RLC Circuits - Part 3C.

My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis. Source of study material: 1). Electric Circuits 6th Ed., Nahvi & Edminister. 2). Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill. Resource: 3). Solutions & Problems of Control Systems, 2nd ed - AK Jairath.

Karl S. Bogha.

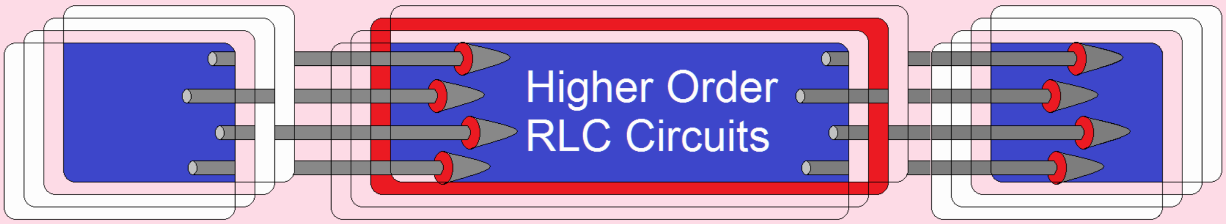
RLC Higher Order Circuits Fully Solved Examples and Problems.

1). Section 1: Schaums Electric Circuits - 90 Pages
2). Section 2: Transfer Functions - 40 Pages
3). Section 3: One Discussion Bonus Problem - 20 Pages

Total: 150 Pages.

Level: Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



Part 3 - C

(Intermediate Level)

Note: Number of solved examples and problems are high.
They try to cover as much areas of the RLC higher order circuits.
Any errors the student/engineer should be able to correct with little effort.
Section 1 solved examples and problems correspond to Part 3B notes.
Section 2 solved example are the same for the transfer functions in RLC circuits though these are from a Controls textbook.
Section 3 problem is a discussion question the solution provided is more toward looking at the solution from multiple perspectives.

Apologies for any errors and omissions.

October 2020.

Engineering college year 2 course of 4 year program OR year 1 of 3 year program. Re-fresher OR Self Study. Graduate Study Review.
May be used in New Zealand, US, Malaysia, India, Pakistan, UK, and other Common Wealth Country engineering colleges.
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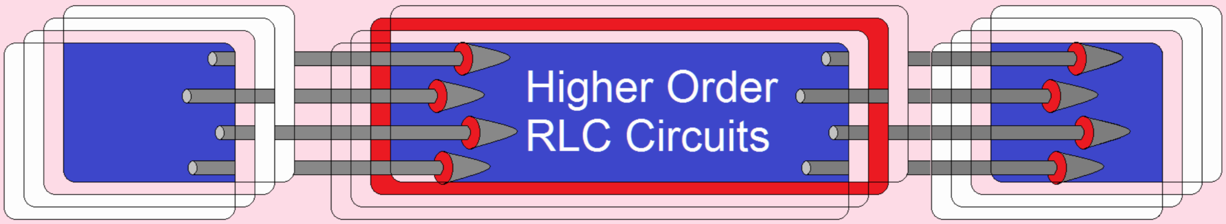
SECTION ONE.

Level: Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



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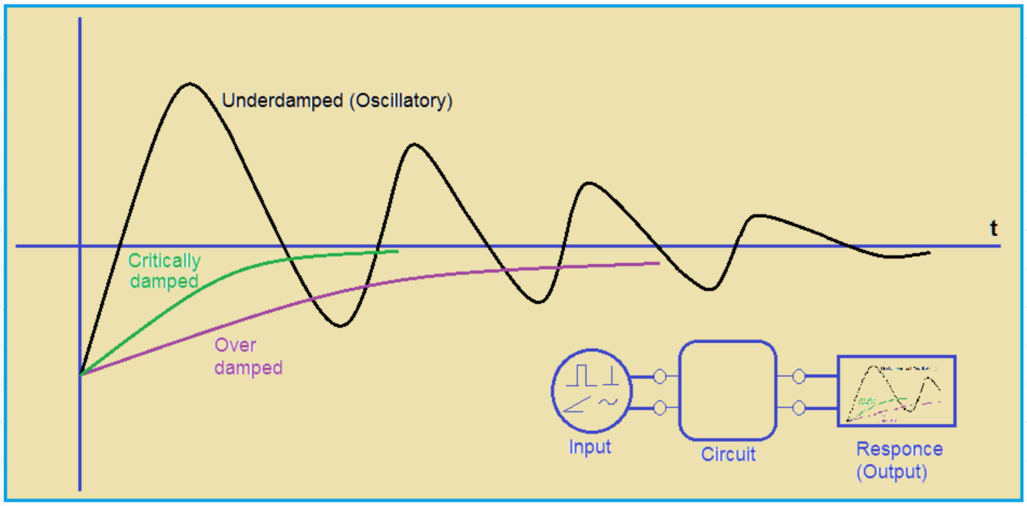
Copy-paste from Part 3A with addition.

Under damped (Oscillatory), Critically damped, and Over damped:

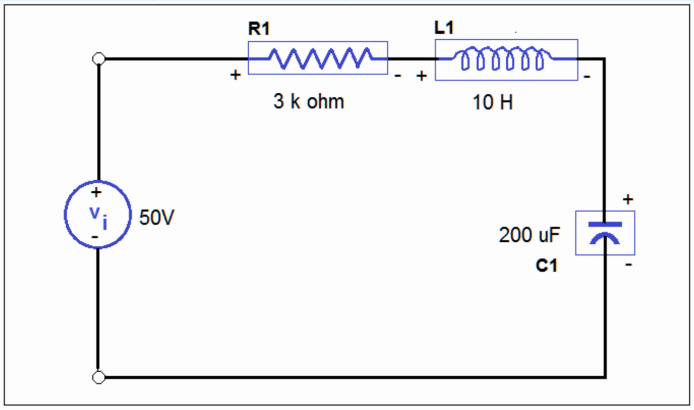


	Series: Current Series RLC	Parallel: Voltage Parallel RLC	
Under damped: (Oscillatory)	$\alpha < \omega_0$	$\alpha^2 < \omega_0^2$	
	$\beta = \sqrt{\omega_0^2 - \alpha^2}$	$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	Damped radian frequency
	$i(t) = e^{-\alpha t}\left(A_1\cos(\beta t) + A_2\sin(\beta t)\right)$	$v(t) = e^{-\alpha t}\left(A_1\cos(\omega_d t) + A_2\sin(\omega_d t)\right)$	
Critically damped:	$\alpha = \omega_0$	$\alpha = \omega_0$	
	$i(t) = e^{-\alpha t}\left(A_1 + A_2 t\right)$	Blank. *Check elsewhere if needed.*	
Over damped:	$\alpha > \omega_0$	$\alpha^2 > \omega_0^2$	
	$\beta = \sqrt{\alpha^2 - \omega_0^2}$	$\beta = \sqrt{\alpha^2 - \omega_0^2}$	
	$i(t) = e^{-\alpha t}\left(A_1 e^{\beta t} + A_2 e^{-\beta t}\right)$	$v(t) = \left(A_1 e^{s_1 t} + A_2 e^{s_2 t}\right)$	
α :	$\left(\frac{R}{2\,L}\right)$	$\left(\frac{1}{2\,RC}\right)$	
ω_0 :	$\left(\frac{1}{\sqrt{LC}}\right)$	$\left(\frac{1}{\sqrt{LC}}\right)$	

Please check with your textbook.

Problem 8.1:

A series RLC circuit with R = 3 k ohm, L = 10 H, and C = 200uF, has a constant voltage source, V = 50V, applied at t=0.



a). Obtain the current transient, if the capacitor has no initial charge?
b). Plot the current?
c). Find the time at which current is maximum from the plot?

Solution:

$R := 3000 \qquad L := 10$

$C := 200 \cdot 10^{-6}$

$\alpha \;=\; \dfrac{R}{2 \cdot L} = 150 \qquad$ s^1.

$\omega \;=\; \dfrac{1}{\sqrt{L \cdot C}} = 22.3606798$

Case	Series RLC	Parallel RLC
Under damped: (Oscillatory)	$\alpha < \omega_0$	$\alpha^2 < \omega_0^2$
Critically damped:	$\alpha = \omega_0$	$\alpha = \omega_0$
Over damped:	$\alpha > \omega_0$	$\alpha^2 > \omega_0^2$
α :	$\left(\dfrac{R}{2\,L}\right)$	$\left(\dfrac{1}{2\,RC}\right)$
ω_0 :	$\left(\dfrac{1}{\sqrt{LC}}\right)$	$\left(\dfrac{1}{\sqrt{LC}}\right)$

Alpha > Omega; 150>22.3, the series RLC circuit is over damped.

$\beta \;=\; \sqrt{\left(a^2\right) - \left(\omega_0^{\,2}\right)}$ <--- Over damped case $\qquad \beta := \sqrt{150^2 - 22.36^2} = 148.3240722$

s1 = - alpha + beta $\qquad$ s2 = - alpha - beta

$s1 := -150 + 148.3 = -1.7 \qquad s2 := -150 - 148.3 = -298.3$

Summary:

1). Decide if its a series or parallel RLC circuit.
2). This leads to selecting the correct alpha.
3). Next calculate omega0.
4). We have alpha and omega0.
Next meet one of the 3 conditions:
5). alpha > omega0 — circuit is overdamped
solve for s1 and s2
natural response fn(t) = A1e^(s1t) + A2e^(s2t)
6). alpha = omega0 — circuit is critically damped
solve for s1 and s2
natural response fn(t) = e^(-alpha)t (A1t + A2)
7). alpha < omega0 — circuit is underdamped
solve for s1 and s2
natural response is:
fn(t) = e^(-alpha)t (A1(cos (wd)t + A2(sin (wd)t)
where (wd) = sqrt(w0^2 - alpha^2)

<---- We had these summary notes in Part 3A page 61. Note: These equations for natural response NOT forced response.

From our previous notes or your textbook, the form of equation we apply is shown in note 5 in figure above.

$$i(t) = A1e^{s1t} + A2e^{s2t}$$

$$i(t) = A1e^{-1.70\,t} + A2e^{-298.3\,t}$$

Next, obvious, we need to solve for coefficients A1 and A2.

What comes to mind? Continuity Condition.

Our circuit was off during t<0.
So no energy built up or storage is found in the inductor and capacitor.

$iL(-0) = 0$ ---> $iL(0) = 0$ ---> $iL(0+.) = 0$

Here at 0+ the inductor is building up current and just past 0 at a low mili or micro second the current will be almost 0 which is practically 0.

In this circuit same for capacitor C:

$iC(-0) = 0$ ---> $iC(0) = 0$ ---> $iC(0+.) = 0$

For our first equation at t=0, plug t=0 in the equation.

$$i(0) = A1e^{-1.70\,(0)} + A2e^{-298.3\,(0)}$$

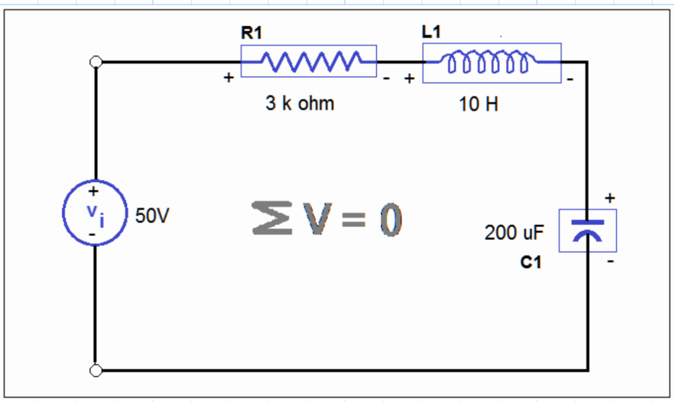
$$0 = A1 + A2 \qquad \text{Eq 1.}$$

Recall we used to take the derivative of the equation, in this circuit we have inductor expression with a first order derivative L(di/dt) that equalled some voltage.

What could this voltage be across the inductor in a series circuit?

I dont know. Each component has a voltage across and their sum equal 50V. However, from my past exercise and theory from part 3A or B, we know THE MATH may provide that solution. We did something were we found di/dt or dv/dt.

We take the equation we have started with i(t) and differentiate it.
The LHS becomes di/dt.
We start with a Voltage Loop Equation (KVL for most - Kickoutt's Voltage Law).
We have V = 50V.



We cant really form an equation because we cannot determine the voltage across each component as we do not have a value for current.

TRUE.

However, I can do a voltage at t=0+, meaning just micro or nano second past t=0, here I can say the voltage is building up and at this time its zero across some or all components.

KVL at t(0+) :

$$R \cdot i(t) + \frac{1}{C}\int i(t)\,dt + L\left(\frac{di}{dt}\right) \quad = \quad V$$

$$0 + 0 + L\left(\frac{di}{dt}\right) \quad = \quad V$$

The reasoning or logic here is the resistor is not gained any voltage the current is too low, the capacitor integral results in same near zero for t=0, both the R and C are practically? zero. However I can form an expression for (di/dt) thru the inductor since the the equation above on the RHS equal V which here is 50V. Merely taking the derivative of current though the current is too low.

$$0 + 0 + L\left(\frac{di}{dt}\right) \quad = \quad 50$$

We just somehow or rather, if you feel comfortable with that way of looking at it, get the expression di/dt solved.

$$0+0+L\left(\frac{di}{dt}\right) = 50$$

$$10\left(\frac{di}{dt}\right) = 50$$

$$\left(\frac{di}{dt}\right) = 5$$

$i(t) = A1e^{-1.70\,t} + A2e^{-298.3\,t}$ Our current expression

We differentiate it:

$\frac{di}{dt} = -1.7\;A1\;e^{-1.70\,t} - 298.3\;A2^{-298.3\,t}$ We plug in (di/dt)

$5 = -1.7\;A1\;e^{-1.70\,t} - 298.3\;A2^{-298.3\,t}$

$t = 0$

$5 = -1.7\;A1\;e^{-1.70\,(0)} - 298.3\;A2^{-298.3\,(0)}$

Next set this equation for t=0, which I question why we take it for t=0 rather than t=0+? Its t=0+, but in the math expression wise e^0 = 1. We normally do not go further in this case to say e^0.00000000...1
Not usually, but you are correct in asking that question. e^0.000000000...1 = 1. Solved that.

$5 = -1.7\;A1 - 298.3\;A2$ Eq 2.

$0 = A1 + A2$ Eq 1.

$Coeff := \begin{bmatrix} 1 & 1 \\ -1.7 & -298.3 \end{bmatrix}$ $RHS := \begin{bmatrix} 0 \\ 5 \end{bmatrix}$

$InvCoeff := Coeff^{-1} = \begin{bmatrix} 1.0057316 & 0.0033715 \\ -0.0057316 & -0.0033715 \end{bmatrix}$ $InvCoeff = \begin{bmatrix} 1.0057316 & 0.0033715 \\ -0.0057316 & -0.0033715 \end{bmatrix}$

$A1A2 := InvCoeff \cdot RHS = \begin{bmatrix} 0.0169 \\ -0.0169 \end{bmatrix}$

$A1 = 16.9 \cdot 10^{-3}$ Or 16.9 mA

$A2 = -16.9 \cdot 10^{-3}$ Or -16.9 mA

$i(t) = A1e^{-1.70\,t} + A2e^{-298.3\,t}$ Substitute A1 and A2

$i(t) = 16.9 \cdot e^{-1.70\,t} - 16.9 \cdot e^{-298.3\,t}$ mA Answer.

$i(t) = 16.9\left(e^{-1.70\,t} - e^{-298.3\,t}\right)$ mA Answer.

To find the time at which current is maximum we need some equation from which we can solve for that time.

How do we get that equation?

The equation $i(t) = 16.9 \cdot e^{-1.70\,t} - 16.9 \cdot e^{-298.3\,t}$ is current relative to time.

The 1st derivative of the equation i(t) gives us the expression providing the maximum current time t. Logic wise what is this? Current per time. Though I do not have a word for it, like velocity, it gives us the maximum value.
ITS THE MATH on the i(t) working for a solution.

At time t=0, i(t) = 0. LHS of equation =0.

$$0 = 16.9 \cdot e^{-1.70\,t} - 16.9 \cdot e^{-298.3\,t}$$

The derivative of above equation:

$$0 = (-1.7)\ 16.9 \cdot e^{-1.70\,t} - (-298.3)\ 16.9 \cdot e^{-298.3\,t}$$

$$-1.70 \cdot 16.9 = -28.73 \qquad -298.3 \cdot 16.9 = -5041.27$$

$$0 = -28.73 \cdot e^{-1.70\,t} + 5041.3 \cdot e^{-298.3\,t}$$ Answer.

Next we need to calculate the time t that gives the maximum current:
Use logarithm to solve for maximum current time t:

$$-28.73 \cdot e^{-1.70 \cdot t} + 5041.3 \cdot e^{-298.\,t} = 0 \qquad \frac{5.16749}{296.6} = 0.0174224$$

$$5041.3 \cdot e^{-298.\,t} = 28.73 \cdot e^{-1.70 \cdot t}$$

$$\frac{5041.3}{28.73} = \frac{e^{-1.70\,t}}{e^{-298.\,t}} \qquad 298.3 - 1.70 = 296.6$$

$$175.47 = e^{(-1.70\,+\,298.3)\,t} = e^{296.6\,t}$$

$$\ln(175.472) = 296.6 \cdot t \qquad \ln(175.473) = 5.16749$$

$$5.16749 = 296.6 \cdot t$$

$$t = \frac{5.16749}{296.6} = 0.01742$$ seconds. Answer.

t = 0.01742 seconds.

t = 17.2 ms - milliseconds.

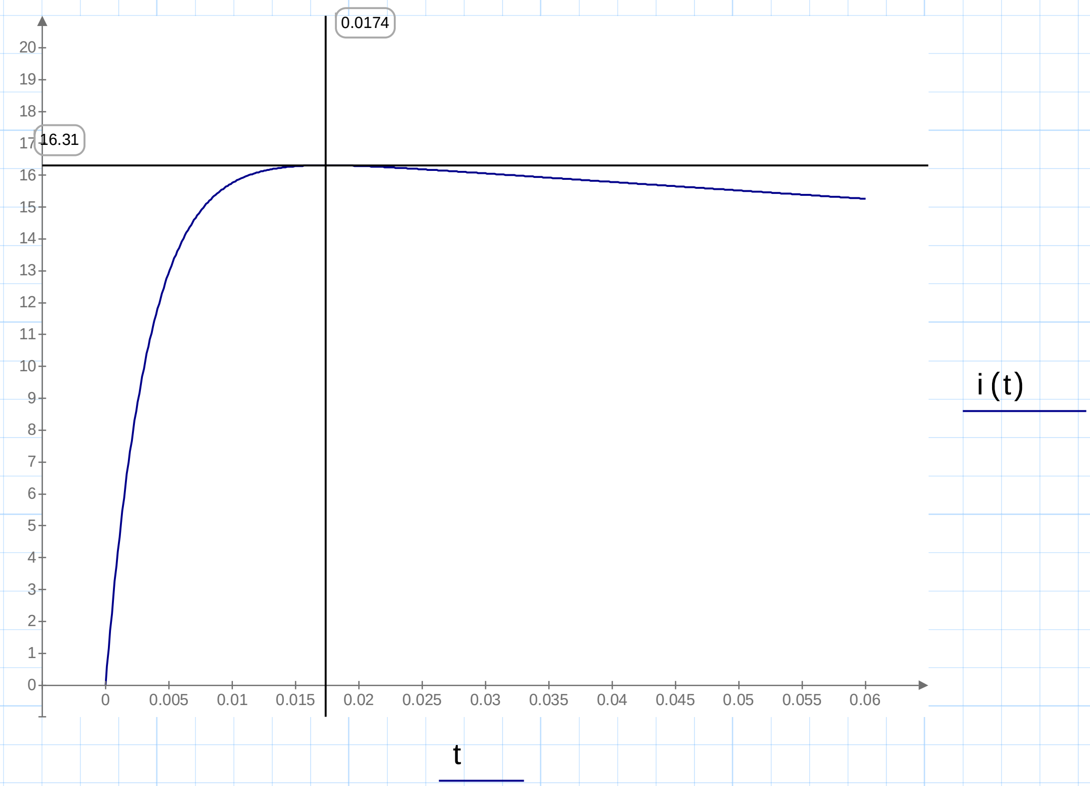
We got the time t for maximum current we plug this into the i(t) equation.
NOT the derivative equation but the i(t) equation.

Next plot.

Note: Plot time t in milliseconds. This from our early calculation for this circuit.

clear (t)

$i(t) := 16.9 \cdot e^{-1.70 \cdot t} - 16.9 \cdot e^{-298.3 \cdot t}$ mA. Note: vertical axis is in mA.

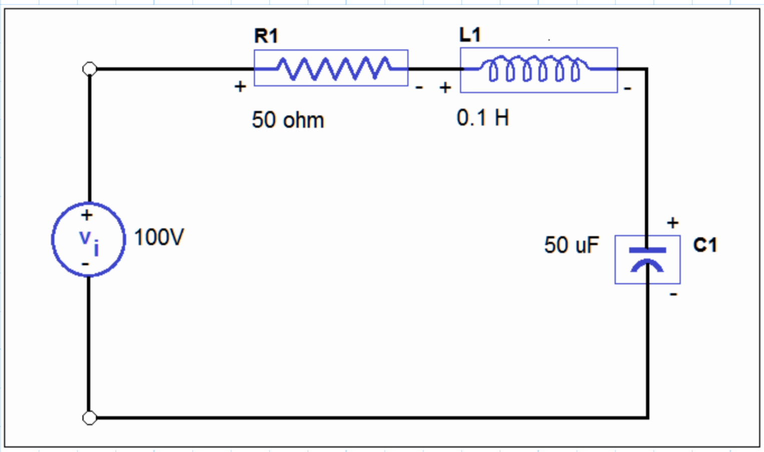


We got time at 17.4 ms and maximum current 16.31 mA. Answer.

Comments: Looked like an easy problem.
Required several different solution methods.
Rewarding for some.

Problem 8.2:

A series RLC circuit with R = 50 ohm, L = 0.1 H, and C = 50 uF,
has a constant voltage source, V = 100V, applied at t=0.



Obtain the current transient, assume zero initial charge charge on capacitor ?

Solution:

Case	Series RLC	Parallel RLC
Under damped: (Oscillatory)	$\alpha < \omega_0$	$\alpha^2 < \omega_0^2$
Critically damped:	$\alpha = \omega_0$	$\alpha = \omega_0$
Over damped:	$\alpha > \omega_0$	$\alpha^2 > \omega_0^2$
α :	$\left(\frac{R}{2L}\right)$	$\left(\frac{1}{2RC}\right)$
ω_0 :	$\left(\frac{1}{\sqrt{LC}}\right)$	$\left(\frac{1}{\sqrt{LC}}\right)$

$R := 50$ $\quad$ $L := 0.1$

$C := 50 \cdot 10^{-6}$

$\alpha = \frac{R}{2 \cdot L} = 250$ $\quad$ s^1.

$\omega = \frac{1}{\sqrt{L \cdot C}} = 447.2135955$

$\beta := \sqrt{250^2 - 447.21^2} = 370.8j$

Alpha < Omega; 250<447, the series RLC circuit is under damped.

Solution takes the form:

$i(t) \quad = \quad e^{-\alpha \cdot t} \cdot \left(A1e^{j\beta t} + A2e^{-j\beta t}\right)$

OR the more common sinusoidal form:

$i(t) \quad = \quad e^{-\alpha \cdot t} \cdot (A1\cos(\beta \cdot t) + A2\sin(\beta \cdot t))$ $\quad$ <---See summary in problem 8.1 for equation.

Roots to the equation above: $\quad s1 = \alpha + j\beta$

$s2 = \alpha - j\beta$

$$i(t) = e^{-250 \cdot t} \cdot (A1 \cdot \cos(370.8 \cdot t) + A2 \cdot \sin(370.8 \cdot t))$$

Circuit initial conditions:
No capacitor stored current to release in circuit since circuit was open for t<0.

Inductor condition at t=0 :

$$iL(0) = 0 = e^{-250 \cdot 0} \cdot (A1 \cdot \cos(370.8 \cdot 0) + A2 \cdot \sin(370.8 \cdot 0))$$

$$0 = A1 \quad \text{Eq 1.}$$

Capacitor has no initial charge means there is no contribution by the capacitor at time t=0+, near 0 on the positive t side of 0.
Leaving us the inductor L derivative term (di/dt) in L(di/dt) = vL.

KVL at t(0+) :

$$R \cdot i(t) + \frac{1}{C} \int i(t)\,dt + L\left(\frac{di}{dt}\right) = V$$

$$0 + 0 + L\left(\frac{di}{dt}\right) = V$$

$$0.1 \cdot \left(\frac{di}{dt}\right) = 100$$

$$\left(\frac{di}{dt}\right) = \frac{100}{0.1} = 1000 \quad \text{A/s.}$$

Next differentiate the equation i(t), and place the value of di/dt above at LHS:

$$i(t) = e^{-250 \cdot t} \cdot (A1 \cdot \cos(370.8 \cdot t) + A2 \cdot \sin(370.8 \cdot t))$$

$$= e^{-250 \cdot t} \cdot A1 \cdot \cos(370.8 \cdot t) + e^{-250 \cdot t} \cdot A2 \cdot \sin(370.8 \cdot t)$$

$$1000 = -250 \cdot e^{-250 \cdot t} \cdot A1 \cdot \cos(370.8 \cdot t) - e^{-250 \cdot t} \cdot 370.8\ A1 \cdot \sin(370.8 \cdot t) +$$

$$-250\ e^{-250 \cdot t} \cdot A2 \cdot \sin(370.8 \cdot t) + e^{-250 \cdot t} \cdot 370.8\ A2 \cdot \cos(370.8 \cdot t)$$

Now let t=0 and evaluate the equation above:

$$1000 = -250 \cdot A1 + 370.8\ A2 \quad \text{Eq 2.}$$

Substitute A1 = 0 in Eq 2.

$$1000 = -250 \cdot A1 + 370.8\ A2$$

$$1000 = 0 + 370.8\ A2$$

$$A2 = \frac{1000}{370.8} = 2.6968716$$

$$A1 = 0$$

$A2 = 2.7$ Substitute in i(t) equation.

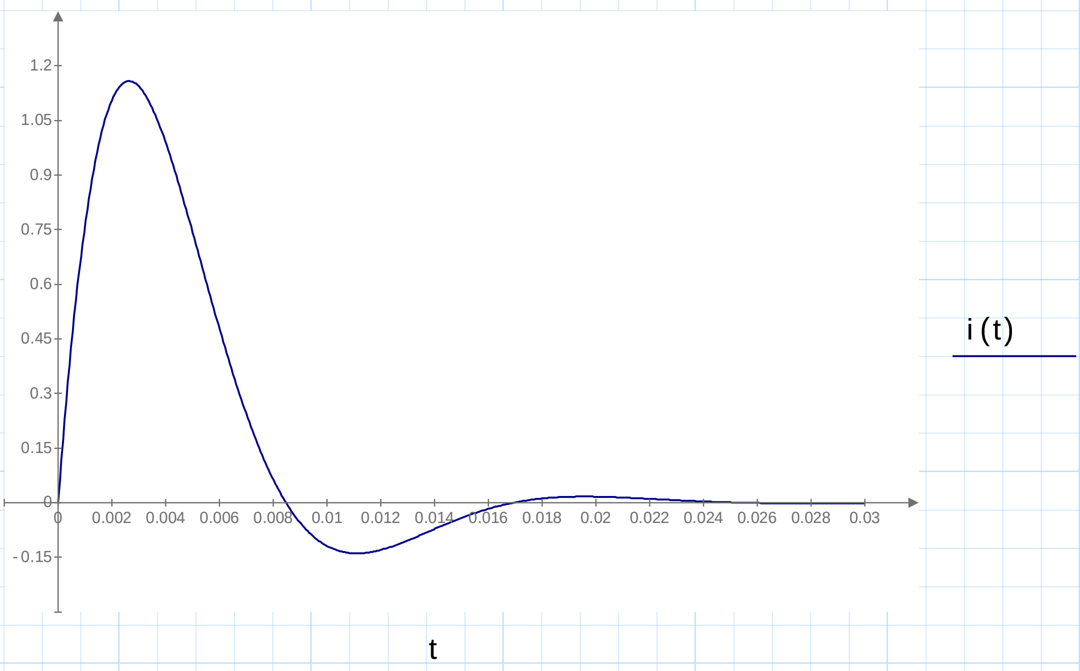
$$i(t) = e^{-250 \cdot t} \cdot A1 \cdot \cos(370.8 \cdot t) + e^{-250 \cdot t} \cdot A2 \cdot \sin(370.8 \cdot t)$$

$$i(t) = e^{-250 \cdot t} \cdot 0 \cdot \cos(370.8 \cdot t) + e^{-250 \cdot t} \cdot 2.7 \cdot \sin(370.8 \cdot t)$$

$$i(t) = e^{-250 \cdot t} \cdot 2.7 \cdot \sin(370.8 \cdot t)$$ A. Answer.

Plot of i(t):

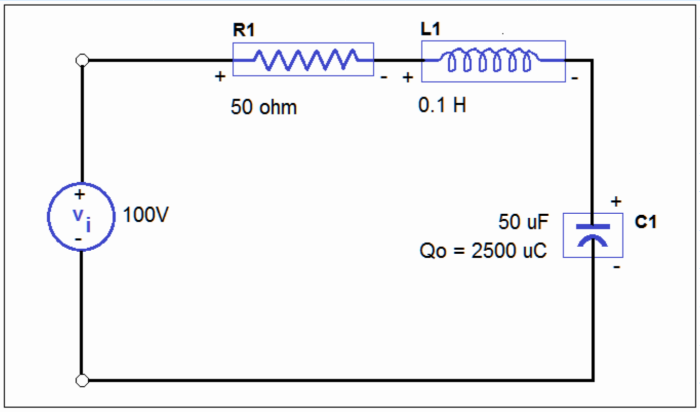
clear (t) $\quad i(t) := e^{-250 \cdot t} \cdot 2.7 \cdot \sin(370.8 \cdot t)$



Comments: Satisfied it is under damped.

Problem 8.3:

Rework Problem 8.2 with the capacitor now having an initial charge Qo = 2500uC.



Solution:

What the problem is stating is we have to consider an initial condition existing when the the circuit came ON at t=0. The condition is that the capacitor is charged to 2500 micro-coulombs. When the switch is closed, not shown, this capacitor also releases energy (charge per unit time ie current) into the circuit.

Capacitor Voltage:

$$Q_o := 2500 \cdot 10^{-6}$$

$$C := 50 \cdot 10^{-6} \qquad C = 0.00005$$

$$v_C := \frac{Q_o}{C} = 50$$

KVL :

$$R \cdot i(t) + \frac{1}{C}\int i(t)\,dt + L\left(\frac{di}{dt}\right) = V$$

$$0 + 50 + 0.1\left(\frac{di}{dt}\right) = 100$$

$$0.1\left(\frac{di}{dt}\right) = 100 - 50 = 50$$

$$\frac{di}{dt} = \frac{50}{0.1} = 500 \quad \text{A/s.}$$

We have a circuit with the same component values. Its one equation, one loop. Our current has reduced by half. In problem 8.2 it was 1000A/s, and in this problem 8.3 its 500A/s. Initial value is half, rest remains the same, so by linearity the solution would be halved for the amplitude, with the rest remaining the same.

$i(t) = e^{-250 \cdot t} \cdot 2.7 \cdot \sin(370.8 \cdot t)$ A. <--- We had this for problem 8.2. Next we half it. The math steps going forward like in problem 8.2 will do the same.

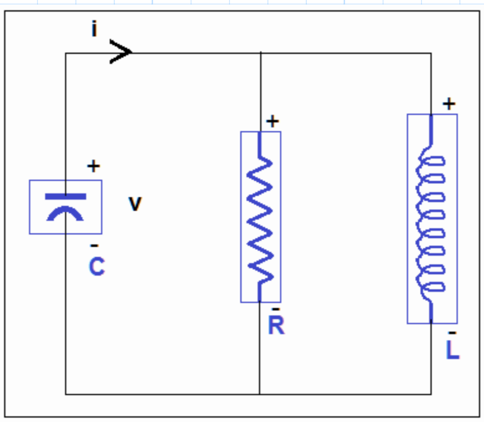
$$i(t) = e^{-250 \cdot t} \cdot \left(\frac{2.7}{2}\right) \cdot \sin(370.8 \cdot t)$$

$i(t) = e^{-250 \cdot t} \cdot 1.35 \cdot \sin(370.8 \cdot t)$ Answer.

Comment: Review problem 8.2, the results here were impacted by half the current per second ie 500 A/s. All other values were the same. Halving from 1000 to 500 halved the amplitude. The 370j was calculated prior to the initial condition, same for the exponential term e^-250t.

Problem 8.4:

A parallel RLC network with R=50 ohm, C=200 uF, and L=55.6 mH, has an initial charge Qo = 5.0 mC on the capacitor.
Obtain the expression for the voltage across the network.



$R := 50$

$C := 200 \cdot 10^{-6}$

$L := 55.6 \cdot 10^{-3}$

$Q_0 := 5 \cdot 10^{-3}$ Coulomb

Solution:
Refer to Example 4 (page 24) and 5 (page 31) for some ideas on this solution in the file Chapter 6 Part 3A.

Case	Series RLC	Parallel RLC
Under damped: (Oscillatory)	$\alpha < \omega_0$	$\alpha^2 < \omega_0^{\ 2}$
Critically damped:	$\alpha = \omega_0$	$\alpha = \omega_0$
Over damped:	$\alpha > \omega_0$	$\alpha^2 > \omega_0^{\ 2}$
α :	$\left(\frac{R}{2\,L}\right)$	$\left(\frac{1}{2\,RC}\right)$
ω_0 :	$\left(\frac{1}{\sqrt{LC}}\right)$	$\left(\frac{1}{\sqrt{LC}}\right)$

Chart on series and parallel and the three cases or conditions.

$\alpha := \dfrac{1}{2 \cdot R \cdot C} = 50$ 1/s $\qquad \alpha^2 = 2500$ 1/s^2

$\omega_0 := \dfrac{1}{\sqrt{L \cdot C}} = 299.880072$ 1/s^2 $\qquad \omega_d = \sqrt{(\omega_0)^2 - (\alpha)^2} = 295.6823592$ rad/s

$\omega_d = 296$ rad/s

Parallel circuit wd instead of Beta B, for underdamped - sinusod wave.

Alpha < Omega, so its under damped.
Most likely I/We use the? Sinusoidal expression for the voltage v(t).
And if so this would results in a sinusoidal expression for i(t).

Solution takes the form: $v(t) = e^{-\alpha \cdot t} \cdot \left(A1 e^{j\beta t} + A2 e^{-j\beta t}\right)$

OR the more common sinusoidal form:

$v(t) = e^{-\alpha \cdot t} \cdot (A1 \cos(\beta \cdot t) + A2 \sin(\beta \cdot t))$

Roots to the equation above: $s1 = \alpha + j\beta$

$s2 = \alpha - j\beta$

$v(t) = e^{-50 \cdot t} \cdot (A1 \cos(296 \cdot t) + A2 \sin(296 \cdot t))$

Since this is a parallel circuit, voltage across capacitor is the voltage for the circuit at time t=0 when the circuit comes on. Capacitor initial condition.
Calculate voltage across capacitor :

$v_C := \dfrac{Q_0}{C} = 25$ V. Next at time t=0, and solve for maybe a coefficient.

$25 = e^{-50 \cdot t} \cdot (A1 \cos(296 \cdot t) + A2 \sin(296 \cdot t))$

$25 = 1\ A1 \cos(0) + A2 \sin(0)$

$25 = A1$ Now updating the voltage response v(t).

$25 = e^{-50 \cdot t} \cdot (25 \cos(296 \cdot t) + A2 \sin(296 \cdot t))$

We want to solve for A2, that would make the solution.
We take the derivative of v(t),
like we did in previous problem taking the derivative of i(t).

$$e^{-\alpha \cdot t} \cdot (A1\cos(\beta \cdot t) + A2\sin(\beta \cdot t))$$

$$e^{-\alpha \cdot t} \cdot A1\cos(\beta \cdot t) + e^{-\alpha \cdot t} \cdot A2\sin(\beta \cdot t)$$ Its the same 1st derivative.

$$\left(-\alpha \cdot e^{-\alpha \cdot t} \cdot A1\cos(\beta \cdot t) - e^{-\alpha \cdot t} \cdot \beta \cdot A1\sin(\beta \cdot t)\right) - \left(\left(\alpha \cdot e^{-\alpha \cdot t} \cdot A2\sin(\beta \cdot t)\right) + e^{-\alpha \cdot t} \cdot \beta \cdot A2 \cdot \cos(\beta \cdot t)\right)$$

Rearrange like terms; alpha and beta:

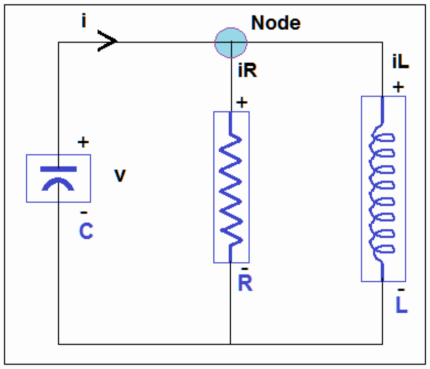
$$-\alpha \cdot e^{-\alpha \cdot t} \cdot (A1\cos(\beta \cdot t) + A2\sin(\beta \cdot t)) + \beta \cdot e^{-\alpha \cdot t} \cdot (-A1 \cdot \sin(\beta \cdot t) + A2 \cdot \cos(\beta \cdot t))$$

Review example 4 page 24 Part 3A for the derivative of voltage on the LHS.

In this circuit parallel RLC, voltage seen across the circuit would be the capacitor initial condition voltage. At t=0, the voltage across the inductor would be zero.
We have a voltage across the resistor.

We seek dv/dt. This can be obtained thru the node equation.

$$i_C(t) = i_R(t) + i_L(t)$$



At this time iL(t) is out its near zero.
We have a series circuit RC for a short while.

Current equation:

$$C\left(\frac{dv}{dt}\right) + \frac{V_o}{R} = 0$$

$$C\left(\frac{dv}{dt}\right) = -\left(\frac{V_o}{R}\right)$$

$$\left(\frac{dv}{dt}\right) = -\left(\frac{V_o}{R}\right) \cdot \left(\frac{1}{C}\right) = \frac{-V_0}{RC} = \frac{-25}{R \cdot C} = -2500\ V/s$$

Next we place dv/dt in the LHS of equation.

$$-2500 = -\alpha \cdot e^{-\alpha \cdot t} \cdot (A1\cos(\beta \cdot t) + A2\sin(\beta \cdot t)) + \beta \cdot e^{-\alpha \cdot t} \cdot (-A1 \cdot \sin(\beta \cdot t) + A2 \cdot \cos(\beta \cdot t))$$

Set t = 0, sine terms equal zero leaving the cosine terms:

$$-2500 = -50 \cdot e^{-50 \cdot 0} \cdot 25\cos(0) + 296 \cdot e^{-50 \cdot 0} \cdot A2 \cdot \cos(0)$$

$$-2500 = -1250 + 296\ A2$$

$$296\ A2 = -2500 + 1250 = -1250$$

$$A2 := \frac{-1250}{296} = -4.222973$$

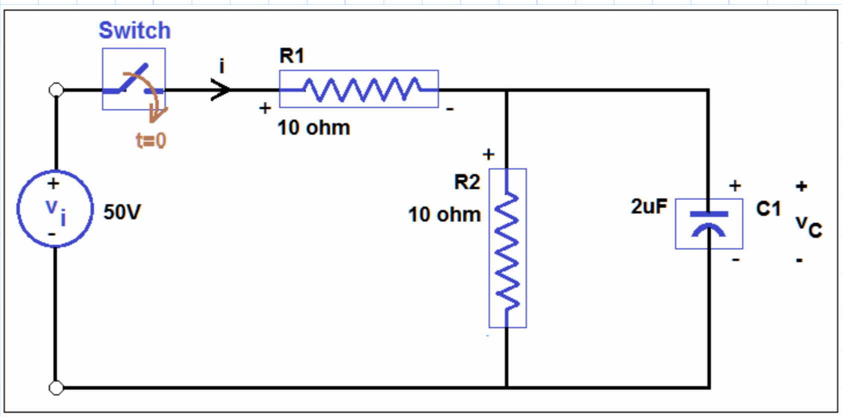
Now we plug in A1 and A2 in the voltage v(t) equation for the solution.

$$v(t) = e^{-50 \cdot t} \cdot (A1\cos(296 \cdot t) + A2\sin(296 \cdot t))$$

$$v(t) = e^{-50 \cdot t} \cdot (25 \cdot \cos(296 \cdot t) - 4.233 \cdot \sin(296 \cdot t))$$ V. Answer.

Problem 8.5 (Two Mesh circuit) :

In the circuit below switch is closed at t=0.
Obtain the current i(t) and the capacitor voltage vC(t) ?



$R1 := 10$

$R2 := 10$

$C1 := 2 \cdot 10^{-6}$

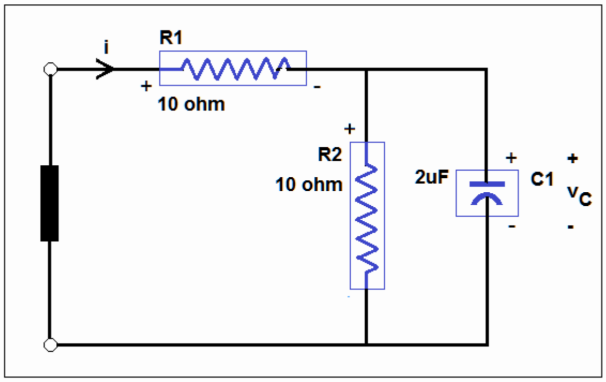
$V_i := 50$

Solution:

The notes relevant to solution may be found on page 36 of Chapter 6 Part 3A. However, here the engineers provides another method to solving it, compared to the differential-integral equation form of two mesh circuit. *From my initial look over it loos like sort of part inspection and a good understanding of initial conditions, solution looks challenging to me.*

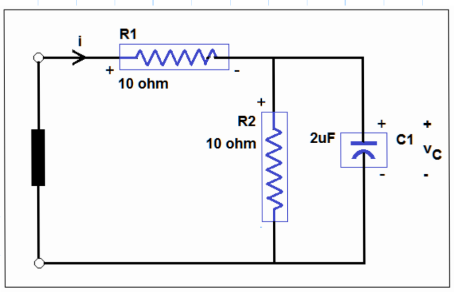
We have the steady state response with voltage source V=50V, and the transient or natural response with no voltage source ie 0V with short circuit at the voltage source. Our solution maybe the combination of both (complete solution).

Transient / Natural response :



The voltage source is pure conductor all voltage, and no resistance, so it can be modelled as a short circuit.
This allows for R1 and R2 be computed for equivalent resistance.

$$R_{eq} := \frac{R1 \cdot R2}{(R1 + R2)} = 5$$ R1 and R2 are parallel.



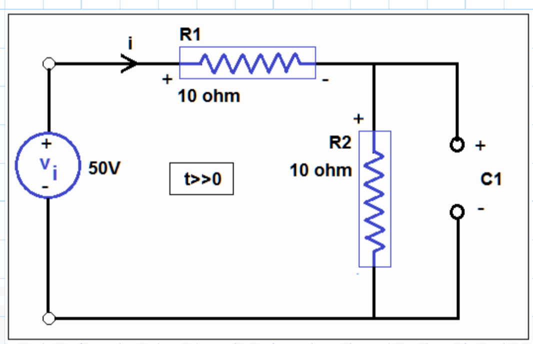
Circuit becomes a series RC circuit with the time constant tau = RC.

$\tau := R_{eq} \cdot C1 = 10 \cdot 10^{-6}$ s. Or 10 micro second.

Initial condition of the capacitor:
Switch is turned on at t=0, therefore t<0 the capacitor voltage = 0V.

Continuity condition: $v_C(0+') \quad = \quad v_C(0-')$

DC 50V voltage source and with t approaching infinity the capacitor eventually turns into an open circuit. Here 50V is in series with 20 ohm. Carefully now, the current we see now is at time t= infinity, with the voltage source supplying 50V, not modelled as short circuit and no equivalent resistance applied here, this here is the steady state condition at time t=infinity for this here circuit.



$R_{series} := R1 + R2 = 20$

$i(\infty) := \dfrac{V_i}{R_{series}} = 2.5$ A. dc steady state condition.

Voltage across the open circuit capacitor is the voltage across R2:

$v_{R2} \;=\; v_{C1}(\infty) \;=\; i(\infty) \cdot 10 = 25$ V. This is the end condition on vC1.

Introduction section 7.3 (Establishing a DC voltage across a Capacitor) in Part 2A. NOT if I remember rather go to page 3 of Part 2B, and the Topic is Response of First Order Circuits to a Pulse. Review example 7.11 on page 9.

Initial condition on vC1 equal 0.

$$v_{C1}(0+') \;=\; v_{C1}(0) \;=\; 0$$

Math of Things (MoT): *Breaking* the expression Vo - Vo(e^(- t/RC1))

Sorting Out that math expression - Initial condition continues to steady state condition when t=infinity, here capacitor is open circuit. Voltage is rising gradually across the capacitor terminals from t=0. That gradual rise is later impacted negatively by - Vo(e^-t/RC) or -Vo(e^-t/tau) <---

Capacitor voltage rises (during storing stage) when full holds full/max voltage, becomes OPEN circuit. C1 discharges current when 50V seen ON in circuit at t=0. See last plot.

$$v_{C1}(0+') \;=\; v_{C1}(0) \;=\; 0 \quad <---$$

$$v_{C1}(0+') \cdot e^{\frac{-t}{RC1}} \;=\; 0 \cdot e^{\frac{-0}{RC1}} \;=\; 0 \quad \text{Correct.}$$

The math using e^-t/RC works at t=0. *Of course it would*. From t=0, vC1(t) rises, this requires a positive exponential term but this rise is shown as Vo - Vo(e^(- t/RC1)) by 2 terms.

$$v_{C1}(t>0) \;=\; v_{C1}(t>0) \cdot e^{\frac{-t}{RC1}} \;=\; -25 \cdot e^{\frac{-t}{RC1}} \quad \text{<--- 2nd term the negative side.}$$

Increasing from -25V to the maximum value of 0V - done by the 2nd term, before open circuit condition where the capacitor is fully charged.
Gradually, time wise, the voltage across the capacitor rises from t=0 to 25V but this is done thru the math expression by adding 25V. Vo = 25V, the first term. Add Vo = 25V to expression - Vo(e^(- t/RC1)), becomes Vo - Vo(e^(- t/RC1)). I have my full expression describing voltage rise of capacitor C1 from 0 to 25V thru the contribution of both 1st and 2nd term. Part 2 topic, revisited.
See the plot next page.

The resistor voltages R1 and R2 were present all the way thru, from t=0 to time t = infinity. Initially voltage across the capacitor cannot be full 25V open circuit voltage till when t=infinity, so vC1(t) gradually increase to 25V ie capacitor open circuit voltage.

$v_{C1}(\infty) \;=\; 25$ V. This is that voltage when the capacitor is fully charged and becomes open circuit.

