## Old Quantum Theory (1900–1925)

$E = hf = \hbar\omega$ $\quad p = \dfrac{h}{\lambda} = \hbar k$ $\quad$ Bohr-Sommerfeld Quantization Rule : $\displaystyle\oint_{\text{one period}} p_q \cdot dq = n_q h$

## Probability

for a probability density $P(x)$ : mean $\langle x\rangle = \int_{x_{\min}}^{x_{\max}} P(x)\, x\, dx$ , variance $\sigma_x^2 \equiv \left\langle \left(x - \langle x\rangle\right)^2 \right\rangle = \langle x^2\rangle - \langle x\rangle^2$ , $\sigma_x \equiv$ standard deviation

Gaussian probability density: $P(x;\, x_0, \sigma) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-(x-x_0)^2/2\sigma^2}$

## Wave Mechanics

probability density $P(x,t)$ or $\rho(\vec{r},t)$ $= \Psi^*\Psi$ $\quad$ prob. current density $\vec{j}(\vec{r},t) = \mathrm{Re}\left[\Psi^* \dfrac{\hat{\vec{p}}}{m}\Psi\right]$ $\quad$ continuity equation : $\vec{\nabla}\cdot\vec{j} = -\dfrac{\partial\rho}{\partial t}$

Schrödinger equation $\quad \hat{E}\Psi = \hat{H}\Psi$ with $\hat{H} = \dfrac{\hat{p}^2}{2m} + V \quad \rightarrow \quad -\dfrac{\hbar^2}{2m}\dfrac{\partial^2\Psi}{\partial x^2} + V\Psi = i\hbar\dfrac{\partial\Psi}{\partial t}$ in 1D

operators: $\hat{E} = i\hbar\dfrac{\partial}{\partial t}$ , $\hat{p}_x = \dfrac{\hbar}{i}\dfrac{\partial}{\partial x}$ , $\hat{x} = x$ $\quad$ expectation value $\langle Q\rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{Q}\, \Psi\, dx$

boundary conditions on wavefunctions:

a. Wavefunctions are always **continuous.**

b. Wavefunctions have **continuous derivatives**, **except** at points where $\boldsymbol{V = \pm\infty}$

where $\displaystyle\lim_{\varepsilon\to 0} \psi'(x+\varepsilon) - \psi'(x-\varepsilon) \;=\; \left(2m/\hbar^2\right) \lim_{\varepsilon\to 0}\int_{x-\varepsilon}^{x+\varepsilon} V(x)\psi(x)dx$

c. Wavefunctions are **zero** in any region where $\boldsymbol{V = \infty}$ .

## Miscellaneous Math

Gaussian Integrals

$$\int_{-\infty}^{+\infty} e^{-ax^2-bx}\, dx = \sqrt{\frac{\pi}{a}}\; e^{\frac{b^2}{4a}} \qquad \int_{-\infty}^{+\infty} x\, e^{-ax^2-bx}\, dx = -\frac{\sqrt{\pi}\, b}{2a^{3/2}}\; e^{\frac{b^2}{4a}} \qquad \int_{-\infty}^{+\infty} x^2\, e^{-ax^2-bx}\, dx = \frac{\sqrt{\pi}}{4a^{5/2}}\left(2a+b^2\right)e^{\frac{b^2}{4a}}$$

Exponential Integrals

$$\int e^{-ax}\, dx = -\frac{e^{-ax}}{a} \qquad \int x\, e^{-ax}\, dx = -\frac{e^{-ax}}{a^2}(ax+1) \qquad \int x^2\, e^{-ax}\, dx = -\frac{e^{-ax}}{a^3}\left(a^2x^2 + 2ax + 2\right)$$

Sinusoidal Integrals

$$\int_0^{\pi} \begin{matrix}\sin^2(a\phi)\\ \cos^2(a\phi)\end{matrix}\; d\phi = \frac{\pi}{2} - \frac{\sin(2\pi a)}{4a} \qquad \int_0^{\pi} \begin{matrix}\sin(n\phi)\sin(m\phi)\\ \cos(n\phi)\cos(m\phi)\end{matrix}\; d\phi = \delta_{nm} \qquad \int_0^{\pi} \sin(n\phi)\cos(m\phi)d\phi = 0$$

Fourier Integrals

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} A(k)e^{ikx}\, dk \quad \text{where} \quad A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} f(x)e^{-ikx}\, dx$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{ik_1x}e^{-ik_2x}\, dx = \delta\left(k_1 - k_2\right) = \begin{cases} 0 & \text{if } k_1 \neq k_2 \\ \infty & \text{if } k_1 = k_2 \end{cases}$$

## Classical Mechanics security blanket ☺

$L\left(q_i, \dot{q}_i, t\right) = T - U$ , $\quad$ Lagrange EOM : $\dfrac{\partial L}{\partial q_i} = \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{q}_i}\right)$

$H \equiv \dot{q}_i(\partial L/\partial \dot{q}_i) - L \qquad dH/dt = -\partial L/\partial t$

Common Forces : $F_{\text{grav}} = \dfrac{Gm_1m_2}{r^2}$ , $F_{\text{elec}} = \dfrac{q_1q_2}{4\pi\varepsilon_0 r^2}$ , $F_{\text{cf}} = \dfrac{mv^2}{r}$

Generalized momentum $p_i \equiv \dfrac{\partial L}{\partial \dot{q}_i}$ , force $Q_i \equiv \dfrac{\partial L}{\partial q_i}$

Hamilton's EOM: $-\dfrac{\partial H}{\partial q_i} = \dfrac{dp_i}{dt}$ , $\dfrac{\partial H}{\partial p_i} = \dfrac{dq_i}{dt}$

Special Relativity: $E^2 = (pc)^2 + (mc^2)^2$

$\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}}$ , $E = \gamma mc^2$ , $p = \gamma mv$ , $v = \dfrac{pc^2}{E}$