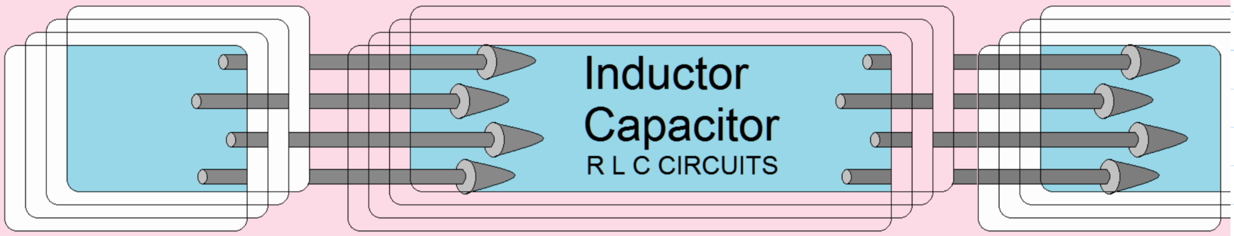
Engineering Circuits Analysis Notes And Example Problems - Schaums Outline 6th Edition.
My Homework. This is a pre-requisite study for Laplace Transforms in circuit analysis. Source of study material: Electric Circuits 6th Ed., Nahvi & Edminister. Engineering Circuit Analysis, Hyatt & Kimmerly 4th Ed. McGrawHill.
Karl S. Bogha.

Part 2 - A.
Chapter 7 Schaums Outlines: Electric Circuits 6th Edition.
Intermediate.

Circuiting PrerequisitesTo Laplace Transform Electric Circuits.



Part 2 - A
(Intermediate Level)

April 2020.

Engineering college year 2 course of 4 year program OR year 1 of 3 year program. Re-fresher OR Self Study. Graduate Study Review.
May be used in New Zealand, US, Malaysia, India, Pakistan, UK, and other Common Wealth Country engineering colleges.
Any errors and omissions apologies in advance.

First Order Circuits.

This is the chapter in Schaum's Outline (Series) where the initial conditions of the L and C come to play in the circuit.

There is lots of explanation in this chapter, chapter 7 in 6th edition of Schaums, and not as in depth as an electrical engineering circuit textbooks. Chapter 7 has at least 1 example after each section. So the purpose here is to attempt the examples in the study material. You need your textbook in hand.

Chapter 7 First Order Circuits - Schaums Outline Electric Circuits 6th Edition:

7.1 Introduction.
7:2 Capacitor discharge in a resistor.
7.3 Establishing a DC voltage across a capacitor.
7.4 The source free RL circuit.
7.5 Establishing a DC current in an inductor.
7.6 The exponential function revisited.
7.7 Complex first order RL and RC circuit.
7.8 DC steady state in inductors and capacitors.
7.9 Transitions at switching time.
7.10 Response of first order circuits to a pulse.
7.11 Impulse response of RL and RC circuits.
7.12 Summary of step and impulse responses in RL and Rc circuits.
7.13 Response of RL and RC circuits to sudden exponential excitations
7.14 Response of RL and RC circuits to sudden sinusoidal excitations
7.15 Summary of forced response in first order circuits
7.16 First order active circuits.

Sections 7.1 - 7.9 in Part A of First Order Circuits file.
Remaining sections 7.10 - 7.16 in Part B.

Too much subject material. Just the examples after each section should do it here. Primarily to get more solving techniques. From my observation on the exercise problems with no solutions, most are similar to the solved ones with need of some changes or tricks in the solution. Schaums variety of solved problems simplify solving exercise problems in other electrical textbooks similarly.

This book is the 6th edition. Some appreciation should be given for as many years it has been in publication, since the mid-60's.

Short Story: Some times I get a lower beneficial reaction from Schaum's example solved problems engineering textbooks, because they seem to leap from one area to another, wide gap. My little experience. This time the experience was different. This edition looked quite comprehensive when I reviewed it for purchase. It is, and covers major areas.

7:2 Capacitor discharge in a resistor.

Example 7.1:

The voltage across a 1 uF capacitor is 10V for t<0.
At t = 0, a 1 M Ohm resistor is connected across the capacitor terminals.

Find the time constant tau, the voltage v(t), and its value at t = 5s.

Solution:

$C := 1 \cdot 10^{-6}$ F $\quad R := 1 \cdot 10^{6}$ Ohm $\quad V_0 := 10$ V

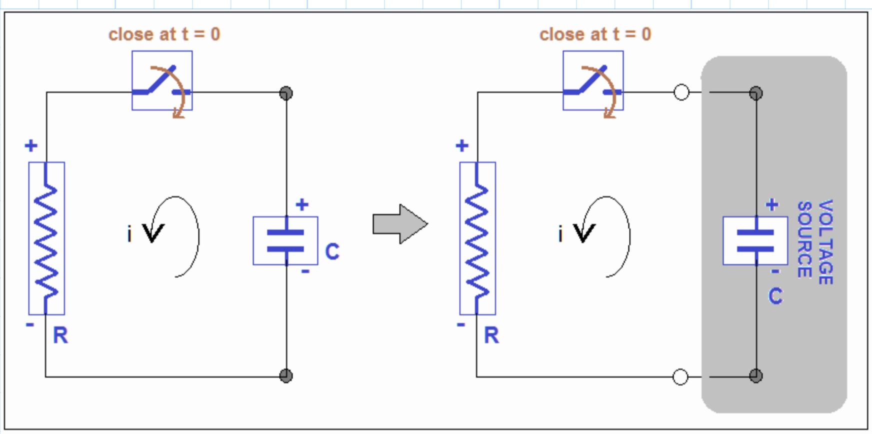
Time constant tau = RC:

$\tau := R \cdot C = 1$ second Answer.

Voltage v(t):

Voltage across the capacitor at t<0 is 10 V.
The capacitor is charged up and that is the voltage Vo = 10 V.
At t = 0 what is the amplitude of the voltage when the resistor is inserted in the circuit?
10V. From -t to t0 its the same amplitude, then, the capacitor starts to discharge into the circuit which has no source. So at t = 0 it is Vo.
Eventually it gets to zero.

This is a parallel circuit from reading of the problem; resistor connected across the capacitor? Its also a series with no source but capacitor C becomes a source. Its a simple series circuit. Parallel would have a branch(s) parallel to the resistor branch.



Sign convention on the circuit, current entering positive terminal of resistor, so here vR = iR.

Current leaves resistor and enters capacitor at -ve terminal. So the current is identified in the negative sign, iC = - C(dv/dt).

Top side of both elements is positive, so their potential is high. Remember when labeling the +ve side relate to high potential. Ground/Earth reference? 0 potential. I dont solve circuits everyday and sign convention is critical.

Voltage across the capacitor is the voltage across the resistor at time t = 0. Using the current sign convention in the figure.

vR = Ri
i = C(dv/dt)

Node equation:
Current entering R = Current leaving capacitor

v/R = -C(dv/dt) ......CORRECT.

(v/R) + C(dv/dt) = 0.

$$\frac{v}{R} + C \cdot \left(\frac{dv}{dt}\right) = 0.$$

Next divide by C to make the coefficient of dv/dt one.
Next rearrange with differential term first.

$$\left(\frac{dv}{dt}\right) + \frac{v}{RC} = 0.$$

Differential equation of the? 1st order.

What should the form of solution take?
We have a current that will eventually settle to 0. So the solution must have that included, and be able to capture the current at time t.

Schaums puts it like this: "The only function whose linear combination with its derivative can be zero is an exponential function of the form Ae^st."

So thats good the linear combination being (v/RC) + (dv/dt). Let v = Ae^st.
The derivative dv/dt of Ae^st = sAe^st. Substitute.

$$s \cdot A \cdot e^{st} + \left(\frac{1}{RC}\right) \cdot Ae^{st} = 0. \qquad A\left(s + \frac{1}{RC}\right) \cdot e^{st} = 0.$$

How do we solve this?

We can set t = 0 thats accepted.
What does A become?
A is the amplitude in the expression Ae^st. So it can be 1.
A = 1, when t = 0.

$$1\left(s+\frac{1}{RC}\right)\cdot e^{s0} = 0.$$

$$\left(s+\frac{1}{RC}\right) = 0.$$

Therefore $s = -\left(\frac{1}{RC}\right)$

Now what do we do with s? Place it back in the expression.
But note at t = 0 the voltage across the capacitor is amplitude A.
We are told that voltage is 10V.
So? A = Vo = 10V.

$$\left(s+\frac{1}{RC}\right) = 0.$$

$$A\left(s+\frac{1}{RC}\right)\cdot e^{st} = 0. \quad \text{---->>} \quad 10\left(\left(-\frac{1}{RC}\right)+\frac{1}{RC}\right)\cdot e^{\left(-\frac{1}{RC}\right)t} = 0.$$

$$10\,(0)\cdot e^{\left(-\frac{1}{RC}\right)t} = 0.$$

LHS = RHS. So we can plug in for t.

Now for v(t) and i(t) for t > 0:

$$v(t) := V_0\cdot e^{\frac{-t}{R\cdot C}}$$ <-----What we were looking for.

$$i(t) := -C\left(\frac{dv}{dt}\right) \qquad \frac{dv}{dt} = \left(\frac{V_0}{RC}\right)\cdot e^{\frac{-t}{RC}}$$

$$i(t) = C\cdot\left(\left(\frac{V_0}{RC}\right)\cdot e^{\frac{-t}{RC}}\right) = \left(\frac{1}{R}\right)\cdot V_0\cdot e^{\frac{-t}{RC}}$$

$$i(t) = \left(\frac{V_0}{R}\right)\cdot e^{\frac{-t}{RC}}$$ <-----What we were looking for.

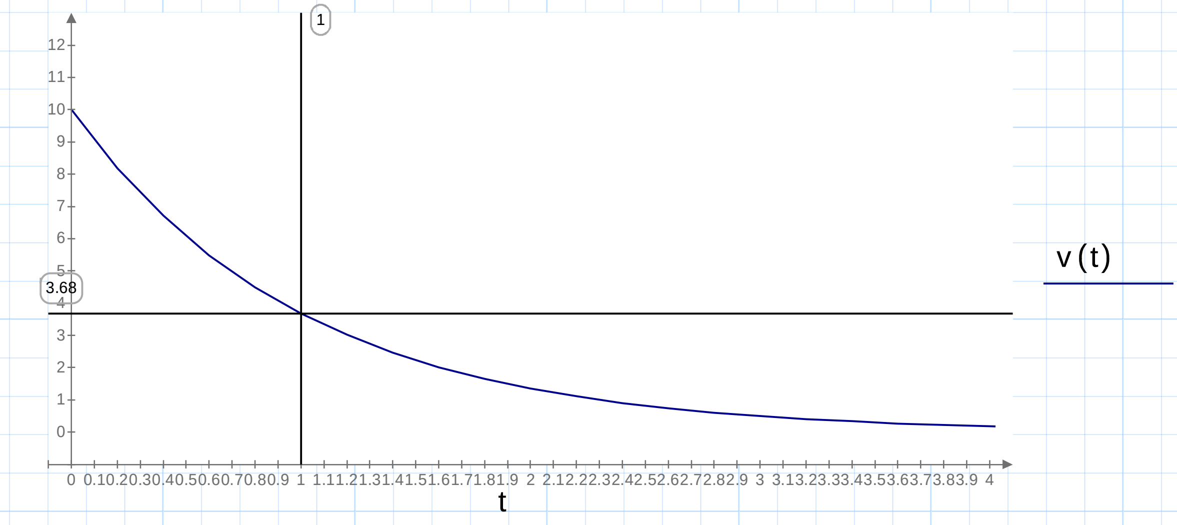
Since v(t) and i(t) are exponential terms the voltage and current will decrease to zero with increasing time (t); -ve t. That is why the solution takes that form.
This is an easy one. I wish and hope we could use this solution more often.

At time t = 5 s, v(t)? **clear** (t)

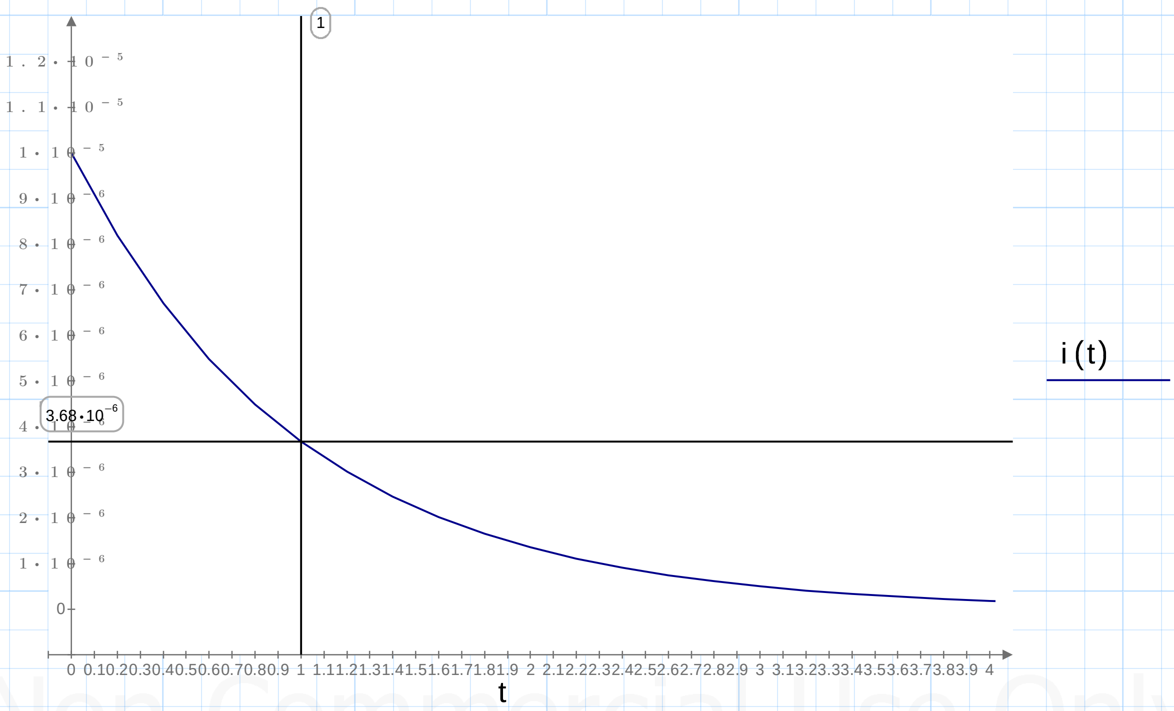
$t := 0, 0.2 .. 5$

$v(t) := V_0 \cdot e^{-\left(\frac{t}{R \cdot C}\right)}$

Managed to get a little subject matter and graphs. X-axis is time t and RC is 1 second. Both plots show the value of v(t) and i(t) at t = 1 sec which is tau = RC = 1 sec.



$i(t) := \left(\frac{1}{R}\right) \cdot V_0 \cdot e^{\frac{-t}{R \cdot C}}$ $\frac{V_0}{R} = 1 \cdot 10^{-5}$ $0.368 \cdot \left(\frac{V_0}{R}\right) = 3.68 \cdot 10^{-6}$ <--See values on y and x axis below.



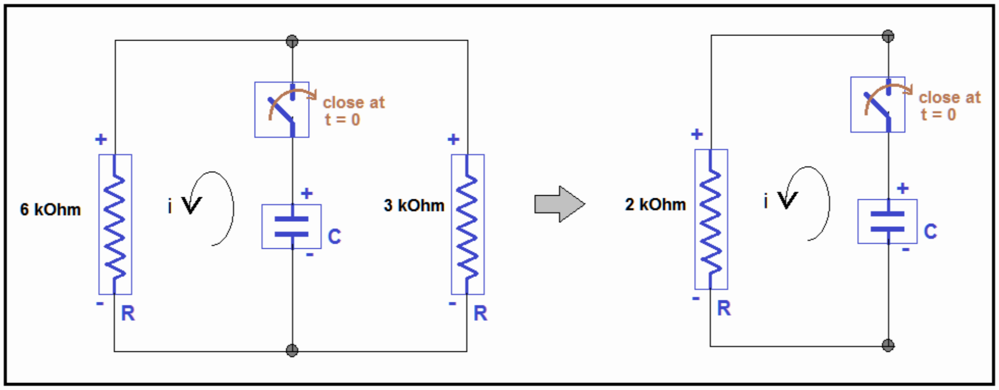
7:2 Capacitor discharge in a resistor.