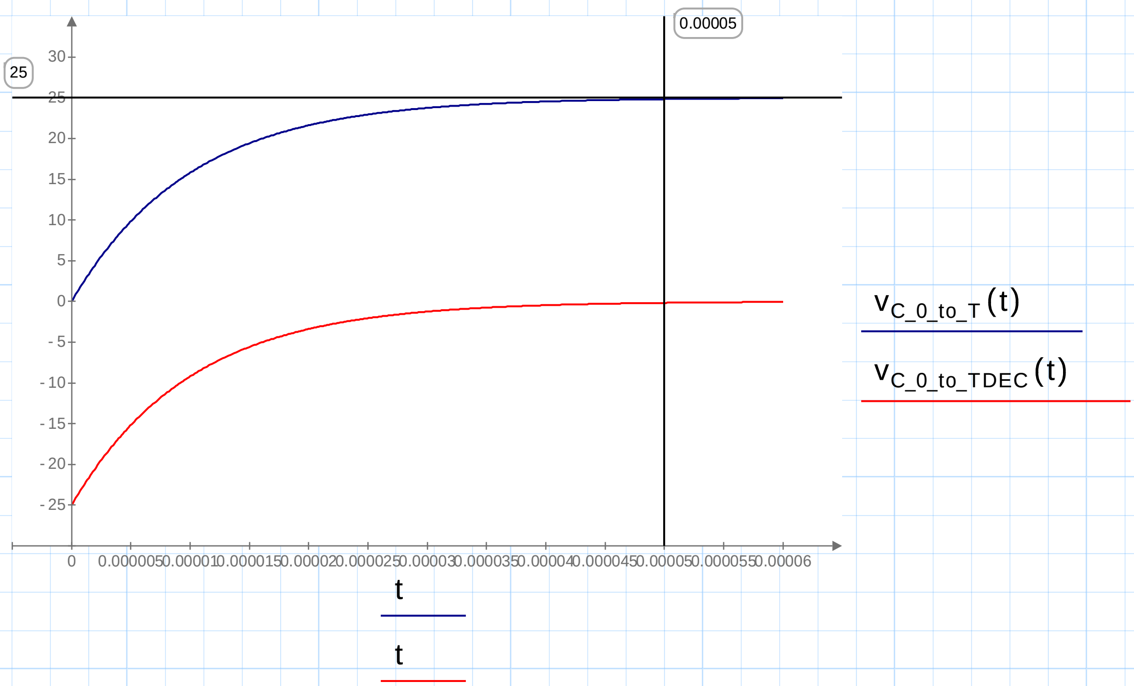
For time t=0 to t>0 to t = infinity :

$$v_{C1}(t) \;=\; \left(v_{C1}(0+')\cdot e^{\frac{-t}{RC1}} - v_{C1}(t>0)\cdot e^{\frac{-t}{RC1}}\right) + v_{C1}(\infty)$$ <--- 3 terms.

Parenthesis term RHS above is capacitor voltage, vC1(t) increasing and later discharging current in transient response. 25V is the steady state end conditon of C1. The first term vC1(0+) is obviouly not necessary because the 2nd term can cover it for t=0 and from t=0 to t>0 - RED CURVE. The 3rd term is the end condition - dc steady state.
Plot similar to below may been done in earlier parts of Part 3.
If not here it is now/again.

clear(t) BLUE CURVE both terms. $\tau := 10\cdot 10^{-6}$ RED CURVE decreasing term

$$v_{C\_0\_to\_T}(t) := 25\cdot\left(1 - e^{\frac{-t}{10\cdot 10^{-6}}}\right) \qquad v_{C\_0\_to\_TDEC}(t) := -25\cdot e^{\left(\frac{-t}{10\cdot 10^{-6}}\right)}$$



25V horizontal marker line, vertical marker shows aprox 50 us the voltage is 25V.

$$v_{C1}(t) = \left(v_{C1}(0+') \cdot e^{\frac{-t}{RC1}} - v_{C1}(t>0) \cdot e^{\frac{-t}{RC1}}\right) + v_{C1}(\infty)$$

Parenthesis part, expression above, is the gradual rise of the C1 voltage.
This *part looked OFF*, *misleading*, because it's 0 - 25e^-(t/RC). Results in - 25 e^-(t/RC).
The negative value is fixed with the end value of 25V ie vC1(t=infinity).

$$v_{C1}(t) = \left(0 - 25\ e^{\frac{-t}{10}}\right) + 25$$

$$v_{C1}(t) = 25 - 25\ e^{\frac{-t}{10}}$$

Thats all you knew it all along. Gets me everytime when I been away from it for a few weeks.

$$v_{C1}(t) = 25 \cdot \left(1 - e^{\frac{-t}{10}}\right)$$

V. Answer. Time t is in micro seconds relative to tau (RC) was in microseconds.

Why did we not write the parenthesis like this it may result in a positive value ?

$$\left(v_{C1}(t>0) \cdot e^{\frac{-t}{RC1}} - v_{C1}(0+') \cdot e^{\frac{-t}{RC1}}\right)$$

That would NOT result in a positive value, since v(0+) = 0, this difference being negative, since the exponent term is negative -t for t>0. Some cases v(0+) may not be 0 because the initial value maybe some numerical value for t<0.

End of Math of Things (MoT).

Current in the capacitor:

$$v_{C1} = \frac{1}{C1} \int i_{C1}(t)\,dt$$

$$C1 \cdot v_{C1} = \int i_{C1}(t)\,dt$$

Taking derivative on both sides.

$$C1 \cdot \frac{dv_{C1}}{dt} = i_{C1}(t)$$

$$v_{C1}(t) = 25 - 25\ e^{\frac{-t}{10}}$$

$$\frac{dv_{C1}(t)}{dt} = \frac{-1}{10}(-25) \cdot e^{\frac{-t}{10}}$$

Remember the 10 is in microseconds.

$$i_{C1}(t) = C1 \cdot \frac{dv_C}{dt} = 2 \cdot 10^{-6} \cdot \left(\frac{1}{10 \cdot 10^{-6}}(25) \cdot e^{\frac{-t}{10}}\right)$$

$$i_{C1}(t) = \left(\frac{25}{5} \cdot e^{\frac{-t}{10}}\right)$$

$$i_{C1}(t) = 5\cdot\left(e^{\frac{-t}{10}}\right) \text{ A. Answer.}$$

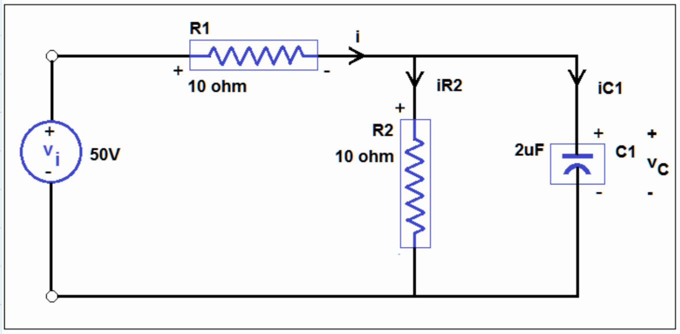
Current in the R2 resistor:
The voltage of which is equal to vC1(t).

$$i_{R2}(t) = \frac{v_{C1}(t)}{R2}$$

$$= \frac{25\cdot\left(1-e^{\frac{-t}{10}}\right)}{10}$$

$$i_{R2}(t) = 2.5\cdot\left(1-e^{\frac{-t}{10}}\right) \text{ A. Answer.}$$

Calculate the current i(t) for the circuit:



$$i(t) = i_{R2}(t) + i_{C1}(t)$$

$$= 2.5\cdot\left(1-e^{\frac{-t}{10}}\right) + 5\left(e^{\frac{-t}{10}}\right)$$

$$= 2.5 - 2.5\, e^{\frac{-t}{10}} + 5\left(e^{\frac{-t}{10}}\right)$$

$$= 2.5 - (2.5-5)\left(e^{\frac{-t}{10}}\right)$$

$$= 2.5 + 2.5\left(e^{\frac{-t}{10}}\right)$$

$$= 2.5 + 2.5\left(e^{\frac{-t}{10}}\right)$$

$$i(t) = 2.5\cdot\left(1+e^{\frac{-t}{10}}\right) \text{ A. Answer.} = 2.5\cdot\left(1+e^{\frac{-t}{10\cdot 10^{-6}}}\right)$$

Plot next page.

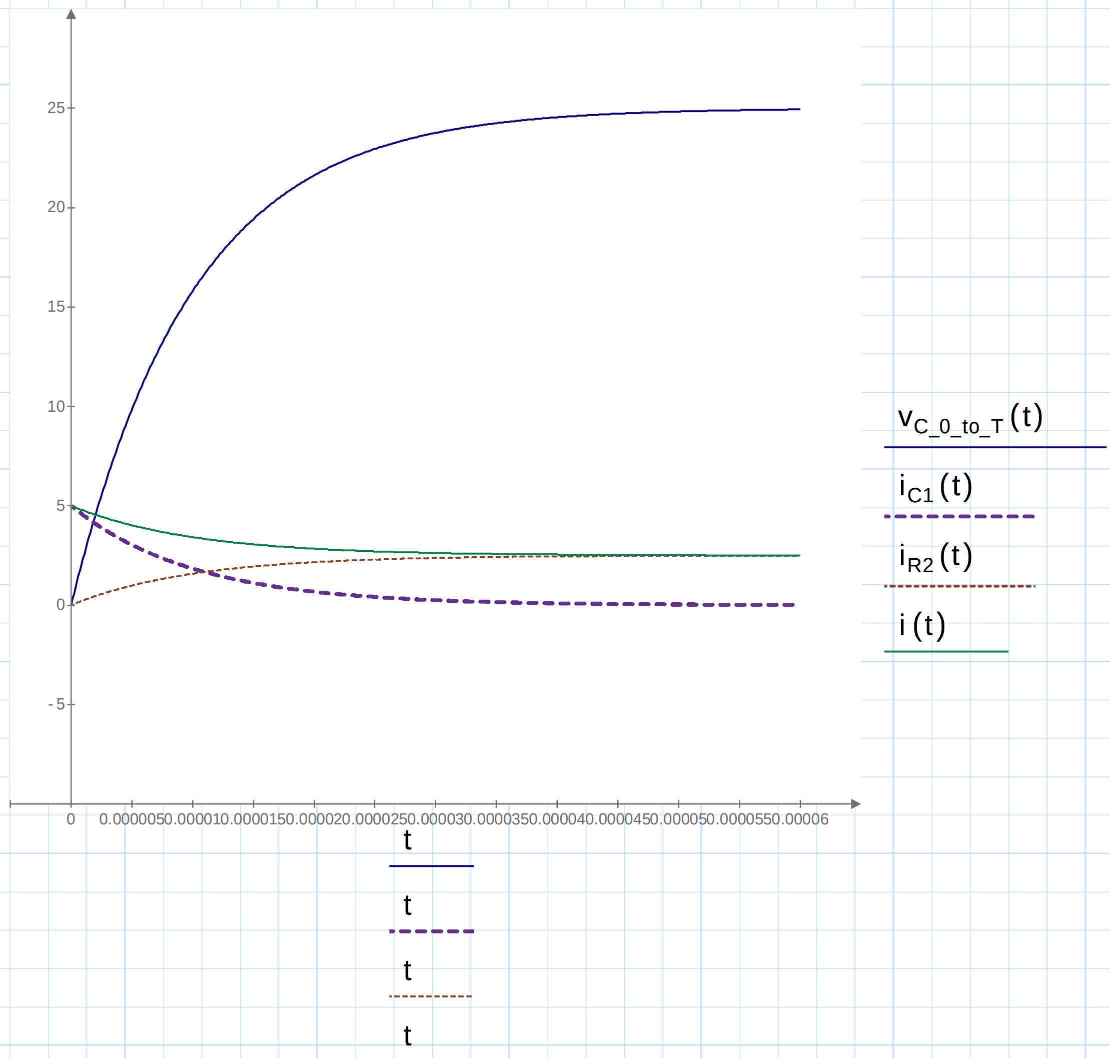
Comments: Requires me to review Part 2 A and B. My explanation may not be satisfactory. One reason I am not satisfied was I saw the circuit charging and discharging in a cycle after a time constant, of what maybe the circuit's period. But such was not the case.....things stay at 25 at time t=infinity. The Capacitor solved examples, in Part 2, circuit response when t=0, then t to T, then t>T those should help......Check with your local lecturer and engineer. Answers are correct to textbook on this one.

clear(t) Time constant of circuit tau: $10 \cdot 10^{-6}$

$$v_{C\_0\_to\_T}(t) := 25 \cdot \left(1 - e^{\frac{-t}{10 \cdot 10^{-6}}}\right) \qquad i_{C1}(t) := 5 \cdot \left(e^{\frac{-t}{10 \cdot 10^{-6}}}\right)$$

$$i_{R2}(t) := 2.5 \cdot \left(1 - e^{\frac{-t}{10 \cdot 10^{-6}}}\right) \qquad i(t) := 2.5 \cdot \left(1 + e^{\frac{-t}{10 \cdot 10^{-6}}}\right)$$

Plots of all relevant values calculated below.
Study the iC1(t) curve, C1 current curve, it starts when the circuit comes ON at t=0.
This is surprising since C1 does not need to be fully charged to the top to 25V.



Maybe simple example problem, and it was, but it was a learning curve for? Me.
Check with your local lecturer and or engineer.

Problem 8.6 and 8.7 We completed in other notes, check Parts 1 or 2.
Problem 6: Given the v(t) or i(t) time function provide the Amplitude and Phase Angle, and the complex frequency s. Table form. Completed.
Problem 7: Given Amplitude and Phase Angle, and complex frequency s, provide the time function v(t) or i(t). Table form. Completed.

Problem 8.8 (Constructing function v(t)) :
Amplitude and phase angle of 10 SQRT(2) at 45 degrees V.
Has an associated complex frequency s = -50 +j100 s^-1.
Find the voltage at t=10 ms.

$\text{Amplitude} = 10 \cdot \sqrt{2} \qquad \text{PhaseAngle} = 45 \text{ deg}$

$\text{Phasor} = 10 \cdot \sqrt{2} \angle 45 \text{ deg}$

$s = -50 + j100$

Solution:

$A = 10 \cdot \sqrt{2}$

$\text{PhAng} = 45 \text{ deg}$

$s = -50 + j100$

$\sigma = -50$

$\omega = 100$

Forming the expression for v(t) : $A \cdot e^{-\sigma \cdot t} \cdot \cos(\omega \cdot t + \theta)$

$$v(t) = 10 \cdot \sqrt{2} \cdot e^{-50 \cdot t} \cdot \cos(100 \cdot t + 45°)$$

Now plug-in the time t for 10 ms:

$t = 10 \cdot 10^{-3}$

$\sigma \cdot t = 50 \cdot 10 \cdot 10^{-3} = 0.5$

$\omega \cdot t = 100 \cdot 10 \cdot 10^{-3} = 1$ rad. $\qquad \omega t_{\_radians} = 1.\ \textbf{rad} = 57.2957795\ \textbf{deg}$

$v(t) = 10 \cdot \sqrt{2} \cdot e^{-0.5} \cdot \cos(57.3\ ° + 45°)$

$= 10 \cdot \sqrt{2} \cdot e^{-0.5} \cdot \cos(102.3\ °)$

This expression can be evaulated for a final numerical value.

$\cos(102.3\ °) = -0.2130304 \qquad e^{-0.5} = 0.6065307 \qquad 10 \cdot \sqrt{2} = 14.1421356$

$v(t) = (14.142) \cdot (0.607) \cdot (-0.213) = -1.8284333$

$v(t) = -1.8284333$ Answer.

Problem 8.9 (Obtain s-domain Impedances) :

A passive network contains resistors, a 70 mH inductor, and a 25 uF capacitor.
Obtain the respective s-domain impedances for a driving voltage

(a): v = 100 sin(300t + 45 deg) V

(b): v = 100e^(-100t) cos (300t) V

Solution:

Network contains resistors, but do they impact the s-domain?
R does not have s assosiated with it like sL and 1/sC.
Resistor is not frequency, w omega, dependent.
So, we work with L and C.

Constant: $j := \sqrt{-1}$ $\quad \frac{1}{j} = -1j$

a): $v(t) = 100\sin(300t + 45\,deg)$

What can we get from this equation?
Amplitude 100
Phase angle 45 degrees
j*w* which is 300
and NO sigma because we do not have an exponential term.
We concentrate on j*w*.

$j\omega t = 300\ t$

$\omega = 300$

$s = \sigma + j\omega = 0 + j300$

$L = 70 \cdot 10^{-3}$ H.

$sL = (0 + j300) \cdot 70 \cdot 10^{-3} = j21$ Answer.

$C = 25 \cdot 10^{-6}$ F

$$\frac{1}{sC} = \frac{1}{(0 + j300) \cdot 25 \cdot 10^{-6}}$$

$$= \frac{1}{j \cdot 7.5 \cdot 10^{-3}} = -133.333j$$

$$\frac{1}{sC} = -j133.3$$ Answer.

(b): $v(t) = 100\, e^{-100\, t} \cos(300\ t)$ V.

What can we get from this equation?
Amplitude 100
Phase angle 0 degrees
j*w* which is 300
s = -100+j300, sigma = -100, *w* = 300
We concentrate on s = sigma + j*w*.

$$\sigma = -100$$
$$j\omega t = 300\ t$$
$$\omega = 300$$
$$s = \sigma + j\omega = -100 + j300$$

$$L = 70 \cdot 10^{-3}\ \text{H.}$$
$$sL = (-100 + j300) \cdot 70 \cdot 10^{-3}$$
$$-100 \cdot 70 \cdot 10^{-3} = -7$$
$$300 \cdot 70 \cdot 10^{-3} = 21$$
$$sL = -7 + j21$$ Answer.

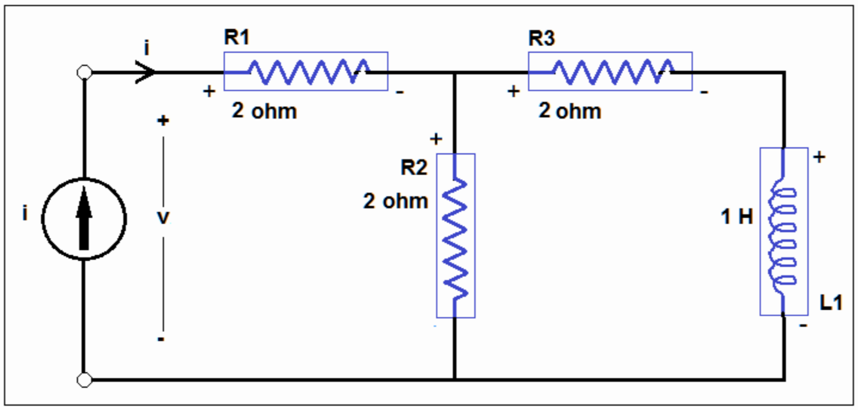
$$C = 25 \cdot 10^{-6}\ \text{F}$$
$$\frac{1}{sC} = \frac{1}{(-100 + j300) \cdot 25 \cdot 10^{-6}}$$
$$(-100) \cdot 25 \cdot 10^{-6} = -0.0025$$
$$(j \cdot 300) \cdot 25 \cdot 10^{-6} = 0.0075j$$
$$= \frac{1}{-0.0025 + j0.0075} = \frac{1000}{-2.5 + j7.5}$$ <---After multiplying by 1000 top and bottom.
$$= \frac{1000 \cdot (-2.5 - j7.5)}{(-2.5 + j7.5)\ (-2.5 - j7.5)} = \frac{-2500 - j7500}{(6.25 + j18.75 - j18.75 + 56.25)}$$
$$= \frac{-2500 - j7500}{62.5}$$
$$\frac{1}{sC} = -40 - j120$$ Answer.

Comment: Good exercise. Some steps from problem 8.6 and or 8.7 applied.

Problem 8.10 (Complex Frequency In Circuits):

For the circuit shown in figure below.
Obtain v at t = 0.1 s for a source current

a). i = 10 cos 2t (A)
b). i = 10 (e ^-t) cos (2t) A



$R1 := 2$

$R2 := 2$

$R3 := 2$

$L1 := 1$

Solution:

First we get the circuit impedance Z, where s is left as s intead of some value. When we get impedance Z computed, then we plug in the s (sigma + jw) for part a and b.

$Z_{R1} := 2 \qquad Z_{R2} := 2 \qquad Z_{R3} := 2 \qquad Z_{L1} = s \cdot L = s1$

L1 and R3 in series: $\quad Z_1 := 2 + s1$

R2 in parallel with Z1: $\quad Z_2 = \dfrac{2 \cdot (2+s)}{2+(2+s)} = \dfrac{2\,s+4}{s+4}$

R1 in series with Z2: $\quad Z_2 = 2 + \left(\dfrac{2\,s+4}{s+4}\right) = \dfrac{2\,(s+4)+(2\,s+4)}{s+4}$

$$= \frac{2\,s+8+2\,s+4}{s+4}$$

$$= \frac{4\,s+12}{s+4}$$

We can factor it like a transfer function with k.

$$Z_{in} = Z_2 = \frac{4\,(s+3)}{s+4}$$

Circuit impedance Z.

We set Z2 equal Zin so its the input impedance, Zin, of the circuit.

a). $i(t) = 10\cos(2t)$ (Note: Don't apply with time t=0.1 here).
In the Part 2 notes: e^st = e^(sigma + jw)t
's' as a complex function we work it indepdent of time t.

$$s = \sigma + jw$$
$$s = 0 + j2 = j2$$

Now we have s, complex frequency, from the current source (given), we plug this in Zin so we get the precise impedance for this circuit for this current source i(t).

$$Z_{in} = \frac{4\,(s+3)}{s+4}$$

$$Z_{in}(j2) = 4\left(\frac{j2+3}{j2+4}\right)$$ Next fix the parenthesis term to a simple complex expression.

$$= \frac{j2+3}{j2+4} = \left(\frac{j2+3}{j2+4}\right)\left(\frac{j2-4}{j2-4}\right) = \frac{-4-8j+6j-12}{-4-8j+8j-16} = \frac{-16-2j}{-20}$$

$$= \left(\frac{-16}{-20}\right) + \left(\frac{-2j}{-20}\right) \;\text{--->}\; \left(\frac{-16}{-20}\right) = 0.8 \quad \frac{2}{20} = 0.1 \;\text{--->}\; 0.8 + 0.1j$$

$$Z_{in}(j2) = 4\cdot(0.8+0.1j) = 3.2 + 0.4j$$

$$Mag\_Z_{in} := \sqrt{(3.2^2) + (0.4^2)} = 3.2249031 \qquad Ang\_Z_{in} := atan\left(\frac{0.4}{3.2}\right) = 7.1250163\ deg$$

$Z_{in}(j2) = 3.225\angle 7.125\ deg$ Answer.

Which Zin is in the complex function s form (s = sigma + jw):

$Z_{in}(s) = 3.225\angle 7.125\ deg$ Answer.

Next we procced using Ohm's Discovery also known as Ohm's Law V=IZ.

$i(t) = 10\cos(2t)$ No phase angle given this means its 0 degreess.

$$I(s) = 10\angle 0$$

$$V(s) = I(s)\cdot Z_{in}(s) = 10\angle 0\cdot(3.225\angle 7.125\ deg)$$

$V(s) = 32.2\angle 7.13\ deg$ Answer.

Next I apply the time t=0.1 s in the time domain after conversion.

$i(t) = 10\cos(2t)$ <--- Our form of v(t) equation for the solution will be similar or same.

$V(s) = 32.2\angle 7.13\ \text{deg}$

Amplitude: 32.2

Phase angle: 7.13 deg

$v(t) = 32.2\cos(2t + 7.13°)$

v(t) at time t = 0.1 s:

$v(t) = 32.2\cos(2(0.1) + 7.13°)$

$= 32.2\cos(0.2 + 7.13°)$

$0.2\ \text{rad} = 11.4591559\ \text{deg}$

$= 32.2\cos(11.46° + 7.13°)$

$v(t) = 32.2\cos(18.59°)$

$\cos(18.59°) = 0.9478241$

$32.2 \cdot 0.948 = 30.5256$

$v(t) = 30.5$ V. Answer.

b). $i(t) = 10 \cdot e^{-t} \cdot \cos(2t)$

$s = \sigma + jw$
$s = -1 + j2$

$$Z_{in}(s) = 4\frac{(s+3)}{(s+4)}$$

Substitute s = -1+j2

$$\frac{s+3}{s+4} = \frac{(-1+j2)+3}{(-1+j2)+4} = \frac{2+j2}{3+j2}$$

$$\left(\frac{2+j2}{3+j2}\right)\left(\frac{3-j2}{3-j2}\right) = \frac{6-j4+j6+4}{9-j6+j6+4} = \frac{10+j2}{13} = 0.7692+j0.1538$$

$$Z_{in}(-1+j2) = 4\cdot(0.7692+j0.1538) = 3.077+j0.615$$

$$Mag\_Z_{in} := \sqrt{(3.077^2)+(0.615^2)} = 3.1378582 \quad Ang\_Z_{in} := \operatorname{atan}\left(\frac{0.615}{3.077}\right) = 11.3027705 \text{ deg}$$

$Z_{in}(-1+j2) = 3.138\angle 11.303$ deg Answer.

Which Zin is in the complex function s form (s = sigma + jw):

$Z_{in}(s) = 3.138\angle 11.303$ deg Answer.

Next we procced using Ohm's Discovery also known as Ohm's Law V=IZ.

$i(t) = 10\cdot e^{-t}\cdot\cos(2\ t)$ No phase angle given this means its 0 degreess.

$$I(s) = 10\angle 0$$

$$V(s) = I(s)\cdot Z_{in}(s) = 10\angle 0\cdot(3.138\angle 11.303 \text{ deg})$$

$V(s) = 31.38\angle 11.303$ deg Answer.

Next I apply the time t=0.1 s in the time domain after conversion.

$i(t) = 10\ e^{-t}\ \cos(2\ t)$ <--- Our form of v(t) equation for the solution will be similar or same.

$$V(s) = 31.38\angle 11.303 \text{ deg}$$

Amplitude: 31.38

Phase angle: 11.303 deg

$$v(t) = 31.38\ e^{-t}\ \cos(2\ t + 11.303\ °)$$

v(t) at time t = 0.1 s:

$$v(t) = 31.38\ e^{-0.1}\ \cos(2\ (0.1) + 11.303\ °)$$

$$(31.38)\cdot(0.905)\cdot\cos(0.2 + 11.303\ °)$$

$$0.2\ \text{rad} = 11.4591559 \text{ deg}$$

$28.399\cos(11.46° + 11.303°)$

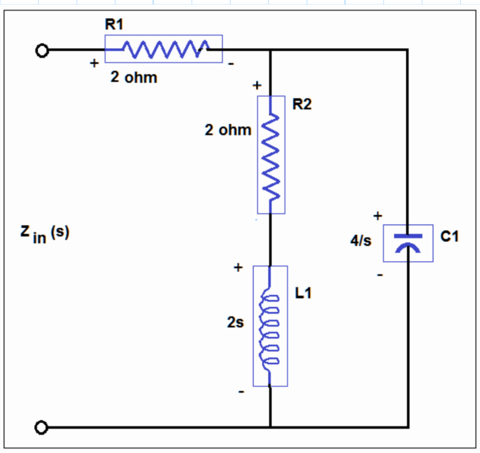
$v(t) = 28.399\cos(22.763°)$

$\cos(22.763\,deg) = 0.9221132$
$28.399 \cdot 0.922 = 26.183878$

$v(t) = 26.2$ V. Answer.

Problem 8.11 (Impedance Z and Complex Frequency In Circuits):
For the circuit shown in figure below.
Obtain the circuit impedance at:

a). s = 0
b). s = j4 rad/s
c). |s| = infinity



$R1 := 2$

$R2 := 2$

$L1 = 2\,s$

$C1 = \frac{4}{s}$

Form the circuit impedance:
C1 parallel to R2 and L1 in series.
Series R2 and C1:

$$Z_{L1R2}(s) = 2 + s2$$

$$Z_{L1R2\_C1} = \frac{\left(\frac{4}{s}\right)(2+s2)}{\left(\frac{4}{s}\right)+2+s2}$$

$$= \frac{\left(\frac{8}{s}\right)+\frac{8\,s}{s}}{\frac{4+s2+s^2\,2}{s}}$$

$$= \frac{\frac{8}{s}+8}{\frac{s^2\ \ 2+s2+4}{s}} = \frac{\frac{8+s8}{s}}{\frac{s^2\ \ 2+s2+4}{s}}$$

$$= \left(\frac{8+s8}{s}\right)\cdot\left(\frac{s}{s^2\ \ 2+s2+4}\right)$$

$$= \left(\frac{8+s8}{s^2\ \ 2+s2+4}\right)$$ Divide by 2 top and bottom ---> $$= \left(\frac{4+s4}{s^2\ +s+2}\right)$$

$$Z_{L1R2\_C1\_R1} = 2+\left(\frac{4+s4}{s^2\ +s+2}\right)$$

$$Z_{in}(s) = 2+\left(\frac{4+s4}{s^2\ +s+2}\right)$$

a). s = 0 :

$$Z_{in}(0) = 2+\left(\frac{4+s4}{s^2\ +s+2}\right) = 2+\left(\frac{4+0\cdot 4}{0^2\ +0+2}\right) = 2+\left(\frac{4}{2}\right)$$

$Z_{in}(0)$ = 4 Ohm Answer.

b). s = j4 :

$$Z_{in}(j4) = 2+\left(\frac{4+(j4)\ 4}{(j4)^2\ +j4+2}\right) = 2+\left(\frac{4+j16}{-16+j4+2}\right)$$

$$= 2+\left(\frac{4+j16}{-14+j4}\right)$$

Next fix the parenthesis term to a simple complex expression.

$$\left(\frac{4+j16}{-14+j4}\right)\left(\frac{-14-j4}{-14-j4}\right) = \frac{-56-j16-j224+64}{196+16} = \frac{8-j240}{212}$$

$$Z_{in}(j4) = 2+\left(\frac{8-j240}{212}\right) = \frac{2\cdot 212+8-j240}{212} = \frac{424+8-j240}{212} = \frac{432-j240}{212}$$

$$= 2.0377-j1.1321$$

$$Mag\_Z_{in} := \sqrt{(2.0377^2)+(1.1321^2)} = 2.331 \qquad Ang\_Z_{in} := atan\left(\frac{-1.1321}{2.0377}\right) = -29.056\ deg$$

$Z_{in}(j4) = 2.33\angle -29.05\ °$ Ohm Answer.

Here s= 0 + j4, so this says w? 4.
Which says 'the voltage source would be something'? OR 'this is the impedance offered to a source': v(t) = sin (4t) OR v(t) = cos(4t).

c). s = infinity :

$$Z_{in}(\infty) = 2 + \left(\frac{4 + s4}{s^2 + s + 2}\right) = 2 + \left(\frac{4 + (\infty)\,4}{(\infty)^2 + \infty + 2}\right)$$

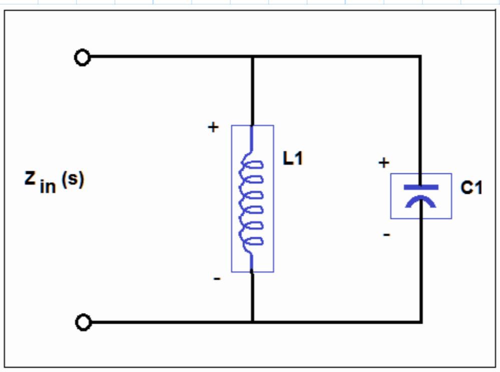
$$= 2 + \left(\frac{(\infty)\,4}{(\infty)^2 + \infty}\right)$$ Parenthesis term approximately equal zero, so let it equal 0.

$$= 2 + 0$$

$$Z_{in}(\infty) = 2 \text{ Ohm}$$ Answer.

Point: At very high frequencies, jw = j (infinity), the capacitance acts like a short circuit across the RL branch (R2 & L1). Here resulting in 2 Ohm from R1, the C1 became short circuit instead of open circuit at infinity, a closed path was required, and that was thru the C1 branch. R2&L1 did not manifest/materialise in the final result, they were 0. Circuit had to be closed somewhere so thats was the C1 shorted.
This was what we saw in this circuits's impedance evaluation with frequency w equal infinity. Very good point to understand, and maybe apply. Similarly other circuit impedance may show similar behaviour or other behaviour when frequency is set to infinity. Something to take into accountt.

Problem 8.12 Circuit Impedance and Frequency :



For the circuit to the right express the impedance Z(s) of the parallel combination of L = 4H and C=1F.

At what frequencies s is this impedance zero or infinite?

Solution:

$L = 4 \text{ H} \qquad Z_L = s4$

$C = 1 \text{ F} \qquad Z_C = \frac{1}{sC} = \frac{1}{s \cdot 1} = \frac{1}{s}$

Zin is the parallel of L and C:

$$Z_{in}(s) = \frac{(s4)\left(\frac{1}{s}\right)}{(s4) + \left(\frac{1}{s}\right)}$$

$$Z_{in}(s) = \frac{(s4)\left(\frac{1}{s}\right)}{\frac{s^2\ 4+1}{s}}$$

$$Z_{in}(s) = \frac{4}{\frac{s^2\ 4+1}{s}} = 4\cdot\left(\frac{s}{s^2\ 4+1}\right) = \frac{4\ (s)}{4\cdot\left(s^2+\frac{1}{4}\right)} = \frac{s}{s^2+\frac{1}{4}}$$

$$Z_{in}(s) = \frac{s}{s^2+0.25}$$ Answer.

Next for the frequencies, w, for which the impedance Zin equal zero or infinity:

$$Z_{in}(0) = \frac{s}{s^2+0.25} = \frac{0}{0+0.25} = \frac{0}{25} = 0$$

The impedance Zin equal 0 at 0 frequency.

$$Z_{in}(\infty) = \frac{\infty}{\infty^2+0.25} = \frac{\infty}{\infty^2} = 0$$

The impedance Zin equal 0 at infinity frequency.

What frequency would it make for Zin equal infinity?
The poles and zeros?
Solving for the poles would provide the highest or peak or infinity response? Yes.
Thats one option.

$$s^2 + 0.25 = 0$$

$$s^2 = -0.25$$

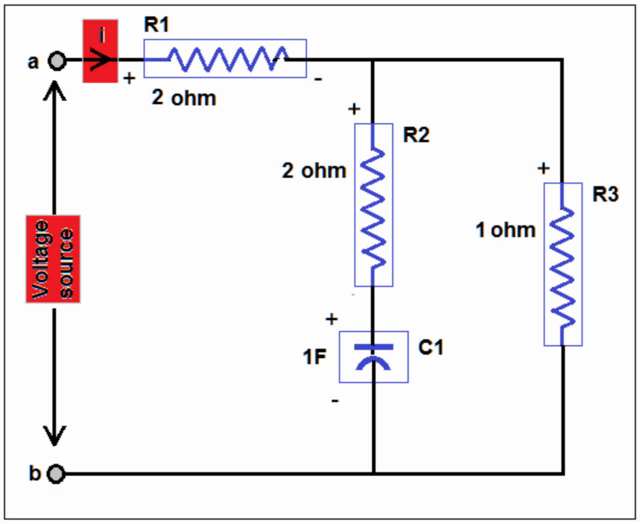
$$s = \sqrt{-25} = +/-\ \ j5$$ ... resulting in an infinite impedance.

$$s = +/-\ \ j5$$ rad/s this is the jw part of s

A source with w = + 5 rad/s will provide an impedance Zin equal INFINITY.
Usually on the frequency we go by the positive, +jw, the real part.
This source may be a sinusoidal driving force of frequency 0.5 rad/s.

Problem 8.13 Circuit Network Function H(s) :

Circuit shown below has a voltage source connected at terminals ab.
The response to the exciation (voltage source) is the input current.
Obtain the appropriate network function H(s).



$R1 = 2 \text{ Ohm}$

$R2 = 2 \text{ Ohm}$

$R3 = 1 \text{ Ohm}$

$C1 = 1 \text{ F} = \frac{1}{sC} = \frac{1}{s \cdot 1}$

$C1 = \frac{1}{s}$

Solution:

$$H(s) = \text{Current } I(s) \text{ response / Voltage excitation } V(s)$$

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)}$$

$$Z_{R2\_C1}(s) = 2 + \frac{1}{s}$$

$$Z_{R2\_C1\_R3}(s) = \frac{\left(2 + \frac{1}{s}\right) \cdot (1)}{\left(2 + \frac{1}{s}\right) + 1} = \frac{\left(2 + \frac{1}{s}\right)}{3 + \frac{1}{s}} = \frac{\left(\frac{2s+1}{s}\right)}{\left(\frac{3s+1}{s}\right)}$$

$$= \left(\frac{2s+1}{s}\right) \cdot \left(\frac{s}{3s+1}\right)$$

$$Z_{R2\_C1\_R3}(s) = \left(\frac{2s+1}{3s+1}\right)$$

$$Z_{R2\_C1\_R3\_R1}(s) = 2 + \left(\frac{2\,s+1}{3\,s+1}\right)$$

$$= \frac{2\,(3\,s+1)+(2\,s+1)}{3\,s+1}$$

$$= \frac{6\,s+2+(2\,s+1)}{3\,s+1}$$

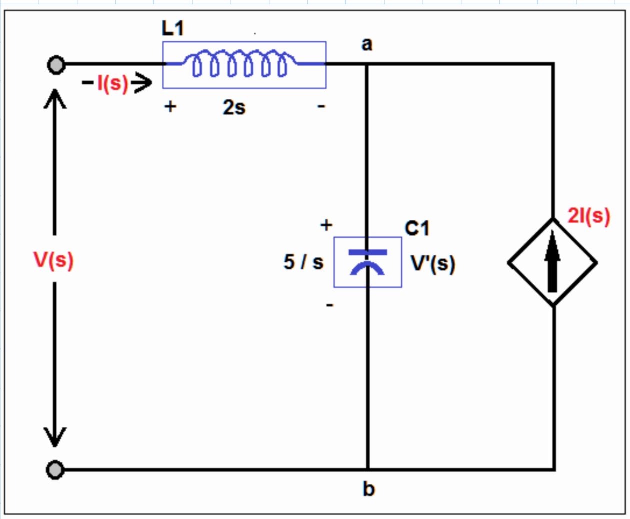
$$Z_{R2\_C1\_R3\_R1}(s) = \frac{8\,s+3}{3\,s+1}$$

$$Z(s) = Z_{R2\_C1\_R3\_R1}(s) = \left(\frac{8\,s+3}{3\,s+1}\right)$$

$$H(s) = \frac{1}{Z(s)} = \frac{1}{\left(\frac{8\,s+3}{3\,s+1}\right)} = \left(\frac{3\,s+1}{8\,s+3}\right)$$ Answer. <--- $\frac{I(s)}{V(s)}$

Problem 8.14 (Transfer Function of Circuit) :

Obtain the H(s) for the network in the circuit below.
Where excitation is the driving current I(s).
Response is the voltage V(s) at the input terminals.



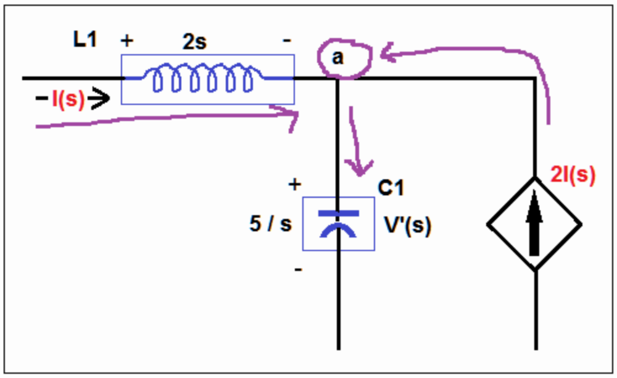
$$L1 = 2\,s$$

$$C1 = \frac{5}{s}$$

$$I_{source} = 2\,I(s)$$

Solution:

Here the excitation is the current source 2I(s).



Form a node (current) equation at node a, commonly known as KCL (Kick-off Current Law):

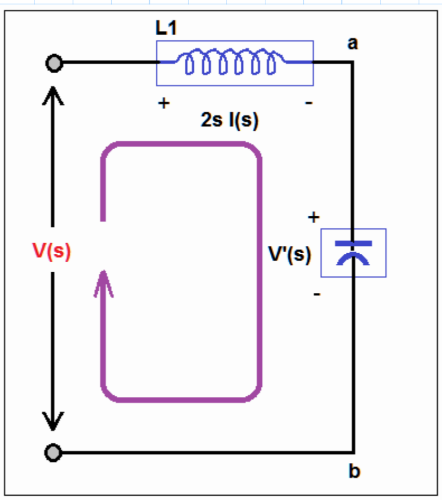
$$I(s) + 2\,I(s) = \frac{V_c(s)}{Z_c(s)}$$

$$I(s) + 2\,I(s) = \frac{V'(s)}{\frac{5}{s}}$$

Solving for V'(s):

$$3\,I(s) = V'(s)\left(\frac{s}{5}\right)$$

$$V'(s) = \left(\frac{15}{s}\right) I(s)$$



Now we do the Kick- off Voltage Law or the simple sum of voltage drops equal zero. Follow the polarity sign.

$$V(s) - 2\,sI(s) - V'(s) = 0$$

$$V'(s) = \left(\frac{15}{s}\right) I(s)$$

$$V(s) - 2\,sI(s) - \left(\frac{15}{s}\right) I(s) = 0$$

$$V(s) - \left(2\,s + \frac{15}{s}\right) \cdot I(s) = 0$$

$$V(s) = \left(2\,s + \frac{15}{s}\right) \cdot I(s)$$

$$V(s) = \left(\frac{2\,s^2 + 15}{s}\right) \cdot I(s)$$

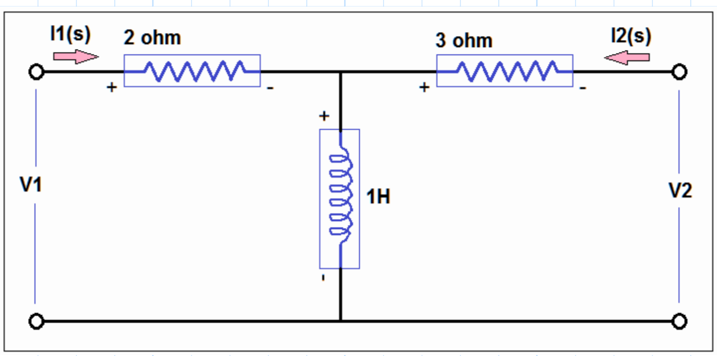
Now we can work on the transfer function H(s),
we have I(s) the driving current
and we got the response which is the voltage V(s)
at the input terminals.

$$H(s) = \frac{V(s)}{I(s)} = \left(\frac{2\,s^2 + 15}{s}\right) \quad \text{Answer.}$$

Comment: Easy to follow. Setting it up takes some getting used too.
The 13 problems for transfer functions from the Problems and Solutions of Control Systems does provide some exercise - Attached at end of this file.
We found a method to get capacitor C voltage V'(s) from a current node equation, then we found a place to fix that V'(s) in the voltage loop equation where we solved for V(s). Then got the transfer function V(s)/I(s), we need to form a mathemtical relationship some where to build toward the solution here it was KCL and KVL. Then merely one divided by the other V(s) over I(s) or I(s) over V(s).

Problem 8.14 Two Port Network :

Review:
Before I start on this probelm. A very short review on two port network.
Its used in power systems course for transmission lines. Transfer function is a good example to apply in two port network, because we got voltages at both end separated by the transmission lines. Its a simplest 2 x 2 matrix. Its a chapter by itself usually after AC power and complex frequency. It uses the transfer function to solve its circuit problems. Easier in comparison to RLC circuits.
It comes before Laplace Transform Method For Electric Circuits.



Circuit to the left is a two port network. V1 one side, and V2 the other, thats the two ports.

Do a KVL for each loop. Set the equation to line up so we can form a matrix.

$$Z_L = s \cdot 1 = 1$$

KVL 1: $V1 = I1(s)\ 2\ \text{ohm} + s\,(I1(s) + I2(s)) = (2 + s)\ I1(s) + (s)\ I2(s)$

KVL 2: $V2 = I2(s)\ 3\ \text{ohm} + s\,(I1(s) + I2(s)) = (s)\ I1(s) + (3 + s)\ I2(s)$

$V1 = (2+s)\ I1(s) + (s)\ I2(s) \quad ---> \quad V_1 = Z_{11}\ I_1 + Z_{12}\ I_2$

$V2 = (s)\ I1(s) + (3+s)\ I2(s) \quad ---> \quad V_2 = Z_{21}\ I_1 + Z_{22}\ I_2$

$Z_{11} = \frac{V_1}{I_1}$ when $I_2 = 0$ $\qquad Z_{12} = \frac{V_1}{I_2}$ when $I_1 = 0$

$Z_{21} = \frac{V_2}{I_1}$ when $I_2 = 0$ $\qquad Z_{22} = \frac{V_2}{I_2}$ when $I_1 = 0$

The coefficients of Zij (matrix size i x j) are called Z-parameters of the network.
Z parameters are also called open circuit impedance parameters since they may be measured at one terminal while the other terminal is open - Schaums page 335 Chapter 13 Two Port Network.

$(2+s)\ I1(s) + (s)\ I2(s)$

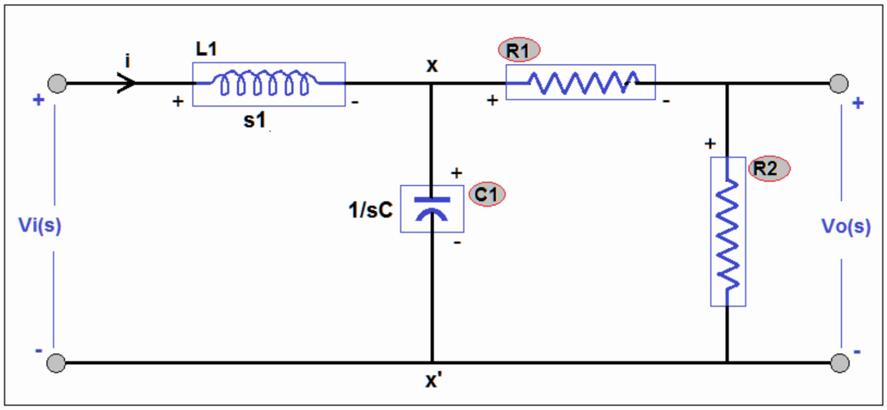
$(s)\ I1(s) + (3+s)\ I2(s)$

---> Form the Z parameters matrix: $Z_{parameters} = \begin{bmatrix} (2+s) & s \\ s & (s+3) \end{bmatrix}$

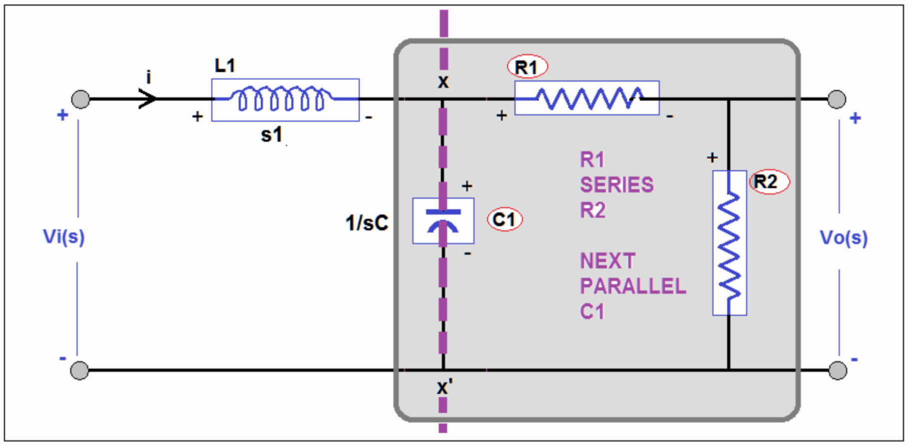
Review completed.

Continuing problem 8:15 :
For the two port network circuit below, find the values of R1, R2, and C.
Given that the voltage transfer function is:

$$H_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{0.2}{s^2 + 3\ s + 2}$$



Somewhere in the circuit we need to have a differentiating line, so we can form the two port network. Is that necessary in all cases? Its circuits! The components RLC in the above circuit could be made into Zequivalent, but the circuit needs a parallel connection to provide a V1 and V2 at either side. In this circuit we use the xx' branch for assisting the solution. Its an example problem. Learning Outcome is circuit analysis method(s).



