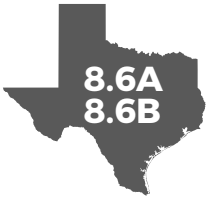
MODELING VOLUME: CYLINDERS AND CONES

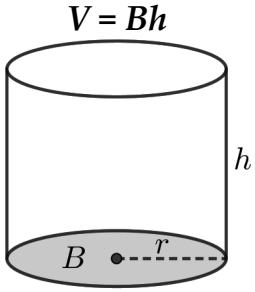


The student is expected to describe the volume formula *V = Bh* of a cylinder in terms of its base area and its height.

The student is expected to model the relationship between the volume of a cylinder and a cone having both congruent bases and heights and connect that relationship to the formulas.

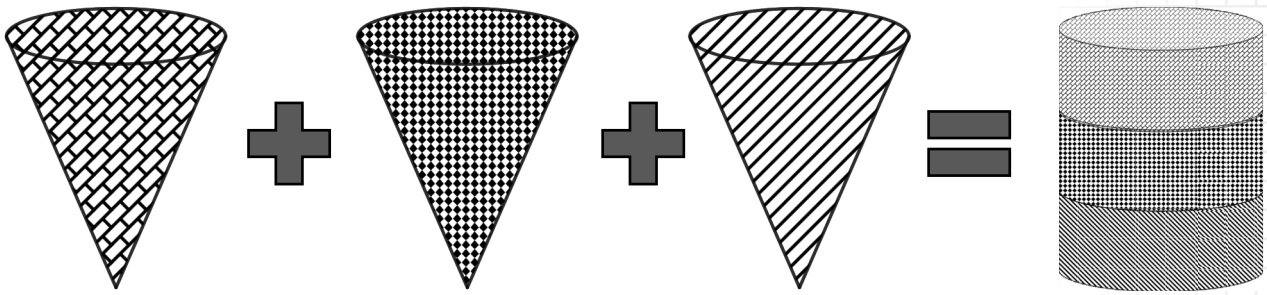
TELL ME MORE...

The **volume** of a figure such as a **prism** or **cylinder** is the amount of space that the figure occupies. As you learned in previous grades, the volume of a prism is the product of the area of the base of the prism and the height of the prism. You can use the volume formula, $V = Bh$, to calculate the volume where B represents the area of the base of the prism and h represents the height of the prism.



The same relationship is true for cylinders. A cylinder, like a prism, has two parallel and congruent bases that are joined with a lateral surface. The volume of a cylinder is the product of the area of the base and the height of the cylinder. But in the case of a cylinder, the base is a circle, so the area of the base is calculated using the formula $B = \pi r^2$, where r represents the radius of the circle creating the base of the cylinder.

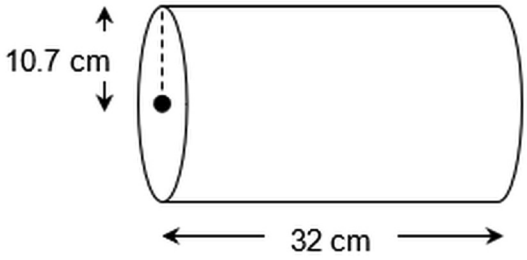
A **cone**, like a cylinder, has a circular base. However, instead of a congruent and parallel second base, the surface of a cone comes to one point called the apex or vertex of the cone. If the height of a cone and cylinder are the same and if the bases of a cone and cylinder are congruent, then the volume of three cones is equal to the volume of one cylinder.



$V = \frac{1}{3}Bh$ $V = \frac{1}{3}Bh$ $V = \frac{1}{3}Bh$ $V = Bh$

EXAMPLES

EXAMPLE 1: A cylinder and its dimensions are shown in the diagram. Write an equation that can be used to find V, the volume of the cylinder in cubic centimeters.



STEP 1 The volume of a cylinder is found using the formula, $V = Bh$. Write an equation for the area of the base of the cylinder. The base of a cylinder is a circle, so use the formula for the area of a circle, $B = \pi r^2$.

- The radius of the base of the cylinder is given in the diagram as 10.7 centimeters.
- Substitute $r = 10.7$ into the area formula.

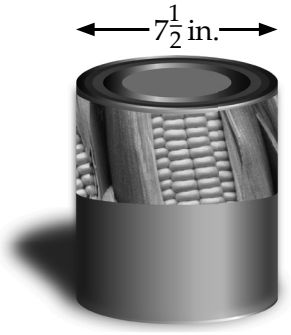
$\boldsymbol{B = \pi(10.7)^2}$

STEP 2 Substitute $B = \pi(10.7)^2$ into the volume formula, $V = Bh$.

- The height of the cylinder is given in the diagram as 32 centimeters.
- Substitute $B = \pi(10.7)^2$ and $h = 32$ into the volume formula.

$\boldsymbol{V = \pi(10.7)^2(32)}$

EXAMPLE 2: A family-sized can of corn is in the shape of a cylinder and has a diameter of $7\frac{1}{2}$ inches. Write an expression that can be used to determine B, the area of the base of this can in square inches.



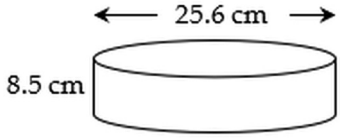
STEP 1 Write an equation for the area of the base of the cylinder. The base of a cylinder is a circle, so use the formula for the area of a circle, $B = \pi r^2$.

- The diameter of the base of the cylinder is given in the diagram as $7\frac{1}{2}$ inches.
- The area formula for a circle requires the *radius*, r, of the circle.
- The radius of a circle is $\frac{1}{2}$ the diameter: $r = \frac{1}{2}d = \frac{1}{2}\left(7\frac{1}{2}\right) = 3\frac{3}{4}$ in.
- Substitute $r = 3\frac{3}{4}$ into the area formula.