Example 7.2:

5 uF capacitor with an initial voltage of 4V.
Capacitor connected across 3 kOhm and 6 kOhm resistors.
Switch closed at t = 0 s.
Find the current in the 6 kOhm resistor?

Solution:

$C := 5 \cdot 10^{-6}$ F $\quad V_0 := 4$ V. $\quad R1 := 3 \cdot 10^3 \quad R2 := 6 \cdot 10^3$ Ohm



Circuit equivalent resistance:

$$R\_eq := \frac{(R1 \cdot R2)}{R1 + R2} = 2000 \quad \text{Ohms}$$

Time constant tau = RC:

$$\tau := R\_eq \cdot C = 0.01 \quad \text{second}$$

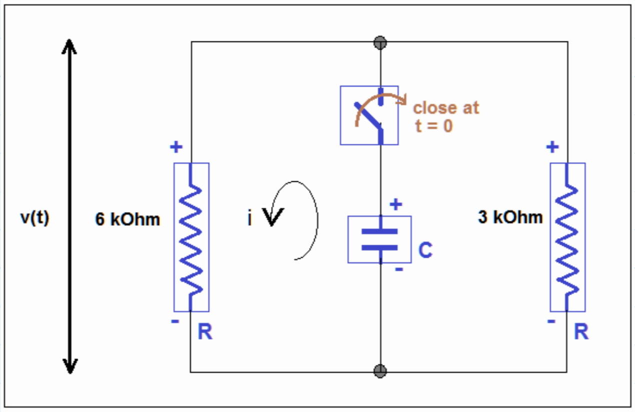
Typical solution for this circuit: $A \cdot e^{st}$ How were we to know this? Previous example, RC circuit.

What is the value of A? 4V.

What is the value of s: $-\left(\frac{1}{R\_eq \cdot C}\right) = -100$

$v(t) := 4 \cdot e^{-100 \cdot t}$ Answer.

We have the voltage v(t), returning to the original circuit for the current in the 6k Ohm resistor.



At t = 0

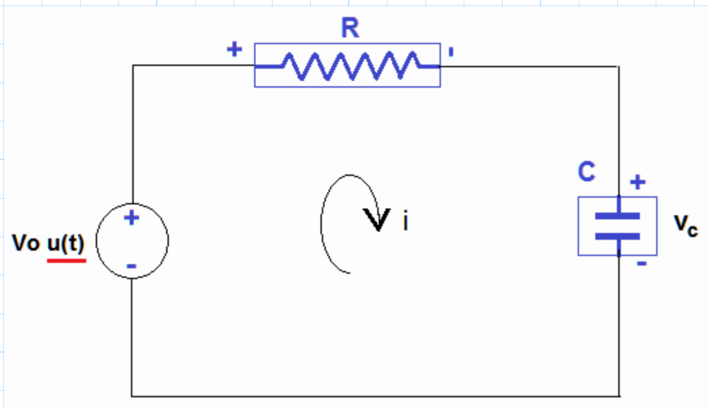
$$v(t\_0) := 4 \cdot e^{-100 \cdot 0}$$

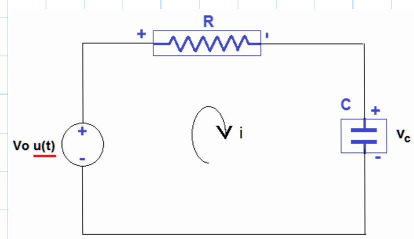
$$v(t\_0) := 4 \qquad v := 4 \qquad V$$

$$i := \frac{v}{6000} \qquad i := \frac{4}{6000} = 0.000667$$ A. The amplitude of the current function.

$$i(t) := 0.667 \cdot e^{-100 \cdot t}$$ mA Answer.

7:3 Establishing a DC Voltage across a Capacitor.





Loop equation: $R \cdot i + v_c = V_0$

Current i: $i = C \cdot \left(\frac{dv}{dt}\right)$ Substitute in for i

$$R \cdot C \cdot \left(\frac{dv}{dt}\right) + v_c = V_0$$ Make dv/dt coefficient = 1

$$\left(\frac{dv}{dt}\right) + \frac{v_c}{RC} = \frac{V_0}{RC}$$ for t>0 ......Eq 1 Correct since capacitor is uncharged at t<0.

v(0+) = v(0-) = 0 ......Eq 2

Voltage across capacitor 0 at t<0, uncharged.
Voltage at t=0 equal 0 for the first few us or ms
v=0, there is an internal resistance, inertia, to overcome by the capacitor before voltage builds up, so at t>0 and just past 0, v = 0. Good enough!

The other way of writing it
v(t>0) = v(t<0) = 0 ......Eq 2

Our solution should satisfy Eq 1 and Eq 2.

For Eq 1 the specific or particular solution for Vo can be solved by the usual function v(t) or here v_p(t) for the particular time t, example t = 5 seconds, 5 us, 5 ms, etc.

Eq 2 for the initial conditions, we know the charge takes a sudden increase that is represented by the exponential term. So here the solution's form we seen prior Ae^-st should work, v_adj(t) = Ae^-st, where A the amplitude can represents Vo thru adjustments of the exponential term e^-st. <--- My wording.

Engineering textbooks usually name them the
'particular v_p + homogeneous v_h' solution <---From Differential Eq World!

Combining both v(t) = v_p(t) + vh(t) = Vo + Ae^-st.

Some simplifications from the initial conditions shown below again.

v(0+) = v(0-) = 0 ......Eq 2

v(0+) = Vo + A = 0 .....this is in the time t>0...waveform dies out

Rearranging
Vo = - A OR
A = -Vo

So we are getting closer to the complete solution.
Remember the input is a unit step DC VOLTAGE. Shown in the figure above
u(t) = 0 when t < 0
u(t) = 1 when t = 0
u(t) = 1 when t > 0

$$v(t) = \left(V_0 - A \cdot e^{\frac{-t}{RC}}\right) \cdot u(t)$$ plug in the tau; -t/tau = -t/RC

$$v(t) = V_0 \cdot \left(1 - e^{\frac{-t}{RC}}\right) \cdot u(t)$$ Here A = Vo. <---Expression we were seeking.

Now how for the current i(t):

$$i(t) = \left(\frac{V_0}{R}\right) - \left(\frac{V_0}{R}\right) \cdot e^{\frac{-t}{RC}} \cdot u(t) = \left(\frac{V_0}{R}\right) \cdot e^{\frac{-t}{RC}} \cdot u(t)$$

1st term is in the 2nd term when t=0, Yes. i(t) decreases anyway, so no need -ve sign.

i = Vo/R that is the current through the resistor, the voltage across the resistor is the voltage produced by the battery/source which charged the capacitor from 0 to Vo. Hence, the voltage in the circuit is Vo e^-st. The exponential term showing the gradual rise in voltage in the circuit when the capacitor was charged - see coming example plot. Maximum voltage cannot be greater than Vo which is the maximum battery voltage.

7:3 Establishing a DC Voltage across a Capacitor.

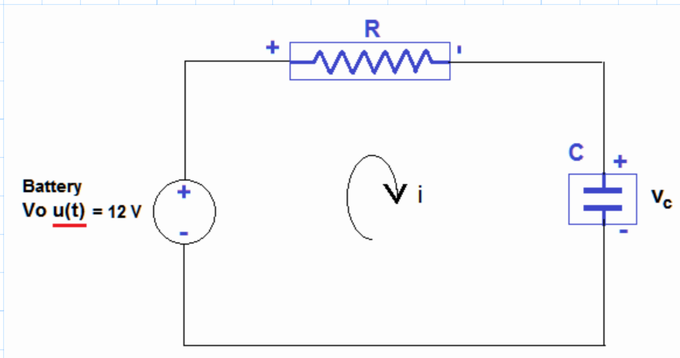
Example 7.3:

4 uF capacitor with an initial voltage of v(0-) = 2V.
Connected to a battery through a resistor R = 5 kOhm at time t = 0.
Find the voltage across the capacitor for t>0?
Find the current through the capacitor for t>0?

Solution:

$C := 4 \cdot 10^{-6}$ F $\quad v_c(0^-) = 2$ V, we saying v(t<0) = 2

$V_0 := 12$ V battery DC voltage. $\quad R := 5 \cdot 10^3$ Ohm



$Tau := R \cdot C = 0.02$ s.

$s := -\left(\frac{1}{R \cdot C}\right) = -50$

$v(t) = V_0 \cdot \left(1 - e^{\frac{-t}{RC}}\right) \cdot u(t)$ <---Expression we found previously.

Initial conditions:

$v_c(0^-) = 12 + A = 2$ 12 V from u(t) battery, and A volts from the capacitor.

$A = 2 - 12 = -10$ t>0
This has to be for t>0 since for t<0 there is only the capacitor voltage 2V.

$v(t) = V_0 \cdot \left(1 - e^{\frac{-t}{RC}}\right) \cdot u(t) = V_0 \cdot u(t) - V_0 \cdot e^{\frac{-t}{RC}} \cdot u(t)$

The 2nd term's Vo = A.

$$v(t) = V_0 \cdot u(t) - A \cdot e^{\frac{-t}{RC}} \cdot u(t)$$

We can remove u(t) since here it would not impact the results, voltage given is unit*Amplitude or sort of DC for the Vo and A.

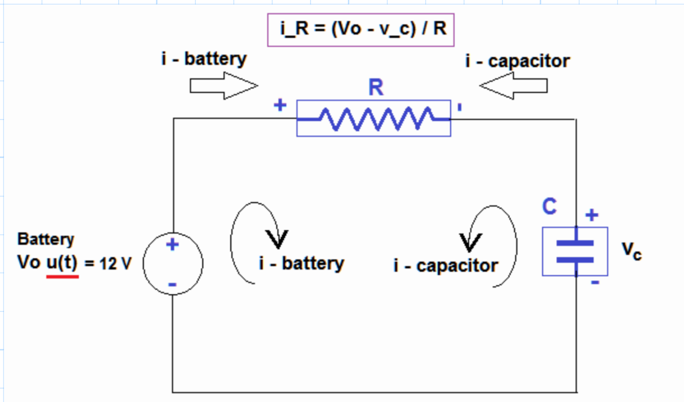
$$v(t) = V_0 - A \cdot e^{\frac{-t}{RC}}$$

substitute s for (-1/RC)

$$v(t) = 12 - 10 \cdot e^{-50\,t}$$

V. Voltage across the resistor. Answer.

Continuing for the current i(t):



Discussion: Attempt here to show the current contribution in the resistor. May not be correct?
But its just like a loop two currents in opposite direction thru a component *but we have ONE loop instead of TWO*. So that is why I said may not be correct. You may change the polarity on the capacitor it may add the currents, yet similar approach. Correct.

$$i(t) = \frac{V_0 - v_c}{5000}$$

Initial voltage in capacitor is 2V.
u(t) source comes in adds 12 V to the circuit at t>0.
So at time t>0 the voltage difference between the u(t) source and capacitor is equal to 12V-2V = 10V.

$$i(t) = \left(\frac{V_0}{5000}\right) - \left(\frac{A \cdot e^{-50\,t}}{5000}\right)$$

$$i(t) = \left(V_0 - v\left(0^{'+'}\right)\right) \cdot \left(\frac{e^{-50\,t}}{5000}\right)$$

$$i(t) = (12-2) \cdot \left(\frac{e^{-50\,t}}{5000}\right)$$

$$i(t) = \left(\frac{10}{5000}\right) \cdot \left(e^{-50\,t}\right) = 2 \cdot 10^{-3} \cdot \left(e^{-50\,t}\right)$$

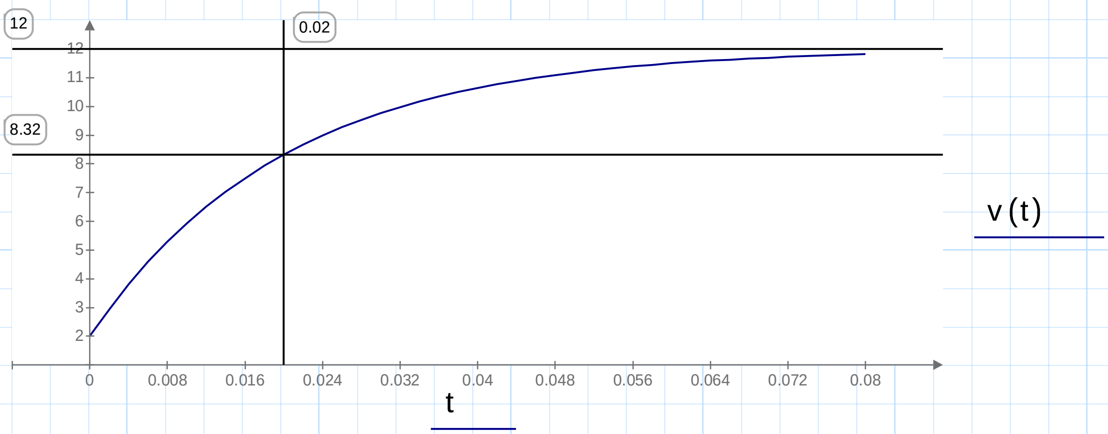
A. Answer.

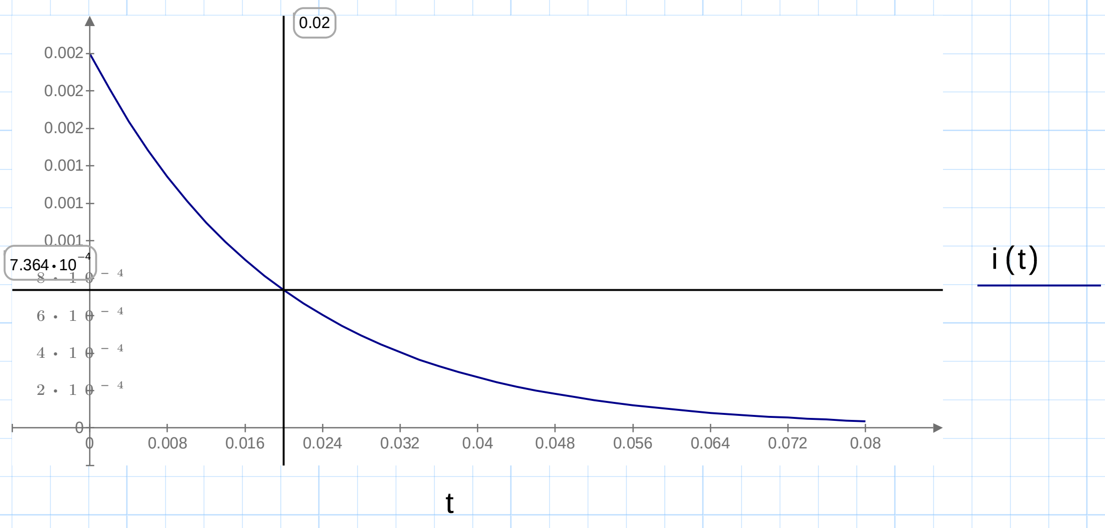
Lets plot the graphs for v(t) and i(t) for a period of at least tau (RC).

clear (t, v, i)

$t := 0.0, 0.002 .. 0.08$

$v(t) := 12 - 10 \cdot e^{-50 \cdot t}$ $\quad i(t) := 2 \cdot 10^{-3} \cdot \left(e^{-50\ t}\right)$ $\quad R \cdot C = 0.02$



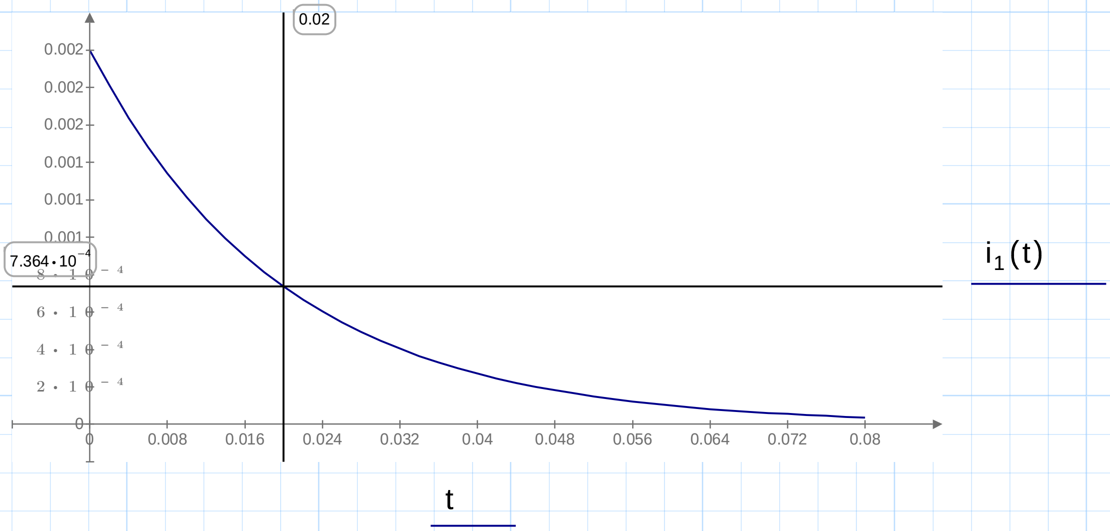


Comments:
The voltage rises at tau (0.02 s) and the current drops.
Voltage starts at 2V and rises to 12 V, with a time constant of 0.02s.
Current drops from 2 mA to zero.
Current i maybe calculated from C(dv/dt).