The impedance looking into xx', here for the right side off the xx' branch including the capacitor on xx'. This mostly covers the impedance of the network on the right side. Reminds me of Thevenin's Equivalent. Not to rush.

$$Z_{R1\_R2} = R1 + R2$$

$$Z_{R1\_R2\_C1} = \frac{\left(\frac{1}{sC}\right)\cdot(R1+R2)}{\left(\frac{1}{sC}\right)+(R1+R2)} = \frac{\left(\frac{1}{sC}\right)\cdot(R1+R2)}{\left(\frac{1}{sC}\right)+(R1+R2)}$$ Multiply by sC

$$Z_{R1\_R2\_C1} = \frac{R1+R2}{1+sC\,(R1+R2)}$$

Impedance seen on xx' from the left, so it becomes the impedance of the output side.

$$\text{Let} \quad Z' = Z_{R1\_R2\_C1} = \frac{R1+R2}{1+sC\,(R1+R2)}$$

Total circuit impedance of the network will be the input impedance (looking from Vi) :

$$Z_i = Z_L + Z' = s\cdot 1 + Z'$$
$$= s + Z'$$

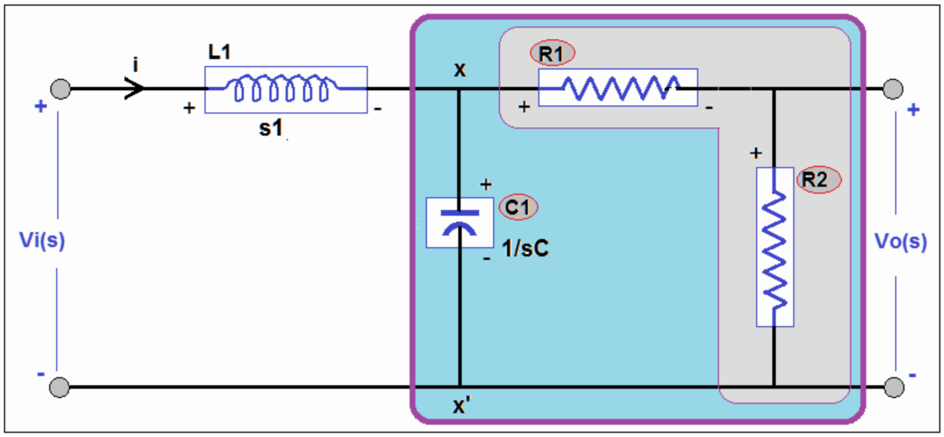
We understand up to here, usual impedance calculation. Here we separated the left from the right at xx'. And we see next why we choose this separation, and why impedances to the left and right help this solution.

$$\left(\frac{V_o}{V_{xx'}}\right)\left(\frac{V_{xx'}}{V_i}\right) = \frac{V_o}{V_i}$$

This can be called voltage division. The method is in representing voltages in terms of impedances, voltage division, respective to their relationship in the equation.

We relate:
1.). Vxx' to Z'
2). Vi to Zi
3). Vo to ? Coming next.

$$\frac{V_o}{V_i} = \left(\frac{V_o}{V_{xx'}}\right)\left(\frac{V_{xx'}}{V_i}\right)$$



The shaded grey part is part of the blue shown in circuit above.
The blue part is the Z' we calculated.
Now if we wanted the impedance representative of Vo, could be by taking a proportion of Z' by multiplying (R2/R1+R2)?

The dividing line is xx'. We have Z' for the total impedance of the left side port. For the impedance on representing Vo we can voltage division, R2 divided by R1 plus R2, then multiply it to the total resistance of the right side of impedance seen at xx'.

$$Z_o = Z' \cdot \left(\frac{R2}{R1 + R2}\right)$$

Now we have:
1.). Vxx' to Z'
2). Vi to Zi
3). Vo to Zo

$$\left(\frac{V_o}{V_{xx'}}\right)\left(\frac{V_{xx'}}{V_i}\right) = \frac{V_o}{V_i} = \frac{Z_o}{Z_i}$$

So we solve for Zo/Zi next.

$$Z_o = Z' \cdot \left(\frac{R2}{R1+R2}\right)$$

$$Z_i = s + Z'$$

$$\frac{Z_o}{Z_i} = \frac{Z' \cdot \left(\frac{R2}{R1+R2}\right)}{s+Z'} = \left(\frac{Z'}{s+Z'}\right) \cdot \left(\frac{R2}{R1+R2}\right)$$

$$= \left(\frac{\frac{R1+R2}{1+sC\,(R1+R2)}}{s+\frac{R1+R2}{1+sC\,(R1+R2)}}\right) \cdot \left(\frac{R2}{R1+R2}\right)$$

Multiply by 1/s :
$$= \frac{\frac{1}{s}\left(\frac{R1+R2}{1+sC\,(R1+R2)}\right) \cdot \left(\frac{R2}{R1+R2}\right)}{\frac{1}{s}\,(s) + \frac{1}{s} \cdot \left(\frac{R1+R2}{1+sC\,(R1+R2)}\right)}$$

$$= \frac{\left(\frac{R1+R2}{s+s^2 \cdot C\,(R1+R2)}\right) \cdot \left(\frac{R2}{R1+R2}\right)}{1+\left(\frac{R1+R2}{s+s^2 \cdot C\,(R1+R2)}\right)}$$

$$= \frac{\left(\frac{R1+R2}{s+s^2 \cdot C\,(R1+R2)}\right) \cdot \left(\frac{R2}{R1+R2}\right)}{\frac{s+s^2 \cdot C\,(R1+R2)}{s+s^2 \cdot C\,(R1+R2)} + \left(\frac{R1+R2}{s+s^2 \cdot C\,(R1+R2)}\right)}$$

$$= \frac{\left(\frac{R1+R2}{s+s^2 \cdot C\,(R1+R2)}\right) \cdot \left(\frac{R2}{R1+R2}\right)}{\frac{s+s^2 \cdot C\,(R1+R2)+R1+R2}{s+s^2 \cdot C\,(R1+R2)}}$$

$$= \left(\frac{R1+R2}{s+s^2 \cdot C\,(R1+R2)}\right) \cdot \left(\frac{R2}{R1+R2}\right) \cdot \left(\frac{s+s^2 \cdot C\,(R1+R2)}{s+s^2 \cdot C\,(R1+R2)+R1+R2}\right)$$

$$= \left(\frac{R1+R2}{1}\right) \cdot \left(\frac{R2}{R1+R2}\right) \cdot \left(\frac{1}{s+s^2 \cdot C\,(R1+R2)+R1+R2}\right)$$

$$= \frac{R2}{(R1+R2)} \cdot \left(\frac{R1+R2}{s+s^2 \cdot C(R1+R2) + R1 + R2} \right)$$

$$= \left(\frac{R2}{s+s^2 \cdot C(R1+R2) + R1 + R2} \right)$$

$$= \left(\frac{R2}{s^2 \cdot C(R1+R2) + s + (R1+R2)} \right)$$

Lets make the coefficient of the 2nd order term unity. Divide by C(R1+R2).

$$= \left(\frac{\frac{R2}{(C(R1+R2))}}{\frac{s^2 \cdot C(R1+R2)}{C(R1+R2)} + \frac{s}{C(R1+R2)} + \frac{(R1+R2)}{C(R1+R2)}} \right)$$

$$= \left(\frac{\frac{R2}{(C(R1+R2))}}{s^2 + \frac{1}{C(R1+R2)} s + \frac{1}{C}} \right)$$

Next we equate the given transfer function H(s) to the above :

$$H_v(s) = \frac{V_o(s)}{V_i(s)} = \frac{0.2}{s^2 + 3s + 2} = \left(\frac{\frac{R2}{(C(R1+R2))}}{s^2 + \frac{1}{C(R1+R2)} s + \frac{1}{C}} \right)$$

$$H_v(s) = \frac{0.2}{(s+2)(s+1)} = \left(\frac{\frac{R2}{C(R1+R2)}}{s^2 + \frac{1}{C(R1+R2)} s + \frac{1}{C}} \right)$$

Numerator
<--- As in Schaums.
Denominator

Numerator term:

$$0.2 = \frac{R2}{C(R1+R2)} \qquad = \qquad 0.2\ C(R1+R2) = R2$$

$$0.2\ CR1 + 0.2\ CR2 = R2$$

$$0.2\ CR1 + 0.2\ CR2 - R2 = 0$$

$$C = \frac{1}{2} = 0.5$$

Go to next page the denominator terms C was solved there.
You can see it here from inspection also.

$$0.1\ R1 + 0.1\ R2 - R2 = 0$$

$$0.1\ R1 - 0.9\ R2 = 0$$

$$0.1\ R1 = 0.9\ R2$$

$$R1 = \left(\frac{0.9}{0.1}\right) R2$$

$$R1 = 9\ R2$$

Denominator terms: $s^2 + 3\,s + 2 = (s+2)\,(s+1)$

$s^2 = 1$ 2nd order term here did not apply in transfer function equating of terms.

$$s = 3\,s = \frac{1}{C\,(R1 + R2)}\,s = 3\,CR1 + 3\,CR2 = 1$$

$$2 = \frac{1}{C} = C = \frac{1}{2} \quad \text{--->} \quad \left(\frac{3}{2}\right)\cdot R1 + \left(\frac{3}{2}\right)\cdot R2 = 1$$

$$3\,R1 + 3\,R2 = 2$$

$$R1 = 9\,R2$$

$$27\,R2 + 3\,R2 = 2$$

$$30\,R2 = 2$$

$$R2 = \frac{2}{30}$$

$$R2 = \frac{1}{15}$$

$$R1 = (9)\left(\frac{1}{15}\right) = \frac{9}{15} = \frac{3}{5}$$

Briefly, recap, the values of the components R L and C:

$R1 = \frac{3}{5}$ ohm Answer.

$R2 = \frac{1}{15}$ ohm Answer.

$C = \frac{1}{2}$ F Answer.

Comments:

This was a good problem to work, for me, it was a recap of past math skills.
Took me longer because of several careless mistakes. Good starter problem.
Applicable for electrical courses at year 2 of 3 and year 3 of 4 engineering programs.

Problem 8.16 (Pole Zero Plot) :

Construct the pole zero plot for the transfer admittance function :

$$H(s) = \frac{I_o(s)}{V_i(s)} = \frac{s^2 + 2s + 17}{s^2 + 3s + 2}$$

Solution:

Start with factoring the admittance function.
The admittance function is Y = 1/Z = I/V.
and the impedance function is Z.
The function here is admittance, treat it the same way
start with factoring to get the zeros and poles.

$$s^2 + 3s + 2 = (s+2)(s+1)$$

$s^2 + 2s + 17$ <--- this equation isnt easy to factor.

$$\begin{aligned} (s+4)(s-4) &= s^2 - 4s + 4s - 16 \\ &= s^2 - 16 \\ (s+j4)(s-j4) &= s^2 - j4s + j4s - j^2 \cdot 16 \\ &= s^2 + 16 \end{aligned}$$

$$\begin{aligned} (s+1+j4)(s+1-j4) &= s^2 + s - j4s + s + 1 - j4 + j4s + j4 - j^2\ 16 \\ &= s^2 + s + s + 1 - j^2\ 16 \\ &= s^2 + 2s + 1 + 16 \\ &= s^2 + 2s + 17 \end{aligned}$$ <--- What we were looking for.

$(s + 1 + j4)$ <--- Has three terms, s 1 and j4, that does not mean it would have a 3rd order term resulting. By adding a 1 in (s+j4) and (s-j4) it improved things. It requires some inspection, guess work, trial and error, some continued effort to get the factors. *Factoring is not my favourite math.*

$$H(s) = \frac{(s+1+j4)(s+1-j4)}{(s+1)(s+2)}$$ Now factored for zeros and poles.

Zeros:

$(s + 1 + j4)$ <--- For this expression ---> $(s + 1 - j4)$
to equal 0, s must equal what?

$s = -1 - j4$ $\qquad$ $s = -1 + j4$

Poles:

$(s + 2)$ $\qquad$ $(s + 1)$

$s = -2$ $\qquad$ $s = -1$

We have:

poles: $s_{p1} = -1 - j4$

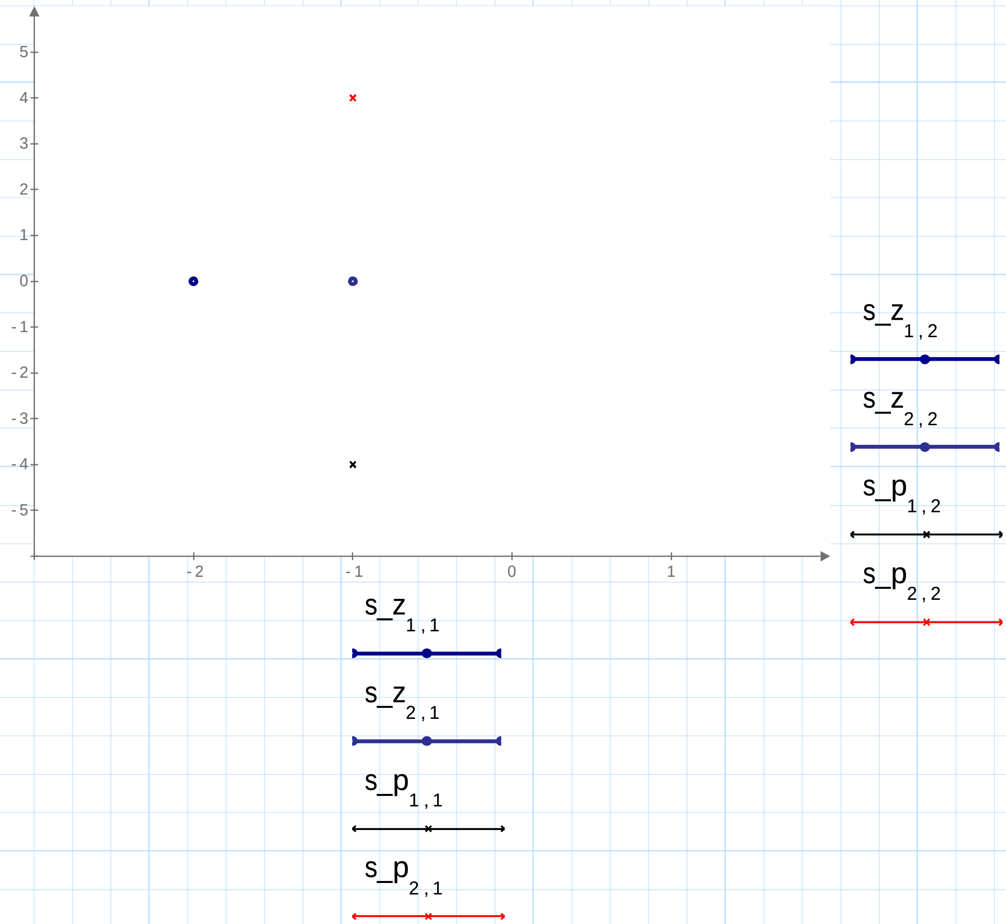
$s_{p2} = -1 + j4$

zeros: $s_{z1} = -2$ jw term = 0 no j term.

$s_{z2} = -1$ jw term = 0 no j term.

Origin(1,1)

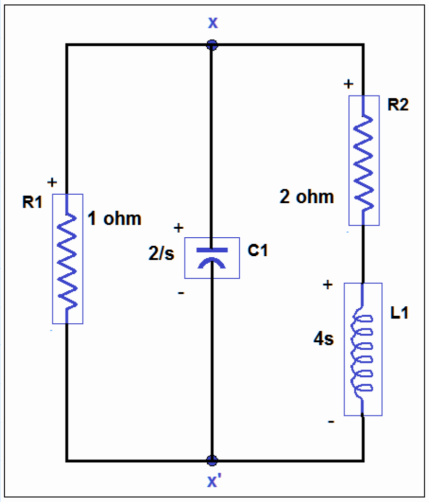
$$s\_p := \begin{bmatrix} -1 & -4 \\ -1 & 4 \end{bmatrix} \qquad s\_z := \begin{bmatrix} -2 & 0 \\ -1 & 0 \end{bmatrix}$$



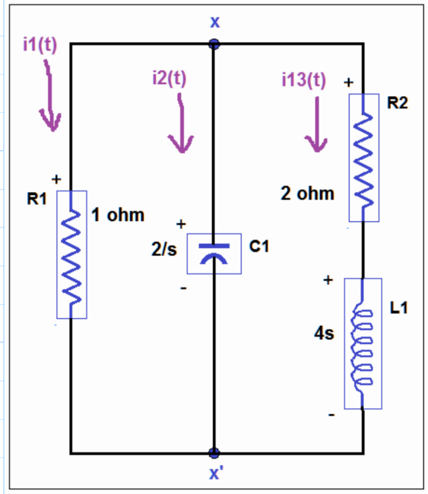
Answer. Plot above.

Problem 8:17 (Find the natural frequencies - s) :

Obtain the natural frequencies of the network, for the circuit shown, below left, by driving it with a conveniently located current source.



Branch currents noted: --->



Solution:

Discuss:
The circuit is a parallel circuit.
The current source can be placed anywhere in the circuit but we want to find the natural frequencies (s), so the placement of the current source must assist in this purpose. The natural frequencies to be found from a transfer function H(s).
And H(s) is V(s) / I(s). So now if I place the current source where I can get a transfer function for the circuit that may help!

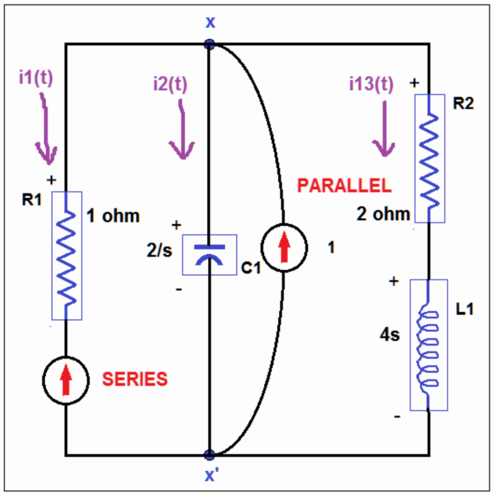
We have a branch with C, a branch with RL, and a branch with R.

$v_{R1} = i1(t)\,R1$ <---Current thru R1 will provide a voltage.

$v_C = \frac{1}{C}\int i2\,dt$ <---Current thru C will provide a voltage.

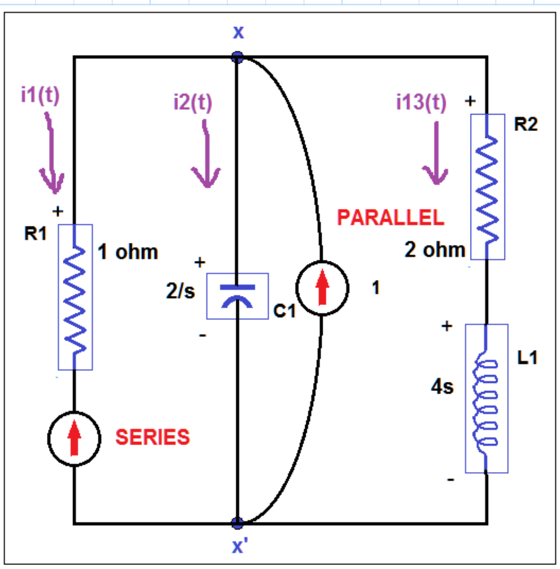
$v_{L\_R2} = L\left(\frac{di3}{dt}\right) + i3(t)\,R2$ <---Current thru L will provide a voltage.

From the above I cannot see readily where to place the current source. If the source is placed in series to other components in a branch it would be a tedious form for the transfer function. In comparison to if I place the current source in parallel in a new branch, and at the strategic location xx', then I see some profit in it. The current would split at node x. At least I have a node equation here.



Circuit to the left shows a current source in series and one in parallel. The series one would not be easy for the tranfer function derivation but the parallel seem easier.

The engineers created the problem with the position xx' given. I was trying to understand if that was arbitrary or a better choice.
So we place the current source across the middle branch which is parallel to all three branches.



Now I realise one major or key benefit of placing the current source parallel is the voltage across all three branches will be the same.

Regardless of the component values in each branch and the currents thorugh it, the voltage is the same across each branch. This does seem to lead to an easy benefit for the transfer function H(s).

With current source added the circuit has a forced response added, with the natural responce considered for both with and without current source.

$$R1 = 1 \qquad C = \frac{2}{s}$$

$$R2 = 2 \qquad L = 4\,s$$

$$H(s) = \frac{V(s)}{I(s)}$$

Here I(s) is the driving current, input, and V(s) the response.

$$V(s) = I(s)\ Z(s)$$

$$H(s) = \frac{V(s)}{I(s)} = \frac{1}{Z(s)}$$

So we start with the circuits impedance Z.

$$\frac{1}{Z(s)} = \frac{1}{Z_{R1}} + \frac{1}{Z_C} + \frac{1}{Z_{R2} + Z_L}$$

Impedance in parallel. The basic form for calculating parallel impedance, got all 3 branches in at once.

$$= \frac{1}{1} + \frac{1}{\left(\frac{2}{s}\right)} + \frac{1}{(2+4\ s)}$$

$$= 1 + \left(\frac{s}{2}\right) + \frac{1}{(2+4\ s)}$$

$$= 1 + \frac{s}{2} + \frac{1}{(2+4\ s)}$$

$$= \frac{1\cdot(2)\ (2+4\ s) + s\cdot(2+4\ s) + 1\cdot(2)}{(2)\ (2+4\ s)}$$

$$= \frac{4+8\ s+2\ s+4\ s^2+2}{4+8\ s}$$

$$= \frac{4\ s^2+10\ s+6}{4+8\ s}$$

Simplify this with a unity coefficient for s^2

$$\frac{1}{Z(s)} = \frac{\frac{4\ s^2}{4} + \frac{10\ s}{4} + \frac{6}{4}}{\frac{4}{4} + \frac{8\ s}{4}} = \frac{s^2+2.5\ s+1.5}{1+2\ s}$$

Unity coefficient for 2s in the denominator will be helpful

$$\frac{1}{Z(s)} = \frac{s^2+2.5\ s+1.5}{2\ (0.5+s)} = \left(\frac{1}{2}\right)\cdot\frac{s^2+2.5\ s+1.5}{(0.5+s)}$$

$$Z(s) = (2)\ \frac{s+0.5}{s^2+2.5\ s+1.5}$$

Next the other hard part to factor this, which in this solution the Engineers provided it. WORSTCASE I would solve using the roots of quadratic equation method !

$$Z(s) = 2\cdot\frac{(s+0.5)}{(s+1)\ (s+1.5)}$$

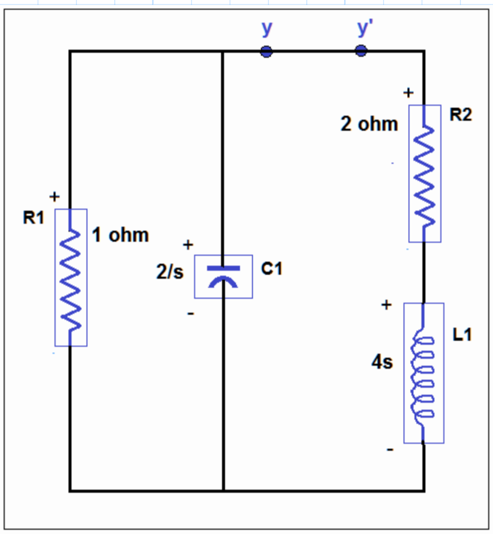
Now we can identify the natural frequencies:

Zero : $s_{z1} = -0.5$ Answer.

Poles : $s_{p1} = -1$ Answer.
$s_{p2} = -1.5$ Answer.

Problem 8.18 (Find the natural frequencies -s) :

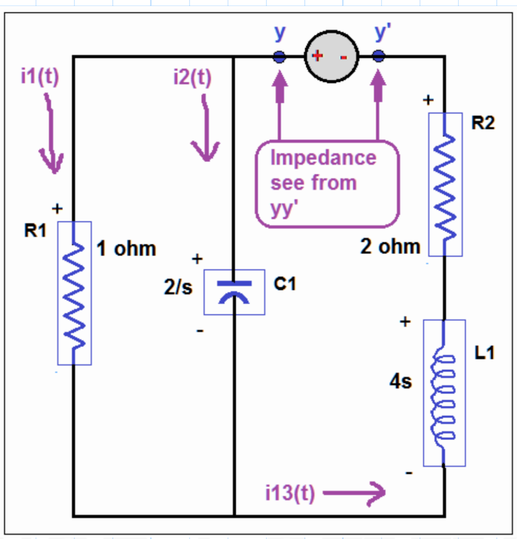
Repeat problem 8.17, this time driving the network with a conveniently located voltage source. Obtain the natural frequencies of the electrical network.



Circuit with the location for voltage source insertion given at yy'.

Solution:

Discuss:
I explained in the previous problem to apply the current source in parallel. Obviously, that is not suited for the voltage source, because we just done a parallel. Why?
Why can we not place the voltage source in the same position as the current source, that makes the voltage across all the brances the same. Maybe my argument is good, however, solution places the voltage source in location yy' in series to the left branch.



However, from our previous experience, we merely need to generate the transfer function, Z(s), that solved for the natural frequencies.

Same here with the voltage source in series we calculate the impedance Z(s). Then plug that in H(s) = I(s)/V(s) here I(s) is the response and V(s) the driving source, which makes H(s) = 1/Z(s).

$$Z_{R2\_L} = R2 + sL = 2 + 4\,s \quad \text{Series}$$

$$\frac{1}{Z_{R1\_C}} = \frac{1}{R1} + \frac{1}{\frac{2}{s}} = \frac{1}{1} + \frac{s}{2} = \frac{2+s}{(1)\,(2)} = \frac{2+s}{2} \quad \text{Parallel}$$

$$Z_{R1\_C} = \frac{2}{2+s} \quad \text{Inverted}$$

$$Z(s) = (2+4\,s) + \left(\frac{2}{2+s}\right) \quad \text{Series}$$

$$Z(s) = (2+4\,s) + \left(\frac{2}{2+s}\right) = \frac{(2+4\,s)}{1} + \left(\frac{2}{2+s}\right)$$

$$= \frac{(2+4\,s)\,(2+s) + (2)\cdot(1)}{(1)\,(2+s)}$$

$$= \frac{4 + 2\,s + 8\,s + 4\,s^2 + 2}{(2+s)}$$

$$= \frac{4\,s^2 + 10\,s + 6}{(2+s)} \quad \text{Factor out 4 for numerator}$$

$$Z(s) = \frac{4\left(s^2 + 2.5\,s + 1.5\right)}{(s+2)}$$

$$\frac{1}{Z(s)} = \left(\frac{1}{4}\right)\cdot\frac{s+2}{s^2 + 2.5\,s + 1.5}$$

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)} = \left(\frac{1}{4}\right)\cdot\frac{(s+2)}{(s+1)\,(s+1.5)}$$

Now we can identify the natural frequencies:

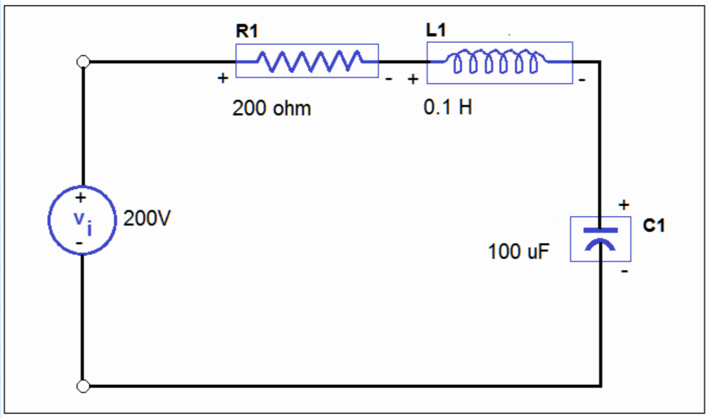
Zero : $s_{z1} = -2$ Answer.

Poles : $s_{p1} = -1$ Answer.
$s_{p2} = -1.5$ Answer.

Supplementary Problems 8.23 (RLC Circuit) :

A series RLC circuit, with R = 200 ohm, L = 0.1 H, and C = 100 uF, has a voltage source of 200V applied at t=0.

Find the current transient, assuming zero initial charge on the capacitor.



Solution:

Case	Series RLC	Parallel RLC
Under damped: (Oscillatory)	$\alpha < \omega_0$	$\alpha^2 < \omega_0^2$
Critically damped:	$\alpha = \omega_0$	$\alpha = \omega_0$
Over damped:	$\alpha > \omega_0$	$\alpha^2 > \omega_0^2$
α :	$\left(\frac{R}{2\,L}\right)$	$\left(\frac{1}{2\,RC}\right)$
ω_0 :	$\left(\frac{1}{\sqrt{LC}}\right)$	$\left(\frac{1}{\sqrt{LC}}\right)$

$R := 200 \qquad L := 0.1$

$C := 100 \cdot 10^{-6}$

$$\alpha = \frac{R}{2 \cdot L} = 1000 \qquad \text{s\^{}1.}$$

$$\omega = \frac{1}{\sqrt{L \cdot C}} = 316.227766$$

$$\omega^2 = \frac{1}{L \cdot C} = 100 \cdot 10^3$$

$$\beta := \sqrt{\left(1 \cdot 10^3\right)^2 - (316.228)^2} = 948.683$$

Alpha > Omega; 1000>316.2, the series RLC circuit is over damped.

s1 = - alpha + beta $\qquad$ s2 = - alpha - beta

$s1 := -1000 + 948.683 = -51.317$ $\qquad$ s^1.
$s2 := -1000 - 948.683 = -1948.683$ $\qquad$ s^1.

From our previous notes or your textbook, check please,
the form of equation we apply is shown below.

$$i(t) = A1e^{s1t} + A2e^{s2t}$$

$$i(t) = A1e^{-51.317\,t} + A2e^{-1948.683\,t}$$

Next, obvious, we need to solve for coefficients A1 and A2.

What comes to mind? Continuity Condition.

Our circuit was off during t<0.
So no energy built up or storage is found in the inductor and capacitor.

$iL(-0) = 0$ ---> $iL(0) = 0$ ---> $iL(0+.) = 0$
Here at 0+ the inductor is building up current and just past 0 at a low mili or micro second the current will be almost 0 which is practically 0.

In this circuit same for capacitor C:

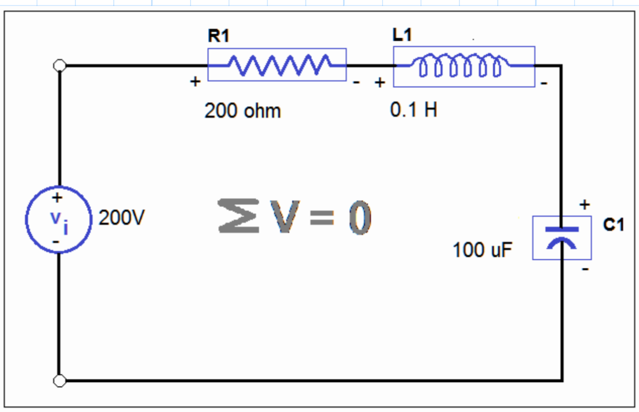
$iC(-0) = 0$ ---> $iC(0) = 0$ ---> $iC(0+.) = 0$

For our first equation at t=0, plug t=0 in the equation.

$$i(0) = A1e^{-51.317\,0} + A2e^{-1948.683\,0}$$

$$0 = A1 + A2 \qquad \text{Eq 1.}$$

We start with a Voltage Loop Equation (KVL for most Voltage Loop Law VLL or V Double L).
We have V = 200V.



We can do a voltage at t=0+, meaning just micro or nano second past t=0, here we can say the voltage is building up and at this time its zero across some or all comnponents.

KVL at t(0+) :

$$R \cdot i(t) + \frac{1}{C}\int i(t)\,dt + L\left(\frac{di}{dt}\right) = V$$

Current is rising, and same current thru R, but for C since its an integral we have the integral lower limit at t=0, here the vC(0) will be zero. Agree.

$$0 + 0 + L\left(\frac{di}{dt}\right) = V$$

$$0 + 0 + L\left(\frac{di}{dt}\right) = 200$$

$$0.1\left(\frac{di}{dt}\right) = 200$$

$$\left(\frac{di}{dt}\right) = \frac{200}{0.1} = 2000$$

$i(t) = A1e^{-51.317\,t} + A2e^{-1948.683\,t}$ Our current expression

We differentiate it:

$\frac{di}{dt} = -51.3\ A1\ e^{-51.3\,t} - 1948.7\ A2^{-1948.7\,t}$ We plug in (di/dt)

$$2000 = -51.3\ A1\ e^{-51.3\,t} - 1948.7\ A2^{-1948.7\,t}$$
$$t = 0$$
$$2000 = -51.3\ A1 \cdot e^{-51.3 \cdot 0} - 1948.7\ A2 \cdot e^{-1948.7 \cdot 0}$$

We have our 2 equations to solve:

$0 = A1 + A2$ Eq 1.

$2000 = -51.3\ A1 - 1948.7\ A2$ Eq 2.

$$Coeff := \begin{bmatrix} 1 & 1 \\ -51.3 & -1948.7 \end{bmatrix} \quad RHS := \begin{bmatrix} 0 \\ 2000 \end{bmatrix}$$

$$InvCoeff := Coeff^{-1} = \begin{bmatrix} 1.027 & 0.0005 \\ -0.027 & -0.0005 \end{bmatrix} \quad InvCoeff = \begin{bmatrix} 1.027 & 0.0005 \\ -0.027 & -0.0005 \end{bmatrix}$$

$$A1A2 := InvCoeff \cdot RHS = \begin{bmatrix} 1.0541 \\ -1.0541 \end{bmatrix}$$

$A1 = 1.0541$ A.
$A2 = -1.0541$ A.

$i(t) = 1.0541 \cdot e^{-51.317\,t} - 1.0541 \cdot e^{-1948.683\,t}$ Substitute A1 and A2

$i(t) = 1.0541\left(e^{-51\,t} - e^{-1949.\,t}\right)$ A Answer.

Added BONUS part to question:
To find the time at which current is maximum we need some equation from which we can solve for that time.
How do we get that equation?

The equation $i(t) = 1.0541 \cdot e^{-51\,t} - 1.0541 \cdot e^{-1949.\,t}$ is current relative to time.

At time t=0, i(t) = 0. LHS of equation =0.
$0 = 1.0541 \cdot e^{-51\,t} - 1.0541 \cdot e^{-1949.\,t}$

The derivative of above equation:

$0 = (-51)\,(1.0541) \cdot e^{-51\,t} - (-1949.)\,(1.0541) \cdot e^{-1949.\,t}$

$0 = -53.759 \cdot e^{-51\,t} + 2054 \cdot e^{-1949\,t}$

Next we need to calculate the time t that gives the maximum current:
Use logarithm to solve for maximum current time t:

$-28.73 \cdot e^{-1.70 \cdot t} + 5041.3 \cdot e^{-298.\,t} = 0$

$5041.3 \cdot e^{-298.\,t} = 28.73 \cdot e^{-1.70 \cdot t}$

$\frac{2054}{53.759} = \frac{e^{-51\,t}}{e^{-1949\,t}}$

$\frac{2054}{53.759} = 38.2075559$

$1949 + 51 = 2000$

$38.208 = e^{(-51 - (-1949)) \cdot t} = e^{1898\,t}$

$\ln(38.208) = 1898 \cdot t$

$\ln(38.208) = 3.64304$
$3.64304 = 1898 \cdot t$

$t = \frac{3.64304}{1898} = 0.00192$ seconds.

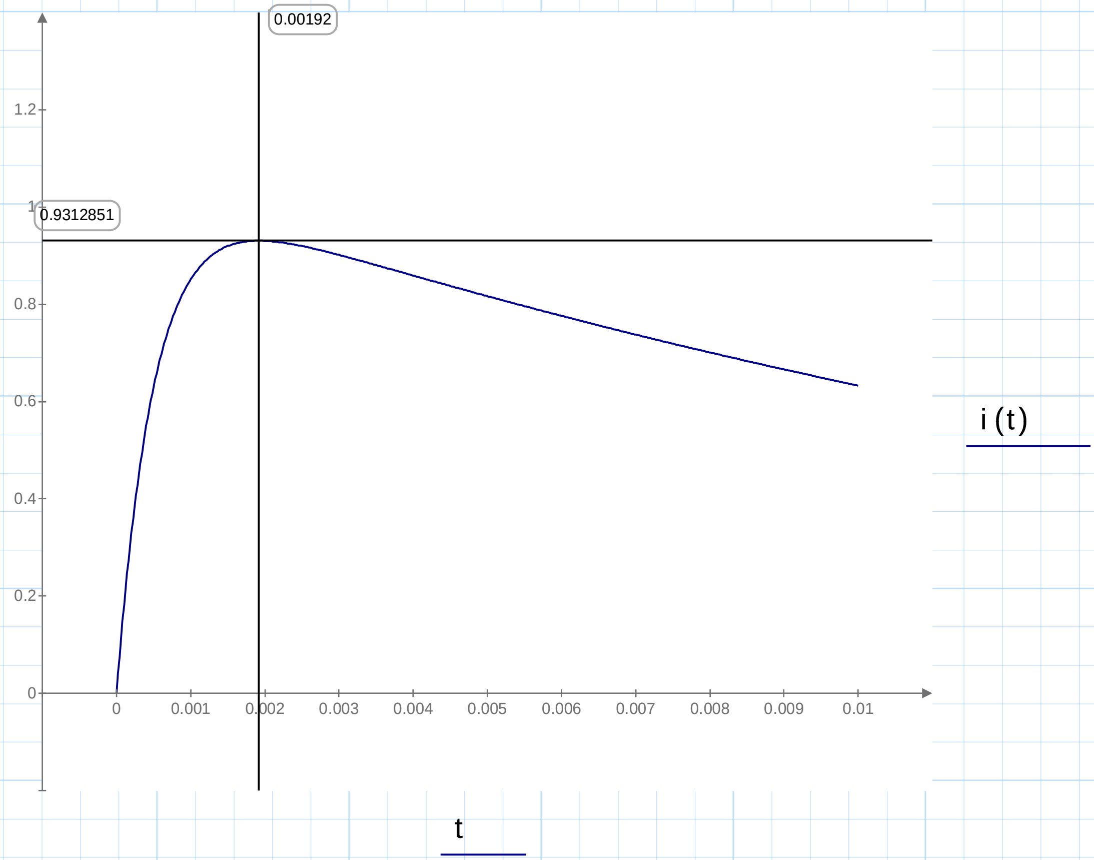
t = 0.00192 seconds. Answer.

We were not calculating the condition of the exponent's +ve or -ve sign rather where t will give that maximum current.

Next plot.
Note: Plot time t in milliseconds. This from our early calculation for this circuit.

clear (t)

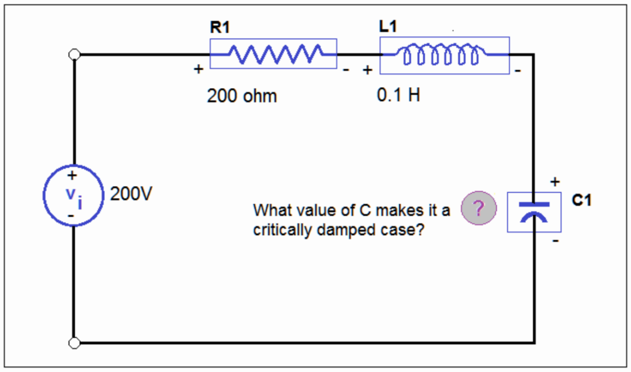
$i(t) := 1.0541 \cdot e^{-51\,t} - 1.0541 \cdot e^{-1949.\,t}$ mA. Note: vertical axis is in mA.



Time at t 1.92 ms and maximum current 0.93 A. Answer.

Supplementary Problem 8.24 (RLC circuit) :

What value of capacitance, in place of the 100 uF in problem 8.23, results in the critically damped case?



Solution:

Case	Series RLC	Parallel RLC
Under damped: (Oscillatory)	$\alpha < \omega_0$	$\alpha^2 < \omega_0^2$
Critically damped:	$\alpha = \omega_0$	$\alpha = \omega_0$
Over damped:	$\alpha > \omega_0$	$\alpha^2 > \omega_0^2$
α :	$\left(\frac{R}{2L}\right)$	$\left(\frac{1}{2RC}\right)$
ω_0 :	$\left(\frac{1}{\sqrt{LC}}\right)$	$\left(\frac{1}{\sqrt{LC}}\right)$

Critically damped requires alpha = omega.

$R := 200$ Ohm

$L := 0.1$ H

$$\alpha := \frac{R}{2 \cdot L} = 1000$$

$$\alpha_{squared} = 1000^2 = 1000000 \qquad \omega_{0\_squared} = \frac{1}{L \cdot C} = \frac{1}{0.1\ C}$$

$$\alpha_{squared} = \omega_{0\_squared}$$

$$1 \cdot 10^6 = \frac{1}{0.1\ C}$$

$$C = \frac{1}{(0.1) \cdot 1 \cdot 10^6} = 0.00001 \quad \text{OR} \quad = 10 \cdot 10^{-6} = 10\ \text{uF} \quad \text{Answer.}$$

Supplementary Problem 8.25 (RLC circuit) :

Find the natural resonant frequency, Beta, of a series RLC circuit with R = 200, L = 0.1, and C = 5 uF.

Solution:

Case	Series RLC	Parallel RLC
Under damped: (Oscillatory)	$\alpha < \omega_0$	$\alpha^2 < \omega_0^2$
Critically damped:	$\alpha = \omega_0$	$\alpha = \omega_0$
Over damped:	$\alpha > \omega_0$	$\alpha^2 > \omega_0^2$
α :	$\left(\frac{R}{2\,L}\right)$	$\left(\frac{1}{2\,RC}\right)$
ω_0 :	$\left(\frac{1}{\sqrt{LC}}\right)$	$\left(\frac{1}{\sqrt{LC}}\right)$

$R := 200$ Ohm

$L := 0.1$ H

$C := 5 \cdot 10^{-6}$ F

$$\alpha := \frac{R}{2 \cdot L} = 1000 \qquad \omega_0 := \frac{1}{\sqrt{L \cdot C}} = 1414.2$$

Alpha < Omega; 1000 < 1414.2; under damped case.

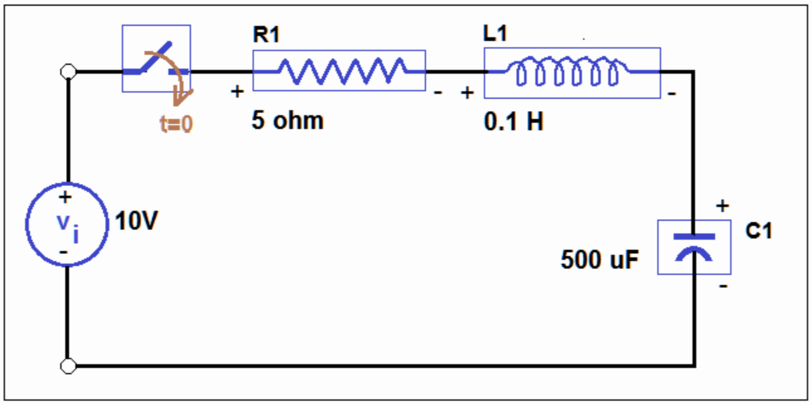
$$\beta \quad = \quad \sqrt{\left(\omega_0^{\,2}\right) - \left(\alpha^2\right)}$$ <--- Under damped case.

$$\beta := \sqrt{\omega_0^{\,2} - \alpha^2} = 1000$$ rad/s Answer.

Supplementary Problem 8.26 (Series RLC) :

A voltage of 10V is applied at t=0 to a series RLC circuit with R=5 ohm, L=0.1H, and C=500uF.

Find the transient voltage across the resistance?



Solution:

$R1 := 5$ $\quad L1 := 0.1$ $\quad C1 := 500 \cdot 10^{-6}$

$\alpha := \dfrac{R1}{2 \cdot L1} = 25$ $\quad \omega_0 := \dfrac{1}{\sqrt{L1 \cdot C1}} = 141.4213562$

$\alpha < \omega_0$ under damped case (oscillatory case). $\beta := \sqrt{\omega_0^{\,2} - \alpha^2} = 139.194$

$i(t) \quad = \quad e^{-25\,t} \cdot (A1\cos(139\ t) + A2\sin(139\ t))$

At time t=0 $\quad 0 \quad = \quad A1 \quad$ Eq 1

KVL at t(0+) : $\quad i_{C1}(0+.) \ = \ i_{C1}(-0.) = 0 \qquad i_{L1}(0+.) \ = \ i_{L1}(-0.) = 0$

$R \cdot i(t) + \dfrac{1}{C}\int i(t)\,dt + L\left(\dfrac{di}{dt}\right) \ = \ V \quad$ ---> $\quad 0 + 0 + L\left(\dfrac{di}{dt}\right) \ = \ V$

$0.1 \cdot \left(\dfrac{di}{dt}\right) \ = \ 10$

$\left(\dfrac{di}{dt}\right) \ = \ \dfrac{10}{0.1} = 100 \quad$ A/s.

$$i(t) = e^{-25t}\cdot(A1\cdot\cos(139\ t) + A2\cdot\sin(139\ t))$$ <---Differentiate

$$\frac{di(t)}{dt} = -25\cdot e^{-25\cdot t}\cdot A1\cdot\cos(139\cdot t) - e^{-25\cdot t}\cdot 139\ A1\cdot\sin(139\cdot t)$$

$$-25\ e^{-25\cdot t}\cdot A2\cdot\sin(139\cdot t) + e^{-25\cdot t}\cdot 139\ A2\cdot\cos(139\cdot t)$$

$$100 = -25\cdot e^{-25\cdot t}\cdot A1\cdot\cos(139\cdot t) + e^{-25\cdot t}\cdot 139\ A2\cdot\cos(139\cdot t)$$

At t=0 :

$$100 = -25\ A1 + 139\ A2$$ Eq 2

$$0 = A1$$ Eq 1

$$A2 = \frac{100}{139} = 0.7194$$

$$i(t) = e^{-25\ t}\cdot(A1\cos(139\ t) + A2\sin(139\ t))$$

$$i(t) = e^{-25\ t}\cdot(0\cdot\cos(139\ t) + 0.719\cdot\sin(139\ t))$$

$$i(t) = e^{-25\ t}\cdot(0.719)\cdot\sin(139\ t) = 0.719\cdot e^{-25\ t}\cdot\sin(139\ t)$$

$$v_{R1}(t) = R1\cdot\left(e^{-25\ t}\cdot(0.719)\cdot\sin(139\ t)\right)$$

$$v_{R1}(t) = 5\cdot\left(e^{-25\ t}\cdot(0.719)\cdot\sin(139\ t)\right)$$

$$v_{R1}(t) = 3.595\ e^{-25\ t}\ \sin(139\ t)$$

$$v_{R1}(t) = 3.60\ e^{-25\ t}\ \sin(139\ t)$$ Answer. Transient voltage across resistor.

Continuing for vL1(t): *Continuing on for the other voltages not part of question.*

$$i(t) = e^{-25\ t}\cdot(0.719)\cdot\sin(139\ t)$$

$$\frac{di(t)}{dt} = -25\cdot 0.719\cdot e^{-25\cdot t}\cdot\sin(139\cdot t) + (0.719)\ (139)\cdot e^{-25\cdot t}\cdot\cos(139\cdot t)$$

$$L1\cdot\frac{di(t)}{dt} = 0.1\left(-17.975\cdot e^{-25\cdot t}\cdot\sin(139\cdot t) + 99.941\cdot e^{-25\cdot t}\cdot\cos(139\cdot t)\right)$$

$$v_{L1}(t) := -1.79\cdot e^{-25\cdot t}\cdot\sin(139\cdot t) + 9.99\cdot e^{-25\cdot t}\cdot\cos(139\cdot t)$$ Inductor voltage.

Capacitor Equation: $v_{C1}(t) = \frac{1}{C1} \cdot \int i_{c1}(t)\, dt$

$$= \frac{0.719}{C1} \int_0^t e^{-25t} \cdot \sin(139\, t)\, dt$$

Integrating i(t) is a mess not doing it. By parts! I done exp and sine term before.
i_c1(t) = i(t). Too long and messy.

Lets try Prime evaluation for the integral term *I was just about to go on the internet.*

$$\int_0^t e^{-25t} \cdot \sin(139\, t)\, dt \rightarrow \lim_{t \to t^-} -\frac{e^{-25 \cdot t} \cdot (139 \cdot \cos(139 \cdot t) + 25 \cdot \sin(139 \cdot t))}{19946} + \frac{139}{19946}$$

$$\int_0^\infty e^{-25t} \cdot \sin(139\, t)\, dt \rightarrow \frac{139}{19946}$$

<--- Remove the approximation to infinity from the above expression. Otherwise the capacitor voltage does not terminate to zero.

$$v_{C1}(t) = \left(\frac{0.179}{500 \cdot 10^{-6}}\right) \cdot \left(-\frac{e^{-25 \cdot t} \cdot (139 \cdot \cos(139 \cdot t) + 25 \cdot \sin(139 \cdot t))}{19946}\right)$$

Lets expression is acceptable. We have voltage of C1 in terms of time t. vC1(t) eventually dies out, becomes open circuit. Since C1 is in series voltage across its terminals is zero, unlike if it were in a parallel branch and it may not be same then.

Variables we have below, caculated, are the transient values for the plot:

clear(t)

$V_s(t) := 10\, (t) \cdot \left(\frac{1}{t}\right)$ This gives Vs=10 for t>0 constant.
Not transient constant voltage.

