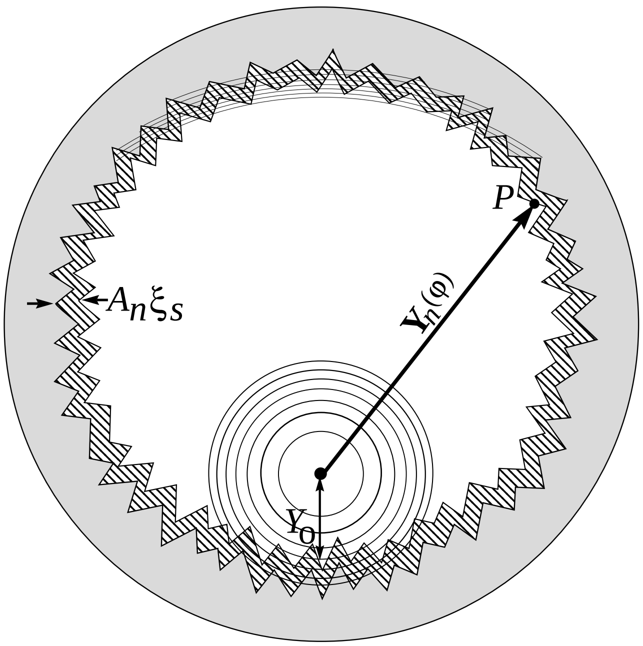

FIG. 4. The pattern of Fresnel zones around the main beam axis, corresponding to the transverse phase factor (5) in formula (4) for baffle diffraction noise.

$$\tilde{h}_{\rm diff} = \frac{\alpha}{\sqrt{3}}\frac{\sqrt{\lambda L}}{4\pi R}\frac{\lambda\bar{A}\tilde{\xi}_s}{LR}\sqrt{N_B}G(\rho)$$
$$= \frac{1.2\times10^{-25}{\rm Hz}^{-1/2}}{(f/10{\rm Hz})^2}\left(\frac{N_B}{220}\right)^{1/2}\frac{G(\rho)}{11}$$
$$\times\frac{\tilde{\xi}_s}{10^{-7}{\rm cm}/\sqrt{\rm Hz}(10{\rm Hz}/f)^2}\ . \qquad (7)$$

Here $\bar{A}(f)$ is the radial motion amplification factor at freqency f, averaged over the edges of the baffles; and the function $G(\rho)$ embodies the dependence of the noise on the mirrors' offset from the beam tube center: ρ is the fractional offset, i.e. $\rho = 1-Y_o/(R-H) \simeq 1-Y_o/R$, where Y_o is the distance of the main beam axis to the nearest baffle point (Fig. 4) and $R-H = 54{\rm cm} \simeq R = 60{\rm cm}$ is the radius of the baffle edges. (Throughout we ignore the difference between R and $R-H$.) $G(\rho)$ is given by

$$G(\rho) = \frac{\sqrt{2(1+6\rho^2+\rho^4)}}{\sqrt{\rho}(1-\rho^2)^2}$$

$$\simeq \frac{1}{(1-\rho)^2} = \left(\frac{R}{Y_o}\right)^2 \quad \text{for } 1-\rho \ll 1\ . \qquad (8)$$

For $\rho = 2/3$ (i.e., main beam axis a distance $Y_o \simeq 20$cm from the nearest baffle), $G(\rho)$ is 11. This is the fiducial value used in Eq. (7).

As the main beam axis is pushed closer and closer to the baffle tops (as Y_o decreases), this diffraction noise rises more rapidly (approximately $\propto 1/Y_o^2$) than does baffle backscatter noise (approximately $\propto 1/Y_o$).

The diffraction noise plotted in Fig. 2 is that for an off-center mirror with $Y_o = 20$cm and the parameter values shown in Eq. (7). At $f = 10$Hz, this noise is $0.03\tilde{h}_{\rm SQL}$.

3. Centered Mirrors With Unserrated Baffles

Turn, now, to an interferometer whose mirrors are at the center of the beam tube. Because of the resulting axisymmetry, such an interferometer is in maximal danger of coherently superposing scattered light that travels along different routes, and correspondingly in danger of coherently magnifying its diffraction noise.