$v(t) := 12 - 10 \cdot e^{-50 \cdot t}$ $\quad \frac{dv}{dt} = -50 \cdot -10 \cdot e^{-50 \cdot t} = 500 \cdot e^{-50 \cdot t}$

$i(t) = \left(4 \cdot 10^{-6}\right) \cdot 500 \cdot e^{-50 \cdot t} = 0.002 \cdot e^{-50 \cdot t}$ Lets plot this graph.

$i_1(t) := 0.002 \cdot e^{-50 \cdot t}$



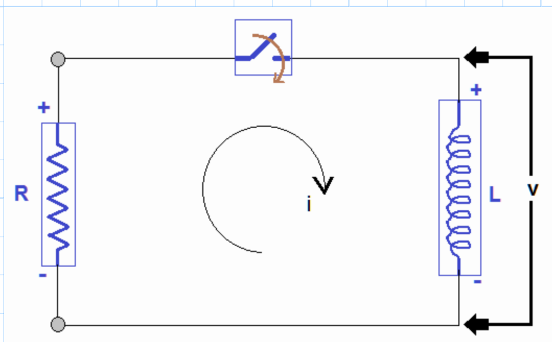
CORRECT. Same graph.

7.4 The source free RL circuit.

Why is there NO source free RC circuit?

Ask and you shall receive! <--- Maybe?

The capacitor in the RC circuit does play the role of a source when it discharges 'charge' into the circuit. Its a source. So that is why its not called a source free RC circuit. You discuss it.



At time t = 0 the switch is closed. This circuit has NO source. The inductor magenetism drops, this is same as saying electric field drops (correct?), this drop in energy has to be transferred or transmitted somewhere. It goes into the circuit's components. The engineer said there is an initial current **Io** in the circuit at time t =0.

Now for time t>0 there must be some equation that describes the behaviour of the current. We know eventually in a matter of miliseconds or more the current will decrease to zero. We have a current in the circuit we have a voltage across the inductor and resistor.

Lets do a loop equation for time greater than t = 0, t>0:

$$v = R \cdot i + L \cdot \left(\frac{di}{dt}\right)$$

The voltage v would drop to zero when the current drops to zero. So how far down the time scale are we looking at? *Tricky.*

From a purely mathematical perspective when dealing with equations and assumptions, we are trying to reach to a situation whereby some variable can be computed. Here the variable voltage or current. So if its down further along in time thats the condition where v = 0. Correct? Think it over or else what is the voltage at t>>>>0? The whole idea was to set up an equation given some current will be there at time t =0 then 0 at t>0.

$$R \cdot i + L \cdot \left(\frac{di}{dt}\right) = 0$$ Progress!

First order equation as before, and this time again we use Ae^st.

$$i(t) = A \cdot e^{s \cdot t}$$

$$\frac{di}{dt} = A \cdot s \cdot e^{st}$$

$$R \cdot \left(A \cdot e^{s \cdot t}\right) + L \cdot \left(A \cdot s \cdot e^{st}\right) = 0$$

$$R \cdot \left(A \cdot e^{s \cdot t}\right) + L \cdot \left(A \cdot s \cdot e^{st}\right) = 0$$

$$A \cdot (R + L \cdot s) \cdot e^{st} = 0$$

Which could be zero?
(Ae^st) OR (R + Ls)?
What decides? t
When t = 0 Ae^st = A. This being the solution process, meaning t is an identified variable.
So we have only option left which is (R + Ls) = 0.

$$R + L \cdot s = 0$$

$$s = -\left(\frac{R}{L}\right)$$ Time constant of RL circuit? $$\frac{1}{\left(\frac{R}{L}\right)} = \frac{L}{R}$$

Next initial condition of inductor L for completing the solution.

$$i(0) = I_0$$

$$i(0) = A \cdot e^{s \cdot 0} = A$$

Now we solved for A: $i(0) = I_0 = A.$ Next substitute for s = -(R/L).

$$i(t) = I_0 \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$$ for t>0.

Notice from the previous examples we saw how critical the time constant tau, RC, for the RC circuit was used/located/positioned in the graphs.

It determines the current and voltage curve's values at multiples of RC (seconds).... t = RC value on the x-axis (time t)? Yes among other things.

It was good enough to be positioned where the value of i(RC) and v(RC) was not maximum, and not near zero, but somwhere not far from half way on the y-axis (i, or v). Time constant is not a period.

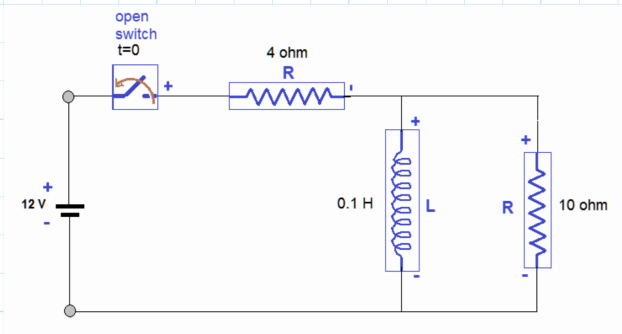
Maybe we see a similar analysis with time constant (R/L) in the RL circuit.

7.4 The source free RL circuit.

Example 7.4:

The 12 V battery is disconnected at time t = 0.
Find the inductor current?
Find the inductor voltage for all times?

Solution:



My thoughts were to start like the previous example(s).

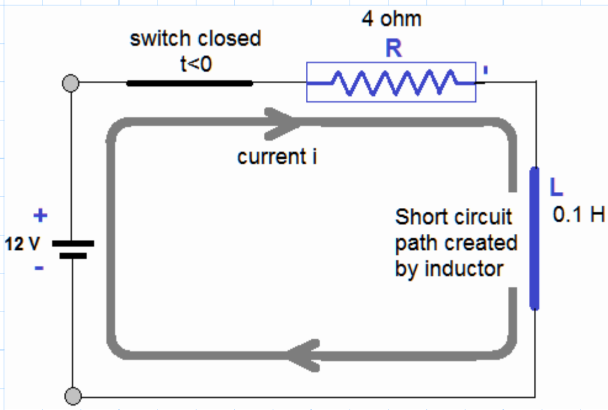
Such was not the case and it has to do with the initial conditions.

Conditions are easily over looked in solving these types of circuit problems.

We are told that to find the inductor current and voltage, these were not going to be generated if the switch was open at time t<0. We need some time to energise the inductor, obviously the switch had to be closed for a time period, in this case a long time period to fully energise the inductor.

We know that when the inductor is fully energised, the inductor will maintain a fixed current. So in the condition the switch is closed in t<0, ie t(-0) the inductor becomes a short circuit because there is no (di/dt), i_L(t) is constant and its derivative is zero. Inductor was fully charged.

Inductor is a short circuit to DC.
The resistor 4 ohm receives all the current because L is short circuited and no current flows thru the resistor 10 ohm.



Time t<0
inductor fully charged by dc current its turned into a short circuit. The other resistor 10 ohm is bypassed not supplied with current.

Now the circuit has a resistor and a dc voltage.

Voltage at t<0 is 12 V.
Current?

$R1 := 4$ $R2 := 10$ Ohm
$L1 := 0.1$ H
$V_{dc} := 12$ V

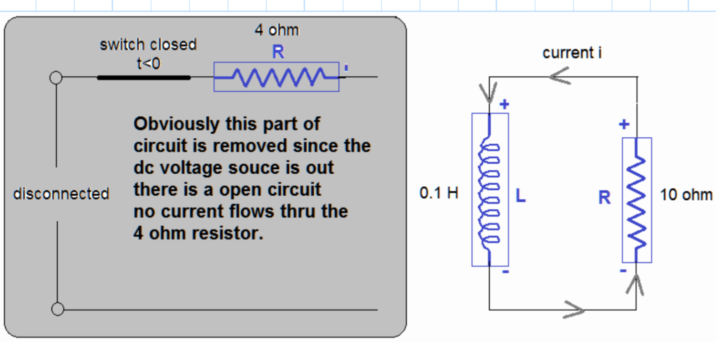
$I_{0\_dc} := \frac{V_{dc}}{R1} = 3$ A. Same current flowing thru inductor during short circuit condition. Answer.

Now for the condition t = 0 the switch remains closed.
The voltage source 12 V dc is removed.

The inductor has voltage across it untill the inductor magnetic circuit losses its field then the voltage drops to zero.

The current at t = 0 is 3 amps passing thru the inductor and it decreases with time to zero.

So it can be said there is a negative (di/dt) because current is decreasing.
Solution for this condition as provided on previous page Ae^st.
Current thru and voltage across inductor?



Solve for the right side circuit

$i(t) := I_{0\_dc} \cdot e^{s \cdot t}$ $\quad s := -\left(\frac{R2}{L1}\right) = -100$ $\quad i(t) := 3 \cdot e^{-100 \cdot t}$ Answer.

$\frac{di}{dt} = -300 \cdot e^{-100 \cdot t}$ $\quad$ Now calculate v(t) = L (di/dt)

$v(t) := L1 \cdot -300 \cdot e^{-100 \cdot t}$

$v(t) := -30 \cdot e^{-100 \cdot t}$ V. Answer.

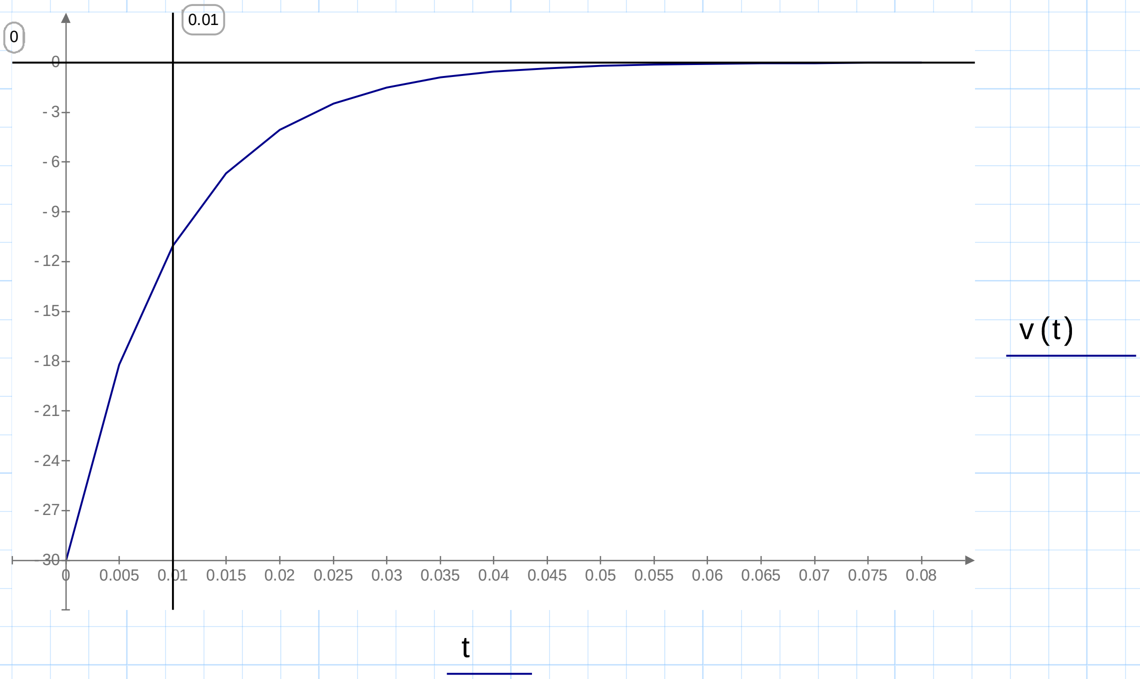
This was a good example simple with a learning outcome.
Lets attempt the graphs.

$\tau_{RL\_circuit} := \frac{L1}{R2} = 0.01$ s.

Used R2 because its in the final circuit when 12V dc was removed, where R1 was not included due to open circuit at voltage source.

clear(t)

$t := 0, 0.005 .. 0.08$ $\quad v(t) := -30 \cdot e^{-100 \cdot t}$

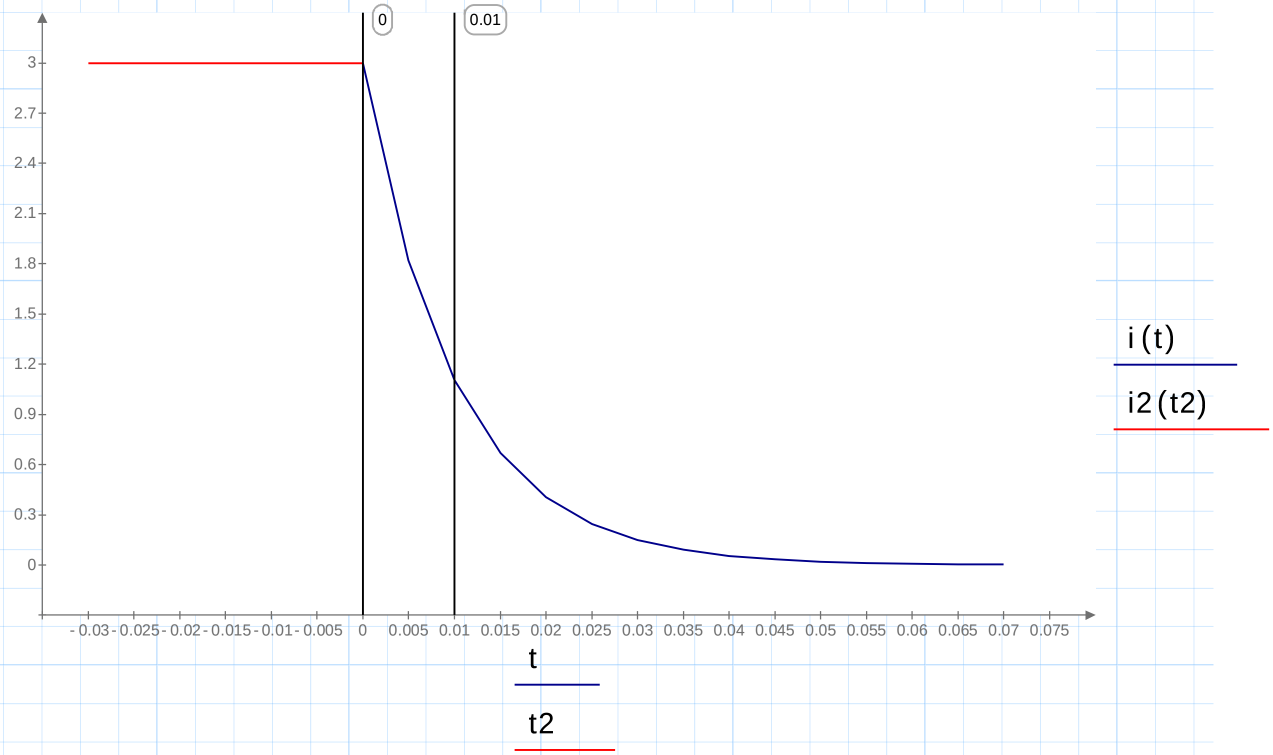


clear(t, t1, t2)

$t := 0, 0.005 .. 0.07$

$t2 := -0.002, -0.0015 .. 0$ $\qquad$ $t2 := 0, -0.0015 .. -0.03$

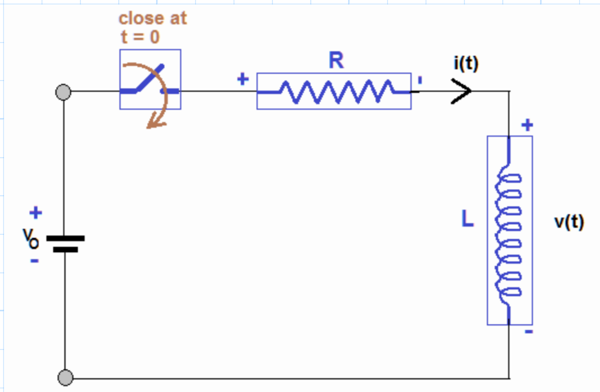
$i(t) := 3 \cdot e^{-100 \cdot t}$ $\qquad$ $i2(t2) := 3$



Current above plotted for for t<0, t=0, and t>0.