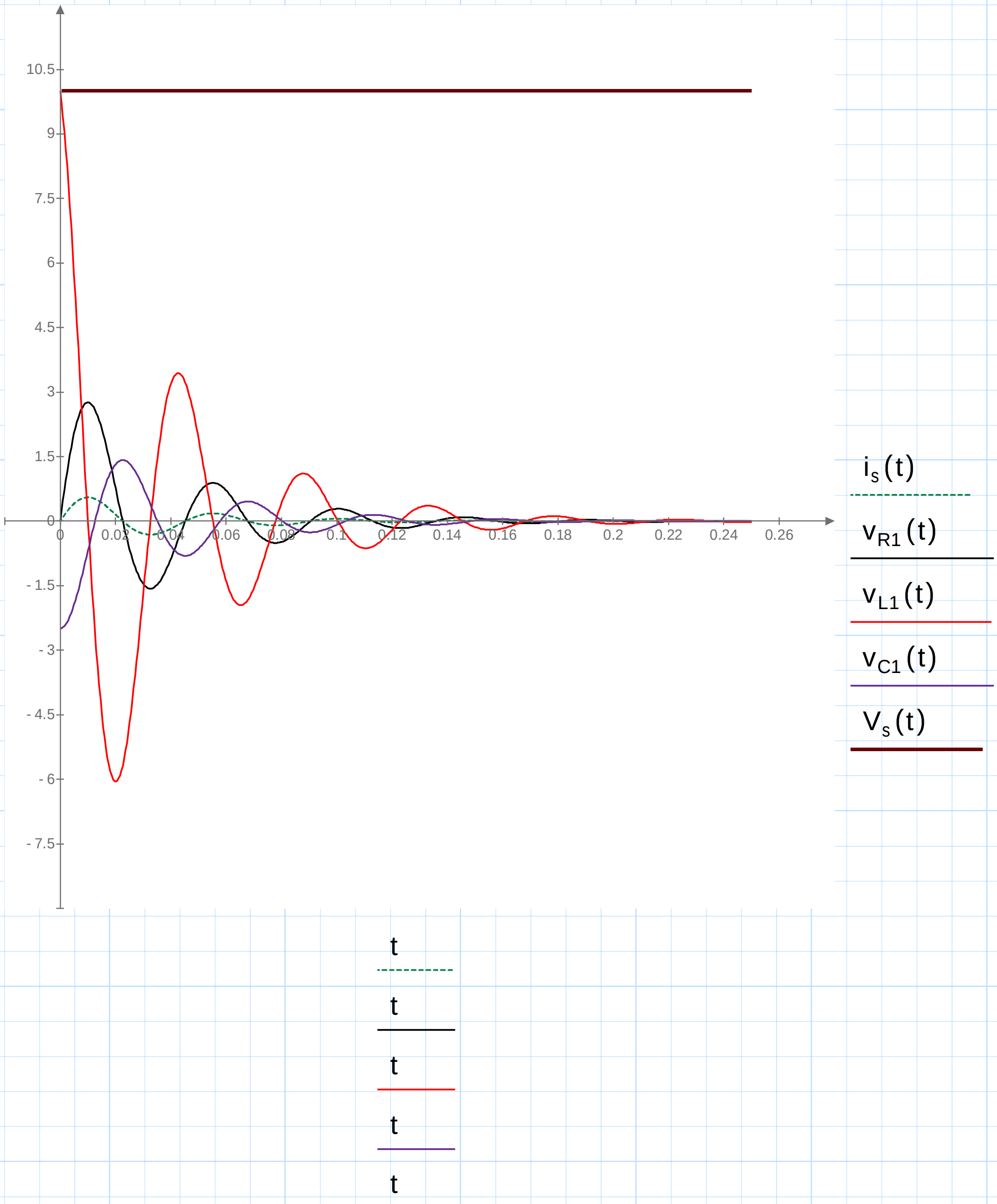
$i_s(t) := 0.719 \cdot e^{-25t} \cdot \sin(139\, t)$ Same current passint thru R1.

$v_{R1}(t) := 3.60\, e^{-25t} \sin(139\, t)$

$v_{L1}(t) := -1.79 \cdot e^{-25 \cdot t} \cdot \sin(139 \cdot t) + 9.99 \cdot e^{-25 \cdot t} \cdot \cos(139 \cdot t)$

$$v_{C1}(t) := -358 \cdot \left(\frac{e^{-25 \cdot t} \cdot (139 \cdot \cos(139 \cdot t) + 25 \cdot \sin(139 \cdot t))}{19946}\right)$$

Plots next page.



Comments: The transient values of voltages and current all die out at around 240*10^-3 seconds or 240 ms. Voltage source at 10V constant. All calculated plots are oscillatory (under damped). Inductor L1 voltage has highest peak, followed by resistor voltage and lastly capacitor C1 voltage. Looks acceptable. Does need your thinking in it. Check with your lecturer or local engineer.

Supplementary Problem 8.28 (Provide the time function of an s-domain voltage) :

A voltage has the s-domain representation 100-30 deg V.
Express the time function for:
a). s = - 2 Np/s
b). s = - 1 + j5 s^-1.

Solution:

a).

$100\angle 30 \text{ deg} \qquad s = -2 + j0$

$\sigma = 2$

$\omega = 0$

Phase angle = 30 deg

Amplitude = 100

$v(t) = Ae^{-\sigma t}\cos(\omega t + \phi)$

$v(t) = 100\, e^{-2\,t}\cos(0\,t + 30)$

$= 100\, e^{-2\,t}\cos(30)$

$\cos(30 \text{ deg}) = 0.866$

$v(t) = 86.6\, e^{-2\,t}$ Answer.

b).

$100\angle 30 \text{ deg} \qquad s = -1 + j5$

$\sigma = -1$

$\omega = 5$

Phase angle = 30 deg

Amplitude = 100

$v(t) = Ae^{-\sigma t}\cos(\omega t + \phi)$

$v(t) = 100\, e^{-1\,t}\cos(5\,t + 30 \text{ deg})$ Answer.

Supplementary Problem 8.29 (Provide the complex frequencies for the current i(t) :

Provide the complex frequencies associated with the current i(t):

$$i(t) = 5.0 + 10\, e^{-3t} \cos(50\, t + 90°)$$

Solution:

$$i(t) = A\, e^{-\sigma t} \cos(\omega\, t + \phi°)$$

$$s = \sigma + j\omega$$

$$i(t) = 10\, e^{-3t} \cos(50\, t + 90°)$$

$s1 = -3 + j50$ Answer.

$s2 = -3 - j50$ Answer.

w = +/- 50 as the frequency is both sides of the centre 0, for an amplitude versus frequency plot.

$i(t) = 5.0$ No complex frequencies because its a constant amplitude A = 5.
No other terms associated to it.

Supplementary Problem 8.30 (Provide current magnitude at time t) :

A phasor current 25-40 deg A has a complex frequency s = -2 + j3 s^-1.
What is the magnitude of i(t) at t=0.2s ?

$i(t) = 25\angle 40\ \text{deg}$ $t = 0.2$

Solution:

$25\angle 40\ \text{deg}$ $s = -2 + j3$

$\sigma = -2$

$\omega = 3$

Phase angle = 40 deg

Amplitude = 25

$$v(t) = Ae^{-\sigma t} \cos(\omega t + \phi) = 25\, e^{-2t} \cos(3\, t + 40)$$

$$v(t) = 25 \cdot e^{-2 \cdot (0.2)} \cdot \cos(3 \cdot 0.2 + 40)$$

$t = 0.2$

$\sigma t = -2 \cdot 0.2 = -0.4$

$\omega t = 3 \cdot 0.2 = 0.6$ rad. $\quad \omega t_{\_radians} = 0.6 \cdot rad = 34.3775$ deg

$v(t) = 25 \cdot e^{-2 \cdot 0.2} \cdot \cos(34.37 + 40)$

25

$e^{-2 \cdot 0.2} = 0.67032$

$\cos(74.37\ deg) = 0.2694241$

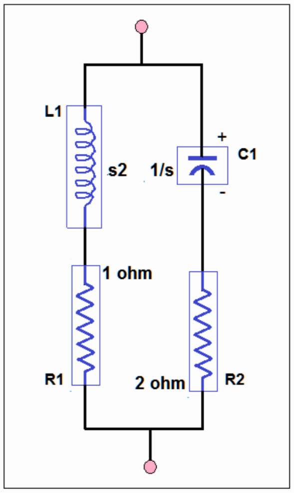
$25 \cdot 0.67 \cdot 0.269 = 4.5058$

$v(t) = 4.51$ V Answer.

Supplementary Problem 8.31 (Calculate Z(s)) :

Calculate impedance Z(s) for the circuit show below at:

a). s = 0
b). s = j1
c). s = j2
d). |s| = Infinity



Solution:

$Z_{R1L1}(s) = (1 + s2)$ Either s2 or 2s, same, to multiply 2s maybe easier to read.

$Z_{R2C1}(s) = \left(2 + \frac{1}{s}\right)$

$$Z(s) = \frac{(1 + s2) \cdot \left(2 + \frac{1}{s}\right)}{(1 + s2) + \left(2 + \frac{1}{s}\right)}$$

$$Z(s) = \frac{2 + \frac{1}{s} + 4s + 2}{3 + s2 + \frac{1}{s}} = \frac{4s + \frac{1}{s} + 4}{s2 + \frac{1}{s} + 3}$$

Numerator: $4\left(s + \frac{1}{4s} + 1\right) = 4\left(\frac{4s^2 + 1 + 4s}{4s}\right) = \frac{4s^2 + 4s + 1}{s}$

Denominator: $\left(s2 + \frac{1}{s} + 3\right) = \frac{2s^2 + 1 + 3s}{s} = \frac{2s^2 + 3s + 1}{s}$

$$Z(s) = \frac{\frac{4s^2 + 4s + 1}{s}}{\frac{2s^2 + 3s + 1}{s}} = \left(\frac{4s^2 + 4s + 1}{s}\right)\left(\frac{s}{2s^2 + 3s + 1}\right)$$

$$Z(s) = \left(\frac{4s^2 + 4s + 1}{2s^2 + 3s + 1}\right)$$

a). $s = 0$

$Z(0) = \left(\frac{4\,0^2 + 4\,0 + 1}{2\,0^2 + 3\,0 + 1}\right) = \frac{1}{1} = 1$ Ohm Answer.

b). $s = j1$

$$Z(j1) = \left(\frac{4j^2 + 4j + 1}{2j^2 + 3j + 1}\right) = \left(\frac{4(-1) + 4j + 1}{2(-1) + 3j + 1}\right) = \left(\frac{-3 + 4j}{-1 + 3j}\right)$$

$-3 + 4j$:

Magnitude: $\sqrt{(3^2) + (4^2)} = 5$ Phase angle: $\operatorname{atan}\left(\frac{4}{-3}\right) = -53.1301$ deg

$-1 + 3j$:

Magnitude: $\sqrt{(1^2) + (3^2)} = 3.1623$ Phase angle: $\operatorname{atan}\left(\frac{3}{-1}\right) = -71.5651$ deg

$\left(\frac{-3 + 4j}{-1 + 3j}\right) = \left(\frac{5}{3.1622}\right)(-53.13 - (-71.565)) = 1.581\angle 18.43$ deg Ohm Answer.

c). s = j2

$$Z(s) = \left(\frac{4s^2 + 4s + 1}{2s^2 + 3s + 1}\right)$$

$$Z(j2) = \left[\frac{4\left(2^2 \cdot j^2\right) + 4(2j) + 1}{2\left(2^2 \cdot j^2\right) + 3\ 2j + 1}\right] = \left(\frac{-16 + 8j + 1}{-8 + 6j + 1}\right) = \left(\frac{-15 + 8j}{-7 + 6j}\right)$$

$-15 + 8j$:

Magnitude: $\sqrt{\left(15^2\right) + \left(8^2\right)} = 17$ Phase angle: $\operatorname{atan}\left(\frac{8}{-15}\right) = -28.0725 \text{ deg}$

$-7 + 6j$:

Magnitude: $\sqrt{\left(7^2\right) + \left(6^2\right)} = 9.2195$ Phase angle: $\operatorname{atan}\left(\frac{6}{-7}\right) = -40.6013 \text{ deg}$

$$\left(\frac{-15 + 8j}{-7 + 6j}\right) = \left(\frac{17}{5.196}\right)(-28.072 - (-40.601)) = 1.84\angle 12.53 \text{ deg}$$ Ohm Answer.

d). $|s| = \infty$ Either $\sigma = \infty$ Or $j\omega = \infty$

Absolute value of s equal infinity. s = sigma + jw, s can have a magnitude and angle based on this, but see it for infinity as the composite absolute value of sigma + jw.

$$Z(s) = \left(\frac{4s^2 + 4s + 1}{2s^2 + 3s + 1}\right)$$

$$Z(\infty) = \frac{4\left(\infty^2\right) + 4(\infty) + 1}{2\left(\infty^2\right) + 3(\infty) + 1} = \frac{4\left(\infty^2\right) + 4(\infty) + 1}{2\left(\infty^2\right) + 3(\infty) + 1}$$

Let $A = 4(\infty) + 1$
$B = 3(\infty) + 1$

A is approximatley equal to B. So we can cancel these part of top and bottom.
The infinity squared term is far greater than infinity. Making A and B almost equal.

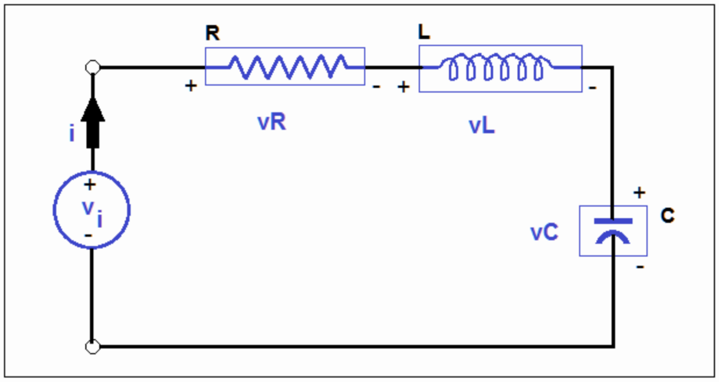
$$Z(\infty) = \frac{4\left(\infty^2\right)}{2\left(\infty^2\right)} = \frac{4}{2} = 2$$ Ohm Answer.

Comment:
d). There is no specific j term. The angle is not determinable, if we consider s = 0 + jw if w = infinty it lies on the y-axis as +/- infinity, and angle wise at sigma=0 it maybe 90 deg or -90 deg because jw = j0. You probably got a better idea there. Something to talk about on this infinity.

Supplementary Problem 8.33 (Transfer functions) :

In the time domain, a series circuit of R L and C has an applied voltage v_i, and element voltages vR, vL, and vC.

Obtain the voltage transfer functions:
a). V_R(s) / Vi(s)
b). V_C(s) / Vi(s)



Solution:

Write the voltage equation for the series circuit.

$$v_i(t) \quad = \quad Ri(t) + L\left(\frac{di(t)}{dt}\right) + \frac{1}{C}\int i(t)\,dt$$

Substitute corresponding Laplace expressions:

$$V_i(s) \quad = \quad RI(s) + LsI(s) + \frac{I(s)}{Cs}$$

a) :

$$V_i(s) \quad = \quad RI(s) + \left(Ls + \frac{1}{Cs}\right) I(s)$$

$$V_R(s) \quad = \quad RI(s) \qquad I(s) \quad = \quad \frac{V_R(s)}{R}$$

$$V_i(s) \quad = \quad V_R(s) + \left(Ls + \frac{1}{Cs}\right) I(s)$$

$$V_i(s) \quad = \quad V_R(s) + \left(\frac{Ls}{R} + \frac{1}{RCs}\right) V_R(s) \quad = \quad V_R(s)\cdot\left(1 + \frac{Ls}{R} + \frac{1}{RCs}\right)$$

$$V_i(s) = V_R(s)\cdot\left(1+\frac{Ls}{R}+\frac{1}{RCs}\right)$$

$$\frac{V_R(s)}{V_i(s)} = \frac{1}{\left(1+\frac{Ls}{R}+\frac{1}{RCs}\right)} = \frac{1}{\left(\frac{RCs+CLs^2+1}{RCs}\right)}$$

$$= 1\cdot\frac{RCs}{\left(RCs+CLs^2+1\right)}$$

$$= \frac{RCs}{\left(RCs+CLs^2+1\right)} = \frac{RCs}{C\cdot\left(Rs+Ls^2+\frac{1}{C}\right)}$$

$$= \frac{Rs}{\left(Rs+Ls^2+\frac{1}{C}\right)} = \frac{Rs}{Ls^2+Rs+\frac{1}{C}}$$

$$= \frac{\frac{Rs}{L}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

Divided by L so the coefficient of s^2 becomes 1.

$$\frac{V_R(s)}{V_i(s)} = \frac{\frac{Rs}{L}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

Answer.

b) :

$$V_i(s) = RI(s)+LsI(s)+\frac{I(s)}{Cs}$$

$$V_C(s) = \frac{I(s)}{Cs} \qquad I(s) = V_C(s)\ Cs$$

$$V_i(s) = RI(s)+LsI(s)+V_C(s)$$

$$V_i(s) = V_C(s)\ Cs\cdot(R+Ls)+V_C(s) = V_C(s)\ \left(RCs+LCs^2\right)+V_C(s)$$

$$V_i(s) = V_C(s)\cdot\left(1+RCs+LCs^2\right)$$

$$V_i(s) = V_C(s)\cdot\left(1 + RCs + LCs^2\right)$$

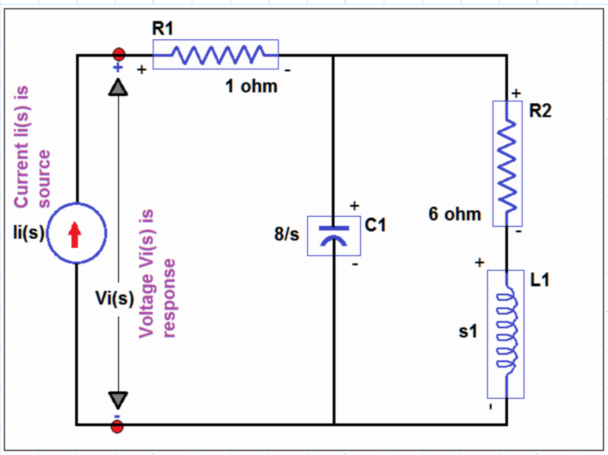
$$\frac{V_C(s)}{V_i(s)} = \frac{1}{\left(LCs^2 + RCs + 1\right)}$$ Divide by LC

$$\frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{LC}}{\left(s^2 + \frac{RCs}{LC} + \frac{1}{LC}\right)}$$

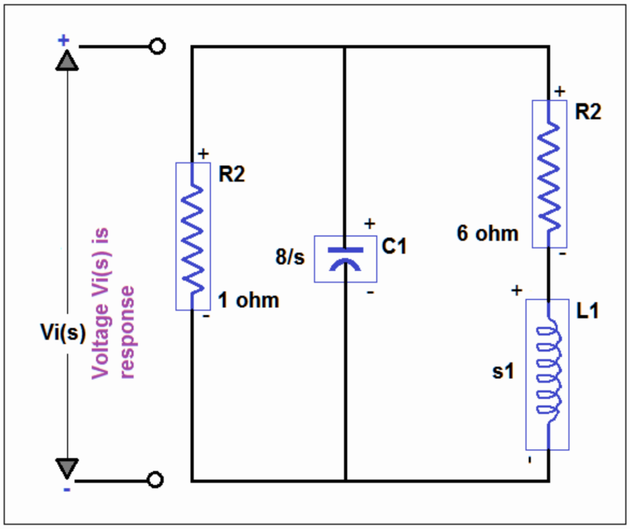
$$\frac{V_C(s)}{V_i(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$ Answer.

Supplementary Problem 8.34 (Transfer function and circuit) :

Obtain the electric circuit network function H(s) for the circuit provided below? The response is the voltage Vi(s).



Solution:
One method, if its correct, is to use Ohm's Idea, V = IZ.
The circuit input source is Ii(s), the response is the voltage across the whole circuit. When the current source is removed, the circuit is a parallel circuit with 3 branches. A re-drawing/sketching of the circuit may show this. Then we do V=IZ, and solve for Z = V/I. We need to get I(s) which we may be able to equate it to something, or substitute it for something. The other way is mesh analysis, two equations for loop voltages. Here, 2 solutions provided, first one turns out wrong, second correct.



This circuit shows the parallel branch connection.
Discussion: By obtaining Zin(s), with I(s) the current passing thru R1, as shown on previous page circuit, this would be Ii(s)R1 = vR1 = Vi(s). vR1 is the voltage across the branch, with current source having 0 impedance - this is not acceptable. If it was a voltage source in series with vR1, and Vi(s) has 0 impedance. No, there is voltage drop across vR1 - would not make branch voltage. I give it a try see what happens.

First Solution.

$$Z_{R2L1} = 6 + 1\,s$$

$$Z_{C1\_R2L1} = \frac{\left(\frac{8}{s}\right)(6+1\,s)}{(6+1\,s)+\left(\frac{8}{s}\right)} = \frac{\frac{48}{s}+8}{\frac{6\,s+1\,s^2+8}{s}} = \frac{\frac{48+8\,s}{s}}{\frac{6\,s+1\,s^2+8}{s}}$$

$$= \left(\frac{48+8\,s}{s}\right)\left(\frac{s}{6\,s+1\,s^2+8}\right) = \left(\frac{48+8\,s}{s^2+6\,s+8}\right)$$

$$Z_{R1\_C1\_R2L1} = \frac{\left(\frac{48+8\,s}{s^2+6\,s+8}\right)\cdot(1)}{\left(\frac{48+8\,s}{s^2+6\,s+8}\right)+1} = \frac{\left(\frac{48+8\,s}{s^2+6\,s+8}\right)}{\left(\frac{48+8\,s+s^2+6\,s+8}{s^2+6\,s+8}\right)} = \frac{\left(\frac{48+8\,s}{s^2+6\,s+8}\right)}{\left(\frac{s^2+14\,s+56}{s^2+6\,s+8}\right)}$$

$$Z_{in} = Z_{R2\_C1\_R2L1} = \left(\frac{48+8\,s}{s^2+6\,s+8}\right)\cdot\left(\frac{s^2+6\,s+8}{s^2+14\,s+56}\right) = \frac{8\,s+48}{s^2+14\,s+56}$$

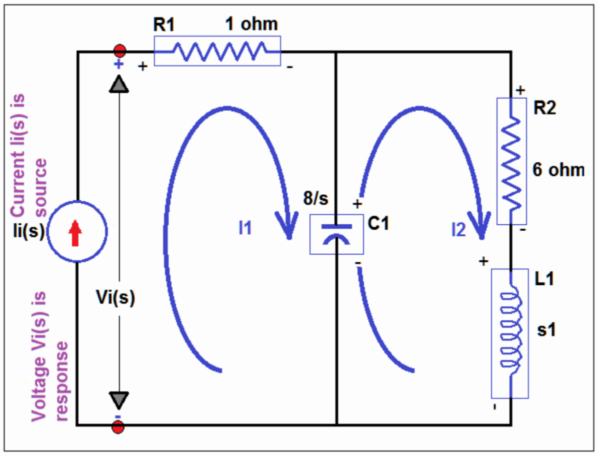
NOT the term!

Came close, looking for this term --->
You can see the cancelled term needed to remain.

$$\frac{s^2+14\,s+56}{s^2+6\,s+8}$$

I get it in the next solution method. Maybe there was an error here you may catch? No.

Conclusion: First solution not possible WRONG. Maybe I got some of my errors sorted. Hope I learnt my mistake. Good! Maybe.



Second solution:

Now I work on the loop equations. Tried the simple way it was wrong and corrupt.

Note: $\left(\frac{1}{C}\right)\int_0^t i1(t) - i2(t)\,dt$

to the s-domain ---> $\frac{1}{Cs}(I1(s) - I2(s))$

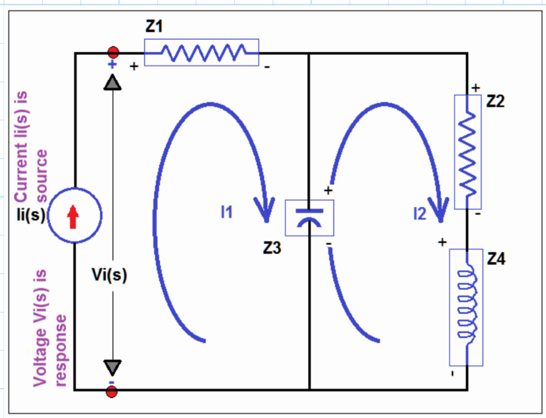
Lets set Z for impedance of components, makes it easier in following the ***Al-Zebra***.

$R1 = 1 = Z1$

$R2 = 6 = Z2$

$C1 = \frac{8}{s} = Z3$

$L1 = s1 = Z4$



Loop 1:

$$V_i(s) = Z1 \cdot I_1(s) + Z3\,(I1(s) - I2(s))$$

$$V_i(s) = Z1 \cdot I_1(s) + Z3 \cdot I1(s) - Z3 \cdot I2(s) \quad \text{Eq-1}$$

Current I1(s) passing thru Z1 is the circuit's I(s) current.

Loop 2:

$$0 = Z3\,(I2(s) - I1(s)) + Z2 \cdot I2(s) + Z4 \cdot I2(s)$$

Find an expression for I2(s) to substitute into the loop 1 equation.

$$0 = Z3\ I2(s) - Z3 \cdot I1(s) + Z2 \cdot I2(s) + Z4 \cdot I2(s) \quad \text{Eq-2}$$

$$Z3 \cdot I1(s) = I2(s) \cdot (Z3 + Z2 + Z4)$$

$$I1(s) = I2(s) \cdot \left(\frac{Z3 + Z2 + Z4}{Z3}\right)$$

$$I2(s) = I1(s) \cdot \left(\frac{Z3}{Z3 + Z2 + Z4}\right)$$

Substitute in Eq-1

$$V_i(s) = Z1 \cdot I1(s) + Z3 \cdot I1(s) - Z3 \cdot I2(s)$$ Eq-1...substitute for I2(s).

$$V_i(s) = Z1 \cdot I1(s) + Z3 \cdot I1(s) - Z3 \cdot \left(I1(s) \cdot \left(\frac{Z3}{Z3 + Z2 + Z4} \right) \right)$$

$$V_i(s) = I1(s) \cdot \left(Z1 + Z3 - \frac{Z3^2}{Z3 + Z2 + Z4} \right)$$

$$\frac{V_i(s)}{I1(s)} = Z1 + Z3 - \left(\frac{Z3^2}{Z3 + Z2 + Z4} \right)$$ Reduce the RHS term.

Remember I1(s) passing thru R1 is I(s) so we can substitute I(s) for I1(s)

Substitute the components values of Zs.

$R1 = 1 = Z1 \qquad R2 = 6 = Z2$

$C1 = \frac{8}{s} = Z3 \qquad L1 = s1 = s = Z4$

$$\frac{V_i(s)}{I1(s)} = 1 + \frac{8}{s} - \left(\frac{\left(\frac{8}{s}\right)^2}{\frac{8}{s} + 6 + s} \right) = 1 + \frac{8}{s} - \left(\frac{\left(\frac{8}{s}\right)\left(\frac{8}{s}\right)}{\frac{8 + 6s + s^2}{s}} \right)$$

$$= 1 + \frac{8}{s} - \left(\frac{8}{s}\right)\left(\frac{8}{s}\right) \cdot \left(\frac{s}{8 + 6s + s^2} \right)$$

$$= \left(\frac{s+8}{s}\right) - \left(\frac{64}{s}\right) \cdot \left(\frac{1}{8 + 6s + s^2}\right)$$

$$= \left(\frac{s+8}{s}\right) - \left(\frac{64}{8s + 6s^2 + s^3}\right)$$

$$= \frac{(s+8)\left(8s + 6s^2 + s^3\right) - 64(s)}{(s) \cdot \left(8s + 6s^2 + s^3\right)}$$

$$= \frac{\left(8s^2 + 6s^3 + s^4\right) + \left(64s + 48s^2 + 8s^3\right) - 64s}{8s^2 + 6s^3 + s^4}$$

$$= \frac{s^4 + 14s^3 + 56s^2}{s^4 + 6s^3 + 8s^2}$$ Divide by s^2

$$= \frac{s^2 + 14s + 56}{s^2 + 6s + 8} = \frac{s^2 + 14s + 56}{(s+2) \cdot (s+4)}$$

$$H(s) = \frac{V_i(s)}{I1(s)} = \frac{V_i(s)}{I(s)} = \frac{s^2 + 14\,s + 56}{(s+2)\cdot(s+4)}$$ Answer Correct.

Textbook answer had the numerator further factored, which is difficult.
You may find a technique in the Math textbook. Or software may do it.
You got to realise as the Engineer author you can make the math first then the circuit - working backward, sure, and here is my 2nd prize!

$$H(s) = \frac{V_i(s)}{I1(s)} = \frac{V_i(s)}{I(s)} = \frac{(s+7-j2.65)\;(s+7+j2.65)}{(s+2)\cdot(s+4)}$$

Textbook Answer.
In this case required answer.

Expansion proof below:

$(s+7-j2.65)\cdot(s+7+j2.65)$

$= \quad s^2 + 7\,s + j2.65s + 7\,s + 49 + j18.55 - j2.65s - j18.55 - j^2\;7.023$

$= \quad s^2 + 7\,s + j2.65s + 7\,s + 49 + j18.55 - j2.65s - j18.55 + 7.023$

$= \quad s^2 + 14\,s - j0 + 49 - j0 + 7.023$

$= \quad s^2 + 14\,s - j0 + 49 - j0 + 7.023$

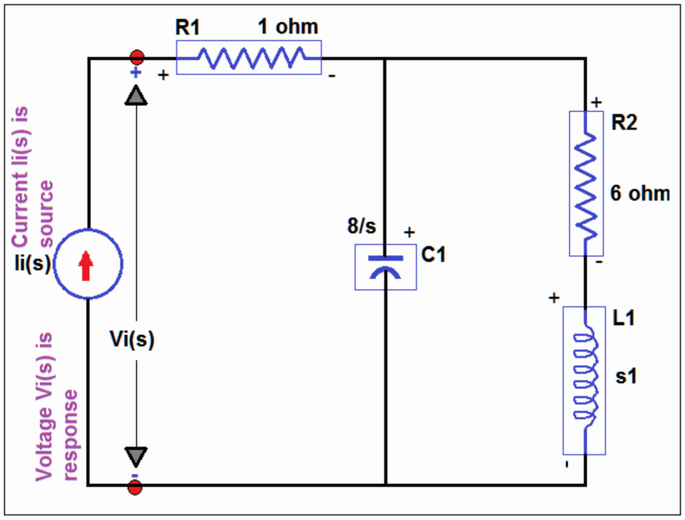
$= \quad s^2 + 14\,s + 56.023$ Close enough.
Admirable the use of the j term in the expansion.
TOUGH!
I take the 2nd prize.

There is a manual technique you can find in the maths textbook on how to factor tough higher order expressions.
Very surprising for me, hope not for you, the solution above with the j term is important in the next problem. We have basically the zeros and poles in the transfer function. So if we can factor them its helps. If we can get a j term in there that benefit is seen in the next problem.

Comments: My UG degree was General Engineering degree with a concentration in electrical engineering. ABET Accreditted - US. I had core courses in Strength of Materials and Dynamics, these were tough courses, the problems in Dynamics were 3 dimensional. Here in electric circuits the math takes such a strong position in comparison to visualising the problem, in mechanical courses visualistion plays a critical role. I say now for electric circuits the visualisation is in the math side of things the flow of the math, rather than 3-d object visualiation, maybe whats needed in electric circuits. You cannot do a physical visualisation of electrical objects but the visualisation of the waveforms expected....connected to the math. Math Visualisation. Electromagnetic Fields sure visualisation is good but where visualisations in the textbook? They were so poor in mine next to nil. 3D is helpful in fields.

Supplementary Problem 8.35 (Transfer function and circuit) :

Construct the s-plane plot for the transfer function of problem 8.34.
Evaluate H(j3) from the plot.



Note: j3 is the test point. See Part 3 A or B notes on the construction of zero and pole plot. Where the test point can move up and down the j-axis.

Solution:

$$H(s) = \frac{V_i(s)}{I(s)} = \frac{s^2 + 14s + 56}{(s+2)\cdot(s+4)}$$

$$H(s) = \frac{V_i(s)}{I(s)} = \frac{(s+7-j2.65)\ (s+7+j2.65)}{(s+2)\cdot(s+4)}$$

Find the zeros and poles:

Zeros: $(s+7-j2.65)\ (s+7+j2.65)$
What does s has to equal for each factor to equal 0.

$(s+7-j2.65)$: $s_{01} = -7+j2.65$

$(s+7+j2.65)$: $s_{02} = -7-j2.65$

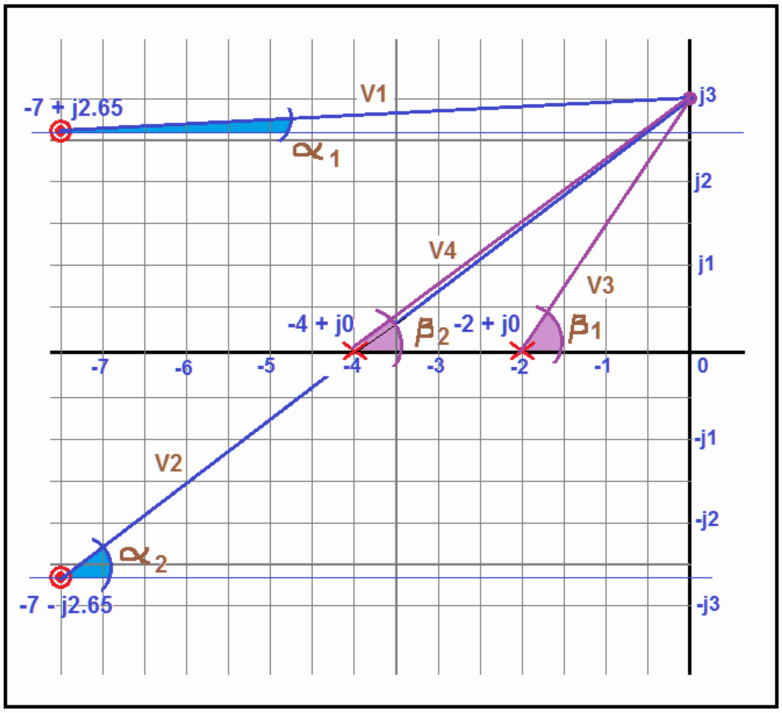
Poles: $(s+2)\cdot(s+4)$

What does s has to equal for each factor to equal 0.

$(s+2)$
Same as saying $(s+2+j0)$: $s_{p1} = -2+j0$

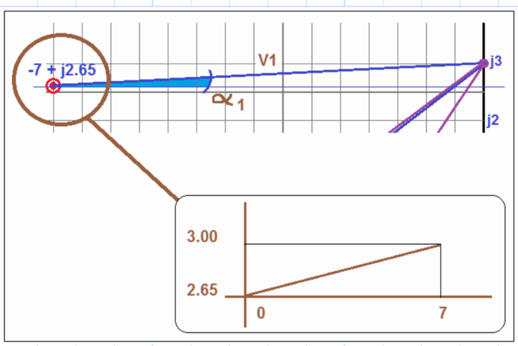
$(s+4)$
Same as saying $(s+4+j0)$: $s_{p2} = -4+j0$

Next we create the graph with the zeros and poles.
This shows their location.
We locate H(j3), ie j3, on the graph. This is (0, j3) on the graph.
Next we find the magnitude and angle from each zero and pole to (0, j3).
Then we evaluate for H(j3).



V1 :

$$\sqrt{(7-0)^2 + (3-2.65)^2} = 7.0087 \qquad V1_{ang} := \operatorname{atan}\left(\frac{3-2.65}{7-0}\right) = 2.8624 \text{ deg}$$



The figure on the right serve to show the reference for x and y axis so we get the correct sign on the angle.

Here I see its in reference to the point of the vector start to 0+j3. Otherwise the angle may be negative.

V2 :

$$\sqrt{(7-0)^2 + (2.65+3.00)^2} = 8.9957 \qquad V2_{ang} := \operatorname{atan}\left(\frac{5.65}{7}\right) = 38.9085 \text{ deg}$$

$\beta 1$:

$$\sqrt{(4)^2 + (3.00)^2} = 5 \qquad V2_{ang} := \operatorname{atan}\left(\frac{3}{4}\right) = 36.8699 \text{ deg}$$

$\beta 2$:

$$\sqrt{(2)^2 + (3.00)^2} = 3.6056 \qquad V2_{ang} := \operatorname{atan}\left(\frac{3}{2}\right) = 56.3099 \text{ deg}$$

Plug in the vectors we calculated:

$$H(s) = \frac{(s+7-j2.65)\,(s+7+j2.65)}{(s+2)\cdot(s+4)}$$

$$H(s) = \frac{(7.009\angle 2.862)\,(8.996\angle 38.908)}{(5\angle 36.87)\,(3.606\angle 56.31)}$$

$$= \frac{63.053\angle 41.77}{18.03\angle 93.18}$$

$$= 3.497\angle -51.41$$

$$H(s) = 3.5\angle -51.41 \text{ deg}$$ Answer. Same as textbook.

Discussion: If we wanted to show this on the graph where and whats its orientation? Can we convert it to cartesian, x-y axis, and place the location on the graph?
I give it a try.

x: $3.5\cdot\cos(-51.41 \text{ deg}) = 2.1831012$

y: $3.5\cdot\sin(-51.41 \text{ deg}) = -2.7357027$

The point (2.183 -j2.736) on the graph dont mean anything to me in relation to the vectors we placed. It looks OFF. But could this be the circuit's impedance at the given values of j3? The transfer function is Vi(s)/I(s) is really Zin(s). So that transfer function is the circuit's impedance. Which in this circuit the transfer function was Zin.

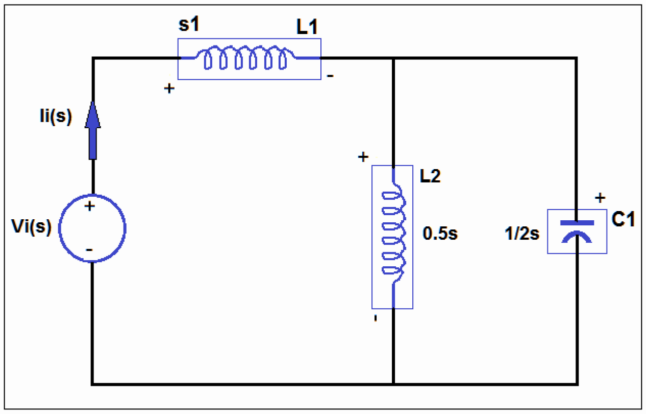
$$H(s) = \frac{V_i(s)}{I(s)} = Z_{in}(s) = \frac{(s+7-j2.65)\,(s+7+j2.65)}{(s+2)\cdot(s+4)}$$

$$H(s) = Z_{in}(s) = 3.5\angle -51.41 \text{ deg} = 2.183 - j2.736 \quad \text{at } s = 0 + j3.$$

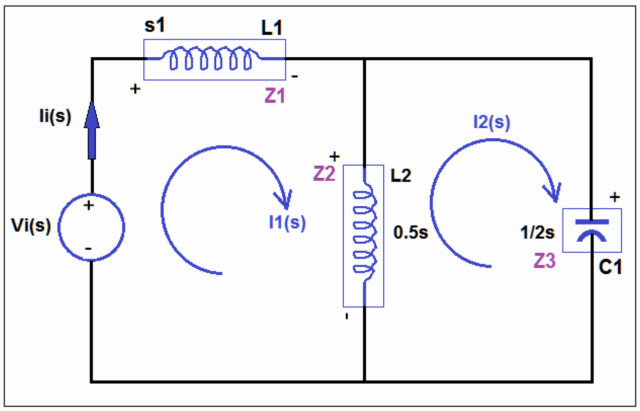
You are encouraged to conclude and or discuss with your local lecturer or engineer.

Supplementary Problem 8.36 (Circuit and transfer function) :

Obtain H(s) = Vi(s) / Ii(s) for the circuit shown below, and construct the pole-zero plot.



Solution:



$$L1 = s1 = Z1$$

$$L2 = 0.5\ s = Z2$$

$$C1 = \frac{1}{2\ s} = Z3$$

Loop 1 voltage equation:

$$Vi(s) = I1(s)\ Z1 + Z2\,(I1(s) - I2(s))$$

$$Vi(s) = I1(s)\ Z1 + I1(s) \cdot Z2 - I2(s) \cdot Z2$$

$$Vi(s) = I1(s)\ (Z1 + Z2) - I2(s) \cdot Z2 \qquad \text{Eq-1}$$

Loop 2 voltage equation:

$$0 = Z2\,(I2(s) - I1(s)) + I2(s)\ Z3$$

$$0 = I2(s)\ Z2 - I1(s)\ Z2 + I2(s)\ Z3 \qquad \text{Eq-2}$$

Solve for I2(s) in terms of I1(s) :

$$I1(s)\ Z2 = I2(s)\ (Z2+Z3)$$

$$I1(s) = \frac{I2(s)\ (Z2+Z3)}{(Z2)}$$

$$I1(s) = I2(s)\left(\frac{Z2+Z3}{Z2}\right)$$

$$I2(s) = I1(s)\left(\frac{Z2}{Z2+Z3}\right) \qquad \text{Eq-3}$$

Substitute I2(s) in Eq-1:

$$Vi(s) = I1(s)\ (Z1+Z2) - I2(s)\cdot Z2$$

$$Vi(s) = I1(s)\ (Z1+Z2) - I1(s)\cdot\left(\frac{Z2^2}{Z2+Z3}\right)$$

$$Vi(s) = I1(s)\left((Z1+Z2) - \left(\frac{Z2^2}{Z2+Z3}\right)\right)$$

$$Ii(s) = I1(s)$$ Current coming out of Vi(s) and goes into R1 is Ii(s).

$$\frac{Vi(s)}{Ii(s)} = H(s) = \left((Z1+Z2) - \left(\frac{Z2^2}{Z2+Z3}\right)\right)$$

Substitute component values for impedances.

$$L1 = s = Z1$$

$$L2 = 0.5\ s = \frac{s}{2} = Z2$$

$$C1 = \frac{1}{2\ s} = Z3$$

$$H(s) = \left(s+\frac{s}{2}\right) - \left(\frac{\left(\frac{s}{2}\right)^2}{\frac{s}{2}+\frac{1}{2\ s}}\right)$$

$$H(s) = \left(s+\frac{s}{2}\right)-\left(\frac{\frac{s^2}{4}}{\frac{s}{2}+\frac{1}{2\,s}}\right) = \left(\frac{2\,s+s}{2}\right)-\left(\frac{\frac{s^2}{4}}{\frac{2\,s^2+2}{4\,s}}\right)$$

$$= \left(\frac{2\,s+s}{2}\right)-\left(\frac{s^2}{4}\right)\left(\frac{4\,s}{2\,s^2+2}\right) = \left(\frac{2\,s+s}{2}\right)-\left(\frac{s^3}{2\,s^2+2}\right)$$

$$= \frac{(2\,s+s)\left(2\,s^2+2\right)-2\,s^3}{(2)\left(2\,s^2+2\right)} = \frac{\left(4\,s^3+4\,s+2\,s^3+2\,s\right)-2\,s^3}{\left(4\,s^2+4\right)}$$

$$= \frac{\left(4\,s^3+6\,s\right)}{\left(4\,s^2+4\right)} \quad \text{Divide by 4.} \quad = \frac{\left(s^3+1.5\,s\right)}{\left(s^2+1\right)}$$

$$H(s) = \frac{s\left(s^2+1.5\right)}{\left(s^2+1\right)}$$

Correct to textbook. Finally after several wrong substitutions.
And wrong signs. *May been attacked by hacker.*

Zeros and Poles:

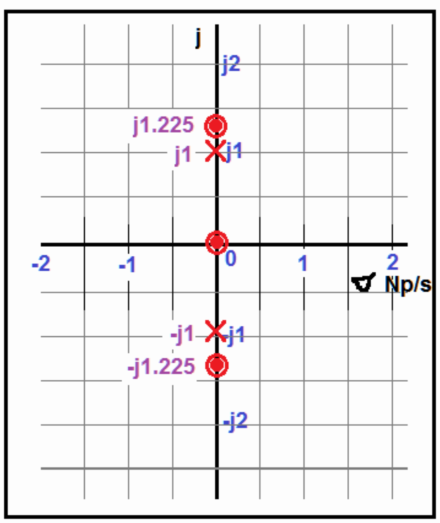
Zeros: $s_{z1} = 0$ $\quad s_{z2\_z3} = \sqrt{-1.5} = \sqrt{j1.5} = \sqrt{1.5} = 1.2247$

$s_{z2} = j1.225$ $\quad s_{z3} = -j1.225$

Poles: $s_{p1\_p2} = \sqrt{-1} = \sqrt{j1} = \sqrt{1} = 1$

$s_{p1} = j1$ $\quad s_{p2} = -j1$

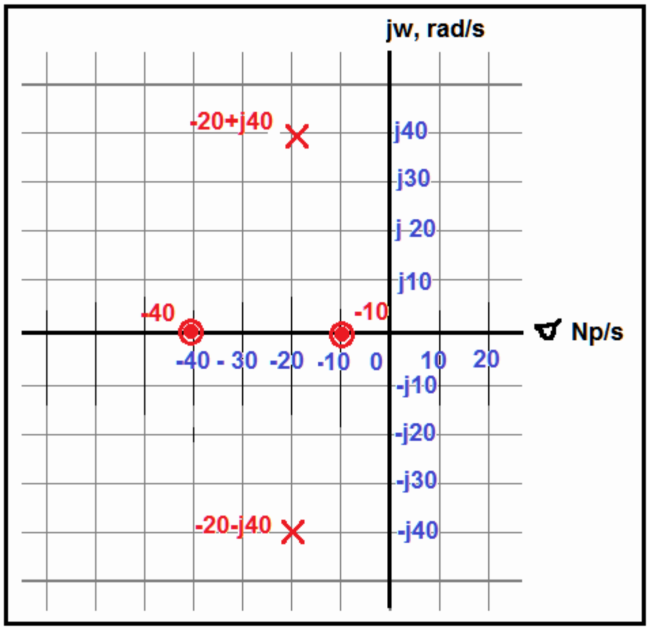
Next just place these zeros (O) and pole (X) on a graph.



Required graph/sketch/plot.
Answer.

Supplementary Problem 8.37 (Transfer function pole zero) :

Write the transfer function H(s) whose pole-zero plot is given below.



Solution:

From the plot list the pole and zero locations:

Zeros:

$s_{z1} = -10 + j0$

$s_{z2} = -40 + j0$

Poles:

$s_{p1} = -20 + j40$

$s_{p2} = -20 - j40$

Remember they were zeros and poles because it was the case how does s result in 0 in the factor(s). So these values make those factor 0. Those factors make up the transfer function. So we need to make the sign opposite in the process going in reverse.

Since there are 2 zeros and 2 poles the factor should result in a quadratic equation.

Zero expression: $(s + 10)\ (s + 40) = s^2 + 40\ s + 10\ s + 400$

$= s^2 + 50\ s + 400$ Answer.

Pole expression are conjugate : $-20 + j40$ and $-20 - j40$

These change sign $20 - j40$ and $20 + j40$

My method next is trial and error. You may have a better approach.

Points to consider:
1. Sign of the factor changes back, since what we have is how to you make s equal zero.
2. If its two parentheses, and two elements in the factor, (s+10) and (s+20), at most we expect 2nd order expression.
3. When I get one or two of the constants, it may lead me to get the rest thru trial and error.
4. *And in this case having the answer provided helps make certain its correct.*

$$(20-j40)\,(20+j40) \;=\; 400+800j-800j-j^2\;1600 \;=\; 400+1600 \;=\; 2000$$

Here 2000 is that value we use to make simpler the complex term.

Example $$\left(\frac{10+j40}{20-j40}\right)\left(\frac{20+j40}{20+j40}\right) = \frac{200+j400+j800+j^2\;1600}{400+800j-800j-j^2\;1600}$$

$$= \frac{-1400+j1200}{2000}$$ <---There is that 2000

Here I am making the case/point that 2000 is the constant term in the quadratic expression I seek.