$\boldsymbol{B = \pi\left(3\frac{3}{4}\right)^2}$.

YOU TRY IT!

A cylinder and its dimensions are shown in the diagram. Write an equation that can be used to find V, the volume of the cylinder in cubic centimeters.



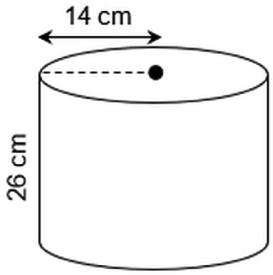
Radius of Base, r = _____

Area of Base, B = _____

Volume of Cylinder, $V = Bh$

V = _____(_____)
B h

EXAMPLE 3: A cone has a base and height that are congruent to the cylinder shown. Write an equation that you could use to determine the volume of the cone.



STEP 1 The volume of a cylinder is found using the formula, $V = Bh$. The volume of a cone with a congruent base and height is found using the formula, $V = \frac{1}{3}Bh$. Write an equation for the area of the base of the cone. The base of a cone is a circle, so use the formula for the area of a circle, $B = \pi r^2$.

- The radius of the base of the cone is the same as the radius of the cylinder given in the diagram as 14 centimeters.
- Substitute $r = 14$ into the area formula.

$\boldsymbol{B = \pi(14)^2}$

STEP 2 Substitute $B = \pi(14)^2$ into the volume formula, $V = \frac{1}{3}Bh$.

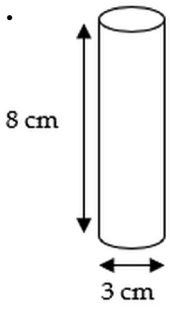
- The height of the cone is the same as the height of the cylinder given in the diagram as 26 centimeters.
- Substitute $B = \pi(14)^2$ and $h = 26$ into the volume formula.

$\boldsymbol{V = \frac{1}{3}\pi(14)^2(26)}$

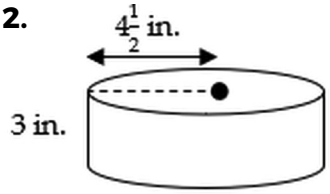
PRACTICE

For questions 1-3, write an equation that could be used to determine the volume of the given figure.

1.



2.



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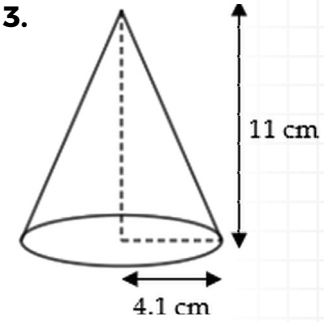
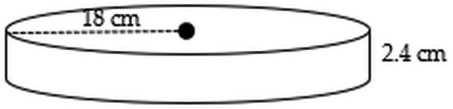
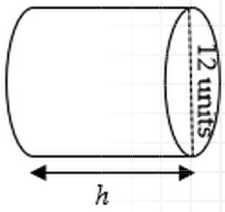


4. The volume of a cylinder can be determined using the formula $V = Bh$ where B is the area of the base of the cylinder and h is the height. Write an equation that can be used to find the value of B, in square centimeters, for the cylinder shown below?

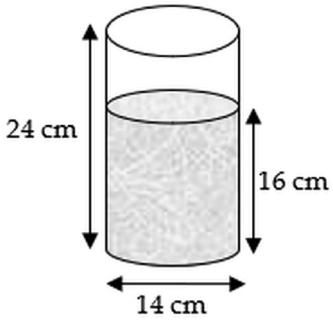


5. Use the formula $V = Bh$, where B is the area of the base of the cylinder and h is the height, to write an equation that can be used to determine the height of the cylinder.



Use the following information to answer questions 6-8.

The cylindrical drinking glass shown has a diameter of 14 centimeters, a height of 24 centimeters, and is partially filled with water. The glass with its dimensions is shown below.

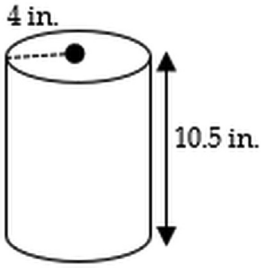


6. What equation can be used to determine *B*, the area of the base of the drinking glass?

7. What equation can be used to determine the volume of water in the glass?

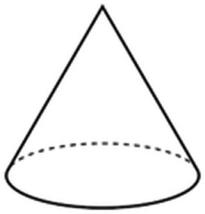
8. What equation can be used to determine the maximum possible volume of the glass?

9. A coffee can is in the shape of a cylinder with a radius of 4 inches and a height of 10.5 inches.



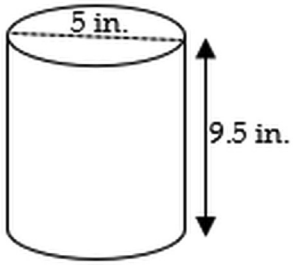
If a cone has the same radius and height as the coffee can, what equation describes the volume of the cone?

10. The cone shown below has a volume that can be determined using the equation $V = \frac{\pi(2)^2 5}{3}$.



If a cylinder has the same radius and height as the cone shown, what equation represents the volume of the cylinder?

11. Michelle eats oatmeal each morning for breakfast. Her favorite brand comes in a cylindrical container. The dimensions of the container are shown below.



What equation can be used to find *V*, the volume of the oatmeal container in cubic inches?

A $V = \pi \times (5)^2 \times (9.5)$

B $V = \pi \times (9.5)^2 \times (2.5)$

C $V = \pi \times (2.5)^2 \times (9.5)$

D $V = \pi \times (9.5)^2 \times (5)$