7.5 Establishing a DC current in an inductor.

We had a topic earlier '7.3 Establishing a DC voltage across a capacitor'.
For inductor the differential term is di/dt, where as the capacitor was dv/dt.
Similar approach here.



I try to not explain and hope the equations are able to make their case. Since we seen one with RC this may work for RL. Try!

DC source Vo suddenly applied to a series RL circuit at rest.
Current increases exponentially from 0 to a CONSTANT value, with time constant L/R.

$$R \cdot i + L \cdot \left(\frac{di}{dt}\right) = V_0$$

for t>0, switch was closed at t =0.
has to be t>0 otherwise no voltage no loop equation.

Current at time zero plus (0+) means past time t =0, and at the very begining of that time the current in the circuit will be 0, because the current has to over come the inertia of the circuit inductor. Inductor internal inertia. *<--- You may discuss.*

$$i = i_h(t) + i_p(t)$$

home-geneous and particular solution.

$$i_h(t) = A \cdot e^{-\left(\frac{R}{L}\right) \cdot t} \qquad i_p(t) = \frac{V_0}{R}$$

One way to remember 'particular solution' its the 'simple' one, here Vo/R. Homogenous the other ie differential form one.

$$i = A \cdot e^{-\left(\frac{R}{L}\right) \cdot t} + \frac{V_0}{R}$$

Find A?

$$i\left(0^{'+'}\right) = A \cdot e^{-\left(\frac{R}{L}\right) \cdot 0} + \frac{V_0}{R} = 0$$

$$i\left(0^{'+'}\right) = A + \frac{V_0}{R} = 0$$

$$A = -\left(\frac{V_0}{R}\right)$$

So what can we do with A, having discovered what the coefficient of A can be?

We perhaps can narrow in to i(t) and v(t), Yes, substitute A in equation:

$$i = -\left(\frac{V_0}{R}\right) \cdot e^{-\left(\frac{R}{L}\right) \cdot t} + \frac{V_0}{R}$$

$$i = \left(\frac{V_0}{R}\right) \cdot \left(1 - e^{-\left(\frac{R}{L}\right) \cdot t}\right)$$

$$i(t) := \left(\frac{V_0}{R}\right) \cdot \left(1 - e^{-\left(\frac{R}{L}\right) \cdot t}\right)$$ for t>0 solved for the current through inductor.

The voltage across the inductor:

$$v = L \cdot \left(\frac{di}{dt}\right)$$

$$di/dt = d/dt \left(\left(\frac{V_0}{R}\right) \cdot \left(1 - e^{-\left(\frac{R}{L}\right) \cdot t}\right)\right)$$

$$di/dt = -\left(\frac{V_0}{R}\right) \cdot \left(-\frac{R}{L}\right) \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$$

$$di/dt = \left(\frac{V_0}{L}\right) \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$$

$$v = L \cdot \left(\frac{V_0}{L}\right) \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$$

$$v(t) := V_0 \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$$ for t>0 solved for the inductor voltage.

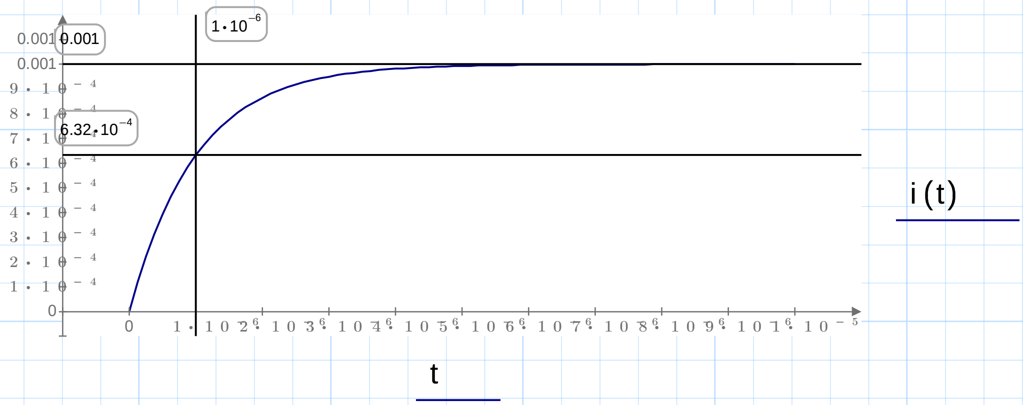
Lets try to get some plots since Schaums Outline done them.

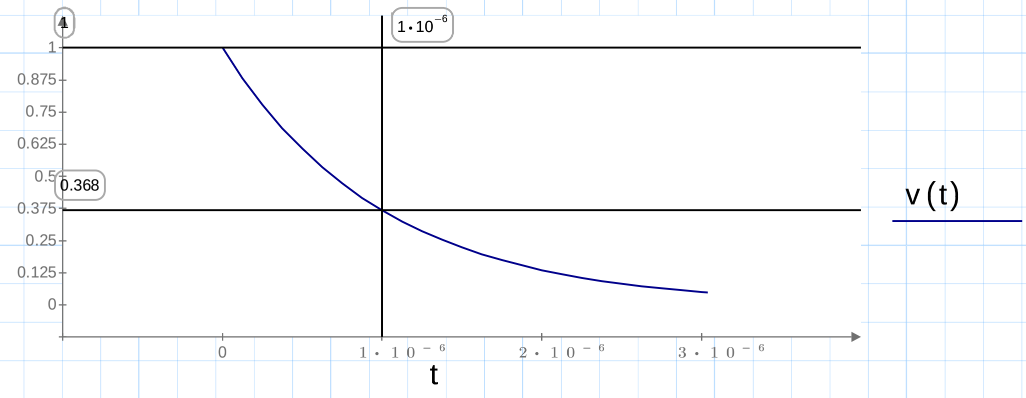
We have i(t) and v(t) we have Vo, R, L, and for time t which we apply the time constant of an RL circuit. We set values of 1 to these variables.

$V_0 := 1$ $R := 1 \cdot 10^{3}$ Ohm $L := 1 \cdot 10^{-3}$ H $\frac{V_0}{R} = 0.001$ $\tau_{RL} := \left(\frac{L}{R}\right) = 1 \cdot 10^{-6}$ s

$t := 0, 0.125 \cdot 10^{-6} .. 10 \cdot 10^{-6}$ $0.632 \cdot \left(\frac{V_0}{R}\right) = 6.32 \cdot 10^{-4}$

$i(t) := \left(\frac{V_0}{R}\right) \cdot \left(1 - e^{-\left(\frac{R}{L}\right) \cdot t}\right)$ $v(t) := V_0 \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$

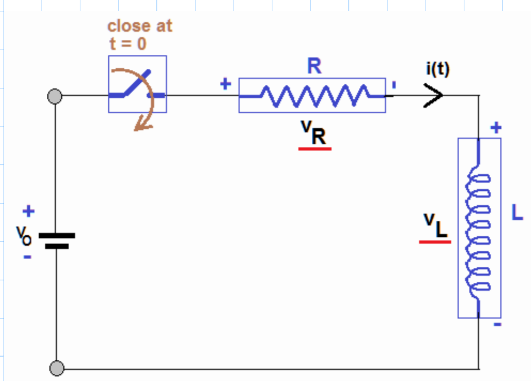




Satisfied with the intersections at tau = L/R for Vo/R and 0.368 Vo.

We will return here, after an example, in section 7.6.

Solved Problem 7.12: Series RL Circuit:



A series RL circuit has a constant voltage Vo,
applied at t = 0.
At what time does vR = vL?

Solution:

From the RL circuit we last saw, in establishing a DC current in an inductor, it showed the current started from 0 then rose to a final value of V/R = 0.001. See plot.

Now given this RL circuit we may apply the same equations for t>0.

$i(t) := \left(\frac{V_0}{R}\right)\cdot\left(1 - e^{-\left(\frac{R}{L}\right)\cdot t}\right)$ current i(t) in the inductor circuit.

$v_L(t) := V_0 \cdot e^{-\left(\frac{R}{L}\right)\cdot t}$ inductor voltage.

The voltage across the resistor vR(t) = Ri(t)

$v_R(t) := (R)\cdot\left(\frac{V_0}{R}\right)\cdot\left(1 - e^{-\left(\frac{R}{L}\right)\cdot t}\right)$

$v_R(t) := V_0\cdot\left(1 - e^{-\left(\frac{R}{L}\right)\cdot t}\right)$

Circuit voltage v_0 = vR + vL.
We notice there are some common terms in the vL and vR, maybe simply further.

$v_R(t) := V_0\cdot\left(1 - e^{-\left(\frac{R}{L}\right)\cdot t}\right)$

$v_L(t) := V_0 \cdot e^{-\left(\frac{R}{L}\right)\cdot t}$ <--- Substitute vL(t) in vR(t).

$v_R(t) := V_0 - v_L(t)$ Rearranging

$V_0(t) := v_R(t) + v_L(t)$

$V_0 = v_R + v_L$ the 2 voltages will be equal when each equal half Vo, ie Vo/2
hence vR = Vo/2 and vL = Vo/2, and (Vo/2)+(Vo/2) = Vo.

$v_R(t) := V_0 - V_0 \cdot e^{-\left(\frac{R}{L}\right)\cdot t}$

$\frac{V_0}{2} = V_0 - V_0 \cdot e^{-\left(\frac{R}{L}\right)\cdot t}$ rearranging---> $V_0 \cdot e^{-\left(\frac{R}{L}\right)\cdot t} = V_0 - \frac{V_0}{2} = \frac{V_0}{2}$ Next cancel Vo

$e^{-\left(\frac{R}{L}\right)\cdot t} = \frac{1}{2}$ solve for (R/L) ie tau---> $e^{\frac{-t}{\tau}} = \frac{1}{2}$ *tau=L/R, 1/tau = (R/L)*

$\left(\frac{t}{\tau}\right) = \ln(2)$

$\ln(2) = 0.693$

$\left(\frac{t}{\tau}\right) = 0.693$

$t = 0.693 \cdot \tau$ This is the time t we seek and it is independent of Vo. Answer.

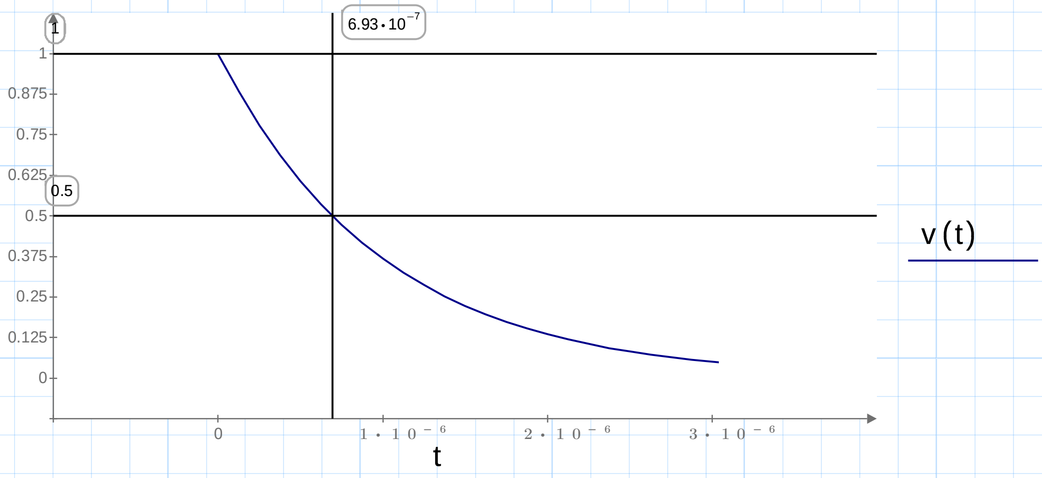
Lets try to plot this. We have some values similar to previous example.

$V_0 := 1$ $R := 1 \cdot 10^{3}$ Ohm $L := 1 \cdot 10^{-3}$ H $\frac{V_0}{R} = 0.001$ $\tau_{RL} := \left(\frac{L}{R}\right) = 1 \cdot 10^{-6}$ s

$t := 0, 0.125 \cdot 10^{-6} .. 10 \cdot 10^{-6}$ $0.632 \cdot \left(\frac{V_0}{R}\right) = 6.32 \cdot 10^{-4}$

$i(t) := \left(\frac{V_0}{R}\right) \cdot \left(1 - e^{-\left(\frac{R}{L}\right) \cdot t}\right)$ $v(t) := V_0 \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$

$t1 := 0.693 \cdot \left(\frac{L}{R}\right) = 6.93 \cdot 10^{-7}$



We see it is 0.5V. The voltage at time t = 0 = 1V,
and at 0.693*10^-6 second is 0.5V.

Sufficient.
This was not exactly the example for 'Establishing a DC current in an inductor' but of good value. I could not find one directly related maybe you could in the solved problems. Problem solving technique increasing.

7.6 The exponential function revisited.

The exponential function came about because it was a solution meeting some conditions.
Specifically in the form Ae^st.
We can now also write it as f(t) = Ae^st. A function in time t which is exponent based.
Tau for RC circuit = RC.
Tau for RL circuit = L/R.

The time constant, tau, shows some promise as an indicator on the plots.
How such a value of RC and L/R could do such a job you investigate. If its a mystery you investiage it. By now we've seen the x-axis (time scale) is numbered in multiples of tau (time constants). So obviously time constant plays some role in circuit analysis, its shown some potential here so do give it due attention.

$A := 1$ <--*for assisting the function below so we dont see a red highlight by Mathcad editor.*

$f(t) := A \cdot e^{\frac{-t}{\tau}}$ <---The general decay function.

RC circuit: $f1(t) := A \cdot e^{\frac{-t}{R \cdot C}}$

RL circuit: $f2(t) := A \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$

For the purpose of generating a plot we use the same values of a previous RL circuit. The plot shown on the next page.