$As^2+Bs+2000$

$s(As+B)+2000 \qquad s = 0 \qquad$ and $\qquad s = \dfrac{-B}{A}$ *For this quadratic equation if it should matter.*

$$s_1 = \frac{-b+\sqrt{b^2-4\,ac}}{2\,a}$$

Try B = 20?
With A = 1 because I assume the coefficient of the highest order is unity, and C = 2000 :

$$\frac{-20+\sqrt{20^2-4\cdot 1\cdot 2000}}{2\cdot 1} = -10+43.589j$$

Try B = 40?
With A = 1 and C = 2000 :

$$\frac{-40+\sqrt{40^2-4\cdot 1\cdot 2000}}{2\cdot 1} = -20+40j$$

$$\frac{-40-\sqrt{40^2-4\cdot 1\cdot 2000}}{2\cdot 1} = -20-40j$$

Looks promising we got the values of the pole(s).

So what we got?

We set A = 1 and that worked.
We got C = 2000 by expanding two factors.
Then I did a trial with 20 and then 40, and 40 resulted with the poles.

$$As^2 + Bs + C = 0$$

$$1\,s^2 + 40\,s + 2000$$

$$s^2 + 40\,s + 2000$$ Answer.

Now we form the transfer function expression:

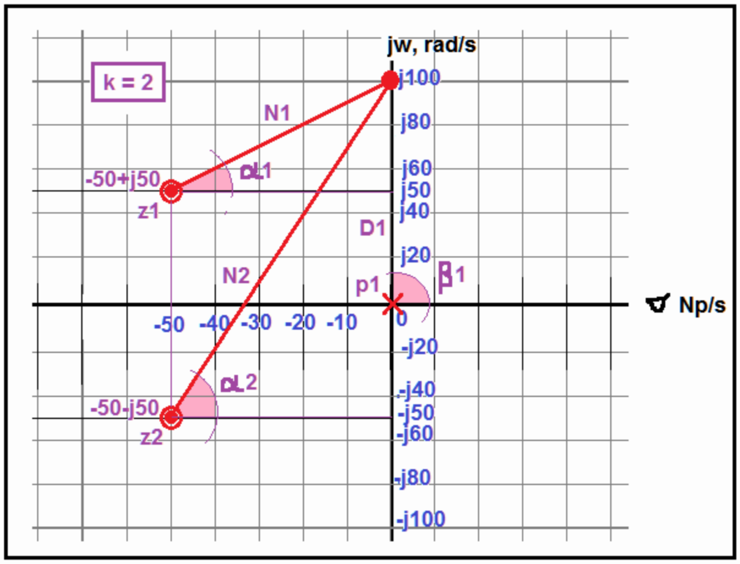
$$H(s) = \frac{s^2 + 50\,s + 400}{s^2 + 40\,s + 2000}$$

When we have our circuit we do lots of factoring including cancelling.
The expression above may had a factor or multiplier in front of it which we generally identifed as k, we place that in.

$$H(s) = k \cdot \frac{s^2 + 50\,s + 400}{s^2 + 40\,s + 2000}$$ Answer.
Final textbook answer.

Supplementary Problem 8.38 (Transfer function and test point) :

The pole zero plot in figure below shows a pole at s=0 and zeros at s = -50 +/- j50.
Use the geometrical method to evaluate the transfer function at the test point j100.



k = 2 given in figure.

Solution:

N1 :

$N1_{mag} = \sqrt{(50-0)^2 + (100-50)^2} = 70.7107$ $\quad N1_{ang} := \operatorname{atan}\left(\frac{50}{50}\right) = 45\ \text{deg}$

N2 :

$N2_{mag} = \sqrt{(50-0)^2 + (100-(-50))^2} = 158.1139$ $\quad N1_{ang} := \operatorname{atan}\left(\frac{150}{50}\right) = 71.5651\ \text{deg}$

D1 :

$D1_{mag} = \sqrt{(100-0)^2 + (0)^2} = 100$ $\quad N1_{ang} := \operatorname{atan}\left(\frac{100}{0.0000001}\right) = 90\ \text{deg}$

To get 90 deg make the denominator close to zero, the software adjust's it for 90 deg for no decimal point.

Zeros:

$D1 := 100\angle 90\ \text{deg}$

Poles:

$N1 := 70.711\angle 45\ \text{deg}$ $\quad N2 := 158.114\angle 71.565\ \text{deg}$

$k := 2$ Given in problem figure.

$$H(s) := k \cdot \left(\frac{(N1 \cdot N2)}{D1} \right) = 223.608 \angle 26.565°$$

$H(s) = 223.61 \angle 26.57°$ Answer.

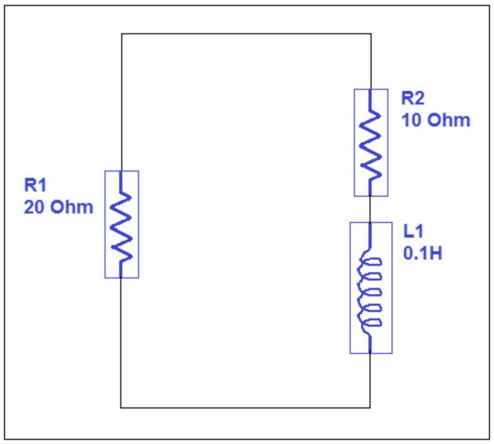
Next supplementary problem start on next page.

Supplementary Problem 8.39 (Circuits and complex frequency) :

A two branch parallel circuit has a resistance of 20 Ohm in one branch and the series combination of R=10 Ohm and L=0.1H in the other.

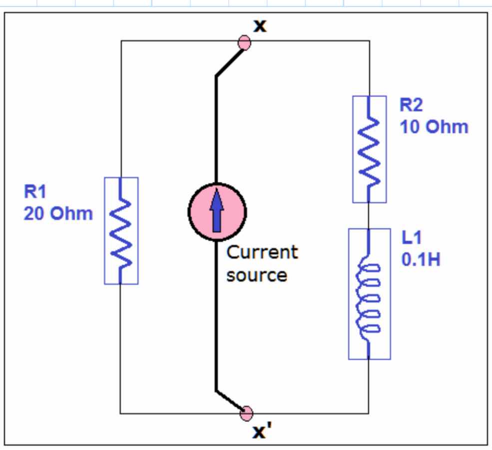
First apply an excitation Ii(s) and obtain the natural frequency from the denominator of the network function. Try different locations for applying the current source.

Second, insert a voltage source, Vi(s), and obtain the natural frequency.



Solution:

I solve this problem using solved problem 8.17 as a guide.
Question looks similar. Is what I am doing cheating?
If it is let others start of fresh. I spent enough time in 8.17 made some notes. Usually its a 100% everyone uses the same solution steps from the guide one they already done.
Make that a solid 100%.



Current source, I placed it in the middle of the series circuit, turned it into a parallel circuits. Makes it easier for the transfer function.
Excitation is I(s) response is V(s).
V(s) is same across parallel branches.

$$Z_{R1} = R1$$

$$Z_{R2L1} = R2 + sL1$$

$$Z_{in} = \frac{(R1)\cdot(R2 + sL1)}{(R1 + R2 + sL1)}$$

$$Z_{in} = \frac{R1R2 + sR1L1}{R1 + R2 + sL1}$$

$$Z_{in} = \frac{R1R2 + sR1L1}{R1 + R2 + sL1} = \frac{(20)\ (10) + s(20)\ (0.1)}{20 + 10 + s \cdot 0.1}$$

$$Z_{in} = \frac{200 + s2}{30 + s0.1}$$ Multiply top and botom by 10

$$Z_{in} = \frac{(2000 + s20)}{(300 + s)}$$

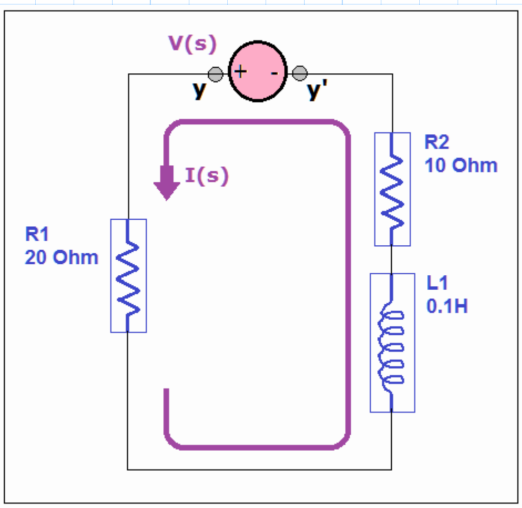
Now we can identify the natural frequencies:

Zero : $20\ s_{z1} = -2000$

$s_{z1} = -100$

Poles : $s_{p1} = -300$ Np/s. Answer.
Here the problem is concerned with the pole, denominaor, the maximum value we get.

Different locations, in parallel position, will result in the same pole frequency.



Place the voltage source in series and solve for a series circuit transfer function. Excitation is V(s), response is I(s) so place it in series same I(s) in series.

$$Z_{in} = R1 + R2 + sL$$

$$Z_{in} = 20 + 10 + s0.1$$

$$= 30 + s0.1$$

$$\frac{I(s)}{V(s0)} = \frac{1}{Z_{in}(s)}$$

Excitation is V(s) so the response is I(s).

$$\frac{1}{Z_{in}(s)} = \frac{1}{30 + s0.1}$$ Multiply top and bottom by 10

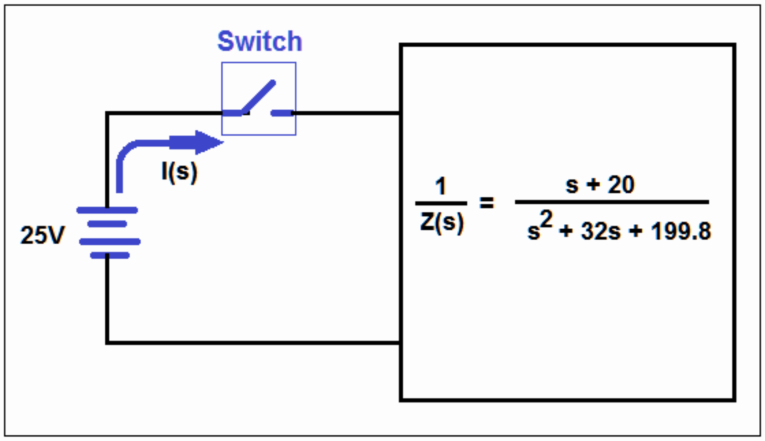
$$\frac{1}{Z_{in}(s)} = \frac{10}{300 + s}$$

Poles :

$s_{p1} = -300$ Np/s. Answer.
Problem is concerned with the pole, ie the denominator.

Supplementary Problem 8.40 (Transfer function and natural frequencies):

In the network shown below the switch is closed at t=0.
At t=0+, i=0, and di/dt = 25 A/s.



Obtain the natural frequencies and the complete current **i = i_n + i_f.**

Solution:

For Me It Was A Tough Solution. Several different tries till I said the solution path is in the equation.

$$V(s) = I(s) \cdot Z(s)$$

$$I(s) = \frac{V(s)}{Z(s)}$$ This is correct but I am looking for I(s) over V(s). V(s) the source, I(s) the response.

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{Z(s)} = \frac{(s+20)}{(s^2 + 32\ s + 199.8)}$$ Given.

Now, applying Ohm's rule:

$$I(s) = \frac{V(s)}{Z(s)} \qquad Z(s) = \frac{(s^2 + 32\ s + 199.8)}{(s+20)}$$

$$I(s) = \frac{V(s)}{Z(s)} = \frac{25}{\frac{(s^2 + 32\ s + 199.8)}{(s+20)}} = 25 \cdot \frac{(s+20)}{(s^2 + 32\ s + 199.8)}$$

I have an expression for current in the s domain.

$$I(s) = \frac{(s25 + 500)}{(s^2 + 32\ s + 199.8)} = \frac{25\ (s+20)}{(s+8.5)\ (s+23.5)}$$

Using solve function in Mathcad.

$$s \cdot 25 + 500 \xrightarrow{\text{solve}} -20$$

$$s^2 + 32 \cdot s + 199.8 \xrightarrow{\text{solve}} \begin{bmatrix} -8.5033340744034751939 \\ -23.496665925596524806 \end{bmatrix}$$

OR using the quadratic roots equation:

$$s_{z1} = \frac{-32 + \sqrt{32^2 - 4 \cdot 1 \cdot 199.8}}{2.1} = -8.0984 \qquad \text{(-8.5) Mathcad solve.}$$

$$s_{z1} = \frac{-32 - \sqrt{32^2 - 4 \cdot 1 \cdot 199.8}}{2.1} = -22.3778 \qquad \text{(-23.49) Mathcad solve.}$$

Close enough, the simple method is not much different compared to the software output. Textbook used the solve method, using some numerical method solution. The simple method is good. I want to stick with the textbook numeircal value, thats all.

Zeros: $s_{z1} = -20$ Zero response at zero(s), (s + 20) s = -20.
Poles: $s_{p1} = -8.5$
$s_{p2} = -23.5$ Maximum response is at poles(s).

Voltage source is 25V dc.
We use the exponential form of response since we have 2 roots for pole, a circuit that may be RL or RLC or RC circuit components. At present we only have the transfer function.

Forced response:
Equation for zero: $(s + 20)$
Excluding the s term? 20 Minimum at s=-20 Constant term.

Equation for pole: $s^2 + 40\,s + 199.8$
Excluding the s terms? 199.8 Approximately: 200 Constant term.

Now the transfer function is: $\dfrac{1}{Z(s)} = \dfrac{20}{200} = \dfrac{1}{10}$

$$Z(s) = 10$$

Voltage applied in circuit dc source: 25 V
Here we have current I(s) equal : $25\ V = I(s)\ Z(s)$

$$25\ V = I(s)\ 10$$

$$I(s) = \frac{25}{10} = 2.5 \quad \text{A. Forced response.}$$

2.5A is for t>0 when the natural response has died out and the forced response steady state continues. *A plot for this solution is provided at end.*

Natural response :

In the time domain $\quad i(t) = A1 \cdot e^{-8.5 \cdot t} + A2 \cdot e^{-23.5 \cdot t}$

Initial condition i(0+) = 0. Applies to both L and C in the network.

$$i(t) = A1 \cdot e^{-8.5 \cdot 0} + A2 \cdot e^{-23.5 \cdot 0}$$

$$0 = A1 + A2 \quad \text{at } t=0$$

Discussion: At t=0 the switch closes and 25V voltage source provides forced excitation, at time t=0, the natural response has not built up its near 0; i(0+) = 0. The forced response has built up and its 2.5 A.
The circuit sees 2.5A. So we place 2.5A for the expression above instead of 0.

$$2.5 = A1 + A2 \quad \text{at } t=0 \qquad \text{Equation 1}$$

Next for the 2nd equation, and the impact of the circuit complexity.

Differentiate i(t): $\quad i(t) = A1 \cdot e^{-8.5 \cdot t} + A2 \cdot e^{-23.5 \cdot t}$

$$\frac{di(t)}{dt} = -8.5\ A1 \cdot e^{-8.5 \cdot t} - 23.5\ A2 \cdot e^{-23.5 \cdot t}$$

$$\frac{di(t)}{dt} = 25 \quad \text{given in problem statement.}$$

$$25 = -8.5\ A1 \cdot e^{-8.5 \cdot t} - 23.5\ A2 \cdot e^{-23.5 \cdot t}$$

$$25 = -8.5\ A1 - 23.5\ A2 \qquad \text{Equation 2} \qquad \text{At } t=0.$$

$$2.5 = A1 + A2 \qquad \text{Equation 1} \qquad \text{At } t=0.$$

$$Coef1 := \begin{bmatrix} 1 & 1 \\ -8.5 & -23.5 \end{bmatrix} \qquad RHS1 := \begin{bmatrix} 2.5 \\ 25 \end{bmatrix}$$

Next the usual matrix/determinant way of solving the simultaneous equations.

$$Coef1inv := Coef1^{-1} = \begin{bmatrix} 1.5667 & 0.0667 \\ -0.5667 & -0.0667 \end{bmatrix}$$

$$A\_Coefs := Coef1inv \cdot RHS1 = \begin{bmatrix} 5.5833 \\ -3.0833 \end{bmatrix}$$

<--- These are NOT the coefficients. WRONG.

I am not looking at the circuit from the Z(s) mathematical equation given complexity.

The expression Z(s) is of the 2nd order.

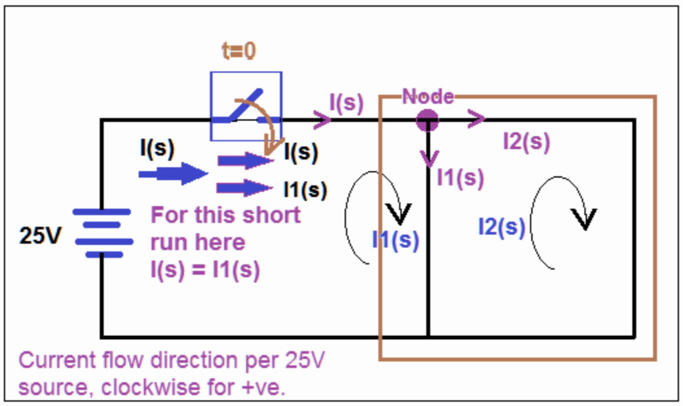
This means in the block diagram there can be a minimum of 2 meshes or loops.

The current has to be looked at from this perspective to make the 2nd equation valid.

So we have a voltage loop equation and current node equation possible.

This is the next step of the solution.

Mesh analysis:



Study the circuit.

Mesh 1 voltage loop:

$$Z(s)\ (I1(s) - I2(s)) = 25$$

$$I1(s) - I2(s) = \frac{25}{Z(s)} = \frac{25}{\dfrac{(s^2 + 32\ s + 199.8)}{(s+20)}} = \frac{25\ (s+20)}{(s+8.5)\ (s+23.5)}$$

Mesh 2 voltage loop:

$$Z(s)\ (I2(s) - I1(s)) = 0$$

$$Z(s)\ I2(s) = Z(s) \cdot I1(s)$$

Usually it is not so simple but here we assume thats the case the Z(s) cancels off.

$$I2(s) = I1(s)$$

Because of the equal condition in mesh 2 it is not be possible to solve for 2 simultaneous equations. Need to do a current equation at node.

Current equation at node:

$$I(s) = I1(s) + I2(s) \qquad \text{Substitute for } I2(s)$$

$$\text{Since} \quad I2(s) = I1(s)$$

$$I(s) = 2\ I1(s)$$

Now we return to our early equations for I(s) and make that 2I1(s).

$$I(s) = \frac{V(s)}{Z(s)} = 25 \cdot \frac{(s+20)}{\left(s^2 + 32\ s + 199.8\right)}$$

$$I(s) = 2\ I1(s) = 25 \cdot \frac{(s+20)}{\left(s^2 + 32\ s + 199.8\right)}$$

Since I am taking into account the circuit's component's natural response supplying current back into the node, from C component in the circuit, the value of I(s) has to be adjusted.

$$I(s) = I1(s) = \left(\frac{25}{2}\right) \cdot \frac{(s+20)}{\left(s^2 + 32\ s + 199.8\right)}$$

divided by 2,
from 2I(s) to I(s).

$$I(s) = (12.5) \cdot \frac{(s+20)}{\left(s^2 + 32\ s + 199.8\right)}$$

$$I(s) = (V(s)) \cdot \left(\frac{1}{Z(s)}\right)$$

di/dt is a gradient, when its y-axis current value is halved, over the same time interval dt - x axis, the rate of current di/dt is halved.

A new value 12.5 A/s is placed in instead of 25 A/s.

$$12.5 = -8.5\ A1 - 23.5\ A2 \qquad \text{At } t>0. \qquad \text{Equation 2 Updated.}$$

Note: Give a look or analyse into the circuitry thru the impedance expression, in this case a 2nd order equation.

$$2.5 = A1 + A2 \qquad \text{Equation 1}$$

$$12.5 = -8.5\ A1 - 23.5\ A2 \qquad \text{Equation 2 Updated.}$$

Using matrix to solve for the values of A1 and A2.

$$Coeff := \begin{bmatrix} 1 & 1 \\ -8.5 & -23.5 \end{bmatrix} \qquad LHS := \begin{bmatrix} 2.5 \\ 12.5 \end{bmatrix}$$

$$Coeff\_Inv := Coeff^{-1} = \begin{bmatrix} 1.5667 & 0.0667 \\ -0.5667 & -0.0667 \end{bmatrix}$$

$$I\_coeff := Coeff\_Inv \cdot LHS = \begin{bmatrix} 4.75 \\ -2.25 \end{bmatrix}$$

These are the natural response coefficients for A1 and A2 but they contain the 2.5A forced response in them. That need to be subtracted, leaving the natural response with the correct sign. Taking into consideration the current directions.

Equation 1

$A1 + A2 \quad = \quad 2.5$

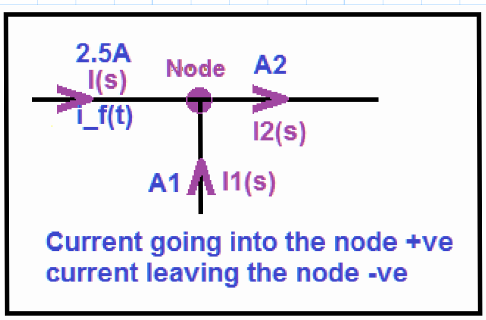
$4.75 + (-2.25) = 2.5$ Satisfied with the forced response 2.5 A.

Equation 2

$-8.5\ A1 - 23.5\ A2 \quad = \quad 12.5$

$-8.5 \cdot (4.75) - 23.5 \cdot (-2.25)$

$-40.375 + 52.875 = 12.5$ Satisfied the derivative equation resulting in 12.5V.



Actual values of A1 and A2:

$A1 \quad = \quad 2.5 - 4.75 = -2.25$

$A2 \quad = \quad 2.5 - 2.75 = -0.25$

Check sum of currents at node equal 0:

$(-2.25) + (-0.25) + 2.5 = 0$

Now maybe some acceptance of the solution by the you reader is there, for me this is the solution until I am told otherwise thru a written solution.

$i(t) = A1 \cdot e^{-8.5 \cdot t} + A2 \cdot e^{-23.5 \cdot t}$ Updating the current equation for A1 and A2.

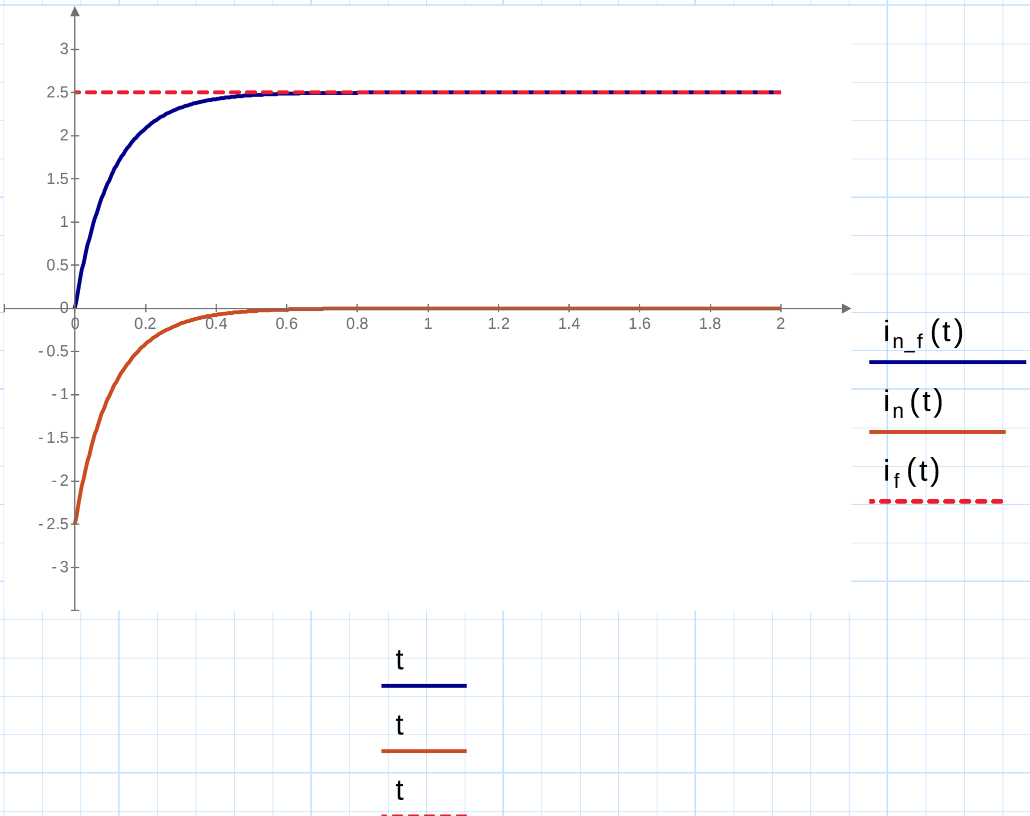
$i(t) = -2.25 \cdot e^{-8.5 \cdot t} - 0.25\, e^{-23.5 \cdot t}$ Add the forced response next.

$i(t) = -2.25 \cdot e^{-8.5 \cdot t} - 0.25\, e^{-23.5 \cdot t} + 2.5$ A. Answer.
Textbook answer.

Comments: I say you may have a better way of solving this problem. The numerical answer is correct. Took me my usual long time to solve it. Check with your local lecturer or engineer.

Plot clear (t) $i_{n\_f}(t) := -2.25 \cdot e^{-8.5 \cdot t} - 0.25 \cdot e^{-23.5 \cdot t} + 2.5$ <--- This function.

$i_n(t) := -2.25 \cdot e^{-8.5 \cdot t} - 0.25 \cdot e^{-23.5 \cdot t}$ $i_f(t) := 2.5$



Comments: The complete solution, i_n_f(t), forced and natural response, starts at 0 and then settles at 2.5A for time t>0.

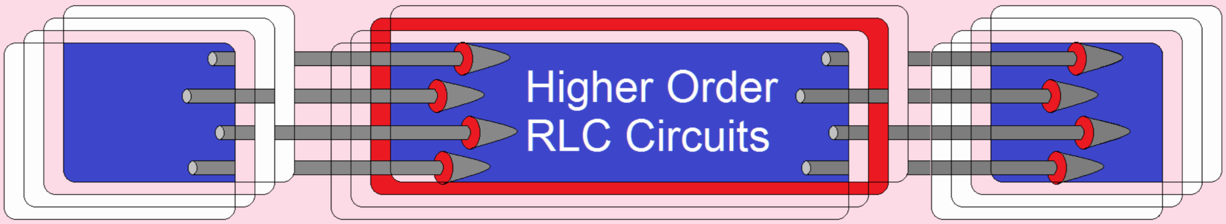
End of this section of Part 3C.

My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis. Source of study material: 1). Electric Circuits 6th Ed., Nahvi & Edminister. 2). Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill. Resource: 3). Solutions & Problems of Control Systems, 2nd ed - AK Jairath.
Karl S. Bogha.

SECTION TWO.

Level: Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



Part 3 - C

(Intermediate Level)

Apologies for any errors and omissions.

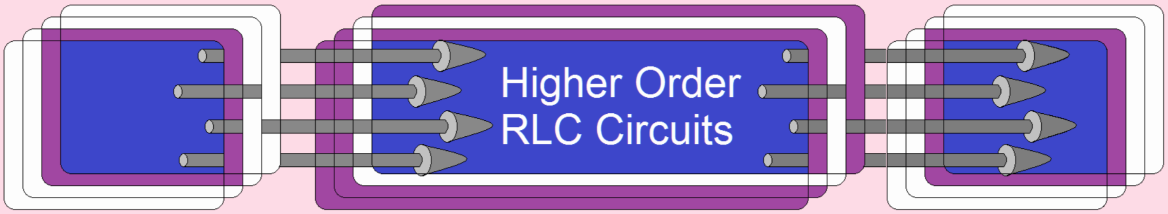
October 2020.

Engineering college year 2 course of 4 year program OR year 1 of 3 year program. Re-fresher OR Self Study. Graduate Study Review. May be used in New Zealand, US, Malaysia, India, Pakistan, UK, and other Common Wealth Country engineering colleges. Any errors and omissions apologies in advance.

Solved Problems In Transfer Functions of RLC circuits.
Resource: Solutions & Problems of Control Systems, 2nd ed - AK Jairath.

Level: Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



Transfer Functions
(Intermediate Level)

Apologies for any errors and omissions.

August 2020.

I selected AK Jairath textbook because it goes back to 1992, when this engineer first published this book. 2nd edition in 1994, and reprinted in 1996.

Solutions & Problems in Control System. May not be in circulation now. Its a small book. Concise similar to Schaums (Supplementary), its not a main textbook. Chapter 1 is Transfer Functions. All the problems in chapter 1 are are made up of R L C components. So this was in line with my/our starting plan to stay within the electric circuits corridor. First keep things simple. So if you asked why, thats the reason I selected this chapter. We did some theory-examples in transfer function at end of Part B, so its best to do them first since these are fresh in minds.

Got an oppurtunity to work with RLC components in the transfer function and secondly control systems context, why waste it. So I did these few example problems.

AK Jairath: The transfer function of a system is the ratio of Lapalce transforms of the output and input quantities, initial conditions being zero. When a physical system is analysed, a mathematical model is prepared by writing differential equations with the help of various laws. An equation describing a physical system has integrals and differentials. The response can be obtained by solving such equations.

The **steps involved** in obtaining the transfer function are:

1. Write differential equations of the system.

2. Replace terms involving $\frac{d}{dt}$ by s and $\int dt$ by 1/s. **<--- Applies to L & C.**
L and C from RLC was worked in electric circuits.
See notes bottom next page.

3. Eliminate all but the desired variable.

See figure next page.

$v(t) = e^{st}$ OR $i(t) = e^{st}$ $L\frac{d\left(e^{st}\right)}{dt} = sL \cdot e^{st}$ $\frac{1}{C}\int e^{st}\,dt = \frac{1}{sC} \cdot e^{st}$

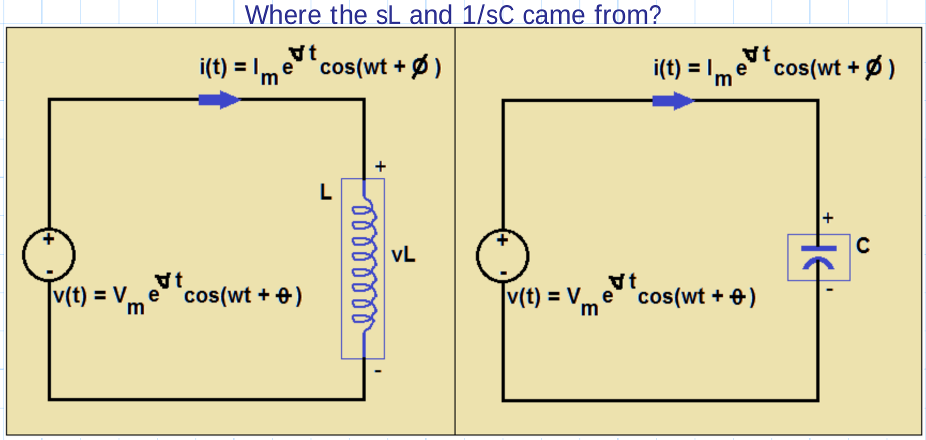
^ | Here*. ^ | Here*.

$L\left(\frac{di}{dt}\right)$ $L : sL$ $\frac{di}{dt} : I(s)$ Inductor current derivative of i(t) - time domain. Its equivalent frequency domain: I(s).

$\frac{1}{C}\int i(t)\,dt$ $\frac{1}{C} : \frac{1}{sC}$ $\int i(t)\,dt : I(s)$ Capacitor current integral over a limit t - time domain. Its equivalent frequency domain: I(s).

*Figure and notes below for reference.

Where the sL and 1/sC came from?



$$v(t) = V_m e^{\sigma t}\cos(\omega t + \theta) \qquad i(t) = I_m e^{\sigma t}\cos(\omega t + \phi)$$

$$s = \sigma + j\omega \qquad\qquad s = \sigma + j\omega$$

$$v(t) = V_m e^{\sigma t}\cos(\omega t + \theta) \qquad\qquad i(t) = I_m e^{\sigma t}\cos(\omega t + \phi)$$

$$= \mathrm{Re}\left(V_m e^{j\theta} e^{st}\right) \qquad\qquad = \mathrm{Re}\left(I_m e^{j\theta} e^{st}\right)$$

Taking the real part of v(t)/i(t)
See notes in Part 3 A and B.

$$= \mathrm{Re}\left(V_m e^{st}\right) \qquad\qquad = \mathrm{Re}\left(I_m e^{st}\right)$$

Inductor:

$$v(t) = L\frac{d\left(I_m e^{st}\right)}{dt} = sL I_m e^{st}$$

$$\mathrm{Re}\left(V_m e^{st}\right) = sL I_m e^{st}$$

$$V_m e^{st} = sL I_m e^{st}$$

$$V_m = sL I_m$$

$$V = sL \cdot I$$

$$V(s) = sL \cdot I(s) \quad \text{<----}$$

Capacitor:

$$v(t) = \frac{1}{C}\int I_m e^{st}\,dt = \frac{1}{sC} I_m e^{st}$$

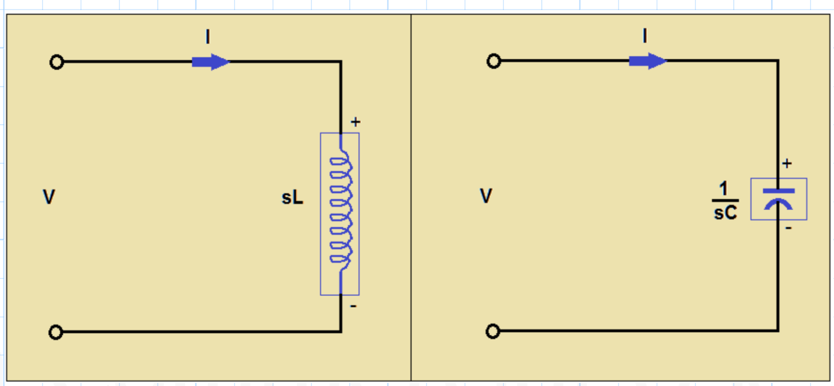
$$\mathrm{Re}\left(V_m e^{st}\right) = \frac{1}{sC} I_m e^{st}$$

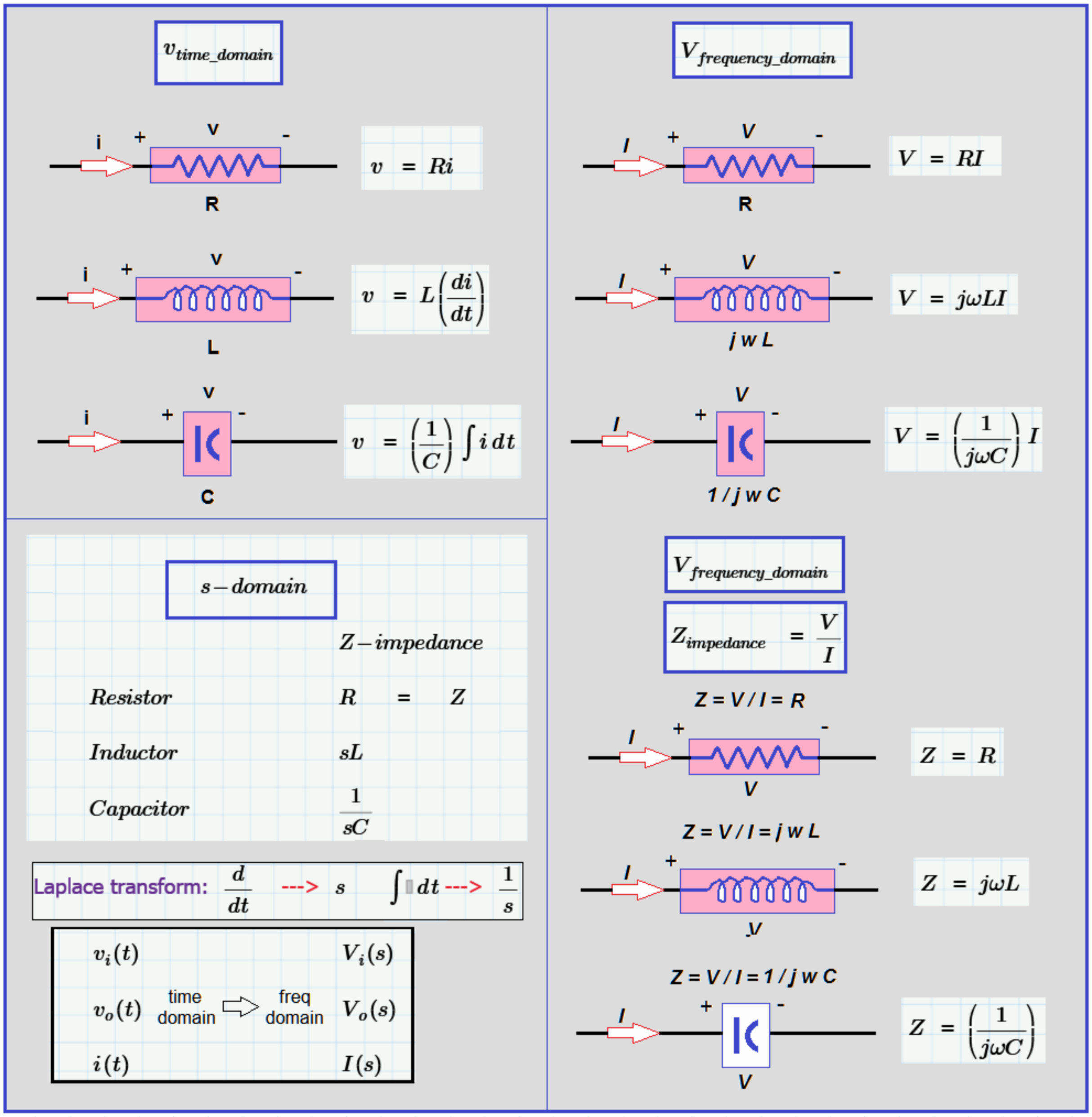
$$V_m e^{st} = \frac{1}{sC} I_m e^{st}$$

$$V_m = \frac{1}{sC} I_m$$

$$V = \frac{1}{sC} I$$

$$V(s) = \frac{1}{sC} I(s) \quad \text{<----}$$





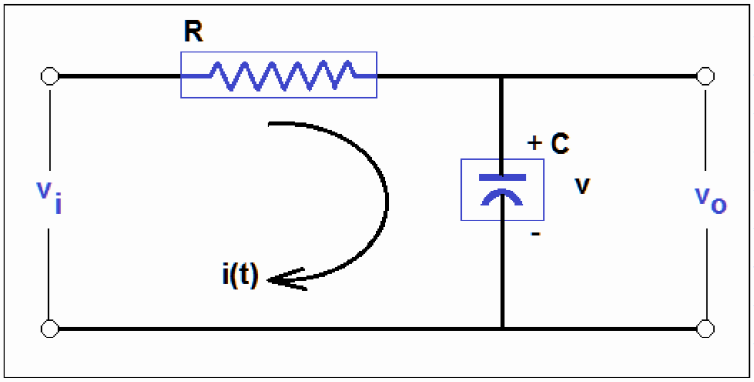
Derivative and Integral substitues for s and 1/s for the component L and C respectively.

$\frac{d}{dt}$ ---> s $\qquad\qquad$ $\int .\,dt$ ---> $\frac{1}{s}$

Inductor: $v_L(t) = L\cdot\frac{di}{dt}$ $\qquad$ Capacitor: $v_C(t) = \frac{1}{C}\int i\,dt$

---> $V_L(s) = Ls\cdot I(s)$ <--- $\qquad$ ---> $V_C(s) = \frac{1}{sC}\cdot I(s)$ <---

Chp 1 Problem 1-1:



Derive the transfer function of the circuit shown in figure to the left.

Solution:

First thing is its a series circuit. We do a voltage conservation. Meaning the sum of voltages add to zero. You call that Kickoff's OR Kickout's Law.

The output is across the capacitor terminals.
The input is supply voltage for the resistor and capacitor.

$v\_i(t) = R \cdot i(t) + v\_C(t)$ i(t) is the circuit's current.

Set $v\_o(t) = v\_C(t) = \frac{1}{C} \int i\, dt$

$v\_i(t) = R \cdot i(t) + v\_o(t)$

Now we convert the expression above to the s-domain.
Which in control systems textbook they say 'Taking the Laplace transform'.
Laplace Transforms starts with transfer functions in the s-plane or in terms of complex frequency. So, thats why we used a Controls textbook. Same.

$V_i(s) = R\,I(s) + V_0(s)$

Vo(s) is that voltage across the capacitor C terminals, which we can set this in the s-domain of the capacitor.

$V_o(s) = \frac{1}{sC} \cdot I(s)$ <--- C: 1/sC, and i(t): I(s).

Its more than forming a loop equation, we want to all the required variables in the expression so we can form that Vo(s)/Vi(s).

How do we know what all terms and their forms we need before we can get to forming a transfer function?

Keep working in more and more example problems, partially looks like a guess, but after a few examples we get the general idea.

The Electrical Engineering expressions for defining components are formed in such a way that they have a future in advanced math where they can be manipulated in various ways to take full benefit of the math resulting in some output that serves a circuit's purpose - Karl Bogha.

$$V_o(s) = \frac{1}{sC} \cdot I(s)$$

$$I(s) = V_o(s) \cdot sC$$

$$V_i(s) = R\,I(s) + V_0(s)$$

<--- Lets plug in or if you prefer substitute the expression we got into this expression we formed earlier.

$$V_i(s) = R\left(V_o(s) \cdot sC\right) + V_o(s)$$

$$= sRC\left(V_o(s)\right) + V_o(s)$$

$$V_i(s) = V_o(s) \cdot (sRC + 1)$$

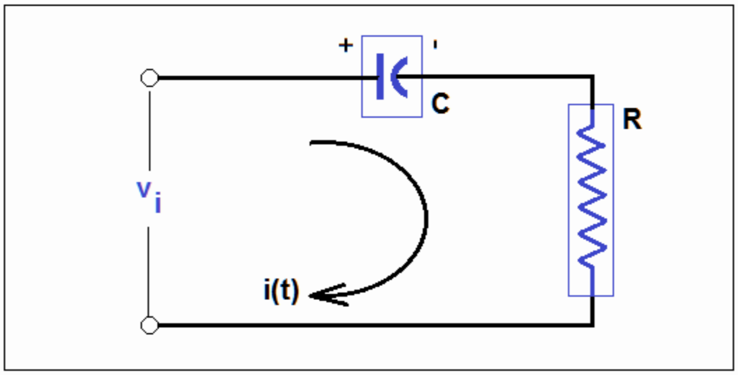
$$= V_o(s) \cdot (1 + sRC)$$

<--- How would we had known that?
Surely had to work examples.
Keep clear of people and peers who say dont do the example go to the end of chapter problems, they lie so they have the edge - Engineer.
In the work place you never ever get problems to solve like hard end of chapter problems in hard core engineering textbooks, fake, it rarely help, most time you got all the time in the world - Karl Bogha.

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sRC}$$

Answer.

Chp 1 Problem 1-2:



We seek the transfer function I(s)/Vi(s) ?

Solution:

First thing is its a series circuit. We do a voltage conservation, meaning the sum of voltages add to zero. You call that *Kickoff's* Law!

$$v\_i(t) = R \cdot i(t) + v\_C(t)$$ i(t) is the circuit's current.

$$v\_C(t) = \frac{1}{C}\int i\,dt = \frac{1}{sC} I(s)$$

$$v\_i(t) = R \cdot i(t) + v\_C(t)$$

$$V_i(s) = R I(s) + \frac{1}{sC} I(s)$$

$$= R I(s) + \frac{1}{sC} I(s)$$

$$= I(s)\left(R + \frac{1}{sC}\right)$$

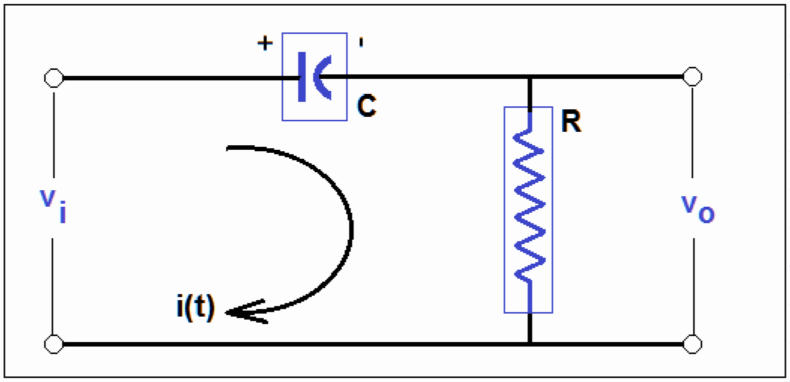
$$\frac{I(s)}{V_i(s)} = \frac{1}{\left(R + \frac{1}{sC}\right)}$$ Simplify this term, multiply by sC/R.

$$\frac{I(s)}{V_i(s)} = \frac{\left(\frac{sC}{R}\right)}{\frac{sC}{R}\left(R + \frac{1}{sC}\right)} = \frac{\left(\frac{sC}{R}\right)}{sC + \frac{1}{R}} = \left(\frac{1}{sC + \frac{1}{R}}\right)\frac{sC}{R} = \left(\frac{sC}{sCR + 1}\right)$$

$$\frac{I(s)}{V_i(s)} = \left(\frac{sC}{1 + sCR}\right)$$ Answer.

Good if we can work the final form of expression like this instead of the one a few steps before. It takes some extra effort to get it in a neat form that is more electric circuit friendly and meaningful.

Chp 1 Problem 1-3:



We seek the transfer function Vo(s)/Vi(s) ?

Solution:

Conservation of voltage means something else?
I am not sure, when conserved it would remain the same.
So the sum equal zero in a loop. That for me is conserved.
Maybe they used it for something else. Usuall I am not the first.

We kickoff with the voltage conservation.

$$V_i(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt$$

$$\frac{1}{C}\int i(t)\,dt = \frac{1}{sC}\cdot I(s)$$

$$V_i(s) = R\,I(s) + \frac{I(s)}{sC}$$

Our circuit identifies voltage across resistor terminals as Vo(t) which now becomes? Vo(s) for the frequency domain.

$$V_o(s) = R\,I(s)$$

$$I(s) = \frac{V_o(s)}{R}$$ Substitute in here: $$V_i(s) = R\,I(s) + \frac{I(s)}{sC}$$

$$V_i(s) = R\left(\frac{V_o(s)}{R}\right) + \left(\frac{V_o(s)}{R}\right)\cdot\frac{1}{sC}$$ Isolate Vo(s)

$$V_i(s) = V_o(s)\cdot\left(1 + \frac{1}{sCR}\right)$$

Next for the required transfer function:

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\left(1+\frac{1}{sCR}\right)}$$

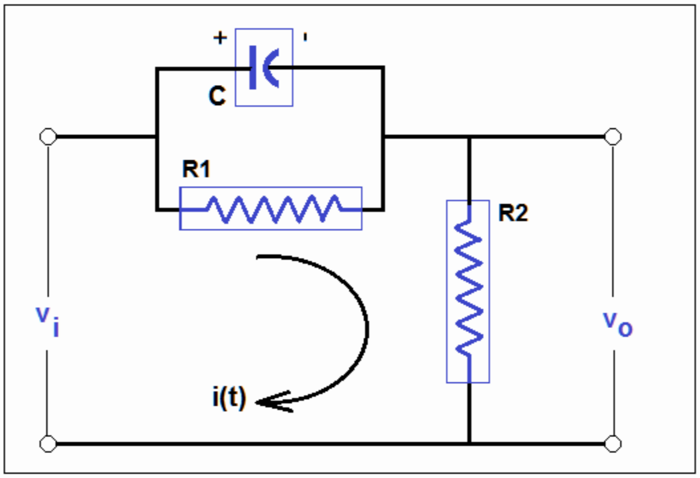
<--- This can be simplified.
Its awkward, that is why we simplify these awkward terms.

Multiply by sCR:

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\left(1+\frac{1}{sCR}\right)} \cdot \frac{sCR}{sCR} = \frac{sCR}{(sCR+1)}$$

Answer.

Chp 1 Problem 1-4:



We seek the transfer function, Vo(s)/Vi(s), of the electrical network shown to the left in phase lead form ?