Slight randomness in the positions of baffles relative to each other will guarantee that the light diffracted from the various baffles superposes incoherently. However, there is danger of coherent superposition for light that diffracts off different regions of the same baffle. The baffle serrations are designed to protect against this.

We shall explore the serrations' role by examining in turn three specific baffle configurations: (i) no serrations (this subsection), (ii) regular serrations (the next subsection), (iii) random serrations (the subsequent subsection).

For unserrated baffles and centered mirrors, the distance from the beam axis to the baffle edge is $Y_n(\varphi) = R$ independent of position φ, so the phase factor $\exp(ikY_n^2/2l_{nR}) = \exp(ikR^2/2l_{nR})$ is constant around the baffle, and the Fresnel zones are parallel to the baffle edge. This means that the Fresnel zone pattern is unable to protect against coherence. The only hope for protection lies in the randomly varying phases of the scattered light factors $f_{\rm sm}(\mathbf{Y}_n)$ and $f_{\rm rm}(\mathbf{Y}_n)$ produced by random deviations of the mirrors from axisymmetry. It is not at all clear how much protection these factors will give; to rely solely on them would be foolish. In the extreme case where (i) they give no significant protection (mirror irregularities nearly axisymmetric so $f_{\rm sm}$ and $f_{\rm rm}$ are constant around the baffle, with moduli approximately unity) and where (ii) each baffle's vibrations are coherent around its circumference (so $A_n\xi_s$ is constant), Eq. (4) gives for the spectrum of the diffraction noise

$$\tilde{h}_{\rm diff}(f) = \frac{\alpha\lambda\bar{A}(f)\tilde{\xi}(f)}{\sqrt{2}LR}\sqrt{N_B}$$
$$= \frac{2.2\times10^{-24}{\rm Hz}^{-1/2}}{(f/10{\rm Hz})^2}\left(\frac{N_B}{220}\right)^{1/2}\bar{A}$$

$$\times \frac{\tilde{\xi}}{10^{-7}\mathrm{cm}/\sqrt{\mathrm{Hz}}(f/10\mathrm{Hz})^2} \ . \quad (9)$$

Here N_B is the total number of baffles in each interferometer arm.

Note that at 10Hz this coherent diffraction noise is $0.5\tilde{h}_{\rm SQL}$, which is unacceptably large.

4. Centered Mirrors with Regularly Serrated Baffles

Suppose, now, that the baffles have regular, triangular serrations (the present design), as depicted in Fig. 5. The present peak to valley serration height in the vertical plane, $\Delta H = 2$mm, is large compared to the width of a Fresnel zone,

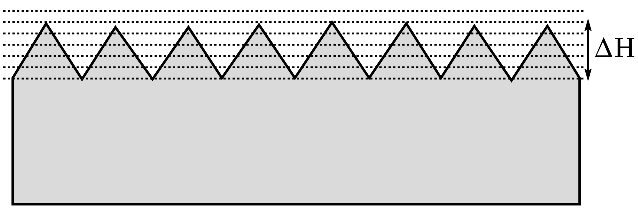

FIG. 5. A segment of a baffle with regular serration of peak to valley height ΔH. The dotted lines are Fresnel zones.

$$w_n = \frac{\lambda l_{nR}}{2R} = \frac{\lambda L}{8R}\beta_n = 0.4\beta_n \mathrm{mm};. \quad (10)$$

Here

$$\beta_n \equiv \frac{4 l_{nR}}{L} = \frac{4 l_n (L - l_n)}{L^2} \quad (11)$$

ranges from unity for baffles near the center of the beam tube to about 0.1 for baffles near the ends. For baffles near the center of the tube, $\Delta H/w_n \simeq 4$; for baffles near the ends, $\Delta H/w_n \simeq 40$. Because $\Delta H/w_n \gg 1$, the serrations average over a number of Fresnel zones thereby reducing the diffraction noise.