$v(t) := V_0 \cdot e^{-\left(\frac{t}{\tau_{RL}}\right)}$ We use this expression the same as f1(t)

$V_0 := 1$ $R := 1 \cdot 10^3$ Ohm $L := 1 \cdot 10^{-3}$ H $\tau_{RL} := \left(\frac{L}{R}\right) = 1 \cdot 10^{-6}$

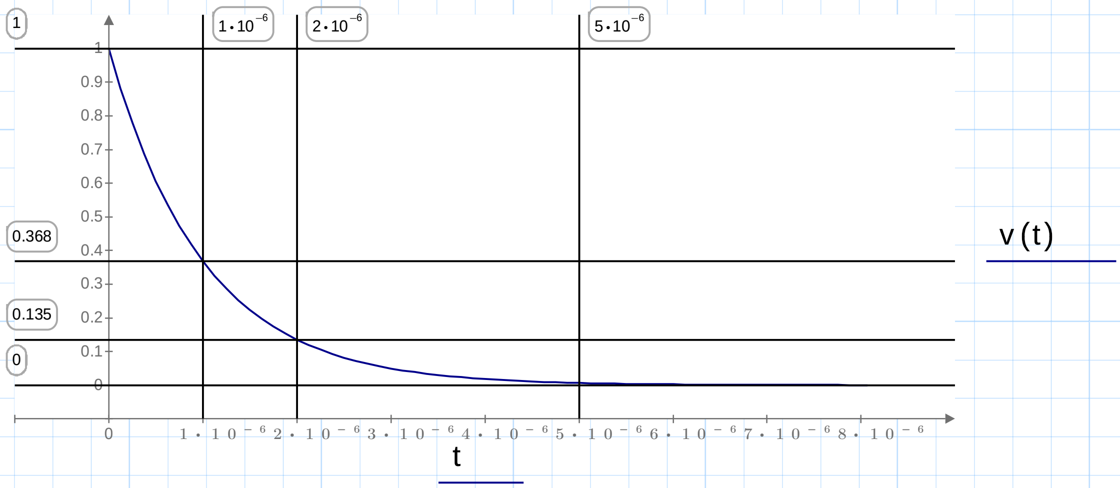
$\frac{V_0}{R} = 0.001$ $0.368 \cdot \left(\frac{V_0}{R}\right) = 3.68 \cdot 10^{-4}$ $0.632 \cdot \left(\frac{V_0}{R}\right) = 6.32 \cdot 10^{-4}$

$v(t) := V_0 \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$ **clear** (t)

$v(t) := V_0 \cdot e^{-\left(\frac{t}{\tau_{RL}}\right)}$ <--- This has tau shown makes more sense here for the plot

$t := 0, 0.125 \cdot 10^{-6} .. 10 \cdot 10^{-6}$

Continued on next page starting with plot.



x-axis we have in multiples of tau, from 1 thru 8.
At tau = 1 function v(t) = 0.368.
Now at t = tau the function is

$$v(t) := V_0 \cdot e^{-\left(\frac{t}{\tau_{RL}}\right)}$$

$$v(t) := V_0 \cdot e^{-\left(\frac{\tau_{RL}}{\tau_{RL}}\right)}$$

$$v(t) := V_0 \cdot e^{-(1)}$$ the power of the exponent is -1

$$1 \cdot e^{-1} = 0.368$$ Since V = 1

Tau = 1 the function v(t) = 0.368 PERCENT of the initial value which is 1 that was 100%.
The x-axis may continue from 8 tau to 100 tau to infinity.
At 1 tau the function v(t) has undergone a change from 1 down to 0.632 or 63.2%.
This change can be written as:

$v(0^{'+'})$ to $v(0^{' 1)\tau '})$ from t = 0 to t = 1 tau.

Because tau is so small 1*10^-6 it can be said the function v(t) has undergone 63.2% of change from

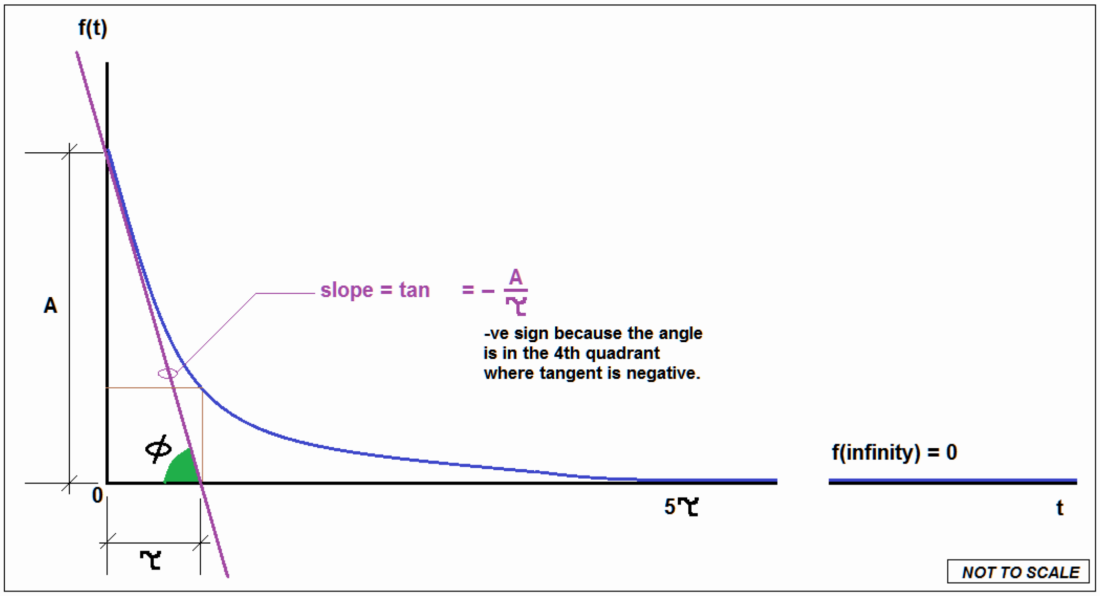
$v(0^{'+'})$ to $v(0^{'infinity'})$

Its said from a practical position the transient is often regarded as over at tau = 5 tau. Same here in the plot at 5 tau the function has a value of 0.0067, far less than 1 percent of initial value.

Obviously some significance of the time constant (tau) is appreciated.
Its a critical player in electrical engineering.

The figure below shows the slope = f ' (0+) = - A/tau.
The apostrophe next to f shows that its the derivative of f.

$$f(t) := A \cdot e^{\frac{-t}{\tau}} \qquad f'(t) := \left(\frac{A}{-\tau}\right) \cdot e^{\frac{-t}{\tau}} \qquad f'(0'^{+}') = \left(\frac{A}{-\tau}\right) \cdot e^{\frac{-0}{\tau}} = -\left(\frac{A}{\tau}\right)$$ <---Slope.



In the lab world or in the field in instrumentation, at times a transient is only partially displayed, (on an oscilloscope or graph-chart), plus the simultaneous values of the function and slope needed in the above method are NOT available.

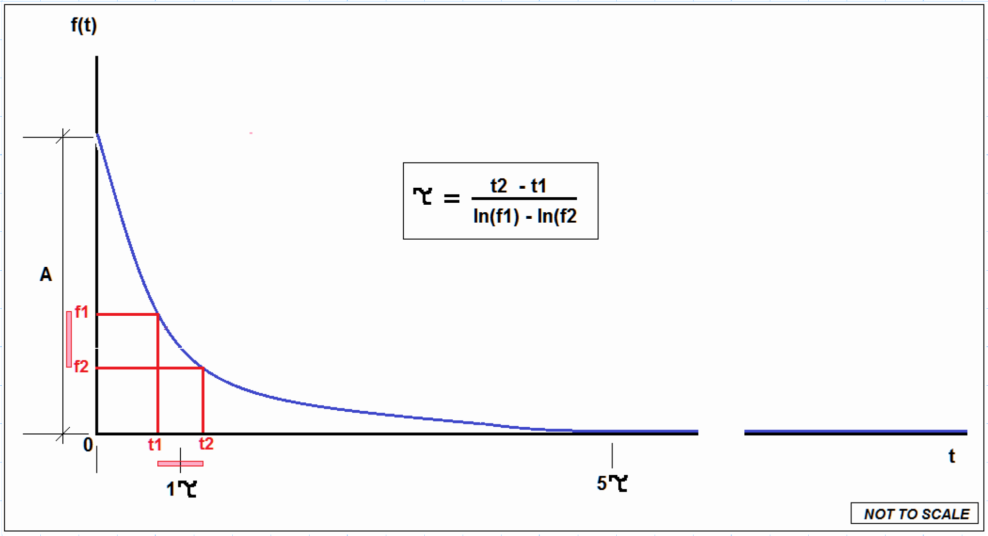
Then use any 'pair of data points' that can be read from instruments in the lab or field to find the equation of the transient. Sounds simple enough. In the lab an oscilloscope or similar device, and in the field a hand held device or data logger.

Basically get the function values at 2 locations.
f(t1) and f(t2). These then solved thru the expression below for tau (time constant).

$$f1(t1) := A \cdot e^{\frac{t1}{\tau}} \qquad f2(t2) := A \cdot e^{\frac{t2}{\tau}}$$

Above 2 solved simultaneously to give $\tau = \dfrac{(t2 - t1)}{\ln(f1) - \ln(f2)}$

Having solved tau, then substitute it in the function for f1 and f2.
See figure below.

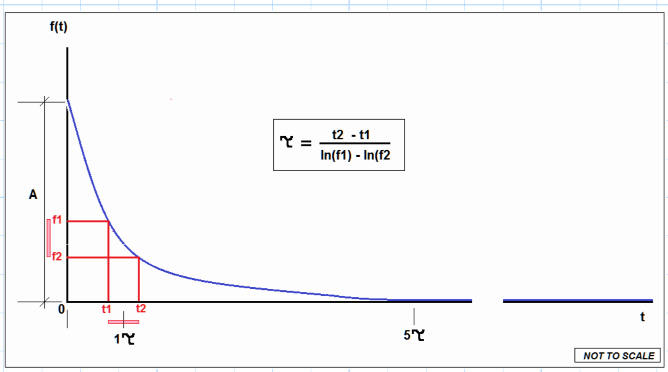


Solved Problem 7.13: RL circuit.

A constant voltage is applied to a series RL circuit at t = 0.
The voltage across the inductance is
20V at 3.46 ms
and
5V at 25 ms.
Find R if L = 2 H.

Solution:

The voltage is decaying from 20V to 5V, from 3.46 to 25 ms. $L := 2\ H$



$t1 := 3.46 \qquad t2 := 25$

$v1 := 20 \qquad v2 := 5$

$$\tau_{RL} := \frac{(t2 - t1)}{(\ln(v1) - \ln(v2))} = 15.54$$

$\tau_{RL\_ms} := 15.54 \cdot 10^{-3}\ s$ <---Slope

$$\tau_{RL\_ms} = \frac{L}{R}$$

$$R := \frac{L}{\tau_{RL\_ms}} = 128.7$$ Ohms. Answer.

Calculation problem with the x-axis time units, when to apply.

$25 - 3.46 = 21.54$ but $25 \cdot 10^{-3} - 3.46 \cdot 10^{-3} = 21.54 \cdot 10^{-3}$

$(\ln(v1) - \ln(v2)) = 1.386$

$\frac{21.54 \cdot 10^{-3}}{1.386} = 0.016$ while ----> $\frac{21.54}{1.386} = 15.541$ *<---Now attach miliseconds. We seek the slope that comes from the value difference without the units and scale relevance; dv/dt. x and y axis are set at different scales and units.*

7.7 Complex first order RL and RC circuit.

Here we are not looking into complex resistor circuits with multiple loops.
We looked at mostly 1 or 2 resistors in a circuit with either an inductor or capacitor.
but what if he had multiple loops, multiple resistors, with an inductor or capacitor?
To solve that apply circuit reduction methods like superposition, Thevnin, Norton, current division, voltage division, etc.

Thats All A Lot of Work.

If you remember from circuits its lots of equations, solving simultaneous equations, then using matrix.

No circuit mushrooms into something so huge that it needs a tracking system thats left to the mail man or courier service.

All circuits are made simple, simplest as possible, serving a purpose in a bigger circuit network. All.

Finally when they get put together, its one output as input into another circuit. This continues untill the objective is achieved.

Put together small circuits to complete a big circuit.

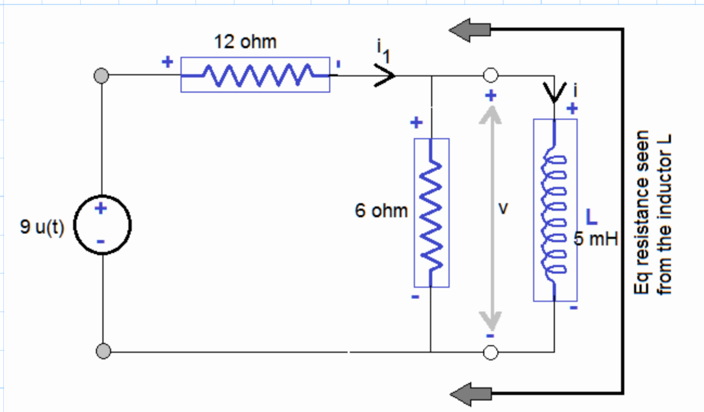
We have one example much bigger than any of the solved problems. Lengthy and tricky.
The next example 7.5 is a simpler one, followed by 7.6 which is a hurdle.

When a source in a circuit is suddenly switched to a dc value, the resulting currents and voltages are exponentials, sharing the same time constants with possibly different initial and final values. The time constants of the circuit is either RC or L/R, where R is the resistance in the Thevenin equivalent of the circuit as seen by the capacitor or inductor - *page 150 Schaums Outline.*

Continued on next page.

Example 7.5

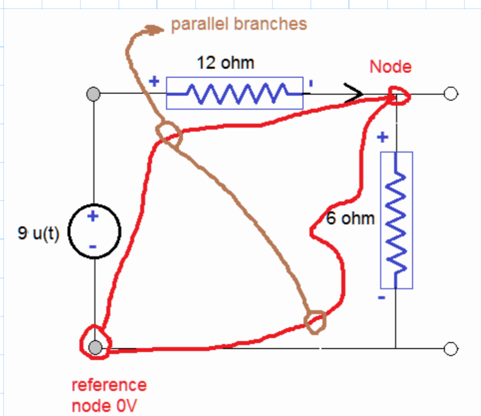
Solve for i, v and i1?



The voltage source at first look is intimidating.

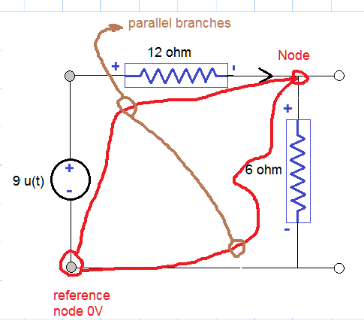
9 times u(t). So u(t) is a step input we seen earlier, comes on at t = 0 and t >0 and equal 1. Here my first assumption is its amplitude is 9 V, 9 x 1V = 9V. Where u(t) = 1V.

Circuit reduction on the circuit above.



Thevenin: An electrical network which contains one or more voltage and current sources can be replaced by a PARALLEL RESISTANCE and a SINGLE VOLTAGE source.

$$R_{th} := \frac{12 \cdot 6}{12 + 6} = 4 \quad \text{Ohm}$$



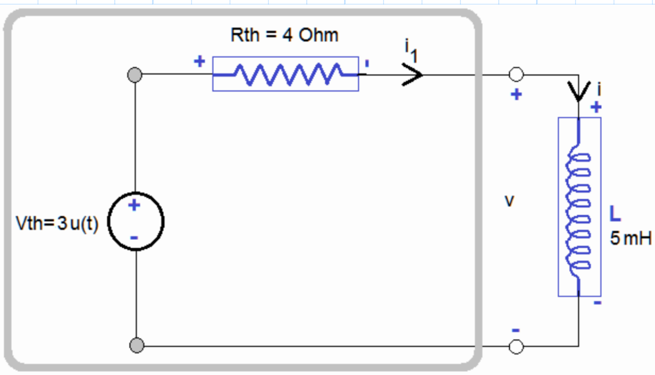
Not an assumption voltage is 9V, it is 9V when t = 0 and t>0.
Solving for i in the non reduced circuit with the open terminals

$$i := \frac{9}{12 + 6} = 0.5 \quad \text{A.}$$

Voltage across the open terminals is equal to the voltage across the 6 ohm resistor.