Solution:

Z1 is the parallell of C and R1:

$$\frac{1}{Z1} = \frac{1}{R1} + \frac{1}{\frac{1}{j\,\omega C}} = \frac{1}{R1} + \frac{1}{\frac{1}{sC}}$$

$$s = \sigma + j\,\omega$$
$$\sigma = 0$$
$$s = j\,\omega$$

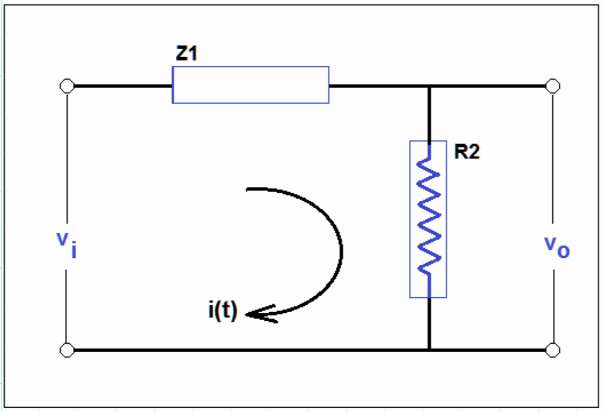
We are concerned with frequency, so we can set sigma = 0.

$$\frac{1}{Z1} = \frac{1}{R1} + sC = \frac{1}{R1} + \frac{sC}{1}$$ multiply by R1

$$\frac{1}{Z1} = \frac{R1}{R1} + \frac{sCR1}{1} = \frac{R1+sCR1}{1 \cdot R1} = \frac{R1+sCR1}{R1}$$

$$\frac{1}{Z1} = \frac{R1}{R1} + \frac{sCR1}{R1} = 1 + \frac{sCR1}{R1} = \frac{1+sCR1}{R1}$$

$$Z1 = \frac{R1}{1 + sCR1}$$ After inverting.



We kickoff with the voltage conservation.

$$v_i(t) = Z1 I(s) + R2 I(s)$$

$$v_o(t) = R2 I(s)$$

Taking the Laplace Transform of the above 2 equation:

$$V_i(s) = Z1 I(s) + R2 I(s)$$

$$V_o(t) = R2 I(s)$$ Plug in equation above

$$I(s) = \frac{V_o(s)}{R2}$$ Plug in equation above

$$V_i(s) = Z1\left(\frac{V_o(s)}{R2}\right) + V_o(s)$$

$$V_i(s) = V_o(s) \cdot \left(\frac{Z1}{R2} + 1\right)$$ Plug in Z1

$$V_i(s) = V_o(s) \cdot \left(\frac{\left(\frac{R1}{1 + sCR1}\right)}{R2} + 1\right)$$

$$= V_o(s) \cdot \left(\frac{\left(\frac{R1}{1 + sCR1}\right)}{R2} + \frac{R2}{R2}\right)$$

$$= V_o(s)\cdot\left(\frac{\left(\frac{R1}{1+sCR1}\right)+R2}{R2}\right)$$

Next rearrange and multiply by ---> $\frac{1+sCR1}{1+sCR1}$

$$= \frac{V_o(s)}{R2}\cdot\left(\left(\frac{R1}{1+sCR1}\right)+\frac{R2\cdot(1+sCR1)}{(1+sCR1)}\right)$$

$$= \frac{V_o(s)}{R2}\cdot\left(\frac{R1+R2+sCR1R2}{1+sCR1}\right)$$

$$= V_o(s)\left(\frac{R1+R2}{R2}\right)\left(\frac{1+sCR1R2}{1+sCR1}\right)$$

...not finished yet in this expression.

$$V_i(s) = V_o(s)\left(\frac{R1+R2}{R2}\right)\left(\frac{1+\frac{sCR1R2}{R1+R2}}{1+sCR1}\right)$$

Place $\frac{1}{R1+R2}$ in there so it cancels the middle term (R1+R2)/R2 when multiplied.

$$\frac{V_i(s)}{V_o(s)} = \left(\frac{R1+R2}{R2}\right)\left(\frac{1+\frac{sCR1R2}{R1+R2}}{1+sCR1}\right)$$

Next invert both sides.

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R2}{R1+R2}\right)\left(\frac{1+sCR1}{1+\frac{sCR1R2}{R1+R2}}\right)$$

As provided in textbook.

$$\frac{V_o(s)}{V_i(s)} = \left(\frac{R2}{R1+R2}\right)\left(\frac{1+sCR1}{1+\left(\frac{R2}{R1+R2}\right)sCR1}\right)$$

Transfer function.

We can simplify a little. Make RC the time constant in a series circuit = tau, and make the constant R2/(R1+R1) = a. OR just any constant T.

$$T = CR1$$

$$a = \frac{R2}{R1+R2}$$

$$\frac{V_o(s)}{V_i(s)} = a\left(\frac{1+sT}{1+asT}\right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{a\cdot(1+sT)}{(1+a\ s\ T)}$$

Answer.

Comment: Previous example problems used T for RC in the final transfer functions.
I left it out because my aim was the approach on how to get the transfer functions. T is not necessarily a time constant for this circuit.
You can verify. We could use P of Q but since its RLC, T or tau makes more sense.

Took time with the algebra otherwise a good easy example for most.

Chap 1 Problem 1.7 :

I jump to problem 1.7 because its the same circuit. This provides a continuity and not having to return later after several problems.

Derive the transfer function of the circuit shown (same circuit of problem 1.4).
If v_i(t) = 8 sin(10t) V, R1 = 50 k Ohms, R2 = 5 k Ohms and C = 1 uF.

Calculate the output voltage in magnitude and phase angle relative to input voltage?

Solution:

Gain $G(s) = \dfrac{V_o(s)}{V_i(s)} = \left(\dfrac{R2}{R1+R2}\right)\left(\dfrac{1+sCR1}{1+\left(\dfrac{R2}{R1+R2}\right)sCR1}\right)$

$k := 10^3$ $\quad M := 10^6$ $\quad u := 10^{-6}$

$R1 := 50\ k$ $\quad R2 := 5\ k$ $\quad C := 1\ u$

Substitute into transfer function:

$$G(s) = \frac{V_o(s)}{V_i(s)} = R2 \cdot \frac{(1+sCR1)}{(R1+R2)+(sCR1R2)}$$

$$= 5000\left(\frac{1+0.05\ s}{55000+250\ s}\right)$$ Divide numerator and denominator by 55,000.

$$= 0.091\left(\frac{1+0.05\ s}{1+0.0045\ s}\right)$$

$$G(s) = 0.01\frac{(1+0.05\ s)}{(1+0.0045\ s)}$$ Constant 0.091 rounded off to 0.01

Zero: $(1+0.05\ s)$
Pole: $(1+0.0045\ s)$

We are interested in s = 0 + jw, where sigma =0.
Hence we can analyse the frequency response.
Substitute s for jw in transfer function.
Now we have 1+0.05s and 1+0.0045s, this gives us the magnitude and angle for both. Since we have a real and imaginary part.

$s = \sigma + j\ \omega$
$\sigma = 0$
$s = 0 + j\ \omega = 0$

$$G(j\ \omega) = 0.01\frac{(1+0.05\ j\ \omega)}{(1+0.0045\ j\ \omega)}$$

Before we can calculate the angles we need the value of w ?

$v(t) \quad = \quad 8\sin(10\cdot t) \quad \text{--->} \quad A\sin(\omega t)$

$\omega \quad = \quad 10$

Zero: $(1+0.05\ j10) \quad = \quad 1+0.5j$
Pole: $(1+0.0045\ j10) \quad = \quad 1+0.045j$

$$Z\_Ang\_G\_s := \operatorname{atan}\left(\frac{0.5}{1}\right) = 26.5651\ \mathbf{deg}$$

$$P\_Ang\_G\_s := \operatorname{atan}\left(\frac{0.045}{1}\right) = 2.5766\ \mathbf{deg}$$

$Ang\_G(s) \quad = \quad 26.565 - 2.577 = 23.988$ degrees. Answer.

Now for the magnitude of the transfer function,
here is where the constant 0.1 is applied.

Magnitude of zero: $\sqrt{1^2 + 0.5^2} = 1.118$

Magnitude of pole: $\sqrt{1^2 + 0.045^2} = 1.001$

Magnitude of G(s): $(0.1)\cdot\left(\frac{1.118}{1.001}\right) = 0.1117$

The input signal is vi(t) = 8 sin (wt)
From which we can obtain the amplitude is 8 V maximum.
We next multiply the magnitude of G(s) to 8V for the maximum output voltage.

$Amplitude := 8.0 \qquad Mag\_G(s) := 0.1117$

$V_o := Amplitude \cdot Mag\_G(\mathbf{s}) = 0.894$ V. Answer.

Good example. Can be found in most circuits and all controls textbook.

Chap 1 Problem 1.8 :

Problem 1.8 is next here because it works on the same transfer function of problem 1.4. This is indicated in the problem statement, exact same circuit.

If C = 1uF in the circuit of problem 1.4.
What values of R1 and R2 will give T = 0.6 sec, and a = 0.1

Solution:

$$G(s) = \left(\frac{R2}{R1+R2}\right)\left(\frac{1+sCR1}{1+\left(\frac{R2}{R1+R2}\right)sCR1}\right) \qquad T = CR1 \qquad a = \frac{R2}{R1+R2}$$

$$G(s) = \frac{a\cdot(1+sT)}{(1+asT)} \qquad C := 1\ u\ F \qquad T := 0.6 \qquad a := 0.1$$

CR1 = 0.6, solve for R1: $CR1 = 0.6$

$$(1\ uF)\ R1 = 0.6$$

$$R1 = \frac{0.6}{1\cdot u} = 6\cdot 10^{5}\ \text{Ohm.} = 0.6\cdot M\ \text{Ohm.}$$ Answer.

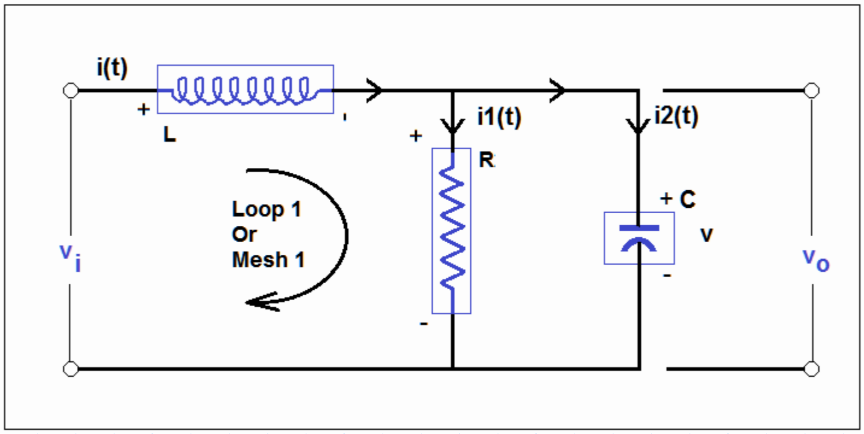
$$a = \frac{R2}{R1+R2} \text{ ---> } 0.1 = \frac{R2}{600000+R2} \text{ ---> } 0.1\ (600000+R2) = R2$$

$$60000 + 0.1\ R2 = R2$$

$$0.9\ R2 = 60000$$

$$R2 = \frac{60000}{0.9} = 66666.7 = 0.066\ M\ \text{Ohms.}$$ Answer.

Chp 1 Problem 1-5:



We seek the transfer function, Vo(s)/Vi(s), of the electrical network shown to the left in phase lead form ?

Solution:

Current at node: $i(t) = i1(t) + i2(t)$

Voltage conservation in loop at left side:

$$v_i(t) = L\frac{di}{dt} + Ri1(t)$$

Next, in a cleaver way, we pull in the v_o(t) relationship thru the capacitor voltage, where C is voltage across resistor R, and we know v_o(t) is the voltage across the capacitor.

$$v_o(t) = Ri1(t) = \frac{1}{C}\int i2(t)\,dt$$

$$v_i(t) = L\frac{di}{dt} + Ri1(t)$$

$$V_i(s) = sLI(s) + RI1(s)$$

$$RI1(s) = \frac{1}{C}\int i2(t)\,dt = \frac{1}{sC}I2(s)$$

$$RI1(s) = V_o(s) = \frac{I2(s)}{sC}$$

Voltage across R: $V_o(s) = \dfrac{I2(s)}{sC} = R\,I1(s)$ thus $I1(s) = \dfrac{V_o(s)}{R}$

We update our I(s)expression here $V_i(s) = sL\,I(s) + R\,I1(s)$

$$i(t) = i1(t) + i2(t)$$

$$I(s) = I1(s) + I2(s)$$

$$V_i(s) = sL\,(I1(s) + I2(s)) + R\,I1(s)$$

$$V_i(s) = sL\,(I1(s) + I2(s)) + V_o(s)$$

Substitute voltage across C for R: $V_o(s) = \dfrac{I2(s)}{sC}$

$$sCV_o(s) = I2(s)$$

$$V_i(s) = sL\left(\frac{V_o(s)}{R} + sCV_o(s)\right) + V_o(s)$$

$$V_i(s) = V_o(s) + sL\left(\frac{V_o(s)}{R} + sCV_o(s)\right)$$

$$V_i(s) = V_o(s) + V_o(s)\cdot\left(\frac{sL}{R} + sCsL\right)$$

$$V_i(s) = V_o(s)\cdot\left(1 + \frac{sL}{R} + s^2\ LC\right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\left(1 + \frac{sL}{R} + s^2\ LC\right)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\left(s^2\ LC + \frac{sL}{R} + 1\right)}$$

Answer. Lots of substitutions.
A compact answer below.

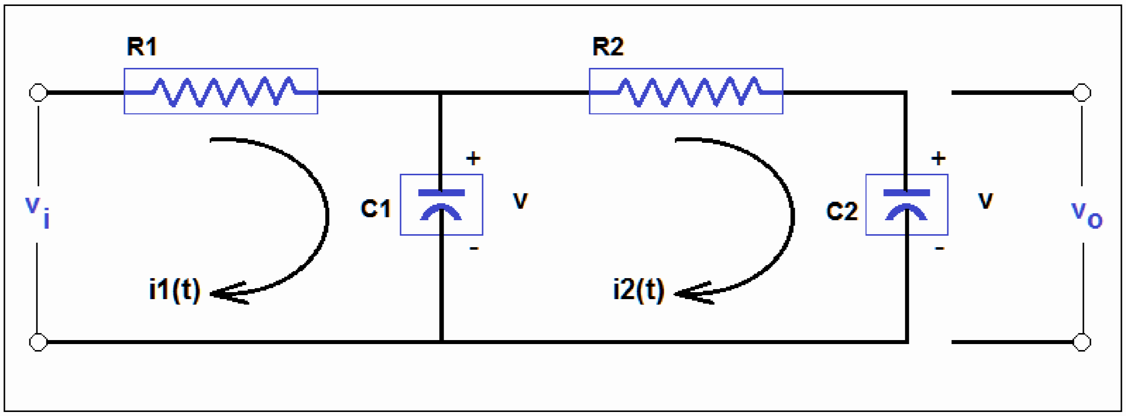
The Engineer makes the expression simpler in appearance, quadratic expression, thru the use of variable T1 and T2. T1 = L/R maybe a time contant but not here. T2 = CR which is NOT a time constant, you verify should it be of concern.

$$T1 = \frac{L}{R} \qquad T2 = CR$$

$$T1T2 = \left(\frac{L}{R}\right)(CR) = LC$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\left(T1T2s^2 + T1s + 1\right)}$$ Answer.

Chp 1 Problem 1-6:



We seek the transfer function, Vo(s)/Vi(s), of the electrical network shown above ?

Solution:

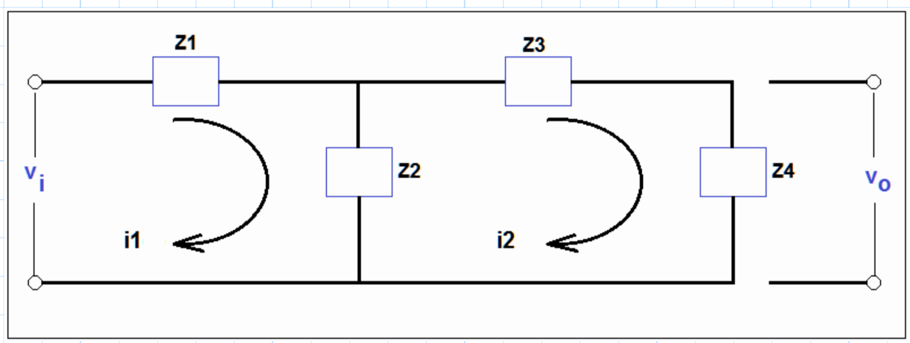
Set up the impedance Z for each component:

$$Z1 = R1$$

$$Z2 = \frac{1}{sC1}$$

$$Z3 = R2$$

$$Z4 = \frac{1}{sC2}$$



Voltage mesh/loop equations in Laplace:

Left loop:

$$V_i(s) = Z1 I1(s) + Z2(I1 - I2)$$

$$V_i(s) = I1(s)\ (Z1 + Z2) - Z2 I2 \qquad \text{...Eq 1}$$

Right loop:

$$0 = Z2(I2 - I1) + Z3 I2(s) + Z4 I2(s)$$

$$0 = -Z2 I1 + I2(s)\ (Z2 + Z3 + Z4) \qquad \text{...Eq 2}$$

Next we form an expression for Vo:

$$V_o(s) = Z4 I2(s) \qquad \text{...Eq 3}$$

If I am correct, from these few examples we seen, we want to place one expression for current, into the the other equation, then work towards the transfer function, provided we have Vo(s) and Vi(s) in that expression to work with.

Here, I1(s) looks the better simpler choice to place in Eq 2.
Because we do not have a voltage source on the RHS.
Then we set Vo(s) for Z4I2(s).
Then work with the equation which can fit-in Vo, Vi, and I1 and I2 in it.
If we dont have it yet continue re-hashing.
What you think, that's the plan? *Of course!*

$$0 = -Z2 I1 + I2(s)\ (Z2 + Z3 + Z4) \qquad \text{...Eq 2}$$

$$I1(s)\ Z2 = I2(s)\ (Z2 + Z3 + Z4)$$

$$I1(s) = \frac{I2(s)\ (Z2 + Z3 + Z4)}{Z2}$$

$$V_i(s) = I1(s)\ (Z1 + Z2) - Z2 I2 \qquad \text{...Eq 1, substitute I1(s)}$$

$$V_i(s) = \frac{I2(s)\ (Z2 + Z3 + Z4)\cdot(Z1 + Z2)}{Z2} - Z2 I2(s)$$ Fix for Z2 at very right of numerator.

$$V_i(s) = \frac{I2(s)\cdot((Z2 + Z3 + Z4)\cdot(Z1 + Z2))\ Z2}{Z2} - \frac{Z2 Z2 I2(s)}{Z2}$$

$$V_i(s) = \frac{I2(s)\cdot\left(((Z2+Z3+Z4)\cdot(Z1+Z2))\ Z2 - Z2^2\right)}{Z2}$$

$$\frac{V_i(s)}{I2(s)} = \frac{((Z2+Z3+Z4)\cdot(Z1+Z2))\ Z2 - Z2^2}{Z2}$$

For me this is new, not a twist but certainly new I dont remember doing a substitution on the LHS! Ok Not typical. Hope I am gaining skills here.

$$V_o(s) = Z4I2(s) \qquad \text{...Eq 3}$$

$$I2(s) = \frac{V_o(s)}{Z4}$$

Substitute this in the expression Vi(s)/I2(s)

$$\frac{V_i(s)}{\left(\frac{V_o(s)}{Z4}\right)} = \frac{\left((Z2+Z3+Z4)\cdot(Z1+Z2) - Z2^2\right)}{Z2}$$

$$\frac{V_i(s)}{V_o(s)} = \frac{\left((Z1+Z2)\cdot(Z2+Z3+Z4) - Z2^2\right)}{Z2\cdot Z4}$$

Invert the expression so we get Vo(s) in the numerator.

$$\frac{V_o(s)}{V_i(s)} = \frac{Z2\cdot Z4}{\left((Z1+Z2)\cdot(Z2+Z3+Z4) - Z2^2\right)}$$

Lets expand the denominator expression:

$$(Z1+Z2)\cdot(Z2+Z3+Z4) = Z1Z2 + Z1Z3 + Z1Z4 + Z2Z2 + Z2Z3 + Z2Z4$$

Now for the full denominator expression:

$$= Z1Z2 + Z1Z3 + Z1Z4 + Z2Z2 + Z2Z3 + Z2Z4 - Z2Z2$$

$$= Z1Z2 + Z1Z3 + Z1Z4 + Z2Z3 + Z2Z4$$

$$\frac{V_o(s)}{V_i(s)} = \frac{Z2\cdot Z4}{Z1Z2 + Z1Z3 + Z1Z4 + Z2Z3 + Z2Z4}$$ The transfer function.

Next we substitute the values of impedances Z1...Z4:

$$\frac{V_o(s)}{V_i(s)} = \frac{Z2 \cdot Z4}{Z1Z2 + Z1Z3 + Z1Z4 + Z2Z3 + Z2Z4}$$

$$Z1 = R1 \qquad Z2 = \frac{1}{sC1} \qquad Z3 = R2 \qquad Z4 = \frac{1}{sC2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC1} \cdot \frac{1}{sC2}}{\frac{R1}{sC1} + R1R2 + \frac{R1}{sC2} + \frac{R2}{sC1} + \frac{1}{s^2\ C1C2}}$$

As usual these types expressions are made simpler, especially in electric circuits.
It helps in building the physical circuit. *Which I almost forgot the true purpose here. We are building circuits and components are to be put together on a bread board for testing. Hello?..true purpose? Why not?*

$$= \left(\frac{1}{\frac{R1}{sC1} + R1R2 + \frac{R1}{sC2} + \frac{R2}{sC1} + \frac{1}{s^2\ C1C2}} \right) \cdot \frac{1}{sC1} \cdot \frac{1}{sC2}$$

$$= \frac{1}{R1sC2 + R1R2 \cdot s^2\ C1C2 + R1sC1 + R2sC2 + 1}$$

Multiplied by sC1 sC2 top and bottom.

$$= \frac{1}{sR1C2 + s^2\ R1R2 \cdot C1C2 + sR1C1 + sR2C2 + 1}$$

$$= \frac{1}{sR1C2 + sR1C1 + sR2C2 + 1 + s^2\ R1R2 \cdot C1C2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + s(R1C2 + R1C1 + R2C2) + s^2\ R1R2 \cdot C1C2}$$

Answer.

The denominator is a neat 2nd order expression.
The circuit is also a practical circuit for application in electric circuits, electronics and other electrical/electronic applications.

Chp 1 Problem 1.11:

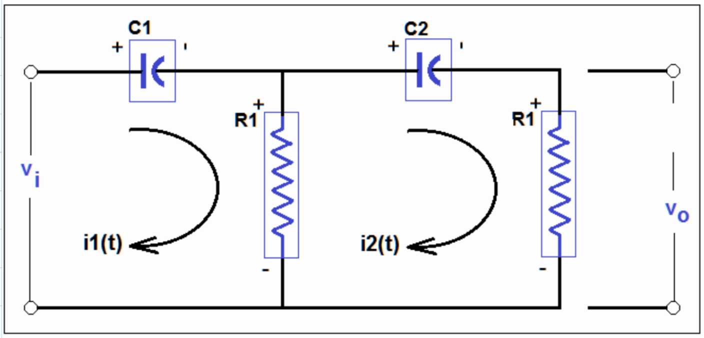
Problem 1.11 comes here because this problem has a similar transfer function to problem 1.6. As indicated in the problem statement of 1.11. The changes being only in the arrangement of components, that being the swap between R and C.

Determine the transfer function relation Vo(s) to Vi(s) for the circuit.
Calculate output voltage t>>0 for a unit step voltage input at t=0.

Solution:

In 1.6 we used the impedance Z to construct the transfer function. Later we plugged in the values for Z's. So thats why this transfer function is relevant.

$$\frac{V_o(s)}{V_i(s)} = \frac{Z2 \cdot Z4}{Z1Z2 + Z1Z3 + Z1Z4 + Z2Z3 + Z2Z4}$$ <--- From problem 1.6



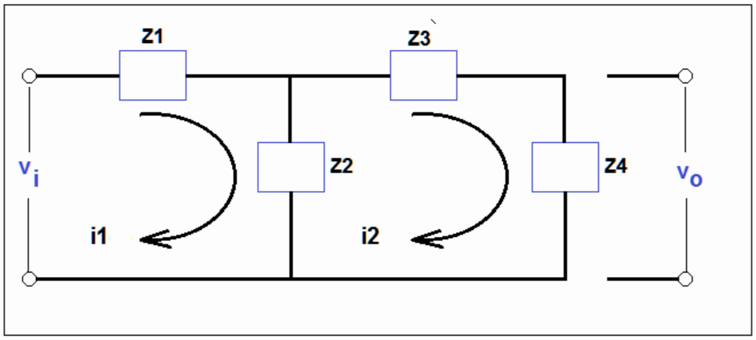
<--- This is the circuit for problem 1.8.

$u := 10^{-6}$ $\qquad M := 10^{6}$

$C1 := 1\ u\ \ F$ $\qquad C2 := 0.5\ u\ \ F$

$R1 := 1\ M$ $\qquad R2 := 1\ M$

The Z impedance circuit becomes:



We make 10^6 the common multiplier for resistors and 10^-6 for capacitor. Now we only need work with the simple numbers.

$Z1 = \frac{1}{s}$ $\qquad Z2 = 1$

$Z3 = \frac{1}{0.5\ s}$ $\qquad Z4 = 1$

$$\frac{V_o(s)}{V_i(s)} = \frac{Z2 \cdot Z4}{Z1Z2 + Z1Z3 + Z1Z4 + Z2Z3 + Z2Z4}$$

$$= \frac{1 \cdot 1}{\left(\frac{1}{s}\right) \cdot 1 + \left(\frac{1}{s}\right)\left(\frac{1}{0.5\ s}\right) + \left(\frac{1}{s}\right)(1) + (1)\left(\frac{1}{0.5\ s}\right) + (1)\ (1)}$$

$$= \frac{1}{\frac{1}{s} + \left(\frac{1}{0.5\,s^2}\right) + \left(\frac{1}{s}\right) + \left(\frac{1}{0.5\,s}\right) + 1}$$ Multiply by s^2

$$= \frac{s^2}{s + 2 + s + 2\,s + s^2} = \frac{s^2}{2 + 4\,s + s^2} = \frac{s^2}{s^2 + 4\,s + 2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{s \cdot s}{s^2 + 4\,s + 2}$$

Unit step voltage comes on at t=0 and is of unit value, ie 1. Vi(s) must equal 1.

$$V_o(s) = \frac{V_i(s) \cdot s \cdot s}{s^2 + 4\,s + 2} = \frac{1 \cdot s \cdot s}{s^2 + 4\,s + 2}$$

$$s_{z1} := 1 \quad V_o(s) = \frac{s}{s^2 + 4\,s + 2}$$

$$ax^2 + bx + c \quad : \quad s^2 + 4\,s + 2$$

$$s1 = \frac{-b - \sqrt{b^2 - 4\,ac}}{2\,a} = \frac{-4 - \sqrt{4^2 - 4\;1\;2}}{2\;1} = -3.4142$$

$$s2 = \frac{-b + \sqrt{b^2 - 4\,ac}}{2\,a} = \frac{-4 + \sqrt{4^2 - 4\;1\;2}}{2\;1} = -0.5858$$

We solved the denominator for the poles. Which math wise were the roots but electrical wise these are the poles.

$$V_o(s) = \frac{s^2}{(s + 3.414) \cdot (s + 0.586)}$$

The poles going back in the transfer function with the opposite sign.

For the pole to be maximum s1 and s2? -3.414 and -0.586

What about the numerator what any value to solve?
Its NOT the numerator its the COEFFICIENTS of Vo(s) and those same for time domain.

At t<0 Vo(<0) = 0, and t>0 Vo(>0) = 0, but for t>>0 Vo(>>0) = 1u(t).
At -0 its near same as 0+ equal 0. So we use continuity here?
No, basically math. To solve for coefficients using the?
Method of proper fractions OR Equating coefficients of like powers.

Next calculate the coefficients.

$$V_o(s) = \frac{s^2}{(s+3.414)\cdot(s+0.586)}$$ Split LHS to solve for coefficients.

$$\frac{s\cdot s}{(s+3.414)\cdot(s+0.586)} = \frac{A}{(s+3.414)} + \frac{B}{(s+0.586)}$$ 2nd order eq.

$$A(s+0.586) + B(s+3.414) = As + 0.586\ A + Bs + B3.414$$

Arrange like terms:

*s below is numerator term in transfer function - s*s split to s*s.*
One 's' for 1 equation (As+Bs) = 1 <---coefficient of s = 1. Like terms.

$As + Bs = s$ ---> $A + B = 1$ Eq 1
$0.586\ A + 3.414\ B = 0$ ---> $0.586\ A + 3.414\ B = 0$ Eq 2

$0.586\ A + 0.586\ B = 0.586$ Eq 1 x 0.586...Eq 3
$0.586\ A + 3.414\ B = 0$ Eq 2

$(0.586 - 3.414)\ B = 0.586$ Eq 3 - 2
$(0.586 - 3.414) = -2.828$
$-2.828\ B = 0.586$

$$B = \frac{0.586}{-2.828} = -0.2072$$

$A + B = 1$
$A - 0.207 = 1$
$A = 1 + 0.207 = 1.207$

The circuit s-domain: $$V_o(s) = \frac{A}{(s+3.414)} + \frac{B}{(s+0.586)}$$

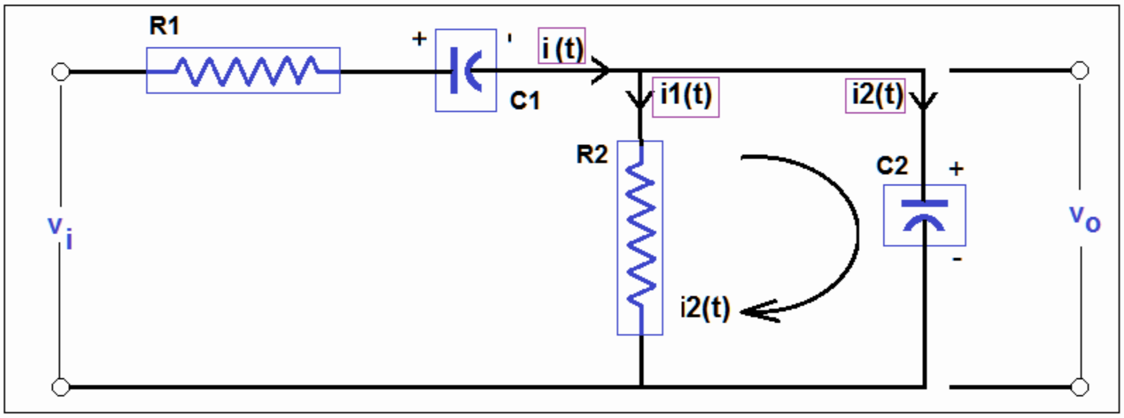
$$V_o(s) = \frac{1.21}{(s+3.414)} - \frac{0.21}{(s+0.586)}$$

The general form of v_o(t): $Ae^{-s1\cdot t} + Be^{-s1\cdot t}$

$$v_o(t) = 1.21\ e^{-3.414\cdot t} - 0.21\ e^{-0.586\cdot t}$$ Answer.

Interesting solution math wise. What math can do for determining coefficients by 'equating coefficients of like powers'.

Chp 1 Problem 1.9:



Find the transfer function of the network shown in figure above.
Plot its poles and zeros for R1 = R2 = 1, and C1 = C2 = 1.

Solution:

Current equation at node:

$i(t) = i1(t) + i2(t)$ Note: Current thru R1 and C1 is i(t).

Voltage mesh equations:

$$v_i(t) = R1i(t) + \frac{1}{C1}\cdot\int i\,dt + R2\cdot((i1(t) - i2(t)))$$

Deviation here: $R2\cdot((i1(t) - i2(t)))$ we neglect i2(t) leaving ---> $R2i2(t)$
Shown later.

Voltage across R2 is Vo(t).
Form the voltage mesh equation using Vo(t).

$$v_o(t) = R2\cdot((i2(t) - i1(t))) + \frac{1}{C2}\cdot\int i2(t)\,dt$$ We may not need this mesh equation.

We did this just so we see the time domain equations, we could have started with to s-domain as we did in other example(s).

Now for converting to s-domain,
in other words taking the Laplace Transform:

$I(s) = I1(s) + I2(s)$

$$V_i(s) = R1\,I(s) + \frac{1}{sC1}\,I(s) + R2\cdot(I1(s) - I2(s))$$

$$V_o(s) = R2\cdot(I2(s) - I1(s)) + \frac{1}{sC2}\cdot I2(s)$$ We may not need this equation.

The voltage across C2 is the same across R2.
This is the voltage v_o(t) or Vo(s).
We can use this voltage expression and plug into the Vi(s) equation.
Obviously we want to plug in for R2I1(s).

$$v_o(t) = \frac{1}{C2}\int i2(t)\,dt$$

$$= R2\cdot i1(t)$$ Here we do not do a mesh method on the current thru R2. We simply identify it to i1(t), since its the voltage across the resistor terminals equated to v_o(t).

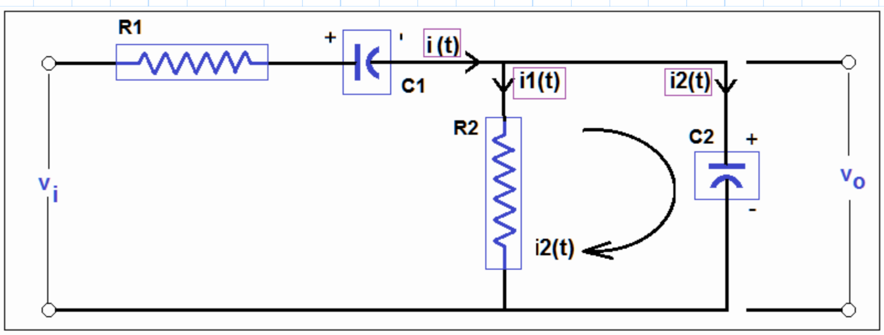
Their Laplace transform:

$$Vo(s) = \frac{1}{sC2}\,I2(s)$$

$$= R2\cdot I2(s)$$

$$V_i(s) = R1\,I(s) + \frac{1}{sC1}\,I(s) + R2\cdot(I1(s) - I2(s))$$ The main equation now.

$$V_i(s) = R1\,I(s) + \frac{1}{sC1}\,I(s) + R2\cdot I1(s) - R2I2(s)$$ Plug in Vo at R2I1(s)



$$V_i(s) = R1\,I(s) + \frac{1}{sC1}\,I(s) + V_o(s) - R2I2(s)$$

Mesh or voltage loop problem, stated earlier below.
Deviation here: $R2\cdot((i1(t) - i2(t)))$ we neglect i2(t) leaving ---> $R2i2(t)$

Few attempts to find a substitute for R2I2(s) was not obtained.
The equation, voltage conservation, by the author-engineer did not include the i2(t) expression for R2. The engineer is taking i1(t) as a known current or on its own.
So there is no need for R2(i1(t) - i2(t)), rather just R2i1(t).

The engineer's solution stated the assumption current distribution as shown below.
I did it taking two loops, mesh equations, until I knew why. Otherwise the assumption would not been clear to me. Thus I leave it as it is, with correction continued below.

The improved or updated voltage equation becomes:

$$v_i(t) = R1i(t) + \frac{1}{C1} \cdot \int i\,dt + R2 \cdot ((i1(t) - i2(t)))$$

$$v_i(t) = R1i(t) + \frac{1}{C1} \cdot \int i\,dt + R2 \cdot i1(t)$$

$$V_i(s) = R1I(s) + \frac{1}{sC1} I(s) + R2I1(s)$$

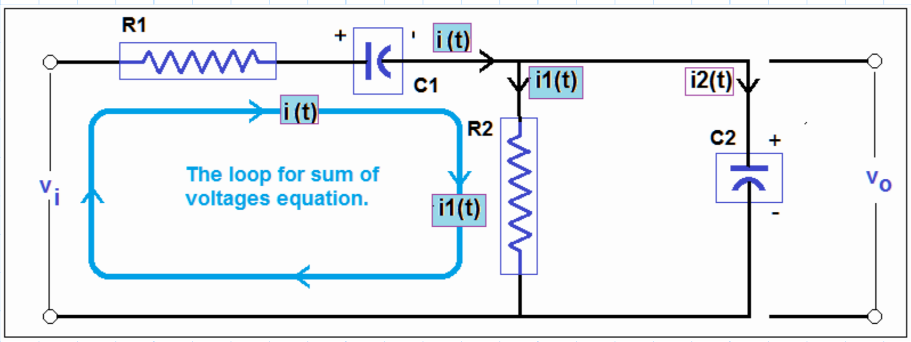


Figure to left is the voltage loop given i1(t) and i2(t) are known values.

$$V_o(t) = R2 \cdot I1(s)$$ Plug in equation above.

$$V_i(s) = R1I(s) + \frac{1}{sC1} I(s) + V_o(s)$$

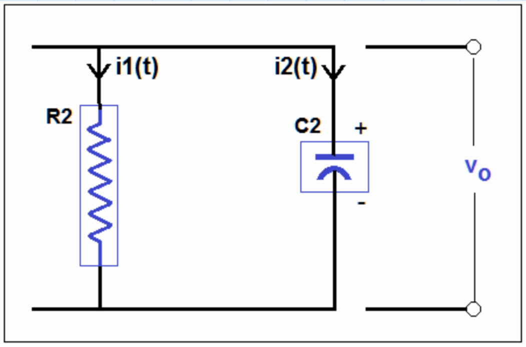
$$I(s) = I1(s) + I2(s)$$ Plug in equation below.

$$V_i(s) = R1(I1(s) + I2(s)) + \frac{1}{sC1}(I1(s) + I2(s)) + V_o(s)$$

Rearranging:

$$V_i(s) = (I1(s) + I2(s)) \left(R1 + \frac{1}{sC1}\right) + V_o(s)$$

I cannot find a Vo(s)/Vi(s) from the above expression.
Cleaver engineer does a substitution for I1(s) and I2(s).



$$V_o(s) = R1\, I1(s)$$

$$I1(s) = \frac{V_o(s)}{R1}$$

$$V_o(s) = \frac{1}{sC2}\, I2(s)$$

$$I2(s) = sC2\ V_o(s)$$

Substitute the expressions for I1(s) and I2(s) into the Vi(s) equation.

$$V_i(s) = (I1(s) + I2(s))\left(R1 + \frac{1}{sC1}\right) + V_o(s)$$

$$V_i(s) = \left(\frac{V_o(s)}{R1} + sC2\ V_o(s)\right)\left(R1 + \frac{1}{sC1}\right) + V_o(s)$$

$$V_i(s) = V_o(s)\cdot\left(\frac{1}{R1} + sC2\right)\left(R1 + \frac{1}{sC1}\right) + V_o(s)$$

Another new trick, maybe not, but not common divide by Vo(s)

$$\frac{V_i(s)}{V_o(s)} = \left(\frac{1}{R1} + sC2\right)\left(R1 + \frac{1}{sC1}\right) + 1$$

Multiply the parenthesis: $\frac{R1}{R1} + \frac{1}{sC1R1} + sC2R1 + \frac{sC2}{sC1}$ Note: C1 = C2

$$= 1 + \frac{1}{sC1R1} + sC2R2 + 1$$

$$\frac{V_i(s)}{V_o(s)} = 1 + \frac{1}{sC1R1} + sC2R2 + 1 + 1$$

$$\frac{V_i(s)}{V_o(s)} = \frac{1}{sC1R1} + sC2R2 + 3$$

Invert for Vo(s)/Vi(s): $\frac{V_o(s)}{V_i(s)} = \frac{1}{\frac{1}{sC1R1} + sC2R2 + 3}$

Next simplify this expression for the purpose of attaining an expression in s form.

The s form of expression we seek where we can identify zeros and poles.

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{\frac{1}{sC1R1} + sC2R2 + 3}$$ Multiply top and bottm by sC1R1

$$= \frac{sC1R1}{1 + (sC1R1)\ (sC2R2) + 3\ (sC1R1)}$$

$$= \frac{sC1R1}{1 + \left(s^2 \cdot C1C2R1R2\right) + 3\ sC1R1}$$

Let C = C1 = C2 = 1
R = R1 = R2 = 1

$$\frac{V_o(s)}{V_i(s)} = \frac{sCR}{1 + \left(s^2 \cdot C^2 \cdot R^2\right) + 3\ sCR}$$

Since R1=R2=C1=C2=1
We substitute for 1.

$$\frac{V_o(s)}{V_i(s)} = \frac{s}{1 + s^2 + 3\ s} = \frac{s}{s^2 + 3\ s + 1}$$ Answer for transfer function.

Zero: 0 Answer.

Pole(s): Solve quadratic equation $s^2 + 3\ s + 1$

$As^2 + Bs + C$ $$s1\ s2 = \frac{-B \quad +/- \quad \sqrt{B^2 - 4\ AC}}{2\ A}$$

$$s1 = \frac{-3 + \sqrt{3^2 - (4 \cdot 1 \cdot 1)}}{2 \cdot 1} = -4 \cdot 10^{-1}$$

$$s2 = \frac{-3 - \sqrt{3^2 - (4 \cdot 1 \cdot 1)}}{2 \cdot 1} = -3$$

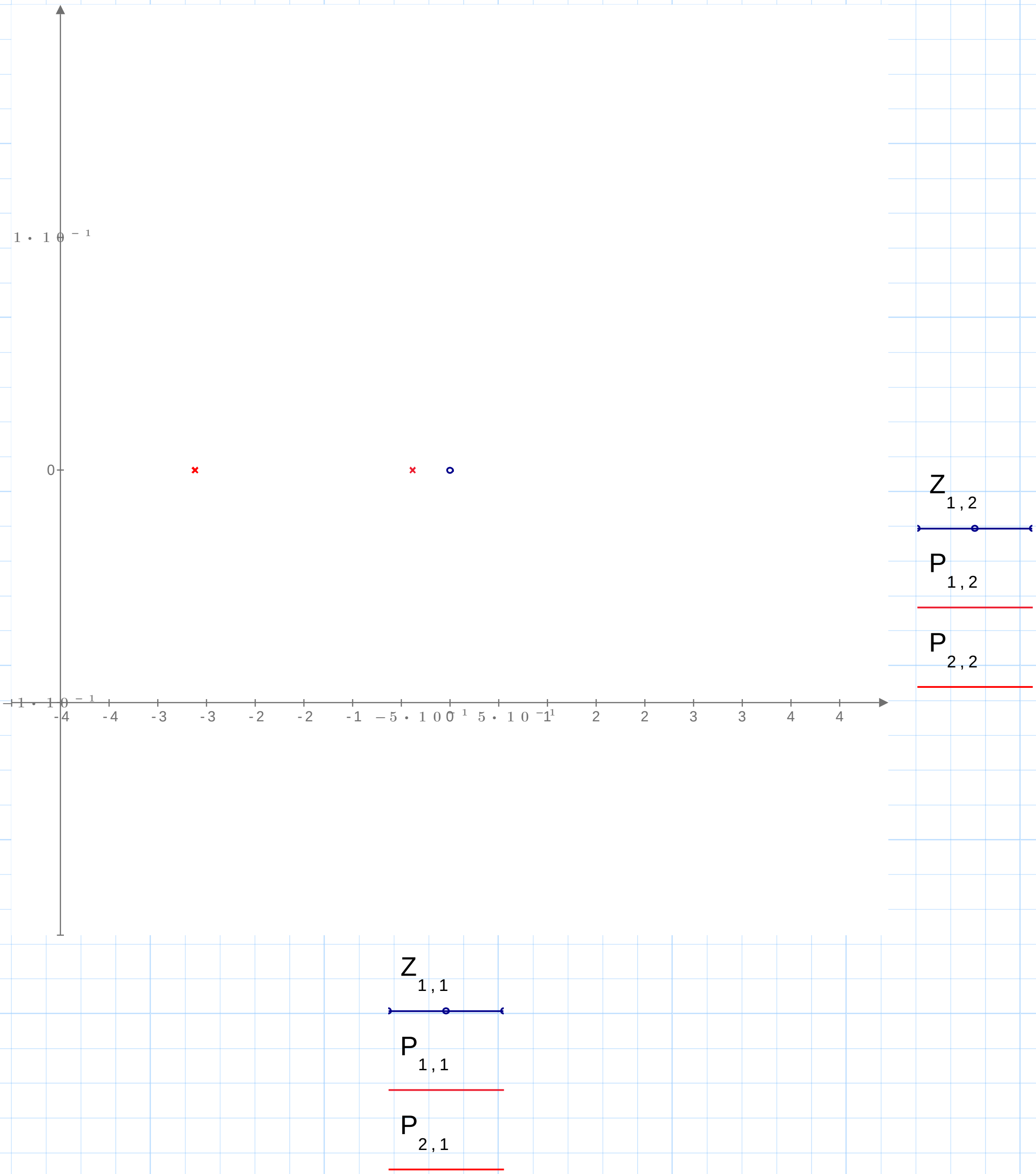
Poles: -0.382 and -2.618 Answer.

The manual plot is easy, real x-axis and imaginary y-axis.
Here all the zero and ploes are on the x-axis at 0, -0.382, and -2.618.

Lets try plotting the functions, numerator and denominator.

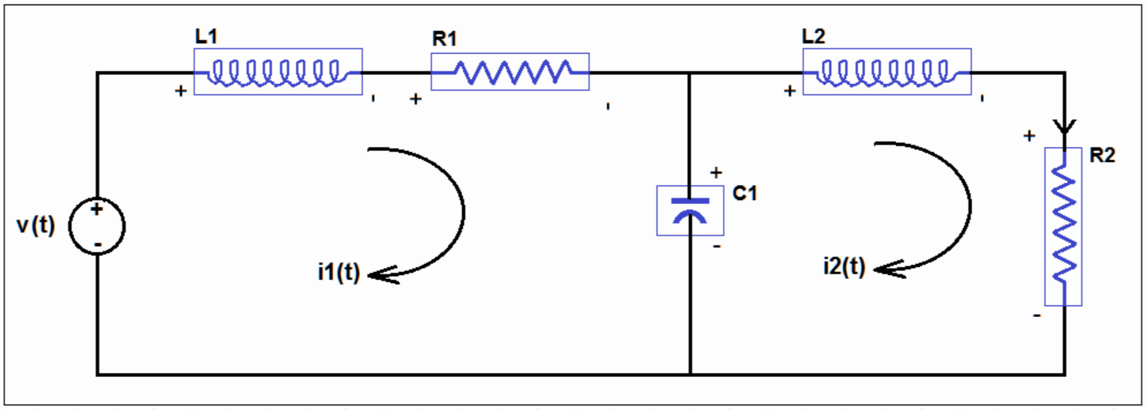
ORIGIN(1,1)   Set start of matrix at 1,1.

$Z := [0 \;\; 0]$ $P := \begin{bmatrix} -0.382 & 0 \\ -2.618 & 0 \end{bmatrix}$ Using matriz Z for zero and P for poles.



Answer. Plot above.

Chp 1 Problem 1.10:



Write the differential equations for the electrical circuit above.

Solution:

I kickoff with the sum of voltage around a loop equal zero.
I do an equation for each loop.

Loop i1(t):

$$v_i(t) = L1\left(\frac{di1(t)}{dt}\right) + R1i1(t) + \frac{1}{C1}\cdot\int i1(t)\,dt - \frac{1}{C1}\cdot\int i2(t)\,dt$$

Loop i2(t):

$$0 = L2\left(\frac{di2(t)}{dt}\right) + R2i2(t) + \frac{1}{C1}\cdot\int i2(t)\,dt - \frac{1}{C1}\cdot\int i1(t)\,dt$$

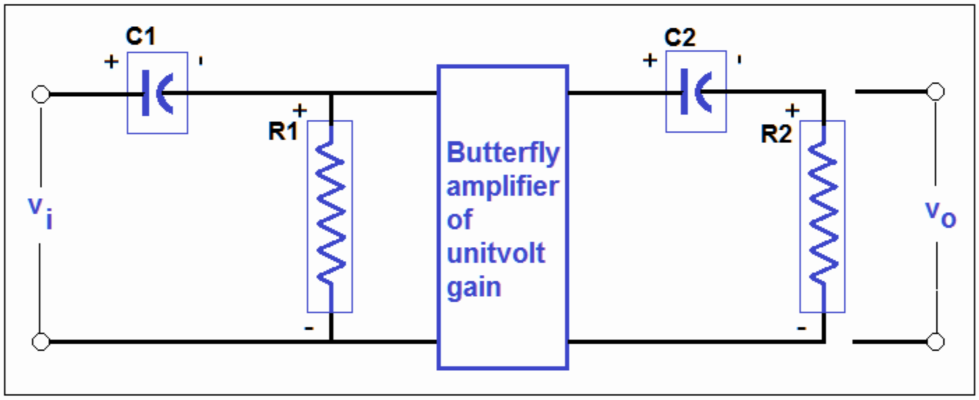
Answer.