More specifically, in our coherent diffraction noise equation (4), with the worst-case assumption of axisymmetric mirrors and vibrational coherence so $f_{\rm sm}$, $f_{\rm rm}$ and A_n are independent of ϕ, and setting $Y_n = R - H+$(triangular serrations of height ΔH) $\simeq R +$ (serrations), the angular integral is easily performed to give

$$\tilde{h}_{\rm diff} = \sum_{\rm mir} \sum_{n=1}^{N_B} \frac{\alpha \lambda A_n \xi_s}{2RL} \Im \left[e^{ikR^2/2l_{nR}} \left(\frac{e^{-i\pi\Delta H/w_n} - 1}{-i\pi\Delta H/w_n} \right) \right] \quad (12)$$

The corresponding noise spectrum [computed taking account of the phase incoherence between baffles and using Eq. (10)] is

$$\tilde{h}_{\rm diff} = \left[\frac{\alpha\lambda \bar{A}\tilde{\xi}_s}{\sqrt{2}RL}\sqrt{N_B} \right] \left[\frac{\sqrt{2}\lambda L}{8\pi R \Delta H} \left(\frac{1}{N_B}\sum_{n=1}^{N_B} \beta_n^2 \right)^{1/2} \right] . \quad (13)$$

The first square bracketed term is the noise without serrations, Eq. (9); the second is the improvement due to regular serrations.

The end families of baffles have $\beta_n \sim 0.01$ and thus contribute negligibly to the noise; almost all the noise comes from the central families, with their β_n ranging from ~ 0.2 to 1. By virtue of their uniform baffle spacing, the central families have many baffles with $\beta_n \sim 1$, and $\sum_{n=1}^{N_B}\beta_n^2$ works out to be $\simeq N_B/2$ (where N_B is the total number of baffles in the arm). *Thus, for the present baffle design, centered mirrors, and worst-case assumptions about coherence (axisymmetry of mirrors and of baffle vibrations), the diffraction noise is*

$$\begin{aligned} \tilde{h}_{\rm diff} &= \left[\frac{\alpha\lambda \bar{A}\tilde{\xi}_s}{\sqrt{2}RL}\sqrt{N_B} \right] \left[\frac{\lambda L}{8\pi R\Delta H} \right] \\ &= \frac{1.5\times 10^{-25}\mathrm{Hz}^{-1/2}}{(f/10\mathrm{Hz})^2} \left(\frac{N_B}{220} \right)^{1/2} \left(\frac{2\mathrm{mm}}{\Delta H} \right) \bar{A} \\ &\quad \times \frac{\tilde{\xi}}{10^{-7}\mathrm{cm}/\sqrt{\mathrm{Hz}}(f/10\mathrm{Hz})^2} \ . \end{aligned} \quad (14)$$

The regular baffle serrations (term in second square bracket) have reduced this worst-case diffraction noise by a factor 15 [cf. Eq. (9)], to $0.04\tilde{h}_{\rm SQL}$ at 10Hz.

To reduce the diffraction noise (14) further, we can remove unnecessary baffles from the central families. This motivates our *Suggestion 4* (Sec. II.B): Make the spacing between baffles be $21\mathcal{N}$m for the central family, in the range of distances $250\mathcal{N}$ to $250(\mathcal{N}+1)$m from the nearest light reflecting mirror. Here $\mathcal{N}$ is an integer. For this choice of spacings, the quantity $\sum_{n=1}^{N_B}{\beta_n}^2$ is about 20, compared to about 110 for the present baffle design; and correspondingly, *the worst-case noise for our proposed new baffle spacing is reduced by about* $\sqrt{20/110}$ *to*

$$\begin{aligned} \tilde{h}_{\rm diff} &= \left[\frac{\alpha\lambda \bar{A}\tilde{\xi}_s}{RL}\sqrt{20} \right] \left[\frac{\lambda L}{8\pi R\Delta H} \right] \\ &= \frac{0.6\times 10^{-25}\mathrm{Hz}^{-1/2}}{(f/10\mathrm{Hz})^2} \left(\frac{2\mathrm{mm}}{\Delta H} \right) \bar{A} \\ &\quad \times \frac{\tilde{\xi}}{10^{-7}\mathrm{cm}/\sqrt{\mathrm{Hz}}(f/10\mathrm{Hz})^2} \ . \end{aligned} \quad (15)$$

This is $0.015\tilde{h}_{\rm SQL}$ at 10 Hz.