$V_{th} := i \cdot 6 = 3$ V.

$V_{th} = 3 \cdot u(t)$ In the same form as the original voltage, unit step.



<--- Thevenin equivalent circuit.

Circuit time constant uses the equivalent Rth for R.
Since its a RL circuit tau = L/Rth.

$L := 5 \cdot 10^{-3}$ H

$\tau := \frac{L}{R_{th}} = 1.25 \cdot 10^{-3}$ s

Comments: In this solution there is no switch. Do we assume the circuit is operating in time t>0? I would. *But that was after looking at the solution!* I raise the question when do we assume its t<0 or t>0? Because there are so many circuit problems and most do not spell out (write) everything clearly. <--- Is that my excuse? Hope not.

Initial conditions apply ONLY to the inductor.

The initial value of the inductor current is zero. <--- Stated in the solution.
So we see the solution beginning at a time when the inductor was not energised, for a unit step voltage this would be at time t<0 where voltage was zero.

So we now are faced with giving the solution process more history.
We say what was the 'history' of the inductor at time t<0?
The inductor was not receving current.

At time t=0 is the same as time t >0 the voltage is 9u(t)V in the original circuit.
The initial condition was t<0, and final condition now is t>0 = 9 u(t) V.
Next we have to ascertain do we use the Thevenin values or the original?

$i(0'^{+}') \quad = \frac{v_{th}}{R_{th}} = 0.75$ A. $\qquad v_{th} = 3 \qquad R_{th} = 4$

Remember circuit analysis method(s) we use the technique's result(s) with new variable(s) and value(s) because the original circuit was not easy to solve.
Stay with these new values until there is reason to go to the original circuit to solve for a variable or constant. Next we plug in the calculated values into the required formulae.

In the 'Establishing the DC current in the inductor' we had:

$$i(t) := \left(\frac{V_0}{R}\right)\cdot\left(1 - e^{-\left(\frac{R}{L}\right)\cdot t}\right) \qquad v(t) := V_0 \cdot e^{-\left(\frac{R}{L}\right)\cdot t}$$

We just calculated Vo/R which was i(0+) equal 0.75.
The current solution form for t>0 should have the u(t).

$$i = \left(\frac{V_0}{R}\right)\cdot\left(1 - e^{-\left(\frac{R_{th}}{L}\right)\cdot t}\right)\cdot u(t) \quad \text{CORRECT.}$$

Plug in R, L, (Vo/R) for i:

$$i = (0.75)\cdot\left(1 - e^{-\left(\frac{4}{5\cdot 10^{-3}}\right)\cdot t}\right)\cdot u(t) \qquad \text{Using Rth for R, } = \left(\frac{4}{5\cdot 10^{-3}}\right) = 800$$

$$i(t) = (0.75)\cdot\left(1 - e^{-800\cdot t}\right)\cdot u(t)$$ A. Current i passing thru the inductor. Answer.

Similarly for the voltage at t>0

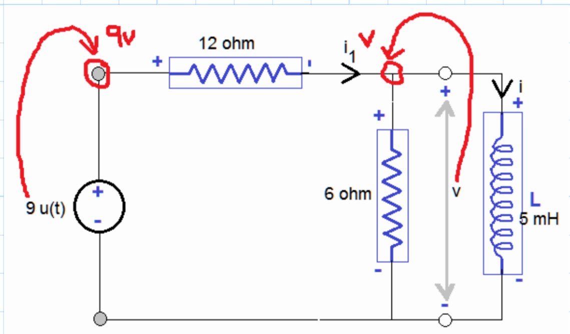
$$v(t) := V_0 \cdot e^{-\left(\frac{R_{th}}{L}\right)\cdot t}$$

We use the v_Thevenin equivalent for Vo.
We are not using L (di/dt) because we do not have di/dt.
Though calculated here v(t) does equal L(di/dt).

$$v = L\cdot\left(\frac{di}{dt}\right) = V_{th}\cdot e^{-\left(\frac{R_{th}}{L}\right)\cdot t}\cdot u(t)$$ Together with the form of voltage u(t).

$$v(t) = 3\cdot e^{-800\cdot t}\cdot u(t)$$ V. Voltage v across the inductor. Answer.

Now to calculate the current i1 passing thru the 12 ohm resistor in the original circuit.

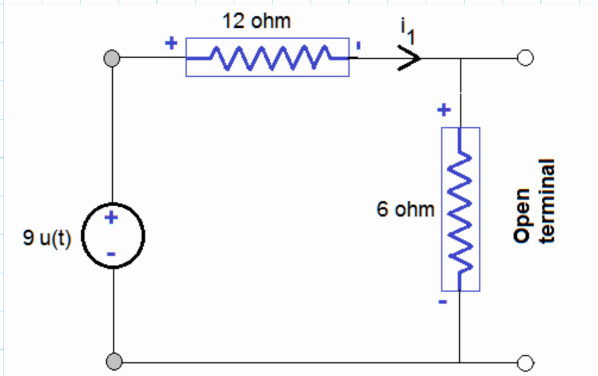


i1 is the current thru the 12 ohm resistor so we have to return to the original (non-reduced) circuit but we apply these new values, *NOT may or if rather will apply.* Study the circuit figure for the solution marked in red.

Current i1 = voltage across 12 ohm resistor divided by resistance value 12 ohm; i = v/R.

$$v_{12ohm} = \frac{9 - v}{12} = \left(\frac{1}{12}\right)\cdot\left(9 - 3\cdot e^{-800\cdot t}\right)\cdot u(t) = \left(\frac{1}{4}\right)\cdot\left(3 - e^{-800\cdot t}\right)\cdot u(t)$$ V. Answer.

Continued on next page for another solution for 12 ohm.



Another way to calculate v across the resistor 6 ohm, which was done early in the example for voltage across the inductor, is using the voltage division.
We have a series circuit.
Voltage source is 9V.

$$v := 9 \cdot \left(\frac{6}{12+6}\right) = 3 \quad V$$

When t = infinity, the voltage across the inductor becomes 0.
Why?
Voltage source is DC, its energised the inductor to the maximum.
Voltage across the inductor v(t) is L(di/dt).

$$v(t) \;=\; 3 \cdot e^{-800 \cdot t} \cdot u(t)$$

Let t = 1000 seconds.

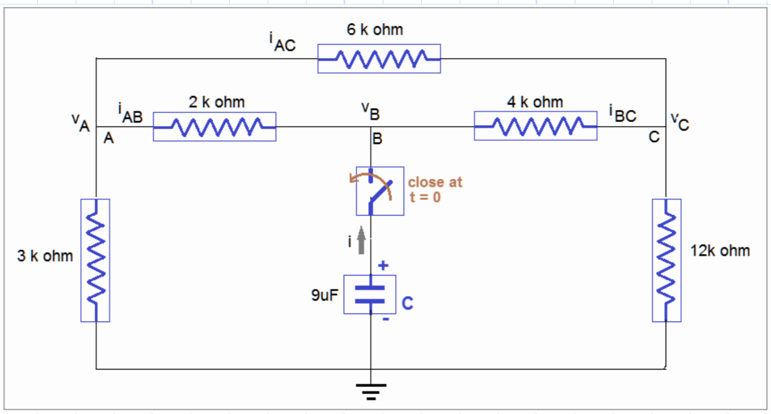
$$3 \cdot e^{-800 \cdot 1000} = 0$$

$$v(t) \;=\; 0 \cdot u(t) \;=\; 0$$

It was a good learning outcome for first order circuit analysis.

Next that small complex circult example mentioned in previous pages.

Example 7.6: Complex 1st Order Circuits.

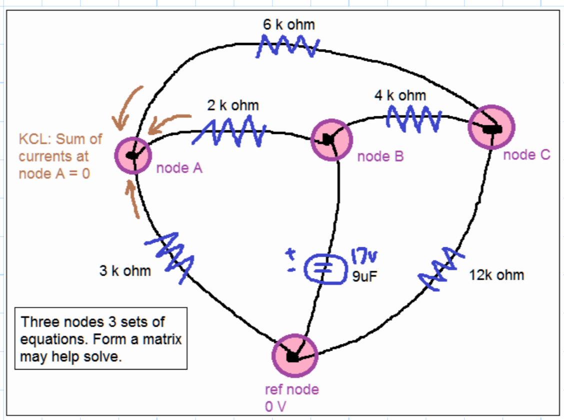


This is a tougher example my explanation may not be complete or accurate you may do better here.

The 9 uF capacitor is connected to the circuit at t = 0.
At this time, capacitor voltage is v_o = 17 V.
Find v_A, v_B, v_C, i_AB, i_AC, and i_BC for t > 0 ?

Solution:

Capacitor is connected at time t = 0, and at that instant the capacitor voltage was 17 V. The capacitor was already charged, placed in the circuit, and at the instant of connection when the switch closed the voltage was 17 V. *Thats how I understand it at this point of time may change later hope not.*



Another way of looking at the same circuit above. Sketch, so to assist the solution. Not really needed but may help when there are more nodes.
First rule in any real serious engineering sketch the problem.
Refresher on circuits.

$k := 10^3$ Apply k for 1000.

KCL at node A:

$$\left(\frac{vA-0}{3\cdot k}\right)+\left(\frac{vA-vB}{2\cdot k}\right)+\left(\frac{vA-vC}{6\cdot k}\right) = 0$$

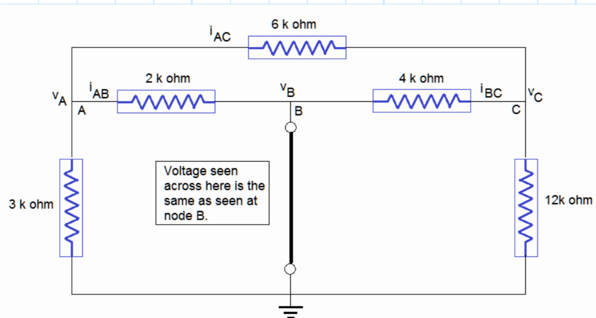
$$\left(\frac{vA}{3\cdot k}\right)+\left(\frac{vA}{2\cdot k}\right)-\left(\frac{vB}{2\cdot k}\right)+\left(\frac{vA}{6\cdot k}\right)-\left(\frac{vC}{6\cdot k}\right) = 0$$

$$\left(\frac{vA}{3\cdot k}\right)+\left(\frac{vA}{2\cdot k}\right)+\left(\frac{vA}{6\cdot k}\right)-\left(\frac{vB}{2\cdot k}\right)-\left(\frac{vC}{6\cdot k}\right) = 0$$ rearranged.

$$\left(\frac{(2+3+1)\ vA}{6\cdot k}\right)-\left(\frac{vB}{2\cdot k}\right)-\left(\frac{vC}{6\cdot k}\right) = 0$$ To simplify further we multiply by 6k.

$$6\cdot vA-3\cdot vB-vC = 0$$ Equation - 1.

We jump to node C since node B has a capacitor its takes more effort to form the equation there. Come back to it after node C.



KCL at node C:

$$\left(\frac{vC-vB}{4\cdot k}\right)+\left(\frac{vC-vA}{6\cdot k}\right)+\left(\frac{vC-0}{12\cdot k}\right) = 0$$

$$\left(\frac{vC}{4\ k}\right)+\left(\frac{vC}{6\cdot k}\right)+\left(\frac{vC}{12\cdot k}\right)-\left(\frac{vB}{4\cdot k}\right)-\left(\frac{vA}{6\cdot k}\right) = 0$$

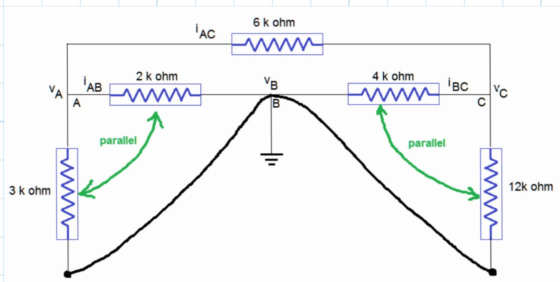
Multiply by 12k

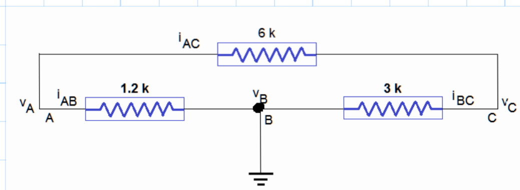
$$-2\cdot vA-3\cdot vB+6\cdot vC = 0$$ Equation - 3.

KCL at node B:

$$\left(\frac{vB-vA}{2\cdot k}\right)+\left(\frac{vB-0}{R_{seen\_by\_C}}\right)+\left(\frac{vB-vC}{4\cdot k}\right) = 0$$

$$\left(\frac{vB}{2\cdot k}\right)-\left(\frac{vA}{2\cdot k}\right)+\left(\frac{vB}{R_{seen\_by\_C}}\right)+\left(\frac{vB}{4\cdot k}\right)-\left(\frac{vC}{4\cdot k}\right) = 0$$





$$R_{right} := \left(\frac{2\cdot 3}{5}\right)\cdot k = 1.2\cdot 10^{3}$$

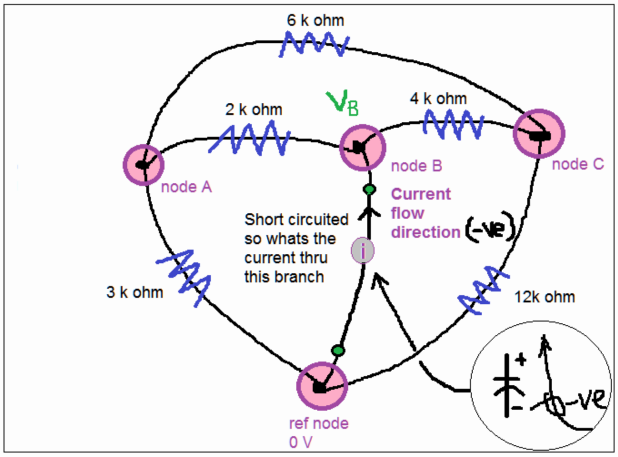
$$R_{left} := \left(\frac{4\cdot 12}{16}\right)\cdot k = 3\cdot 10^{3}$$

$$R_{series\_center} := (1.2+3)\cdot k = 4.2\cdot 10^{3}$$

$$R_{total} := \left(\frac{6\cdot 4.2}{10.2}\right)\cdot k = 2.471\cdot 10^{3}$$

The Rseries_center is the resistance seen at the node B. 6k ohm is not directly connected to the capacitor branch, rather paralled to node A, B, and C. So we use the 4.2k Ohm. Schaum rounds it off to 4.0k. So, we use the 4 k Ohm. So it may NOT impact the results compared to Schaums. *Thats typical in industry to real available components.*

Resistance seen by the capacitor: $R_{seen\_by\_C} := 4000$ ohm.



Discussion: Capacitor is oriented/placed in the opposite direction to current flow with respect to vB, ie capacitor polarity. So the sign on '(vB-0)/RTh' is a -ve sign. Current +ve when flowing into Capacitor +ve terminal, here exiting +ve terminal. *You may have a better explanation here in keeping with Schaums solution. But how about this, higher potential falls to lower potential, what is that fall made of? Waterfall = Current flow. Capacitor is supplying charge into the circuit not to node 0, wasted! So its -ve direction supplying the circuit beneficial.*

What is the Norton's equivalent current seen by the capacitor branch? vB divided by 'R seen by capacitor'.

$$i_{C\_Norton} = \frac{v_B}{R_{seen\_by\_C}}$$

This obviously is not the current running thru each component. Why? When you solve mesh/ loop equations there is a current for each loop.

We return to this later.

What is vB?

Given the voltage across the capacitor is 17 V at time t = 0.
So we may use that for the initial voltage:

$v_{B\_initial} := 17$ V = vB.

Now we have i_C_Norton equivalent: $i_{C\_Norton} := \frac{v_{B\_initial}}{R_{seen\_by\_C}} = 4.25 \cdot 10^{-3}$

Schaums rounded it to 4 mA. $i_{C\_Norton} := 4 \cdot 10^{-3}$ A.