Not part of the question but how if I did a s-domain on the time domain, what the typical controls engineering course will say is taking the Laplace transform? You verify.

$$V_i(s) = sL1\cdot I1(s) + R1\cdot I1(s) + \frac{1}{sC1}\cdot I1(s) - \frac{1}{sC1}\cdot I2(s)$$

$$0 = sL2\cdot I2(s) + R2\cdot I2(s) + \frac{1}{sC1}\cdot I2(s) - \frac{1}{sC1}\cdot I2(s)$$ Answer.

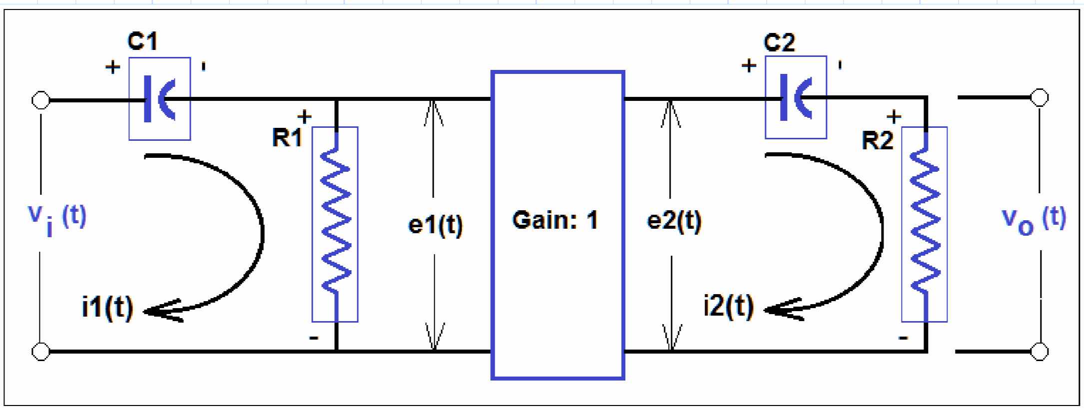
Chp 1 Problem 1.12:



Determine the transfer function relating Vo(s) to Vi(s) for network above.
Calculate the output voltage, t>>0, for a unit step voltage input at t=0, when C1 = 1 uF, R = 1 M Ohm, C2 = 0.5 uF and R2 = 1 M Ohm.

Solution:

Circuit re-sketched for applying sum of voltage in a loop method.
Kickoff's Voltage Law, KVL, usually what the electrical engineer calls.



Amplifier gain e2(t)/e1(t) = 1. Therefore e1(t) = e2(t).
The circuit has a voltage input v_i(t), and to the output side of the amplier is a voltage gained e2(t) this is similar to supplying voltage to the circuit to the right of the ampliffier.

We proceed with KVLoop on the left and right, and we equate the resistor R1 voltage for e1(t).

$$v_i(t) = \frac{1}{C1}\int i1(t)\,dt + R1i1(t)$$

$$e2(t) = \frac{1}{C2}\int i2(t)\,dt + R2i2(t)$$

$e1(t) = R1i1(t)$ Amplifier left side voltage.

$v_o(t) = R2i2(t)$ Amplifier right side voltage.
This being the voltage output v_o(t)

Now we take the Lapalce transforms of the expressions above.
Call it what you want, La Place or No Place, its converting to s-domain.

$$V_i(s) = \frac{I1(s)}{sC1} + R1 \cdot I1(s) \qquad \text{Eq 1}$$

$$E2(s) = \frac{I2(s)}{sC2} + R2 \cdot I2(s) \qquad \text{Eq 2}$$

$$E1(s) = R1 \cdot I1(s) \qquad \text{Eq 3}$$

$$V_o(s) = R2 \cdot I2(s) \qquad \text{Eq 4}$$

METHOD 1:
This by building interconnected relationship, as I done in the past problems here.
The long way and the answer is same as the textbook answer.
If I had not done this then it may remain a mystery!
You may verify.
Method 2 is easy, which was my first re-action to the problem. Just place Vo/Vi, after forming their expression without the usual inter-related quations.
I will do method 2 after method 1 completion.

Rearrange Eq 1:

$$V_i(s) = I1(s) \cdot \left(\frac{1}{sC1} + R1\right) \qquad \text{Eq 5}$$

Rearrange Eq 2:

$$E2(s) = I2(s) \cdot \left(\frac{1}{sC2} + R2\right) \qquad \text{Eq 6}$$

Rearrange Eq 4:

$$I2(s) = \frac{V_o(s)}{R2} \qquad \text{Eq 7...substitute in Eq 6}$$

$$E2(s) = \frac{V_o(s)}{R2} \cdot \left(\frac{1}{sC2} + R2\right) \qquad \text{Eq 8}$$

E2(s) = E1(s): $E1(s) = R1 \cdot I1(s) = E2(s)$

Next substitute E1(s) for E2(s) in Eq 8.

$$E1(s) = E2(s) = R1 \cdot I1(s) = \frac{V_o(s)}{R2} \cdot \left(\frac{1}{sC2} + R2\right) \qquad \text{Eq 9}$$

Substitute Eq 9 for R1I1(s) in Eq 1.

$$V_i(s) = \frac{I1(s)}{sC1} + \frac{V_o(s)}{R2} \cdot \left(\frac{1}{sC2} + R2\right) \qquad \text{Eq 10}$$

How do I substitute for I1(s), try Eq 5, then substitute into eq 10:

$$V_i(s) = I1(s) \cdot \left(\frac{1}{sC1} + R1\right) \qquad \text{Eq 5}$$

$$I1(s) = \frac{V_i(s)}{\left(\frac{1}{sC1} + R1\right)} \qquad \text{Eq 11.....substitute in Eq 10.}$$

$$V_i(s) = \frac{\frac{V_i(s)}{\left(\frac{1}{sC1} + R1\right)}}{sC1} + \frac{V_o(s)}{R2} \cdot \left(\frac{1}{sC2} + R2\right) \qquad \text{Eq 12...looks messy may do it.}$$

$$V_i(s) - \frac{\frac{V_i(s)}{\left(\frac{1}{sC1} + R1\right)}}{sC1} = \frac{V_o(s)}{R2} \cdot \left(\frac{1}{sC2} + R2\right)$$

$$V_i(s) - \frac{V_i(s)}{\left(\frac{1}{sC1} + R1\right)} \cdot \frac{1}{sC1} = \frac{V_o(s)}{R2} \cdot \left(\frac{1}{sC2} + R2\right)$$

$$V_i(s) \cdot \left(1 - \frac{1}{\left(\frac{1}{sC1} + R1\right) \cdot sC1}\right) = \frac{V_o(s)}{R2} \cdot \left(\frac{1}{sC2} + R2\right)$$

$$V_i(s)\cdot\left(1-\frac{1}{(1+sC1R1)}\right) = V_o(s)\cdot\left(\frac{1}{sC2R2}+1\right)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\left(1-\frac{1}{(1+sC1R1)}\right)}{\left(\frac{1}{sC2R2}+1\right)}$$ Transfer function. Need simplifying.

$$\frac{V_o(s)}{V_i(s)} = \frac{\left(\frac{(1+sC1R1)}{(1+sC1R1)}-\frac{1}{(1+sC1R1)}\right)}{\left(\frac{1+sC2R2}{sC2R2}\right)} = \frac{\left(\frac{(sC1R1)}{(1+sC1R1)}\right)}{\left(\frac{1+sC2R2}{sC2R2}\right)}$$

$$= \left(\frac{(sC1R1)}{(1+sC1R1)}\right)\cdot\left(\frac{sC2R2}{1+sC2R2}\right)$$

$$= \left[\frac{\left(s^2\cdot C1C2R1R2\right)}{\left(1+sC2R2+sC1R1+s^2\cdot C1C2R1R2\right)}\right]$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\left(s^2\cdot C1C2R1R2\right)}{\left(1+s(C1R1+C2R2)+s^2\cdot(C1C2R1R2)\right)}$$

Let $A = C1C2R1R2$ $\quad B = C1R1$ $\quad C = C2R2$

$$\frac{V_o(s)}{V_i(s)} = \frac{\left(A\cdot s^2\right)}{\left(1+(B+C)\cdot s+A\cdot s^2\right)}$$ One Transfer Function - METHOD 1.

$C1 := 1\cdot 10^{-6}$ $\quad C2 := 0.5\cdot 10^{-6}$ $\quad R1 := 1\cdot 10^{6}$ $\quad R2 := 1\cdot 10^{6}$

$A := C1\cdot C2\cdot R1\cdot R2 = 0.5$ Or fraction: $\frac{1}{2}$ $\quad B := C1\cdot R1 + C2\cdot R2 = 2$

$$\frac{V_o(s)}{V_i(s)} = \frac{\left(\frac{1}{2}\right)\cdot s^2}{1+\left(\frac{3}{2}\right)\cdot s+\left(\frac{1}{2}\right)\cdot s^2}$$ Multiply by 2.

$$\frac{V_o(s)}{V_i(s)} = \frac{2\left(\frac{1}{2}\right)\cdot s^2}{2\cdot\left(1+\left(\frac{3}{2}\right)\cdot s+\left(\frac{1}{2}\right)\cdot s^2\right)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{s^2}{2+3\cdot s+s^2} = \frac{s^2}{s^2+3\cdot s+2}$$ Answer. SAME AS TEXTBOOK!

Calculate the output voltage, t>>0, for a unit step voltage input at t=0:

Since its unit step voltage input the initial conditions for t<0 = 0.
So i(-0) = i(0+...just near 0) = 0 and
v(-0) = v(0+...just near 0) = 0 v(++) = 1

Comment: How do I get the numerator (zero) = 1 for t>>0 so the Vi(s) = 1 or greater; u(t=0 or t>0) = 1 or u(t) = 1x Constant. But NOT equal 0.

$$\frac{V_o(s)}{V_i(s)} = \frac{s^2}{(s+2)\,(s+1)} \qquad V_o(s) = V_i(s)\cdot\frac{s^2}{(s+2)\,(s+1)}$$

$$V_o(s) = 1\cdot\frac{s\cdot s}{(s+2)\,(s+1)} = \frac{A}{(s+2)}+\frac{B}{(s+1)}$$

To solve for coefficients using the? Method of proper fractions OR Equating coefficients of like powers.

$$s = A(s+1)+B(s+2) = As+A+Bs+2B$$

$$s = s(A+B)+(A+2B)$$

Arrange for like terms:

s : $s(A+B)$ ---> $A+B = 1$ Eq 1

0 : $(A+2B)$ ---> $A+2B = 0$ Eq 2

$B = -1$ Eq 2-1

Substitute B in Eq 1.

$$A+B = 1$$
$$A-1 = 1$$
$$A = 2$$

$$V_o(s) = \frac{A}{(s+2)} + \frac{B}{(s+1)} = \frac{2}{(s+2)} - \frac{1}{(s+1)}$$

Now with the coefficients, zeros, and poles I can form the voltage output in time domain. This will be an exponential equation because the voltage source is a step function, unity, or constant.

$$V_o(s) = Ae^{s1t} + Be^{s2t}$$

$$V_o(s) = -2\,e^{-2\,t} - 1\,e^{-1\,t}$$

Now to convert from s-domain to time domain:

$$v_o(t) = -2\,e^{-2\,t} - 1\,e^{-1\,t}$$

$$v_o(t) = -2\,e^{-2\,t} - e^{-t}$$

Answer. Same as textbook.

Please verify the solution steps and reasoning on the voltage output equation where Vi(s) = 1.

METHOD 2:

Now for Method 2, the supposed to be simpler and shorter solution.

$$V_i(s) = \frac{I1(s)}{sC1} + R1 \cdot I1(s) \qquad \text{Eq 1}$$

$$E2(s) = \frac{I2(s)}{sC2} + R2 \cdot I2(s) \qquad \text{Eq 2}$$

$$E1(s) = R1 \cdot I1(s) \qquad \text{Eq 3}$$

$$V_o(s) = R2 \cdot I2(s) \qquad \text{Eq 4}$$

Set up the transfer function, Vo(s)/Vi(s) based on their respective equations directly:

$$\frac{V_o(s)}{V_i(s)} = \frac{R2 \cdot I2(s)}{\frac{I1(s)}{sC1} + R1 \cdot I1(s)} = \frac{I2(s) \cdot R2}{I1(s) \cdot \left(R1(s) + \frac{1}{sC1}\right)}$$

Eq 5...maybe I2(s) and I1(s) substituion may help.

$$I1(s) = \frac{E1(s)}{R1} \qquad I2(s) = \frac{E2(s)}{\left(R2 + \frac{1}{sC2}\right)}$$

From Eq 2 above.

$$\frac{I2(s)}{I1(s)} = \frac{\dfrac{E2(s)}{\left(R2 + \dfrac{1}{sC2}\right)}}{\dfrac{E1(s)}{R1}} = \frac{E2(s)}{\left(R2 + \dfrac{1}{sC2}\right)} \cdot \frac{R1}{E1(s)}$$

Gain = 1, E2(s)/E1(s) = 1, therefore E1(s) = E2(s).

$$E1(s) = E2(s)$$

Now the current ratio equation becomes: $\dfrac{I2(s)}{I1(s)} = \dfrac{R1}{\left(R2 + \dfrac{1}{sC2}\right)}$

Returning to Eq 5 substitute for I2(s)/I1(s):

$$\frac{V_o(s)}{V_i(s)} = \frac{I2(s) \cdot R2}{I1(s) \cdot \left(R1 + \dfrac{1}{sC1}\right)} \qquad \text{Eq 5}$$

$$= \frac{R1}{\left(R2 + \dfrac{1}{sC2}\right)} \cdot \frac{R2}{\left(R1 + \dfrac{1}{sC1}\right)}$$

$$= \frac{R1R2}{R1R2 + \dfrac{R2}{sC1} + \dfrac{R1}{sC2} + \dfrac{1}{s^2 \; C1C1}}$$

Let: $A = R1 \cdot R2 = 1 \cdot 10^{12}$

$B = \dfrac{R2}{C1} = 1 \cdot 10^{12}$ $\qquad C = \dfrac{R1}{C2} = 2 \cdot 10^{12}$

$D = \dfrac{1}{C1 \cdot C2} = 2 \cdot 10^{12}$

$$\frac{V_o(s)}{V_i(s)} = \frac{1 \cdot 10^{12}}{1 \cdot 10^{12} + \dfrac{1 \cdot 10^{12}}{s} + \dfrac{2 \cdot 10^{12}}{s} + \dfrac{2 \cdot 10^{12}}{s^2}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1 + \dfrac{1}{s} + \dfrac{2}{s} + \dfrac{2}{s^2}} = \frac{1}{1 + \dfrac{3}{s} + \dfrac{2}{s^2}}$$

Multiply by s^2 top and bottom.

$$\frac{V_o(s)}{V_i(s)} = \frac{(s^2)\cdot 1}{(s^2)\cdot\left(1+\frac{3}{s}+\frac{2}{s^2}\right)}$$

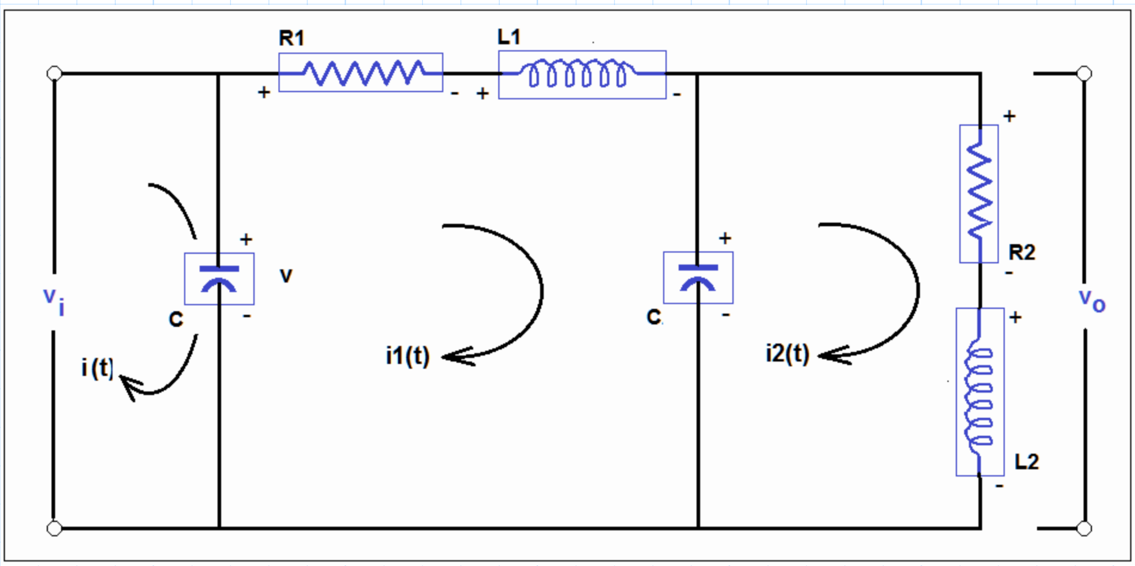
$$\frac{V_o(s)}{V_i(s)} = \frac{s^2}{s^2+3\,s+2}$$

Answer.
Same method used by engineer the faster method.

The short method may give the impression there is no relationship with the components like that established in the longer method. However, the transfer function's definition is just that, output divided by input. Do consider the circuit's components and connections, and carefully construct the equations.

The remaining part on the output voltage same as completed following the long method of the transfer functions.

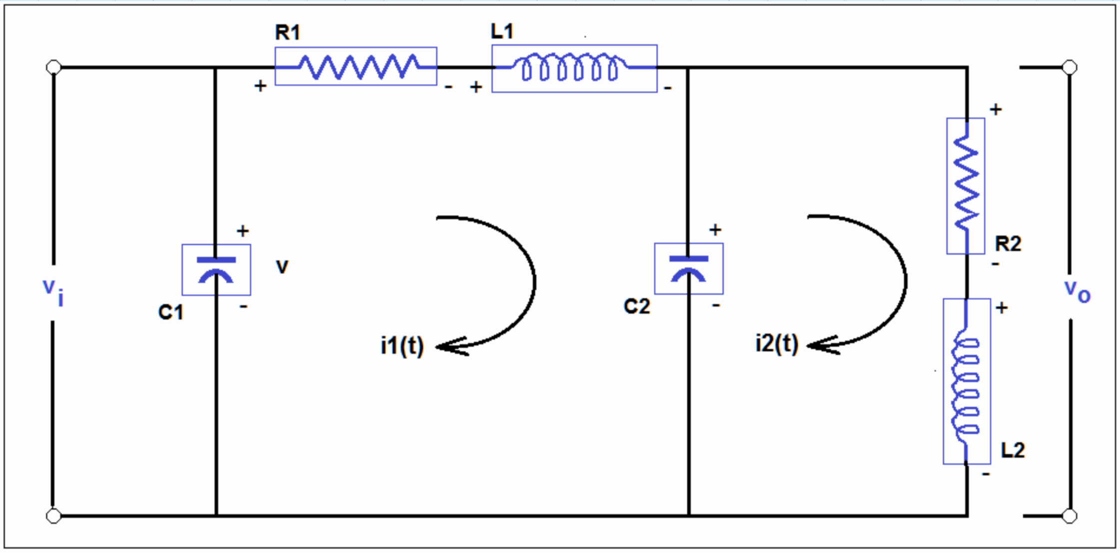
Chp 1 Problem 1.13:



Determine the transfer function of the electrical network above:

Solution:

C1 = C2, the question did not show C1 and C2, rather C.
To ease tracking the solution they were made into C1 and C2.



The basic steps we first started with provided here again, these steps were much the same to what we did in the previous problems.

The **steps involved** in obtaining the transfer function are:

1. Write differential equations of the system.

2. Replace terms involving $\frac{d}{dt}$ by s and $\int \blacksquare \, dt$ by 1/s, for inductor and capacitor respectively.

3. Eliminate all but the desired variable.

Step 1: Check current flow direction. Coming out of C1 -ve terminal -ve voltage.

v_i(t) :

$$v_i(t) = -\frac{1}{C1} \cdot \int i(t)\,dt$$

Check current flow direction. Coming out of C1 - ve terminal -ve voltage (left loop).

$$\frac{1}{C1} \cdot \int (i1(t) - i(t))\,dt = \frac{1}{C1} \cdot \int i1(t)\,dt - v_i(t)$$

This can be written as:

$$v_i(t) + \frac{1}{C1} \cdot \int i1(t)\,dt = 0$$

Sum of voltages, next vi(t) to the LHS, resulting in the same.

$$-v_i(t) = \frac{1}{C1} \cdot \int i1(t)\,dt \quad \text{OR} \quad v_i(t) = -\frac{1}{C1} \cdot \int i1(t)\,dt$$

Step 1:

v_i(t) :

$$v_i(t) = -\frac{1}{C1}\cdot\int i(t)\,dt$$

Check current flow direction. Coming out of C1 -ve terminal -ve voltage.

$$v_i(t) = \frac{1}{C1}\cdot\int i1(t)\,dt$$

Check current flow direction. Coming out of C1 -ve terminal -ve voltage.

This can be written as:

$$v_i(t) + \frac{1}{C1}\cdot\int i1(t)\,dt = 0$$

Sum of voltages, next vi(t) to the LHS, resulting in the same.

$$-v_i(t) = \frac{1}{C1}\cdot\int i1(t)\,dt \quad \text{OR} \quad v_i(t) = -\frac{1}{C1}\cdot\int i1(t)\,dt$$

Centre loop:

$$0 = \frac{1}{C1}\cdot\int i1(t)\,dt + R1i1(t) + L1\left(\frac{di1(t)}{dt}\right) + \frac{1}{C2}\cdot\int (i1(t) - i2(t))\,dt$$

$$-v_i(t) = \frac{1}{C1}\cdot\int i1(t)\,dt$$

Substitute in equation above.

$$0 = -v_i(t) + R1i1(t) + L1\left(\frac{di1(t)}{dt}\right) + \frac{1}{C2}\cdot\int (i1(t) - i2(t))\,dt$$

$$v_i(t) = R1i1(t) + L1\left(\frac{di1(t)}{dt}\right) + \frac{1}{C2}\cdot\int (i1(t) - i2(t))\,dt \qquad \text{Eq 1}$$

v_o(t):

$$v_o(t) = R2\cdot i2(t) + L2\left(\frac{di2}{dt}\right) \qquad \text{Eq 2}$$

Right loop:

$$0 = \frac{1}{C2}\cdot\int (i2(t) - i1(t))\,dt + R2i2(t) + L2\left(\frac{di2(t)}{dt}\right) \qquad \text{Eq 3}$$

Substitute v_o(t) into Right Loop.

Step 2:

Assuming all initial conditions for L and C are zero.
We proceed with taking the? *Laplace Transform*. Convert to s-domain.

$$v_i(t) = R1i1(t) + L1\left(\frac{di1(t)}{dt}\right) + \frac{1}{C2}\cdot\int (i1(t) - i2(t))\,dt \qquad \text{Eq 1}$$

$$V_i(s) = R1I1(s) + sL1\ I1(s) + \frac{1}{sC2}\cdot(I1(s) - I2(s))$$

$$V_i(s) = R1I1(s) + sL1\ I1(s) + \frac{1}{sC2}\cdot(I1(s) - I2(s))$$ Eq 4 <---Same as textbook.

$$v_o(t) = R2\cdot i2(t) + L2\left(\frac{di2}{dt}\right) \qquad \text{Eq 2}$$

$$V_o(t) = R2\cdot I2(s) + sL2I2(s)$$ Eq 5 <----Same as textbook.

$$0 = \frac{1}{C2}\cdot\int (i2(t) - i1(t))\,dt + R2i2(t) + L2\left(\frac{di2(t)}{dt}\right) \qquad \text{Eq 3}$$

$$0 = \frac{1}{sC2}(I2(s) - I1(s)) + R2I2(s) + sL2\cdot I2(s) \qquad \text{Eq 6}$$

Step 3:

$$\frac{V_o(t)}{V_i(s)} = \frac{R2\cdot I2(s) + sL2I2(s)}{R1I1(s) + sL1\ I1(s) + \frac{1}{sC2}\cdot(I1(s) - I2(s))}$$

Same as textbook.
Past this point you have to solve it for the best possible form.

$$\frac{V_o(t)}{V_i(s)} = \frac{I2(s)\cdot(R2 + sL2)}{I1(s)\cdot\left(R1 + sL1 + \frac{1}{sC2}\right) - \frac{1}{sC2}\ I2(s)}$$

Find a substitute for I2 in terms of I1 for the denominator.

$$0 = \frac{1}{sC2}(I2(s) - I1(s)) + R2I2(s) + sL2 \cdot I2(s) \qquad \text{Eq 6}$$

Solve for I2 above:

$$\frac{1}{sC2} I2(s) + R2I2(s) + sL2 \cdot I2(s) = \frac{I1(s)}{sC2} \qquad \text{Multiply by sC2.}$$

$$I2(s) + R2I2(s)\; sC2 + sL2 \cdot I2(s)\; sC2 = I1(s)$$

$$I2(s) \cdot \left(1 + sC2R2 + s^2\; C2L2\right) = I1(s) \qquad \text{Eq 7}$$

$$I2(s) = \frac{I1(s)}{\left(1 + sC2R2 + s^2\; C2L2\right)}$$

$$\frac{V_o(t)}{V_i(s)} = \frac{I2(s) \cdot (R2 + sL2)}{I1(s) \cdot \left(R1 + sL1 + \frac{1}{sC2}\right) - \frac{1}{sC2}\; I2(s)}$$

Substitute I2(s) in denominator above.

$$\frac{V_o(t)}{V_i(s)} = \frac{I2(s) \cdot (R2 + sL2)}{I1(s) \cdot \left(R1 + sL1 + \frac{1}{sC2}\right) - \frac{1}{sC2} \cdot \frac{I1(s)}{\left(1 + sC2R2 + s^2\; C2L2\right)}}$$

$$\frac{V_o(t)}{V_i(s)} = \frac{I2(s) \cdot (R2 + sL2)}{I1(s) \cdot \left(\left(R1 + sL1 + \frac{1}{sC2}\right) - \left(\frac{1}{sC2 + s^2\; C2^2\; R2 + s^3\; C2^2\; L2}\right)\right)}$$

$$\frac{V_o(t)}{V_i(s)} = \left(\frac{I2(s)}{I1(s)}\right) \cdot \frac{(R2 + sL2)}{\left(R1 + sL1 + \frac{1}{sC2}\right) - \left(\frac{1}{sC2 + s^2\; C2^2\; R2 + s^3\; C2^2\; L2}\right)}$$

Find an equation for I2(s)/I1(s) .... Eq 7 below.

$$I2(s) \cdot \left(1 + sC2R2 + s^2\; C2L2\right) = I1(s) \qquad \text{Eq 7}$$

$$\frac{I2(s)}{I1(s)} = \frac{1}{\left(1 + sC2R2 + s^2\ C2L2\right)}$$

Substitute $\frac{I2(s)}{I1(s)}$ in transfer function

$$\frac{V_o(t)}{V_i(s)} = \left(\frac{1}{1 + sC2R2 + s^2\ C2L2}\right)\left[\frac{(R2 + sL2)}{\left(R1 + sL1 + \frac{1}{sC2}\right) - \left(\frac{1}{sC2 + s^2\ C2^2\ R2 + s^3\ C2^2\ L2}\right)}\right]$$

$$\frac{V_o(t)}{V_i(s)} = \frac{(R2 + sL2)}{\left(1 + sC2R2 + s^2\ C2L2\right)\cdot\left(R1 + sL1 + \frac{1}{sC2}\right) - \left[\frac{\left(1 + sC2R2 + s^2\ C2L2\right)}{sC2\left(1 + s\ C2R2 + s^2\ C2\ L2\right)}\right]}$$

Set C1 = C2 = C, as given.

$$\frac{V_o(t)}{V_i(s)} = \frac{(R2 + sL2)}{\left(1 + sCR2 + s^2\ CL2\right)\cdot\left(R1 + sL1 + \frac{1}{sC}\right) - \left(\frac{1}{sC}\right)}$$

Expand the left side terms at the bottom, and set equal to A.
Then the bottom right side term's denominator set to B.

$$\left(1 + sCR2 + s^2\ CL2\right)\cdot\left(R1 + sL1 + \frac{1}{sC}\right) =$$

$$R1 + sL1 + \frac{1}{sC} + sCR1R2 + s^2\ CR2L1 + R2 + s^2\ CR1L2 + s^3\ CL1L2 + sL2 = A$$

$$s^3\ (CL1L2) + s^2\cdot(CR2L1 + CR1L2) + s\left(L1 + \frac{1}{s^2\ C} + CR1R2 + L2\right) + (R1 + R2) = A$$

$$s^3\ (L1L2) + s^2\cdot C\,(R2L1 + R1L2) + s\left(L1 + L2 + CR1R2 + \frac{1}{s^2\ C}\right) + (R1 + R2) = A$$

$$\frac{1}{sC} = B$$

$$\frac{V_o(t)}{V_i(s)} = \frac{(R2 + sL2)}{A - \left(\frac{1}{B}\right)}$$

$$s^3\ (L1L2) + s^2 \cdot C\,(R2L1 + R1L2) + s\left(L1 + L2 + CR1R2 + \frac{1}{s^2\ C}\right) + (R1 + R2) = A$$

In my a equation above there is (1/s^2C) this is not in the textbook anwer.
Textbook answer below does not have B term (1/sC) maybe this was negligible to the overall function because it becomes huge in the denominator, and when it divides the numerator its small or negligible. Usually C is in microFarad units. This may also be the case for (1/s^2C) in the A term. Except for this my result is the same.

$$\frac{V_o(t)}{V_i(s)} = \frac{R2 + sL2}{s^3\ (L1L2) + s^2 \cdot C\,(R2L1 + R1L2) + s\left(L1 + L2 + CR1R2 + \frac{1}{s^2\ C}\right) + (R1 + R2) - \left(\frac{1}{sC}\right)}$$

Neglecting (1/sC) and (1/s^2 C):

$$\frac{V_o(t)}{V_i(s)} = \frac{R2 + sL2}{s^3\ (L1L2) + s^2 \cdot C\,(R2L1 + R1L2) + s\,(L1 + L2 + CR1R2) + (R1 + R2)}$$

My Answer.
You can verify this answer correct it, or present your own. *Here this is as far as I am going.*

Textbook Answer:

$$\frac{V_o(s)}{Vi\,(s)} = \frac{R2 + sL2}{s^3\ CL1L2 + s^2\ C\,(R1L2 + L1R2) + s\,(L1 + L2 + CR1R2) + (R1 + R2)}$$

Transfer function above does look tidy!
You solve it for yourself if you see a need.

You can sort it with your local lecturer/engineer.
Apologies for any errors and omissions.

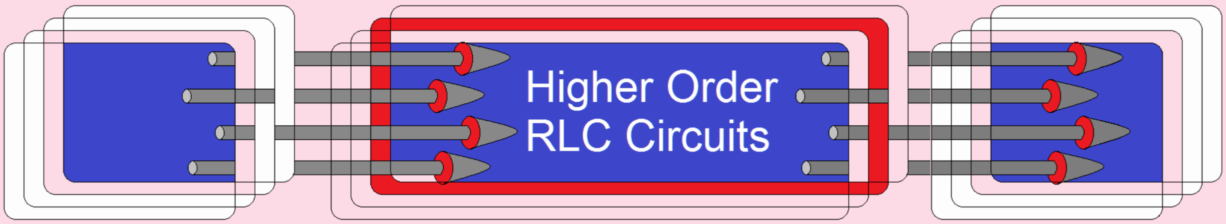
This brings to end the 13 example problems.
Next Schaum's Chapter 8 Solved Problems.

My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis. Source of study material: 1). Electric Circuits 6th Ed., Nahvi & Edminister. 2). Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill. Resource: 3). Solutions & Problems of Control Systems, 2nd ed - AK Jairath.
Karl S. Bogha.

SECTION THREE.

Level: Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



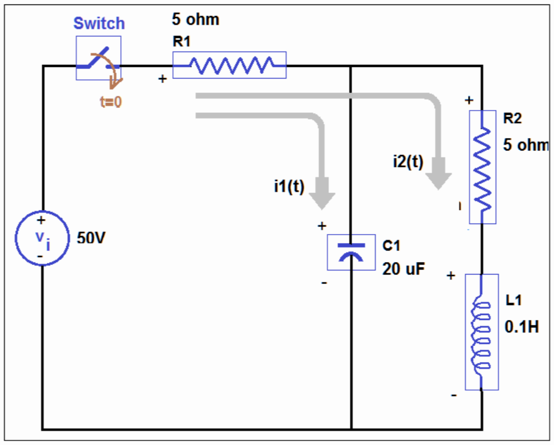
Part 3 - C

(Intermediate Level)

Apologies for any errors and omissions.

October 2020.

DISCUSSION. Supplementary Problem 8.27 (Mesh RLC circuit) :



In the two mesh circuit provided, the switch is closed at t=0. Find i1 and i2 for t>0.

$R1 := 5$ Ohm
$R2 := 5$ Ohm
$C1 := 20 \cdot 10^{-6}$ F
$L1 := 0.1$ H

$$\frac{1}{C1} = 50000$$

$Vi := 50$ V

Solution (Errors and Assumptions) :

What Happened Here?

Over three solution methods attempted. 1 method was creating differential equations and solving them simultaneously, with initial conditions. This did not produce the textbook answer.

There was the question on how to distinguish the time constants which there were two in the solution.

Over/Under/Critical damped conditions considered. Roots, s1 and s2, of equation method did not get to the answers.

That left me with my last option using component initial conditons with voltage and current equations and this was attempted many times until I came to this proposal solution.

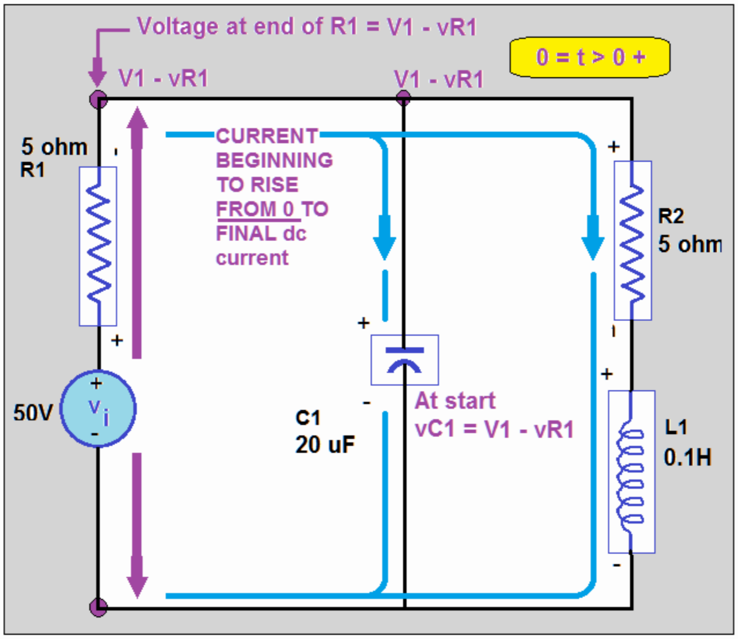
My diffculties may have been solved from the following sketches.

Here I tried to break the circuit into its possible operation at different stages of time. I done this several times until I was closer to where I could call it a proposed solition.

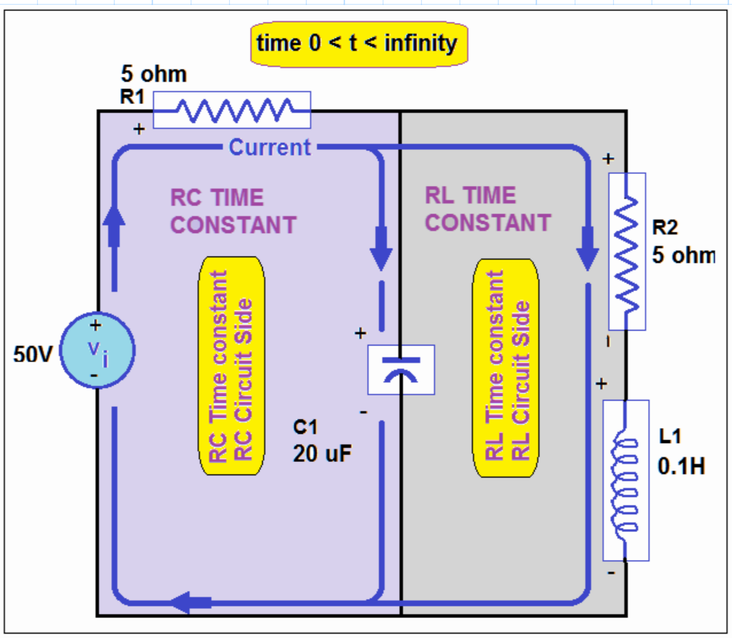
I start my solution with these sketches/figures/circuits then base my solution relative to these circuits/figures.

Any errors and omissions apologies in advance. This I call a **proposal solution which is loose (not firm)** and you may certainly have a better approach.

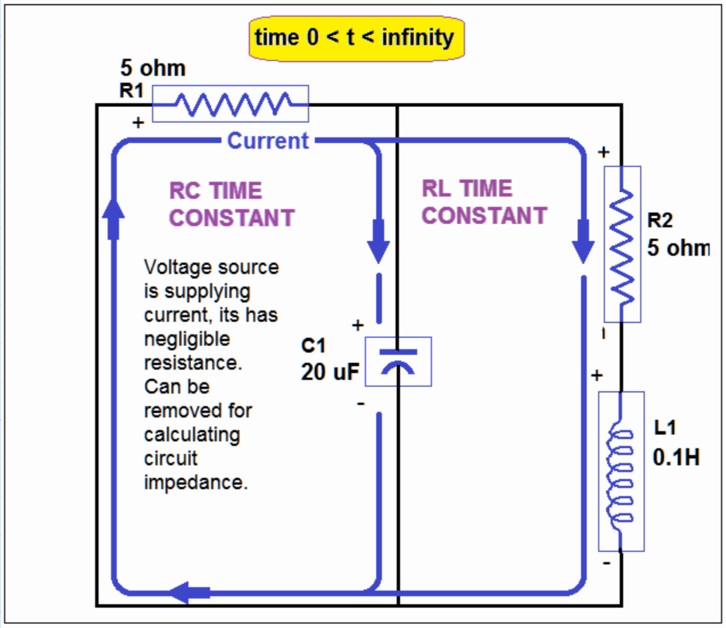
A. Circuit at time t>0+ just past 0.
Read notes in all figures there I tried to explain my understanding of my solution.



B. Circuit at time 0<t<Infinity.



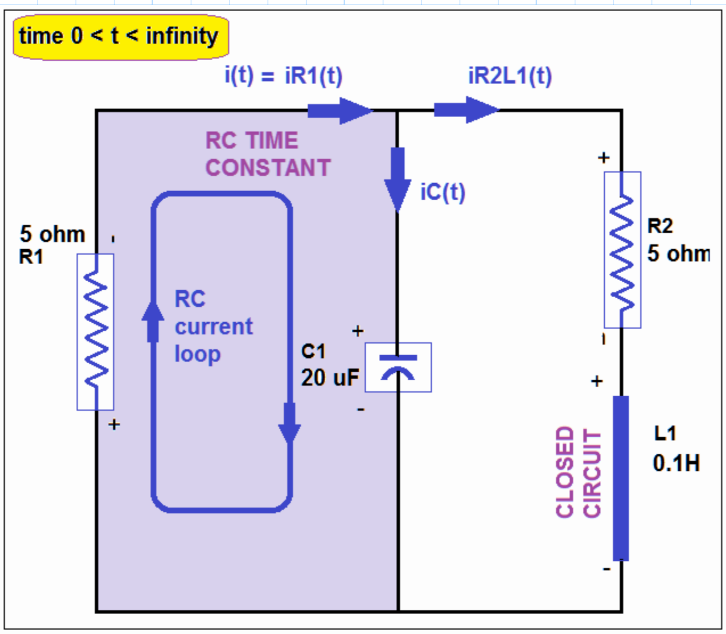
C. Same circuit as B with voltage source removed for calculating impedance(s) relative to the circuit connection.



Identifed a RC mesh and a RL mesh.Each with a separate time constant.

Time frame circuit is operating is shown in yellow in figures.

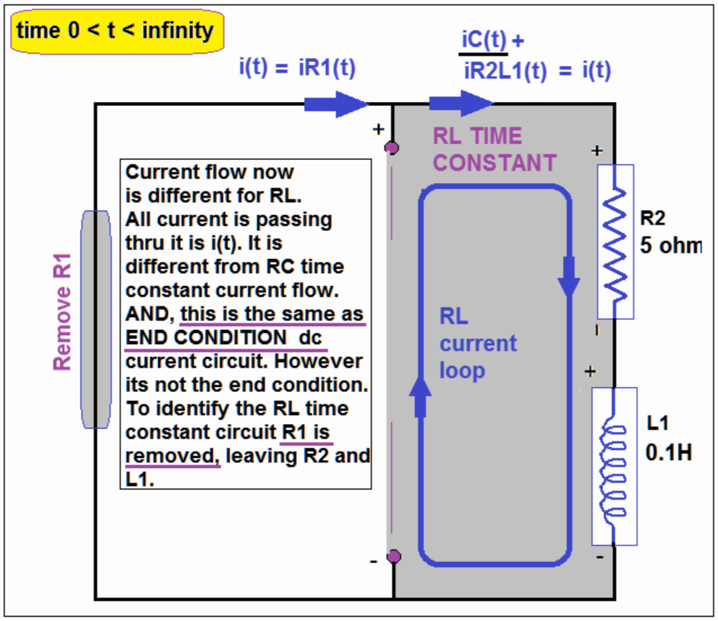
D. What I identify as the RC side of the circuit.
Read notes in figure.



RC time constant.

I emphasise the current node, this is different in the next similar circuit.

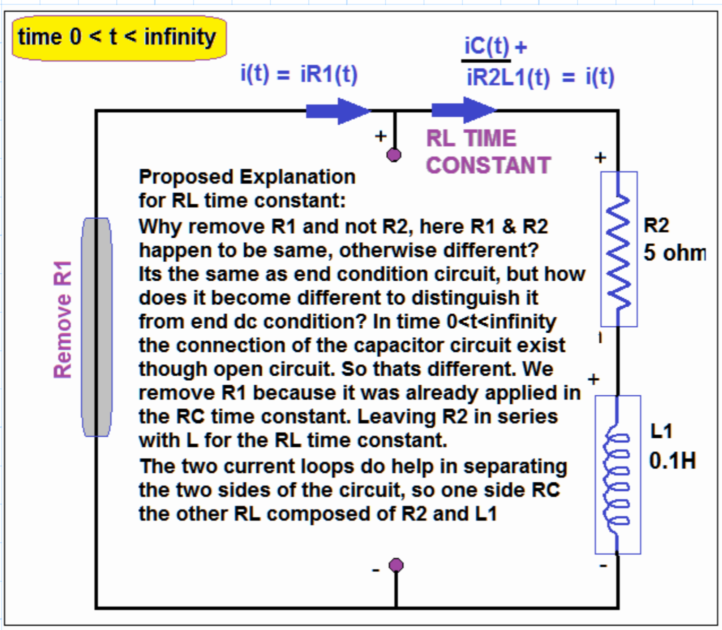
E. What I identify as the RL side of the circuit.
Read notes in figure.



RL time constant.

At current node current now iC(t) is added into the path of iRL1(t), whatever value iC(t) could be from some value to near zero. Because at time t=infinity the C is open circuit, high impedance, with no current flow. At that time its 0, before that C also contributes current into the circuit in the natural or transient state.

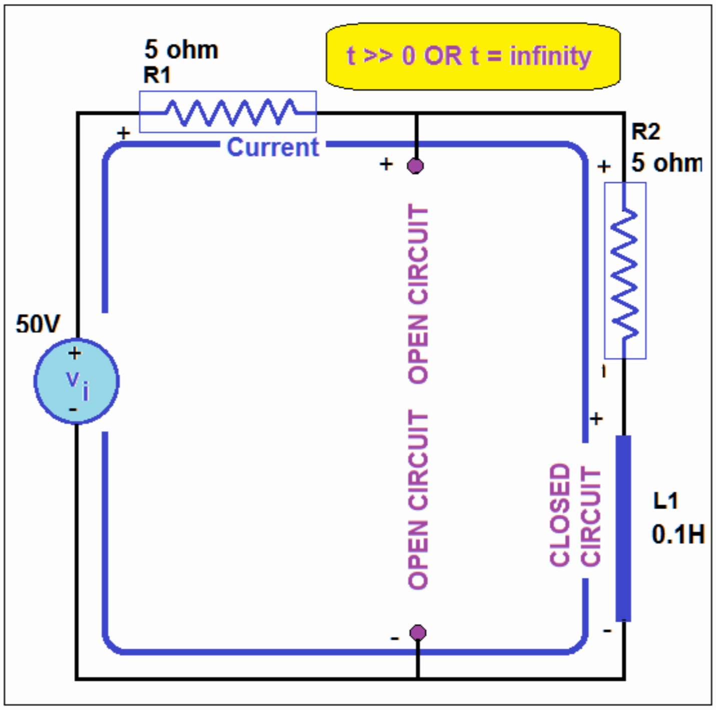
F. For circuit above, figure E, which applies to the RL time constant, R1 or R2 or both? Reference is made to the end condition, dc circuit, at time t=Infinity, which is shown next. So I said R2 is the resistance applied for RL time constant excluding R1.



In this time range, the Rl and RC time constants will impact the as their difference. L and C can be 180 degrees apart dependent on circuit condition. Their currents are not in step/sync the current waveform is impacted by the values of L and c, and the time constant for each helps in determing the time it comes to stable or zero.

Since current is not in step, the phase angle would differ reflected in waveform relative to time constant. So I said the net result is their difference.

G. Now for time t = Infinity which can also be shown as t>>0 meaning much greater than 0. This is where R is open circuit and L is short circuit.

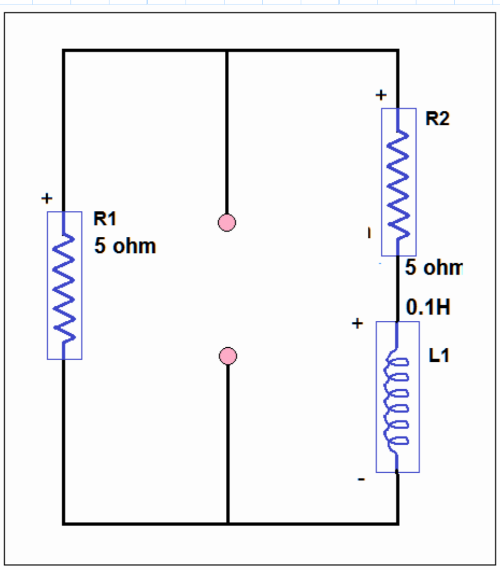


Time t = infinity.

How do I decide on the connection of this circuit time constant wise ? Series Or Parallel RLC? Maybe a mix of both? RL or RC time constant. *Ok I do not have an example, not the first time. My propsed solution take no claim to orginality because I am not 100% sure of its meaning. May back fire on me.*

Now I start with the calculating the different time constants. I start with t=infinity since the dc values apply to the current values in transient condition.