5. Centered Mirrors with Randomly Serrated Baffles

Despite their serrations, the present baffles do *not* protect against coherent superposition: All triangular segments of a given baffle produce scattered, modulated field with identically the same phase, and these fields all superpose coherently (under the worst case assumption of axisymmetric mirrors and baffle vibrations). The serrations, by extending through many Fesnel zones, have substantially reduced the amplitude of the scattered, modulated field from each triangle, but they have not randomized the fields' phases.

To randomize the phases requires that the heights of the baffles' peaks and/or valleys be random by $\gtrsim 2w_n$ (two Fresnel zone widths). For the most dangerous baffles, those near the tube's center, $2w_n \simeq 1$mm. This motivates *Suggestion 5* of Sec. II.B: Make the peaks or the valleys of the baffle serrations random in height by ≥ 1mm in the vertical plane (≥ 1.7mm in the plane of the baffle), or make the peaks and the valleys both random by ≥ 0.5mm.

Denote by σ_B the serration coherence lengthscale, along the baffle's circumference, for these variations of baffle height. Nothing is gained by making this coherence lengthscale shorter than $\sqrt{\lambda l_{\rm nR}}$, because the fields from regions this size merge together as they propagate from mirror to baffle to mirror. For the dangerous central baffles this minimum useful serration coherence scale is $\sqrt{\lambda L/4} \simeq 2.2$cm. Randomizing the serrations on this scale will bring the diffraction noise down from the worst-case level (axisymmetric mirrors and baffle vibrations) to the best-case level (maximum nonaxisymmetry). If the mirrors are maximally nonaxisymmetric, then their $f_{\rm sm} f_{\rm rm}$ will automatically randomize the phases around the baffles on the lengthscale $\sqrt{\lambda l_{\rm nR}}$, and there would be no need to randomize the serrations. However, we cannot rely on the mirrors to be maximally nonaxisymmetric, and we therefore recommend randomizing the baffle serrations.

With the baffles randomly serrated on lengthscales $\sigma_B \lesssim \sqrt{\lambda L/4}$, the diffraction noise gets reduced by 1/(square root of number of phase-incoherent regions), i.e. by $(\sqrt{\frac{1}{4}\lambda L}/2\pi B)^{1/2} = 1/12$. The resulting noise for centered mirrors, if the baffles are changed in accord with our Suggestion 4 (remove unneeded baffles) and Suggestion 5 (randomize serrations), is

$$\begin{aligned}\tilde{h}_{\rm diff} &= \left[\frac{\alpha\lambda\bar{A}\tilde{\xi}_s}{RL}\sqrt{20}\right]\left[\frac{\lambda L}{8\pi R\Delta H}\right]\left[\frac{\sqrt{\lambda L/4}}{2\pi R}\right]^{1/2} \\ &= \frac{0.5\times10^{-26}{\rm Hz}^{-1/2}}{(f/10{\rm Hz})^2}\left(\frac{2{\rm mm}}{\Delta H}\right)\bar{A} \\ &\quad\times\frac{\tilde{\xi}}{10^{-7}{\rm cm}/\sqrt{\rm Hz}(f/10{\rm Hz})^2}\ . \end{aligned} \tag{16}$$

This is $0.0013\tilde{h}_{\rm SQL}$ at 10Hz.

Notice the comparative sizes of the noise improvement from our two proposed baffle changes: a factor 12 from randomizing the serrations, in the worst case of axisymmetric mirrors and baffle vibrations; a factor $\sqrt{110/20} \simeq 2.3$ from getting rid of unnecessary baffles. For highly off-centered mirrors, the serrations buy nothing, while getting rid of unnecessary baffles buys a factor $\sqrt{N_{B\ \rm old}/N_{B\ \rm new}} \simeq \sqrt{2}$.

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[3] Alan W. Greynolds and Gary L. Peterson, "Scatter Analysis of the Basic LIGO Configuration," Breault Research Organization, Inc. Report #2040 (August 28, 1992); and associated unpublished calculations by Stan Whitcomb and Rainer Weiss (Caltech and MIT, 1992).

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