$i_{CNT} := -i_{C\_Norton}$ A. Negative sign from the discussion above.

$i_{CNT} = -4 \cdot 10^{-3}$ A.

$$-\left(\frac{vA}{2 \cdot k}\right) + \left(\frac{vB}{2 \cdot k}\right) + \left(\frac{vB}{4 \cdot k}\right) + \left(\frac{vB}{R_{seen\_by\_C}}\right) - \left(\frac{vC}{4 \cdot k}\right) = 0$$

substitute vB/R_seen.....
...Norton's equivalent I.

$$\left(\frac{vB}{R_{seen\_by\_C}}\right)$$

<---Lets set this term as i since this is the current generated from the voltage at B divided by resistor, its part of the Thevenin-Norton circuit. Capacitor injects current into this node B, then splits to the left and right. At point B the current is assumed to be wholly from the capacitor not spliting to left or right of that point yet.

$$-\left(\frac{vA}{2 \cdot k}\right) + \left(\frac{vB}{2 \cdot k}\right) + \left(\frac{vB}{4 \cdot k}\right) - i - \left(\frac{vC}{4 \cdot k}\right) = 0$$

Next multiply by 4k

$$-(2\cdot vA)+(2\cdot vB)+(vB)-(4\cdot 10^3)\cdot i-(vC) = 0$$

$$-(2\cdot vA)+(2\cdot vB)+(vB)-(vC) = (4\cdot 10^3)\cdot i$$

$$-(2\cdot vA)+(3\cdot vB)-(vC) = (4\cdot 10^3)\cdot i$$ Equation - 2

$$-(2\cdot vA)+(3\cdot vB)-(vC) = (4\cdot 10^3)\cdot i$$ <---Note: Schaums Outline has same.

Why Schaums had ' i ' multiplied to the RHS of equation 2? 'i' represents a current for the 'equation's format'. (vB/Rseen..) is not yet identified as a decaying current of a capacitor. Return later to solve on the decaying current. *You may have a much more appreciated reason.*

The 3 equations here in order to form a matrix:

$$6\cdot vA-3\cdot vB-vC = 0$$ Equation - 1.

$$-(2\cdot vA)+(3\cdot vB)-(vC) = (4\cdot 10^3)\cdot i$$ Equation 2 with i.

$$-2\cdot vA-3\cdot vB+6\cdot vC = 0$$ Equation - 3.

Resistance matrix:

$$v\_ovr\_R := \begin{bmatrix} 6 & -3 & -1 \\ -2 & 3 & -1 \\ -2 & -3 & 6 \end{bmatrix}$$

Inverse resistance matrix:

$$(v\_ovr\_R)^{-1} = \begin{bmatrix} 0.417 & 0.583 & 0.167 \\ 0.389 & 0.944 & 0.222 \\ 0.333 & 0.667 & 0.333 \end{bmatrix}$$

Current matrix; RHS of equation:

$$i_{M\_not\_fixed} := \begin{bmatrix} 0 \\ (4\cdot 10^3)\cdot i \\ 0 \end{bmatrix}$$ First solve for without 'i', later plug in for 'i'.

However we can solve at present for the coefficients of the solution at the node.
Then multiply by i_C of the capacitor what is 'i'.
Discussion: But what then are these values now in iM_not_fixed?
These are currents but not in their final forms. Merely satisfying their position in the 3 equations, 2 values are 0 and 1 is (4x 10^3) that is multiplied by i. Which take on their true current behaviuor later, *we got the skeletal system but the finsihes come later - Karl Bogha.*

$$i_M := \begin{bmatrix} 0 \\ (4\cdot 10^3) \\ 0 \end{bmatrix}$$ Leaving out i.

Solution for the coefficients:

$$v\_over\_R_{coeff} := (v\_ovr\_R)^{-1} \cdot i_M = \begin{bmatrix} 2.333 \cdot 10^3 \\ 3.778 \cdot 10^3 \\ 2.667 \cdot 10^3 \end{bmatrix}$$

Schaum has the same coefficients in fraction form:

$$\frac{7}{3} = 2.333 \quad \frac{34}{9} = 3.778 \quad \frac{8}{3} = 2.667$$

$$v\_over\_R_{coeff} = \begin{bmatrix} 2.333 \cdot 10^3 \\ 3.778 \cdot 10^3 \\ 2.667 \cdot 10^3 \end{bmatrix}$$

<---- to be multiplied by 'i'.

Cofficients at nodes:

$$v\_R_{A\_coeff} := v\_over\_R_{coeff_0} = 2.333 \cdot 10^3$$

$$v\_R_{B\_coeff} := v\_over\_R_{coeff_1} = 3.778 \cdot 10^3$$

$$v\_R_{C\_coeff} := v\_over\_R_{coeff_2} = 2.667 \cdot 10^3$$

Circuit time constant:

Schaum has a time constant of 0.034 s. This was achieved by taking the resistance seen at node B from the coefficients calculated. We do NOT apply Thevenin Resistance or other resistance seen by capacitor at node B to calculate for R in the time constant.

$$v\_R_{B\_coeff} = 3.778 \cdot 10^3$$

$$\tau_{RC} := (3.778 \cdot 10^3) \cdot (9 \cdot 10^{-6}) = 0.034 \text{ s}$$

$$v_C(t) = V_0 \cdot e^{\frac{-t}{\tau}}$$

Schaum multipled the power of the exponential by 1000 so the time constant becomes a whole number instead of decimal.
The Author(s) Engineer(s) said it makes it easier to read and 10^3 is a typical base or reference used in electrical enigneering.
Agree-able as you will or may have seen time constants in many textbook usually are 1000t, 2000t, 3000t,...made you wonder why.

$$v_C(t) = V_0 \cdot e^{\frac{-1000\,t}{1000 \cdot \tau}}$$

Next plug in the values for Vo, and tau for vB.

$$\tau_{RC1000} := \tau_{RC} \cdot 1000 = 34.002 \qquad \tau_{RC} := 34$$

Making it whole number without decimal part.

$$V_0 := 17 \text{ V}$$

$$v_B = 17 \cdot e^{\frac{-1000\,t}{34}} \text{ V.}$$

Answer at t = 0 = 17V.

Capacitor current equation:

$$i_C = -C \cdot \left(\frac{dv}{dt}\right)$$

Negative sign we discussed previously, current out of capacitor moving in the -ve direction into node B, then splits to node A and C, with same -ve sign.

$$\frac{d\left(17 \cdot e^{\frac{-1000\,t}{34}}\right)}{dt} = \left(\frac{-1000 \cdot 17}{34}\right) \cdot e^{\frac{-1000\,t}{34}}$$

$$i_C = -\left(9 \cdot 10^{-6}\right) \cdot \left(\frac{-1000 \cdot 17}{34}\right) \cdot e^{\frac{-1000\,t}{34}}$$

$$= -\left(9 \cdot 10^{-6}\right) \cdot \left(\frac{-1000 \cdot 17}{34}\right) = 4.5 \cdot 10^{-3}$$

$$i_C = 4.5 \cdot 10^{-3} \cdot e^{\frac{-1000\,t}{34}}$$

This is the DECAYING current. Which gives the character to all the branch currents.

$$i = 4.5 \cdot 10^{-3} \cdot e^{\frac{-1000\,t}{34}}$$

<--- This was Schaums idea of multiplying by i, this i.

Discussion: In the i_M matrix we have only one value in equation 2', we had one capacitor. RHS of those equations only had one value in equation 2, the other two were zeros.
If there were more than 1 capacitor? NOT here.! Yes, from this experience, solving for each capacitor current will be challenging. Things don't get that a far...unrealistic the circuit has a purpose. Bigger circuit can be broken to smaller circuits. We don't just thrown stuff in the ciruit.

$$i(t) := 4.5 \cdot 10^{-3} \cdot e^{\frac{-1000\,t}{34}}$$

Set i_C to i(t).

i_0

$$= \quad 4.25 \cdot 10^{-3}$$

at t = 0

$i_C := 4.5 \cdot 10^{-3}$ A <----finally we got i_C

Continued on next page.

The voltages at node A, B. AND C:

$v\_R_{A\_coeff} \cdot i_C = 10.5$

$v\_R_{B\_coeff} \cdot i_C = 17$

$v\_R_{C\_coeff} \cdot i_C = 12$

Voltages with the exponential term (decaying):

$v_A(t) := 10.5 \cdot e^{\frac{-1000\,t}{34}}$

$v_B(t) := 17 \cdot e^{\frac{-1000\,t}{34}}$

$v_C(t) := 12 \cdot e^{\frac{-1000\,t}{34}}$

Next the answers for v_AB, v_AC, v_BC, i_AB, i_AC, and i_BC for t > 0 .

This may look like a vector subraction it may be. I am used to graphing vectors then making mistakes in their analysis, I find them tough. Maybe youre better at it than I am. Lets hope so. It is type of vector in exponential form and the signs on the voltages calculated recently further lends to it, easier compared to the graphing method. WRONG. Exponent power is not degrees here its time but there are those with degrees for the phase angle. Do check the theory on this vector thing and ask your lecturer.

$v_{AB} = 10.5 - 17 = -6.5$

$= (-6.5) \cdot e^{\frac{-1000 \cdot t}{34}}$ V. Answer.

$v_{AC} = 10.5 - 12 = -1.5$

$= (-1.5) \cdot e^{\frac{-1000 \cdot t}{34}}$ V. Answer.

$v_{BC} = 17 - 12 = 5$

$= 5 \cdot e^{\frac{-1000\,t}{34}}$ V. Answer.

Next for the iAB, iAC, and iBC voltages. We know the voltages across these resistors, we divide that voltage by the resistor value for the current. Results keep the same sign, +/-, as the voltage.

$$i_{AB} = \frac{v_{AB}}{2000} = \frac{(-6.5)\cdot\left(e^{\frac{-1000\cdot t}{34}}\right)}{2000} = -3.25\cdot 10^{-3}\cdot e^{\frac{-1000\,t}{34}}$$ A. Answer.

$$i_{AC} = \frac{v_{AC}}{6000} = \frac{(-1.5)\cdot\left(10^{-3}\cdot e^{\frac{-1000\cdot t}{34}}\right)}{6000} = -0.25\cdot 10^{-3}\cdot e^{\frac{-1000\,t}{34}}$$ A. Answer.

$$i_{BC} = \frac{v_{BC}}{4000} = \frac{(5)\cdot e^{\frac{-1000\,t}{34}}}{4000} = 1.25\cdot 10^{-3}\cdot e^{\frac{-1000\,t}{34}}$$ A. Answer.

Schaums Outline had something to say on this, *here in my words*.
Customarily, electrical engineers in manufacturing-laboratory type industries use V, mA, k-Ohm, ms for voltage, current, resistance, and time. In the electrical construction industry, lv/mv/hv work, its sometimes similar but most of the time its kV, A or kA, kOhm per meter, ms/s. In this example we seen some off the usual path of circuit problem solving, and one new technique on the 'i' at least it was for me. The time t was multiplied by 1000, 1000t. We seen textbook examples using the 1000 often the same 1000t. Sometimes assumed in problem solving. Maybe if its not written then its assumed 1000 especially when the time constant is in ? Whole numbers. Like here 34. That may had solved that 1000 mystery on the signal waveform(s). When time constant is a whole number that may lead you to conclude 1000t. Nothing specal here just using the 10^3 but specifically on the these values 1000t, 2000t, 3000t, 4000t,....pull your textbook.

Same answers in the above customary format:

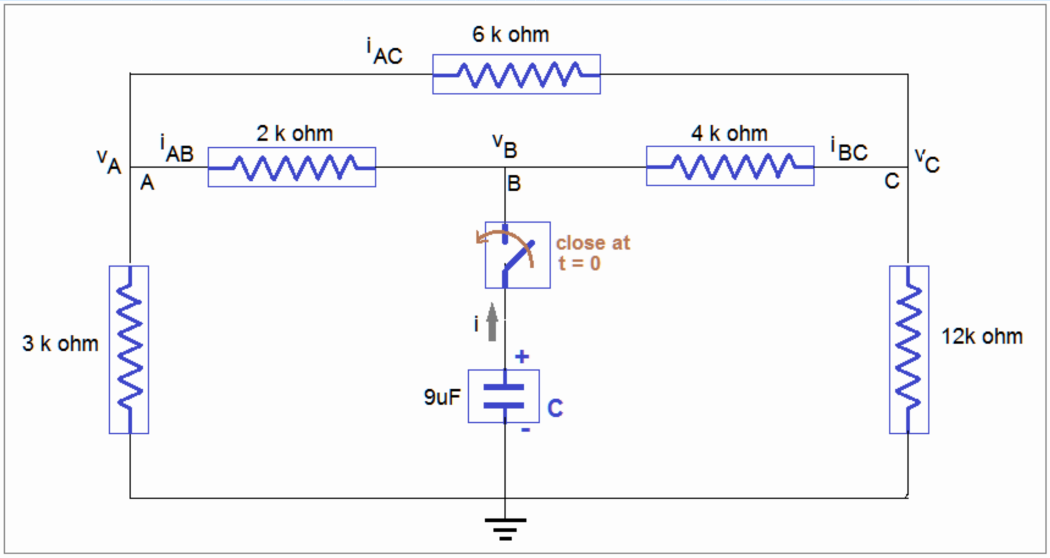
$v_A(t) := 10.5\cdot e^{\frac{-1000\,t}{34}}$ V. $v_{AB}(t) := -6.5\cdot e^{\frac{-1000\,t}{22}}$ V. $i_{AB}(t) := -3.25\cdot e^{\frac{-1000\,t}{34}}$ mA.

$v_B(t) := 17\cdot e^{\frac{-1000\,t}{34}}$ V. $v_{AC}(t) := -1.5\cdot e^{\frac{-1000\,t}{22}}$ V. $i_{AC}(t) := -0.25\cdot e^{\frac{-1000\,t}{34}}$ mA.

$v_C(t) := 12\cdot e^{\frac{-1000\,t}{34}}$ V. $v_{BC}(t) := 5\cdot e^{\frac{-1000\,t}{22}}$ V. $i_{BC}(t) := 1.25\cdot e^{\frac{-1000\,t}{34}}$ mA.

and $i(t) := 4.5\cdot e^{\frac{-1000\,t}{34}}$ mA.

Continued next page for the plots.
Note each variable is on a specific branch connected to a specific component.
Circuit figure re-shown below.



With increasing time, t>0, voltages and currents will die-out.
Since there is no voltage source in the circuit. This is what the plots show.

$\tau_{RC} := 0.034$ Plot between 0 and 5 tau or less.