RL time constant - dc condition Or end condition (t = infinity):



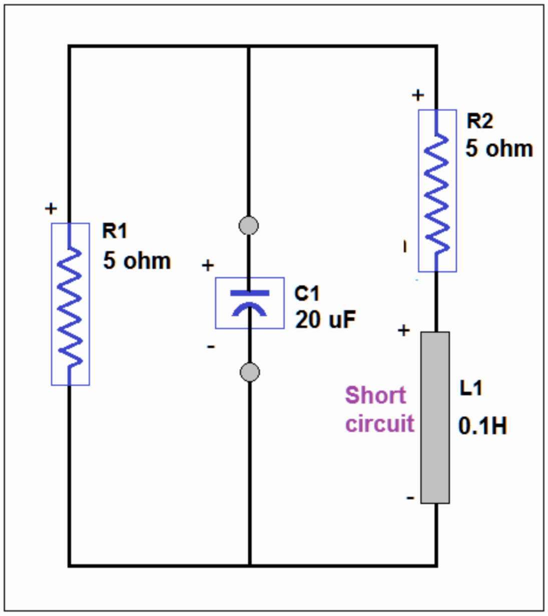
R1 and R2 in series. Vi removed.
At time t>>0 end condition capacitor C1 is open circuit. Inductor shown but it is taken for short circuit it exists in the circuit for continous current.

Here we have an RL series circuit.
Time constant:

$R12 := R1 + R2 = 10$

$\tau_{R1L1} = \dfrac{L1}{R12} = 0.01$

$\dfrac{1}{\tau_{R1L1}} = \dfrac{R12}{L1} = 100$ Steady state condition.



RC circuit (0<t<infinity condition):

For time t: $0 < t < \infty$

R1 and R2 parallel.

At time 0 <<t < infinity inductor is near short circuit - low impedance. Capacitor operating and not open circuit. Suggest C impacting circuit more than L, because C eventually releases current in circuit.

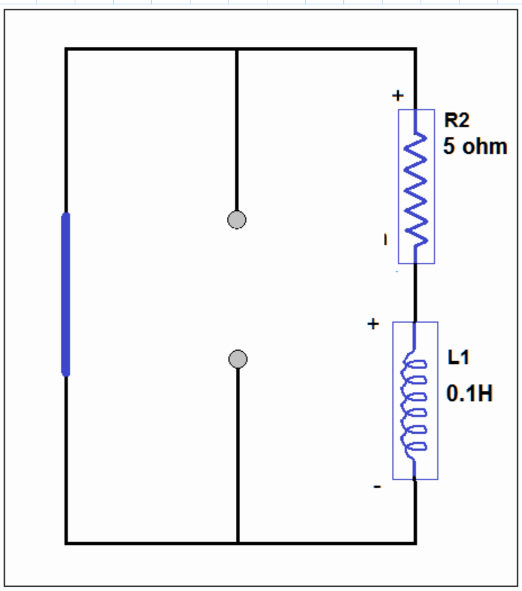
Capacitor plays an increasing impeding role as time approahces infinity. It turns into an open circuit.

I said C impacting the circuit greater than L since L gets closer and closer to short circuit. So I have the RC time constant.

For time t: $0 < t < \infty$

$$\tau_{R1C1} = R1 \cdot C1 = 1 \cdot 10^{-4}$$

$$\frac{1}{\tau_{R1C1}} = \frac{1}{1 \cdot 10^{-4}} = 10000$$



RL circuit (0<t<infinity):

Circuit shown to the left is RL series. R1 ignored as it was already accounted for. Read explanation in Figure E and F. Circuit to the left is RL series.

For time t: $t < \infty$

$$\tau_{R2L1} = \frac{L1}{R2} = 0.02$$

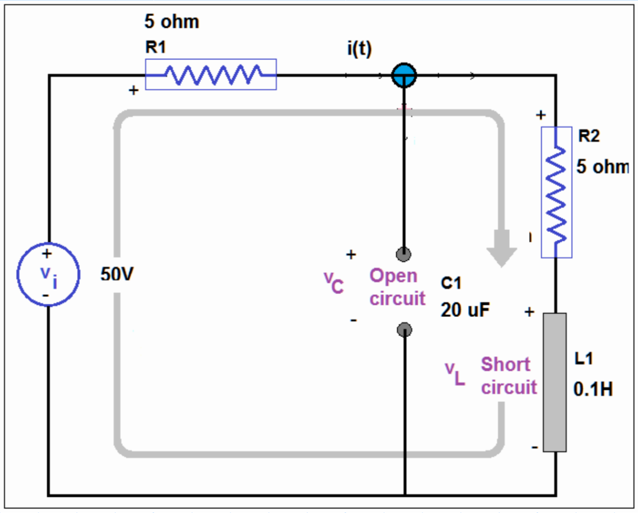
$$\frac{1}{\tau_{R2L1}} = \frac{1}{0.02} = 50$$

Circuit condition above is when there is no external source, and the circuit has 2 time constant connections, RC and RL, both time constants are working on the circuit, the resultant of which is the net difference. One impacting the other, results in net difference. *Phase angle is not in step between L and C, waveform behaviour is dependent on time constant, that I propose results in multiple in time constant.*

$$\frac{1}{\tau_{Net}} = \frac{1}{\tau_{R1C1}} - \frac{1}{\tau_{R2L1}} = 10000 - 50 = 9950 \quad \text{<--- (0< t <infinity)}$$

Next work on the current and voltage based on some conditions.

Based on circuit end conditions :
There is one loop, C1 open circuit, and L1 short circuited, with voltage 50V and 2 resistors are in series. We can calculate current i_end(t) :



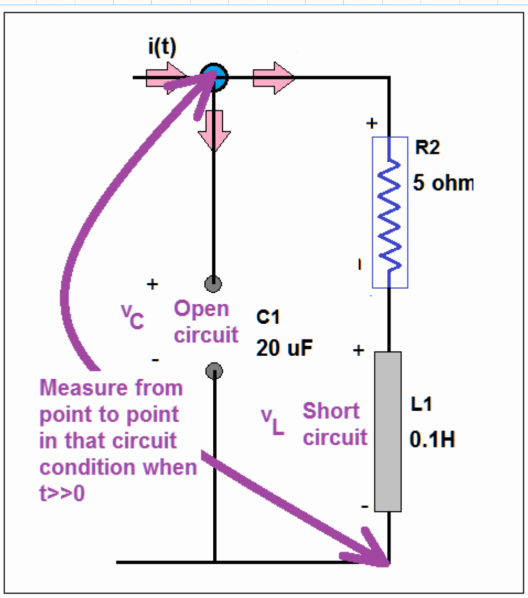
$$i(\infty) := \frac{Vi}{R1 + R2} = 5 \quad \text{A.}$$

Voltage across R2 at t = infinity :

$$v_{R2}(\infty) \ = \ 5 \cdot 5 = 25 \quad \text{V}$$

Also

$$v_{R1}(\infty) \ = \ 5 \cdot 5 = 25 \quad \text{V}$$



From the circuit shown in the left we agree the voltage across the capacitor in time t=infinity will equal that across the branch of the components R2 and L1.
vL1 =0 its shorted, current flows.

$$v_{C\_t\_end}(\infty) \quad = \quad v_{R2}(t) + v_{L1\_end}(t)$$

$$= \quad 25 + 0 \quad \text{V.}$$

$$= \quad 25 \quad \text{V.}$$

Knowing the end condition of C1 voltage, an expression can be written for voltage across C1 branch for t=0, t>0, and t=infinity:

$$v_{C1}(t) \quad = \quad V_m \cdot \left(1 - e^{\frac{-t}{\tau_{R2L1}}}\right)$$

$$v_{C1}(t) \quad = \quad 25 \cdot \left(1 - e^{-100\,t}\right)$$

Not the above expression.

$$v_{C1}(t) \quad = \quad -25\, e^{-100\,t}$$

Voltage v_C1(t) based on the waveform seen across R2 and L1 branch. Using the RL time constant based on RL branch for vC1. Note C1 is open circuit in steady state condition for t=infinity applies.

Not the two part expression just the latter term, because its a loop, 50V - vR1 - vR2 = 50 - 25 - 25 = 0.
<---vC1 or vR2L1 is the negative term.

Capacitor Equation:

$$v_{C1}(t) = \frac{1}{C1}\cdot\int i_c(t)\,dt$$

$$C1\cdot v_{C1} = \int i_{c1}(t)\,dt$$ <--- next the derivative on both sides wrt dt.

$$C1\cdot\frac{dv_C}{dt} = i_{C1}(t) = i_{C1}(t) - i_{C1}(0)$$

$$\frac{dv_{C1}}{dt} = \frac{i_C(t) - i_C(0)}{C1}$$

Take the derivative of the voltage v_c(t) :

$$v_{C1}(t) = -25\,e^{-100\,t}$$ Time constant 100 for end condition (RC).

$$\frac{d\_v_{C1}(t)}{dt} = -25\cdot 100\,e^{-100\cdot t} = -2500\cdot e^{-100\cdot t}$$

Note above ---> $\frac{dv_{C1}}{dt} = \frac{i_C(t) - i_C(0)}{C1}$ Therefore $C1\left(\frac{dv_{C1}}{dt}\right) = i_C(t) - i_C(0)$

iC(t=0+) = iC(t=0) = 0 $i_C(0) = 0$

$$C1\cdot\left(\frac{d\_v_{C1}}{dt}\right) = 20\cdot 10^{-6}\cdot\left(-2500\cdot e^{-100\cdot t}\right) = -0.05\,\left(e^{-100\cdot t}\right)$$

$$i_C(t) - i_C(0) = C1\cdot\frac{d\_v_{C1}}{dt} = -0.05\,\left(e^{-100\cdot t}\right) = -0.05\,\left(e^{-100\cdot t}\right) - 0$$

Lim t to t=0 Lim t to t=0

$$= -0.05\,\left(e^{-100\cdot t}\right)$$

$$i_{C1}(t) - 0 = i_{C1}(t) = i1(t) = -0.05\,\left(e^{-100\cdot t}\right)\ \text{A.}$$

The steady state current.

Inductor Equation (Approach A):

For time t<infinity, capacitor voltage could takes the same wave form as seen across R2 and L1 branch. I give it a try, its part of the circuit's branch.

Is this right equating it to the capacitor branch, instead of the 50V voltage source and resistor R1 branch? Continue on and update as required.

$$vR2 + L1\left(\frac{di2}{dt}\right) = 25 + L1\left(\frac{di2}{dt}\right) = v_{C1}(t) = -25\,e^{-100\,t}$$

$$25 + L1\left(\frac{di2}{dt}\right) = -25\,e^{-100\,t}$$

$$v_{L1}(t) = L1\left(\frac{di2(t)}{dt}\right) = -25 - 25\,e^{-100\cdot t} = -25\,\left(1 + e^{-100\cdot t}\right)$$

$$\left(\frac{di2}{dt}\right) = \frac{v_{L1}(t)}{L1} = -\left(\frac{1}{0.1}\right)25\,\left(1 + e^{-100\cdot t}\right) = -250\,\left(1 + e^{-100\cdot t}\right)$$

$$\left(\frac{di2(t)}{dt}\right) = -250\left(1+e^{-100\cdot t}\right)$$

$$di2(t) = -250\left(1+e^{-100\cdot t}\right)dt$$

Integrating both sides: $\int_0^t i2(t)\,dt = \int_0^t -250-250\cdot e^{-100\cdot t}\,dt$ Lim 0 --> t decaying.

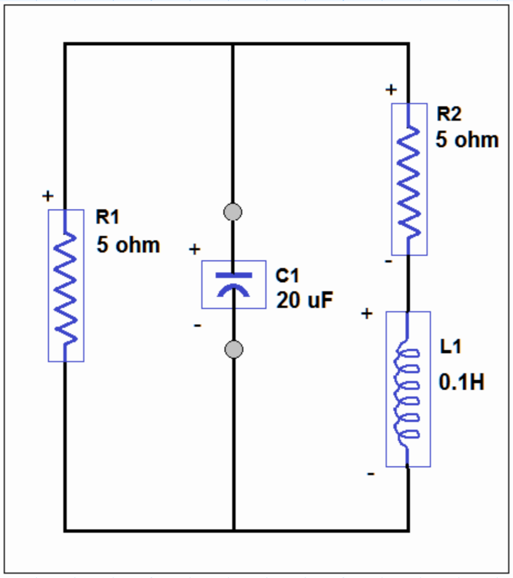
At time t=0, iL(0+) = iL(0) = 0. Therefore i_L1(0) = 0 = i2(0)

$$i2(t)-i2(0) = i2(t)-0 = i2(t) = \int_0^t -250-250\cdot e^{-100\cdot t}\,dt$$

$$= -250\,t-\frac{250}{-100}\,e^{-100\,t} = -250\cdot t+2.5\,e^{-100\,t}$$

$i2(t) = -250\cdot t+2.5\,e^{-100\,t}$ The math expression itself does not lend to a i2(t) current.
Lim t (Upper limit) -250t decreasing linearly continues to do so with no end.
Plots on next page.

This the steady state current? No. I got an expression but the capacitor is a changing waveform it has a rise and decay. Here i2(t) does not decay because of -250t. NEXT what if the voltage source branch was used to equate to the L1 and R2 branch? A more steady waveform from of a source and this circuit's part runs thru circuit end condition.

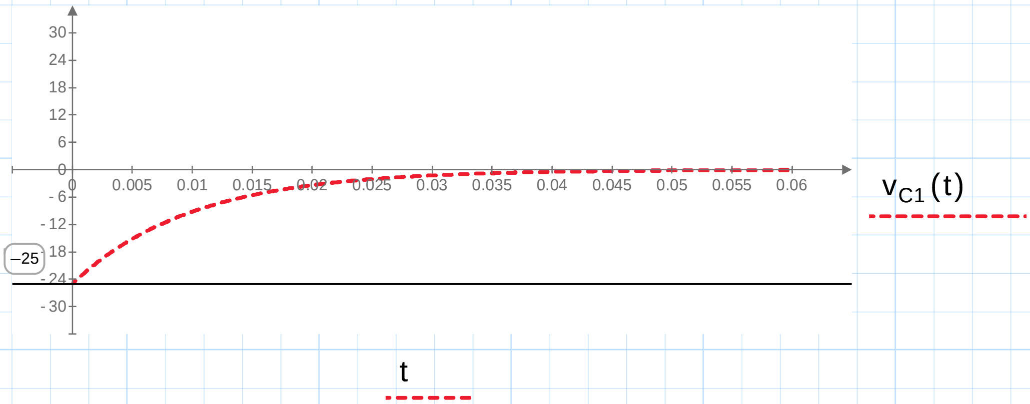


Discussion:
Voltage across the Vi and R1 branch is NOT 50V, there is a voltage drop across resistor R1. Voltage at node is 50-vR1.

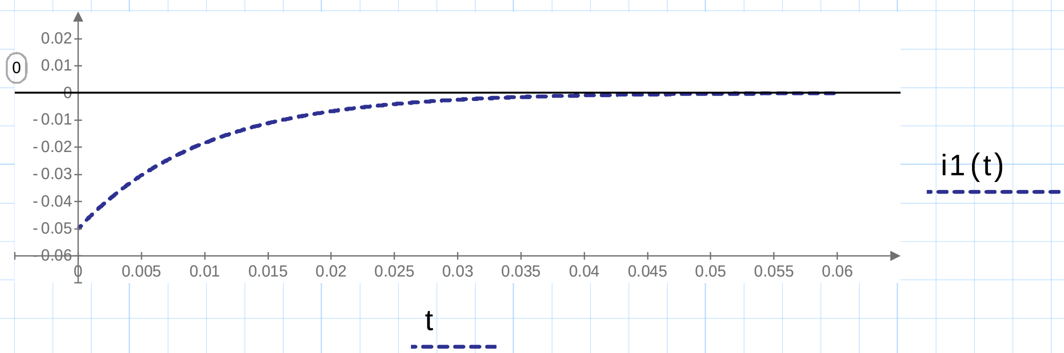
<--- If I look at Vi as all voltage and no impdeance and removed it from the circuit leaving R1 what then is the voltage across R1 in this branch? Maybe 50V? No. That makes that branch voltage a vR1, and used to compute the impedance requirements not voltage or current calculations, when Vi was removed.

clear(t) $\quad v_{C1}(t) := -25 \cdot e^{-100\,t} \quad i1(t) := -0.05 \cdot e^{-100 \cdot t}$

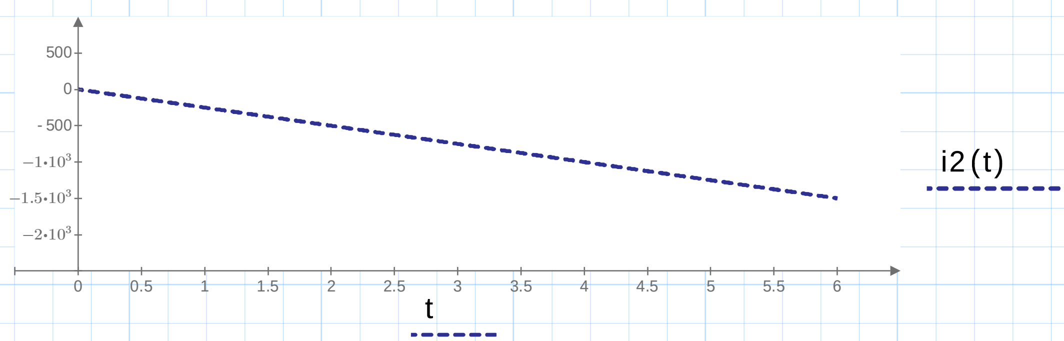
$i2(t) := -250 \cdot t + 2.5\, e^{-100\,t}$ <--- WRONG.



Capacitor voltage vC1 terminates to zero with time. However, there will be voltage across measured across the C1 branch terminal which equals V1 - vR1 OR vR2 + vL1.



Current i1(t) this is the C1 branch, starts at -0.05 (-ve sign flow direction) and settles to 0, this is agreeable for now.



i2(t) is WRONG, decreases linearly, and at a large value near multiples of 250.

Inductor Equation B:
(May NOT be suitable then see C next it does raise an odd situation in B):

$$vR2 + L1\left(\frac{di2}{dt}\right) = vR1 + 50\ V$$ For time t=infinity.

Since R1 = R2 can I cancel them off both sides?
Why Not? End condition is when L1 is shorted but the current for t<infinity is not 5A constant yet. There is the natural response. *That is where the error is here maybe. Anyway I work it to see how the math again tries to steer my solution!*

$$L1\left(\frac{di2}{dt}\right) = 50\ V$$ Here circuit time constant t<infinity made -100. DC circuit, end condition is 100. Approching near RHS expression is 50V.

$$L1\left(\frac{di2}{dt}\right) = 50\cdot\left(1-e^{-100\cdot t}\right)$$ Voltage start at 50 stays constant thru t>>0. Exponential term added for L1 initial and final conditions, forced response.

$$\left(\frac{di2}{dt}\right) = \frac{50\cdot\left(1-e^{-100\cdot t}\right)}{L1} = \frac{50\cdot\left(1-e^{-100\cdot t}\right)}{0.1} = 500\cdot\left(1-e^{-100\cdot t}\right)$$

$$\left(\frac{di2(t)}{dt}\right) = 500\cdot\left(1-e^{-100\cdot t}\right)$$

$$di2(t) = 500\cdot\left(1-e^{-100\cdot t}\right)\ dt$$

Integrating both sides: $$\int_0^t i2(t)\,dt = \int_0^t 500\cdot\left(1-e^{-100\cdot t}\right)dt$$

At time t=0, iL(0+) = iL(0) = 0. Therefore iL1(0) = 0 = i2(0)

$$i2(t) - i2(0) = i2(t) - 0 = i2(t) = \int_0^t 500\cdot\left(1-e^{-100\cdot t}\right)dt$$

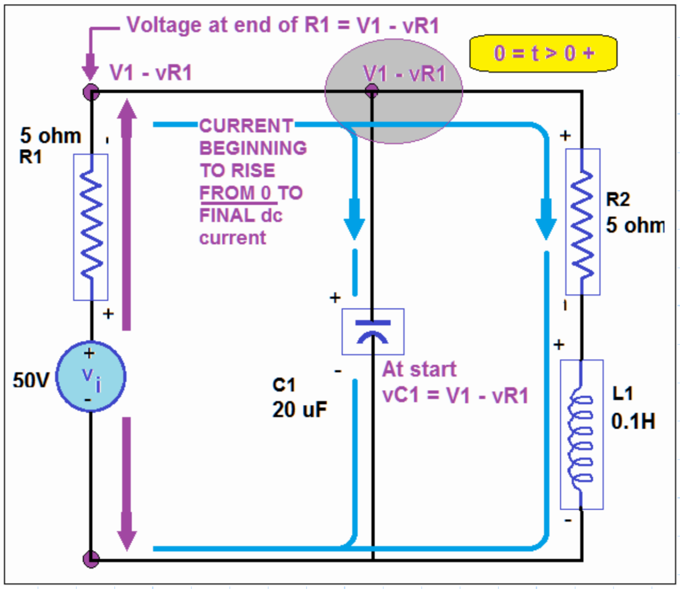
$$i2(t) = 500\ t + \left(\frac{500}{-100}\right)e^{-100\ t} = 500\ t - 5\ e^{-100\ t}$$

$$i2(t) = \left(500\ t - 5\ e^{-100\ t}\right) - 0$$ At time t=0, iL(0+) = iL(0) = 0.

500t ? How do I see that workable? WRONG.

The circuits current cannot increase substantially more than the voltage supply can provide, forced and natural combined, so the 500t term is neglected/discarded/dropped. Why at t=2 s for 500t, approximately current thru L1 = 500 x 2 = 1000 A.

Inductor Equation C (Could be suitable) :



Capacitor C1 voltage:

So that at t=0=-25V,
t>0 is greater than -25V
and t=Infinity = 0V.

$$v_{C1}(t) = -25\, e^{-100\, t}$$

Voltage across the R2 and L1 branch :

$$R2 \cdot i2(t) + L1\left(\frac{di2(t)}{dt}\right) = v_{C1}(t) = -25\, e^{-100\, t}$$

$$L1\left(\frac{di2(t)}{dt}\right)$$ <--- The inductor is short circuited, voltage drop across it is zero.

$$R2(i2(t)) = -25\, e^{-100\, t}$$

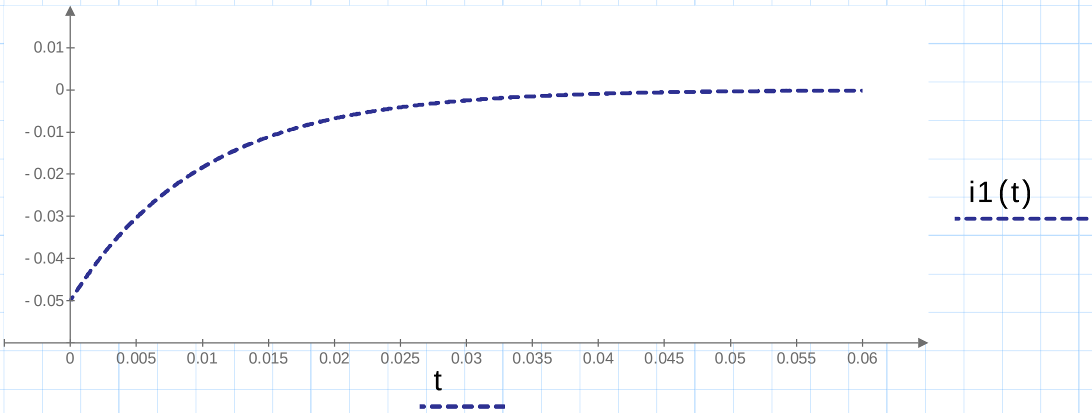
$$i2(t) = -\frac{25}{R2}\, e^{-100\, t} = -\frac{25}{5}\, e^{-100\, t} = -5\, e^{-100\, t}$$

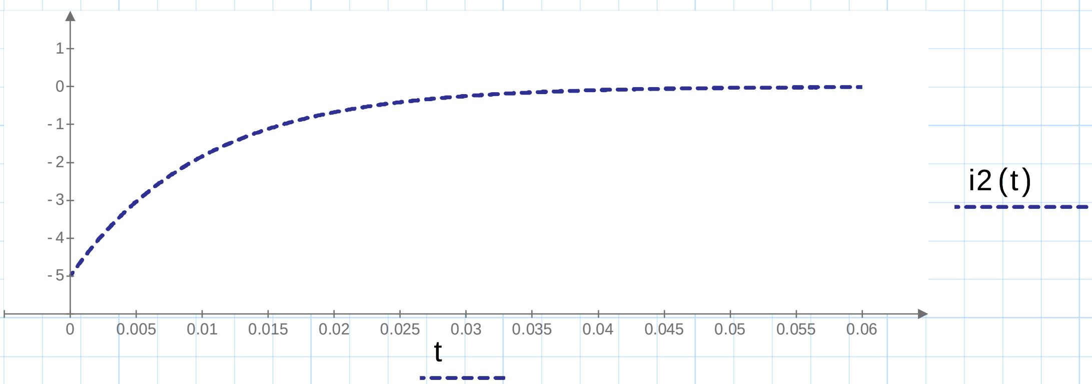
$$i2(t) = -5\, e^{-100\, t}$$

This the steady state current.
Forced response.

Next for a few plots.

clear (t) $i1(t) := -0.05 \cdot e^{-100 \cdot t}$ $i2(t) := -5\ e^{-100\ t}$ <--- Current waveforms.





Lets recap what I got. Both values do settle to zero. They, L1 and C1, are not voltage or current generating sources.

Steady state, decaying, current of C1 (i1(t)) and L1 (i2(t)).

$i1(t) \quad = \quad -0.05\left(e^{-100 \cdot t}\right)$ A.

$i2(t) \quad = \quad -5\ e^{-100\ t}$ A.   Using C1 voltage branch as the voltage across the R2 and L1 branch.

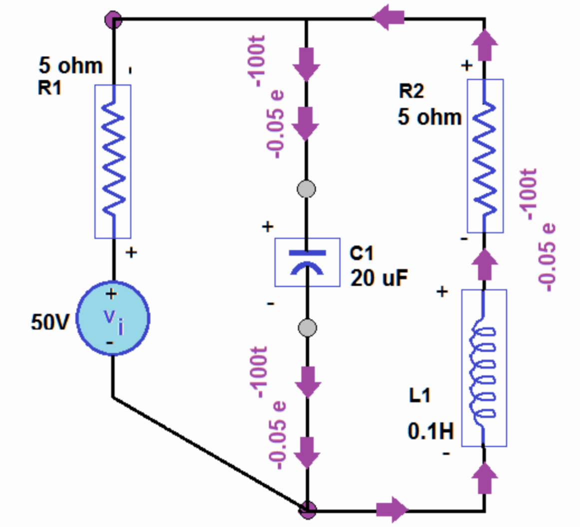
Steady state current for now:

$i1(t) \quad = \quad -0.05\left(e^{-100 \cdot t}\right)$

$i2(t) \quad = \quad -5\ e^{-100\ t}$

Continuing analysis for steady state condition or end condition:

Current from capacitor C1 in will discharge current into R2 and L1 branch, so the current for i2(t) steady state should include this.



As t approaches infinity L1 approaches 0V, current -0.05 see's equal resistance on either branch ie 5 Ohm each, R1 and R2,. So, current flows into L1 -ve terminal that soon becomes short circuited. This would be forced response condition with the 50V included.

For the time
$0 < t <$ infinity.

See my explanation in the figure to the left.

$$i1(t) = -0.05\left(e^{-100\cdot t}\right)$$

$$i2(t) = -5\,e^{-100\,t}$$

$$i2(t) = \left(-0.05\,e^{-100\cdot t}\right) + \left(-5\,e^{-100\cdot t}\right)$$

$$i2(t) = -5.05\cdot e^{-100\,t}$$

My steady state currents ADD the dc 5A end condition for i2(t):

$$i1(t) = -0.05\,e^{-100\cdot t}$$

$$i2(t) = -5.05\cdot e^{-100\,t} + 5$$

Textbook answers steady state currents:

$$i1(t) = -0.05\,e^{-100\cdot t}$$

$$i2(t) = -5.05\cdot e^{-100\,t} + 5$$

i1(t) branch is not part of the end condition since C1 becomes open circuit.

i2(t) is part of the path of the end condition path so here I added 5A.

Next the transient response also called the natural response. Here I remove the voltage source 50V. The flow of current is from the capacitor, its fully charged and discharges into the circuit.

The time constant for steady state was 100, where t was considered equal infinity, next in transient state, the time constant will be 9950 where 0<t<infinity.

Since the current here must eventually die out for the natural response, I place the condition for iL(0+) = 0 since iL(0) = 0, and inserting the exponential term. Here time constant for 0<t<infinity will be -9950t.

Natural response without the 50V in the circuit:

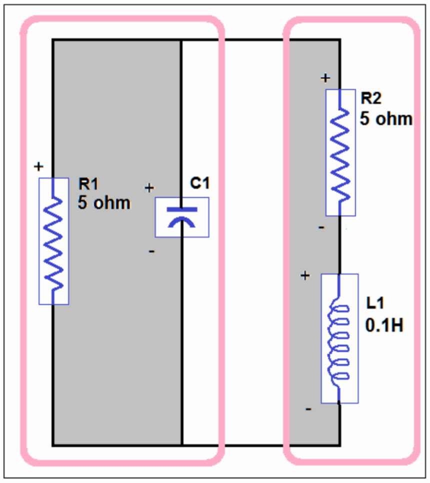
$v_{C1}(t) = -25\, e^{-100\, t}$ Steady state at 25V, transient the exponential term.

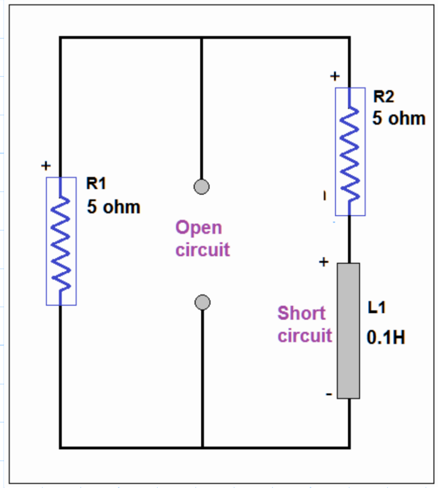
$i_{C1}(t) := -0.05\, e^{-100 \cdot t}$ Transient term settles to zero for large t.

Discussion: *In either case, our experssion will have the exponential for i1(t) because its the RC side of the circuit. Current passing thru capacitor. So remove the thought that there will be a constant term. Eventually the current would die out with t>infinity, for the transient condition, without the V=50 supply for the capacitor. NOT the inductor that would have a constant 5A passing thru when it is shorted. The inductor will have a exponential term for the 0<t<infinity, and at infinity it dies out.*

Discussion To Force My Solution To The Answer: Capacitor C1 and Inductor L1 are not operating in sync, one charges, the other energises. So they impact each other with a net difference result. In Physics two body collision, momentum is added, here is they are not colliding, one lending +/- to the other and vice-versa. I gave the phase angle difference between L and C for the cause early on. My thinking you probably got better idea.

Joke: I may be in error for creating fake new properties and characteristics for the capacitor and inductor also known as 'fake engineering', don't get left out here either - Karl Bogha.

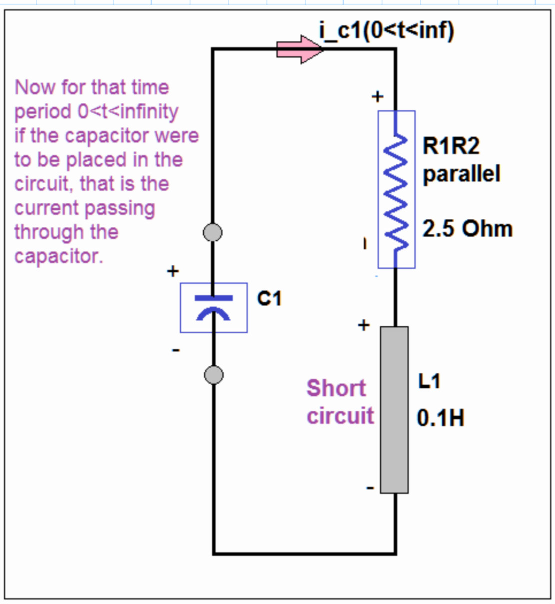




Two circuits side by side; RL and RC - figure to left. Do we work them together or individually for the 9950 time constant? Lets try individually first. Maybe the only way since I am not doing a voltage loop of current node.
The left side RC parallel can be seen as a current source when it dischages current.

$$\frac{1}{\tau_{Net}} = \frac{1}{\tau_{R1C1\_parallel}} - \frac{1}{\tau_{R2L1\_series}} = 10000 - 50 = 9950$$

The time constant applied here.



$$R_{paralel} := \frac{R1 \cdot R2}{R1 + R2} = 2.5$$

No external voltage source.
Total resistance is the parallel resistance of R1 and R2.

Voltage across the branches is the voltage across the capacitor.

$$v_{C1}(0 < t < \infty) = -25\, e^{-9950\, t}$$

Time t of concern: $v_{C1}(0 < t < \infty)$

$$v_{C1}(0 < t < \infty) = -25\, e^{-9950\, t}$$

When capacitor voltage is divided by the resistance, parallel resistance seen by C1, it gives the current discharged from C1.

$$i1(o < t < \infty) = i_{C1}(0 < t < \infty) = \frac{-25\, e^{-9950\, t}}{2.5} = -10\, e^{-9950\, t} \quad \text{A.}$$

Sign on i1(t) = -10.0e(^-9950t) is -ve, going from + to -ve of C1.

The sign needs be made positive for 0<t<inf. Why?

Current is flowing out of C1's -ve to +ve into R2 and L1.

$$i1\,(o < t < \infty) \quad = \quad i_{C1}\,(0 < t < \infty) \quad = \quad 10\ e^{-9950\ t} \quad \text{A.} \qquad \text{<--- +ve.}$$

Sign on i1(t) = -0.05e(^-100t) is -ve, decaying exponential term, from + to -ve of C1.

The sign needs to be made positive for 0<t<inf. Why?

It was in the forced response but it is dying down in the natural response too but now at the -9950 time constant.

Here there is no Vin source, so the current reverses direction in the natural response because the driving source is the capacitor C1.

$$i_{C1}\,(t) \quad = \quad C1 \cdot \frac{d\_v_{C1}}{dt} \quad = \quad -0.05\ \left(e^{-100 \cdot t}\right) \quad \text{A.}$$

$$i_{C1n}\,(t) \quad = \quad C1 \cdot \frac{d\_v_{C1}}{dt} \quad = \quad 0.05\ \left(e^{-9950 \cdot t}\right) \quad \text{A.} \qquad \text{<---Changes too.}$$

This need now need be added for the natural response.

Discussion: My solution conditon impacted twice in i1(t); once for dc and the other for transient where the driving source is C1.

This is where my make it fit solution has cause for you to verify and seek a solution from the lecturer or engineer. Dont want fake engineering.

$$i1_n\,(t) \quad = \quad 0.05 \cdot e^{-9950\ t} + 10\ e^{-9950\ t} \quad = \quad 10.05\ e^{-9950 \cdot t} \quad \text{A}$$

Natural response without voltage source.

Continued next page.

Discussion:

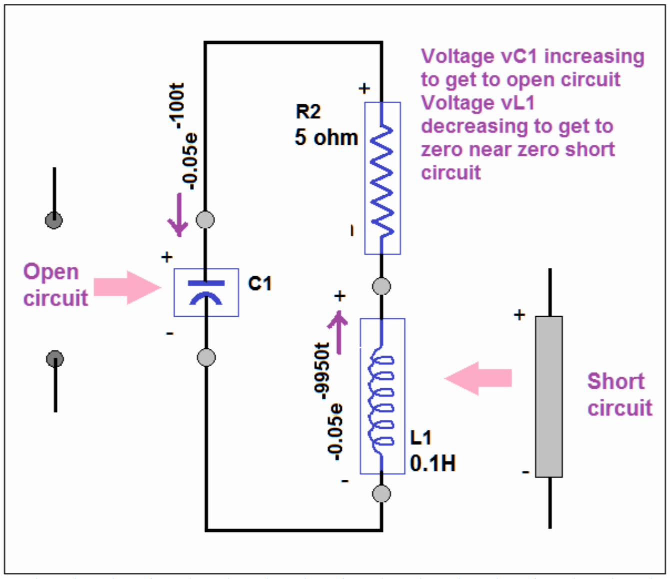
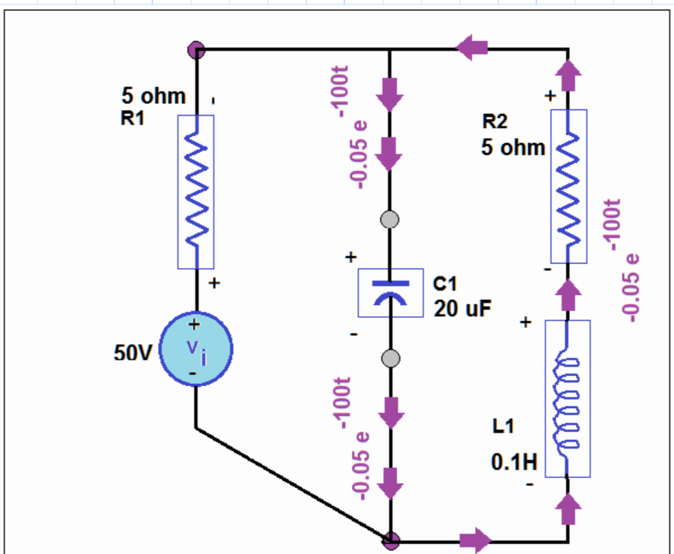


Figure to the left shows the -0.05 e^-100t entering C1.

Since time t is increasing, the voltage acros C1 and L1 too are reaching their final conditions.

The current terms is time dependent it exist in both the forced and natural response. It is also entering the inductor L1 coming out of the +ve terminal, so it should be labelled positive.

2 pages ago in 'Natural Response Without 50V' solution of current i1(t) was found a transient value of ic1(t), exponential term, see figure below. -0.05e^-100t flowing into Vi 50V -ve terminal. Since, vL1 near 0, aprox 0, this current can flow into L1 -ve terminal.



As t approaches infinity L1 approaches 0V, current -0.05 see's equal resistance on either branch ie 5 Ohm each, R1 and R2,. So, current flows into L1 -ve terminal that soon becomes short circuited. This would be forced response condition with the 50V included.

This current going thru the inductor from the -ve to +ve terminal the sign of the current in positive, and the 9950 time constant is the relevant time constant here.

$$i2_n(t) \quad = \quad 0.05\left(e^{-9950 \cdot t}\right) \text{ A.}$$

So there is that current flowing from C1, with Vi removed when vL1 is near 0 equal 0, into L1.

Next recap the transient values and plots.

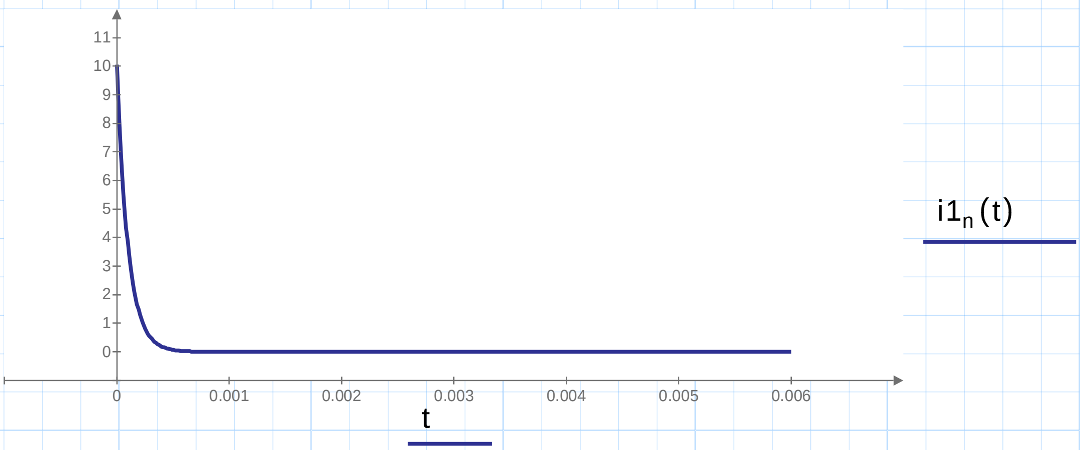
Recap natural response:

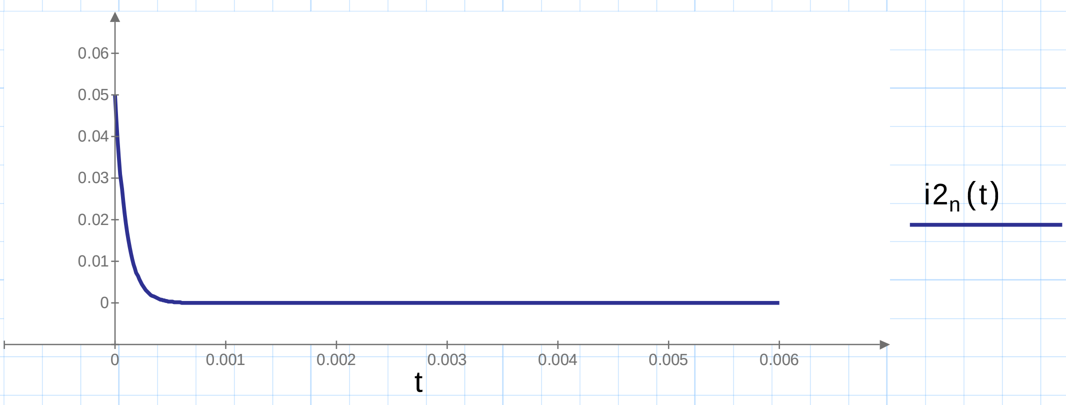
$$i_1(o<t<\infty) \;=\; i_{C1}(0<t<\infty) \;=\; \frac{-25\,e^{-9950\,t}}{2.5} \;=\; -10\,e^{-9950\,t} \;=\; 10\,e^{-9950\,t}\ \text{A.}$$

$$i1_n(t) \;=\; 0.05\cdot e^{-9950\,t} + 10\,e^{-9950\,t} \;=\; 10.05\,e^{-9950\cdot t}\ \text{A}$$ Total natural response without voltage source.

$$i2_n(t) \;=\; 0.05\left(e^{-9950\cdot t}\right)\ \text{A.}$$ Natural response without voltage source when vL1 (near infinity) equal 0V.

$\text{clear}(t)$ $\qquad i1_n(t) := 10.05\,e^{-9950\cdot t}$ A. $\qquad i2_n(t) := 0.05\left(e^{-9950\cdot t}\right)$ A.





Both i1n(t) and i2n(t) natural response dies out faster than previous forced responses. Here almost at same time. The time constant much higher 9950 so obviously the curve will be steeper and settles to zero faster. Settles to zero as required.

Also the dc end condition was 5A.
This needs to be added for the complete solution.

Forced response:

$i1(t) = -0.05\, e^{-100 \cdot t} + 0.05$ A. <--- Remove 0.05

$i1_f(t) = -0.05\, e^{-100 \cdot t}$ A.

$i2(t) = -5.05 \cdot e^{-100\, t} + 5$ <---Remove 5, and then ADD the dc t=infinity 5A. When C is open circuit and L short circuit.

$i2(t) = -5.05 \cdot e^{-100\, t}$ A.

$i2_f(t) = -5.05 \cdot e^{-100\, t} + 5$ A with dc 5A added for t end condition.

Natural/Transient response:

$i1_n(t) = 10.05\, e^{-9950 \cdot t}$ A.

$i2_n(t) = 0.05\left(e^{-9950 \cdot t}\right)$ A.

Now for my trying to make it work solution which can be wrong you verify with your lecturer and local engineer.
Lots of twists and turns for me I could not keep up with my own solution.

$i1(t) = i1_f(t) + i1_n(t)$

$i1(t) = -0.05\, e^{-100 \cdot t} + 10.05\, e^{-9950 \cdot t}$ My Answer.
Match the textbook. You verify.

$i2(t) = i2_f(t) + i2_n(t)$

$i2(t) = -5.05 \cdot e^{-100\, t} + 5 + 0.05\left(e^{-9950 \cdot t}\right)$ My Answer.
Match the textbook. You verify.

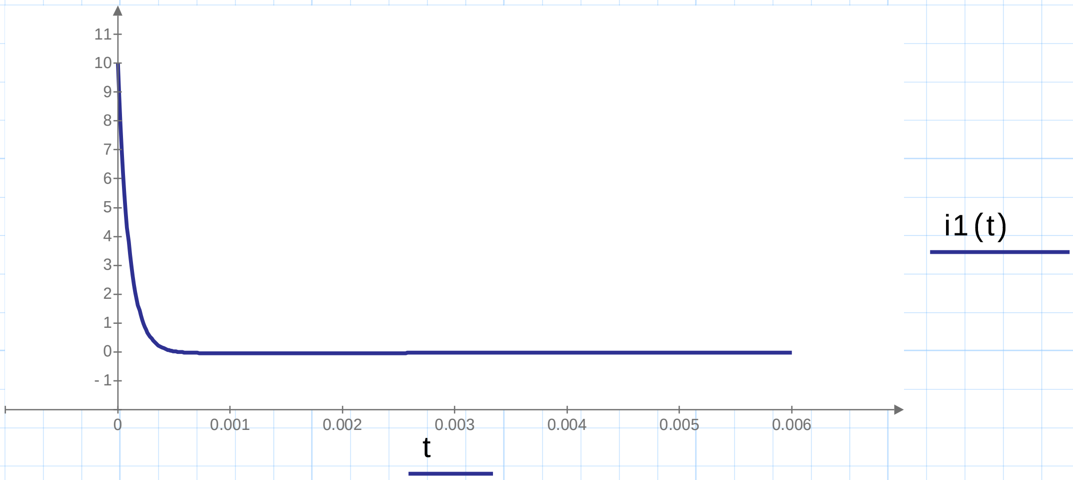
Textbook answers: $i1(t) = -0.05\, e^{-100 \cdot t} + 10.05\, e^{-9950 \cdot t}$

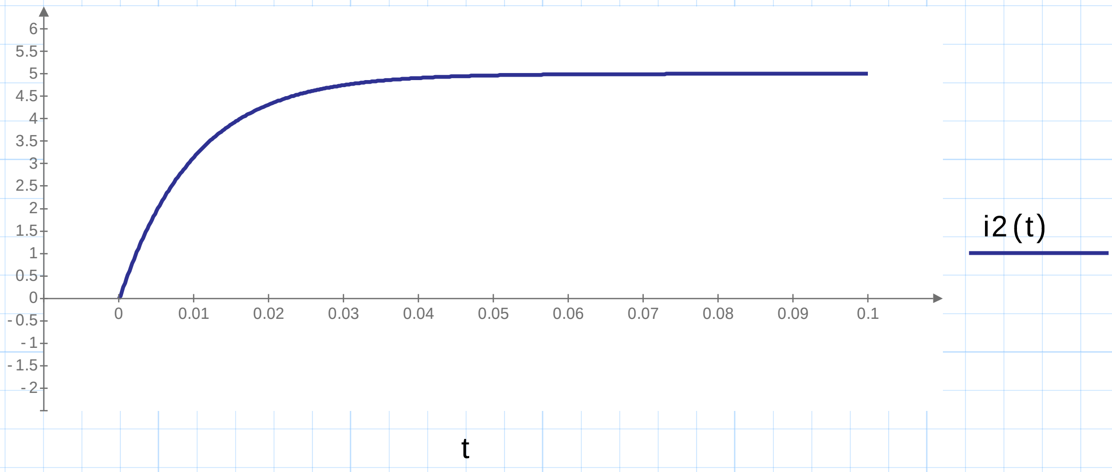
$i2(t) = -5.05 \cdot e^{-100\, t} + 5 + 0.05\left(e^{-9950 \cdot t}\right)$

Plots for the final current expressions : clear (t)

$i1(t) := -0.05 \cdot e^{-100 \cdot t} + 10.05 \cdot e^{-9950 \cdot t}$

$i2(t) := -5.05 \cdot e^{-100\, t} + 5 + 0.05 \cdot \left(e^{-9950 \cdot t}\right)$





Plot discussion:
i1(t) settles to zero since this is where the capacitor C1 is open circuit.
i2(t) continues with 5A here the inductor is short circuit.
Both currents do not start at zero. i2(t) starts from 0A.
i1(t) starts at 10.05A, then drops to zero. This is where capacitor charged up and a current of 10.05 A is seen, and then capacitor discharges current into the circuit to 0A.
The inductor branch is the dc circuit path here the current builds up from 0A by the voltage source and remains steady state at 5A at end condition.
Apologies for all errors and omissions in advance for supplementary problem 8.27.