$2 \cdot \tau_{RC} = 0.068$ $3 \cdot \tau_{RC} = 0.102$ $4 \cdot \tau_{RC} = 0.136$ $5 \cdot \tau_{RC} = 0.17$ **clear** (t)

clear (t)

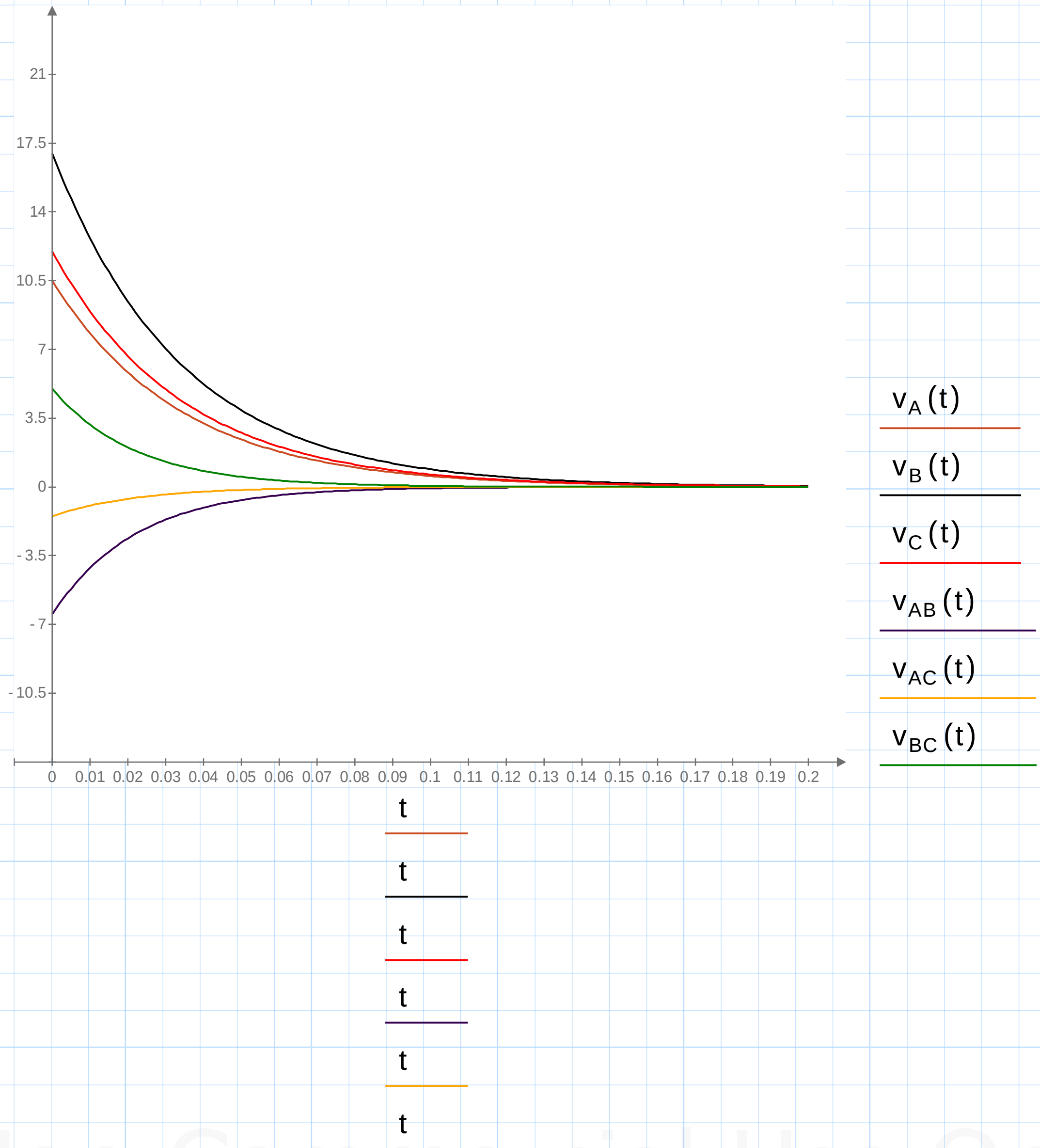
$t := 0.0, 0.001 .. 0.2$

As expected all the voltage and current plots settle to zero with t>0 in a source free circuit.

$v_A(t) := 10.5 \cdot e^{\frac{-1000\,t}{34}}$ V. $\quad v_{AB}(t) := -6.5 \cdot e^{\frac{-1000\,t}{22}}$ V.

$v_B(t) := 17 \cdot e^{\frac{-1000\,t}{34}}$ V. $\quad v_{AC}(t) := -1.5 \cdot e^{\frac{-1000\,t}{22}}$ V.

$v_C(t) := 12 \cdot e^{\frac{-1000\,t}{34}}$ V. $\quad v_{BC}(t) := 5 \cdot e^{\frac{-1000\,t}{22}}$ V.

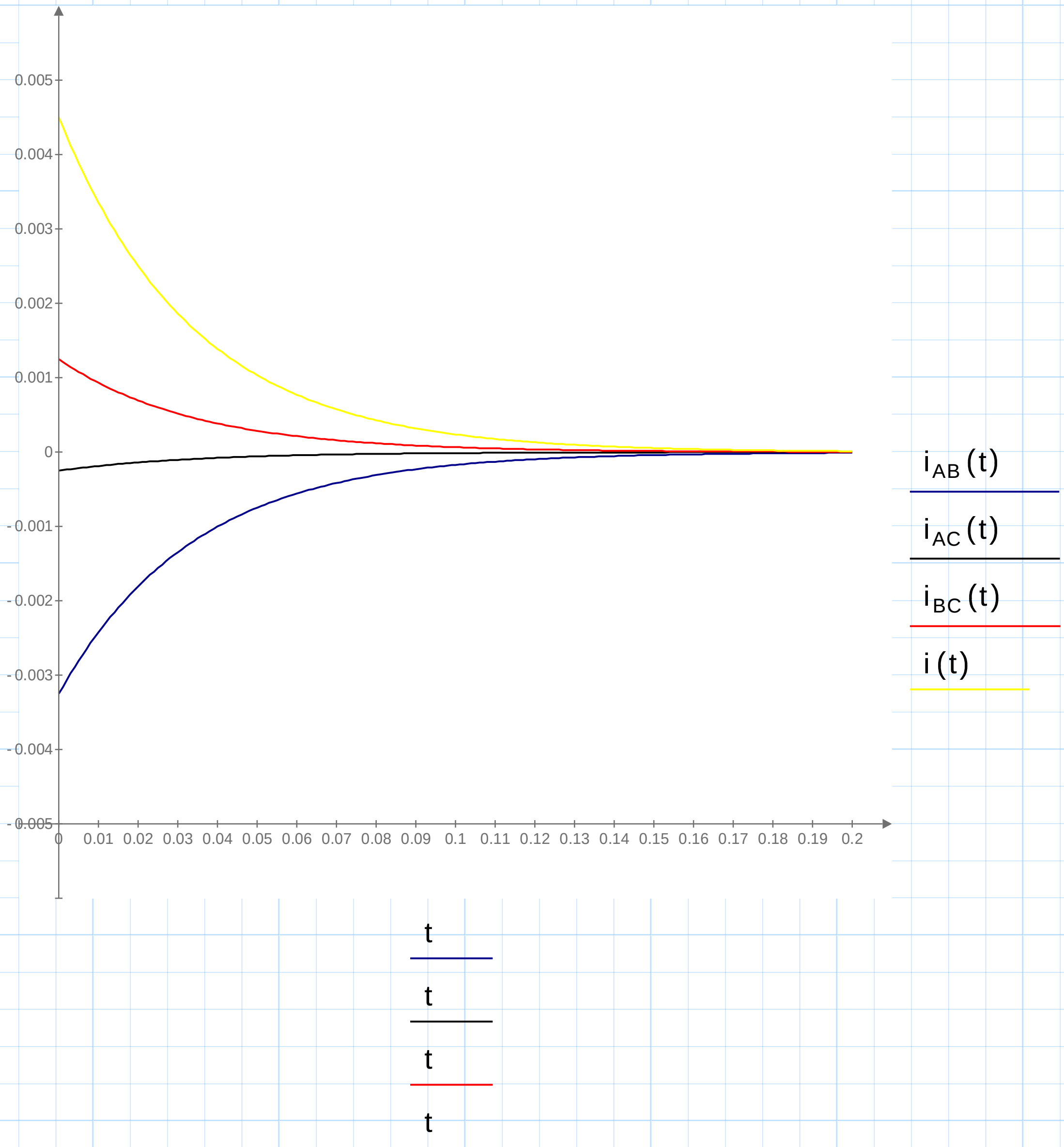


$i_{AB}(t) := -3.25 \cdot 10^{-3} \cdot e^{\frac{-1000\,t}{34}}$ A.

$i(t) := 4.5 \cdot 10^{-3} \cdot e^{\frac{-1000\,t}{34}}$ A.

$i_{AC}(t) := -0.25 \cdot 10^{-3} \cdot e^{\frac{-1000\,t}{34}}$ A.

$i_{BC}(t) := 1.25 \cdot 10^{-3} \cdot e^{\frac{-1000\,t}{34}}$ A.



End of example 7.6.
A super good example, you improve on it for any errors and omissions.

7.8 DC Steady State in Inductors and Capacitors.

How could there be a steady state in a source free RC and RL circuit?
Cannot be. The current and voltage would die out with time.

Only if there is a DC source which would be a dc voltage or dc current source.
So when we have the dc sources, the transient part dies out the dc steady state continues.

Question: Is the voltage and current at the time when the transient exist added on to the dc values? This needs to be found out. OR do the dc steady state current and voltages start after the transients die out. This is what Schaums has to say, next.

Schaums: The natural exponential component of the response of RL and RC circuits to step inputs diminishes as time passes. At t=infinity, the circuit reaches steady state, and the response is made up of the forced dc component only.

At the dc steady state of RLC circuits all currents and voltages are constant.
This may pose a operating problem to the inductor and capacitor.

Why?

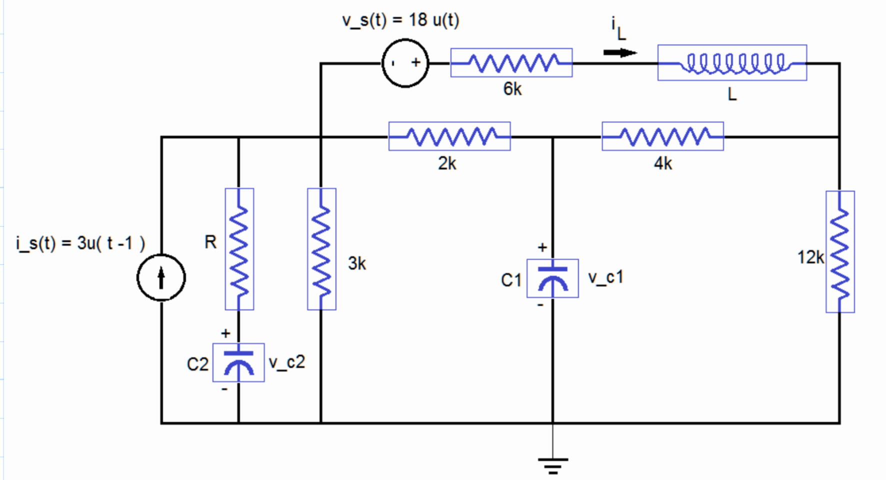
v_L = L (di/dt) constant current means no di/dt = 0 so voltage = 0.
voltage = 0 means there is a? SHORT CIRCUIT across the inductor.
VOLTAGE AT BOTH ENDS OF THE INDUCTOR IS THE SAME (ie 0)
so that part of the connection is brought to the same point (ie the terminals of the inductor) which makes the circuit shorted. Current does flow thru the short circuit, the inductor is no longer operating as an inductor, its shorted. *Current flows so shorted.*

i_C = C (dv/dt) constant voltage means no dv/dt = 0 so current = 0.
current = 0 means there is a? OPEN CIRCUIT across the capacitor.
CURRENT AT BOTH ENDS OF THE INDUCTOR IS THE SAME (ie 0)
so that part of the connection is separated betwen the two points (ie the terminals of the capacitor) which makes the open circuit.
Voltage does not appear across the terminals, opened, so no current flows through the two terminals, the capacitor is no longer operating as an capacitor, its opened. *Voltage is across so opened.*

So what now? We analyse the circuit with short circuit and open circuit in the electric circuit. The circuit without the inductor and capacitor is all resistive circuit. NO DIFFERENTIAL EQUATIONS INVOLVED because NO inductor and capacitors.
You better study thru this with your lecturer and textbook.

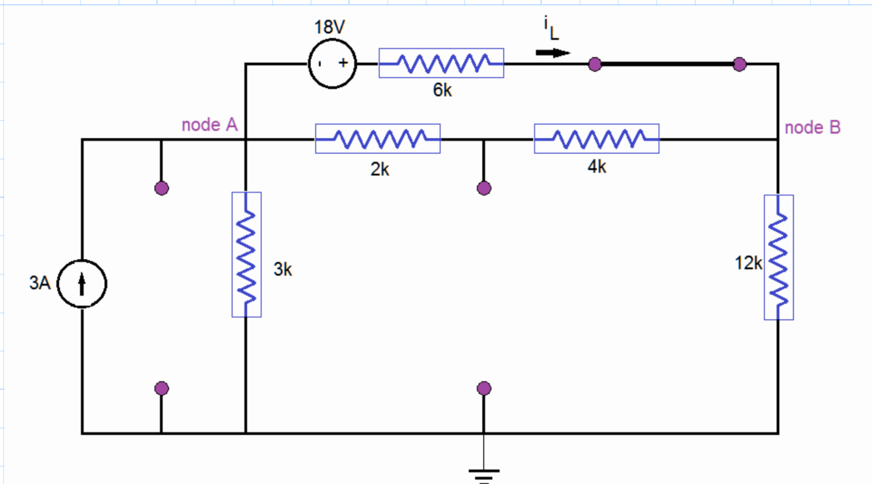
Example 7.7: DC Steady State In Inductors and Capacitors.

Find the steady-state values of i_L, v_C1, and v_C2 in the circuit below.



Solution:

The transient state has passed the circuit has reached the steady state.
In steady state the voltage and current are constant.
So? Inductor turned into a short circuit AND capacitor turned into a open circuit.
Next we update the circuit diagram reflecting this. Shown below.



Time t>0. The voltage is unit step u(t) its now 18 Volts.
The current is also unit step with a 1 second delay or behind the voltage (t-1)
but in the steady state (t>0) we see/notice it had reached 3A.

The voltage appears across a component, thats how we measure it? Yes. So if thats a unit step, surely the current will be of some similar shape, unless we processed it to something else first, which we did NOT.

We have 2 nodes in the new circuit.

Apply KCL or KVL?

For KVL we have 3 loops/mesh so that results in 3 equations.

For KCL we have 2 nodes so that results in 2 equations.
Decision: KCL.

This may not be the reasoning for every circuit solution. Do NOT hold on to it like its law, there maybe cases where KVL is more suitable.
Since all the resistors are in k ohm we can cancel it to single digit.

KCL at node A:

$$3-\left(\frac{vA}{3}\right)-\left(\frac{vA-vB}{2+4}\right)+\left(\frac{vA-vB+18}{6}\right) = 0$$

Current source only source into node the rest of branches current is leaving so -ve sign that.

$$\left(\frac{vA}{3}\right)+\left(\frac{vA-vB}{2+4}\right)+\left(\frac{vA-vB+18}{6}\right) = 3$$

$$\left(\frac{vA}{3}\right)+\left(\frac{vA-vB}{6}\right)+\left(\frac{vA-vB+18}{6}\right) = 3$$ multiply by 6

$$(2\cdot vA)+(vA-vB)+(vA-vB+18) = 18$$

$$(4\cdot vA)-2\cdot vB+18 = 18$$

$$2\cdot vA-vB = 0$$ Equation 1.

KCL at node B:

$$\left(\frac{vB}{12}\right)+\left(\frac{vB-vA}{6}\right)+\left(\frac{vB-vA-18}{6}\right) = 0$$ Note: -18.

The voltage source was +ve in the previous equation, its negative here because the current would be flowing in the opposite direction in (vb - vA) compared to (vA - vB), so the voltage is -18.

Multiply by 12.

$$(vB)+(2\cdot vB-2\cdot vA)+(2\cdot vB-2\cdot vA-36) = 0$$

$$-4\cdot vA+5\cdot vB = 36$$ Equation 2.

$2 \cdot vA - vB \quad = \quad 0$ Equation 1.

$-2 \cdot vA + 5 \cdot vB \quad = \quad 36$ Equation 2.

Equation 1 add to 2.

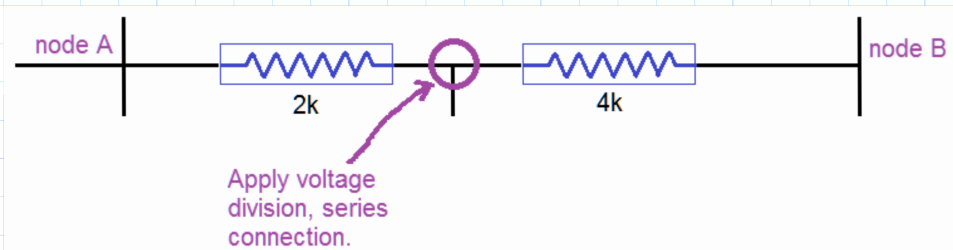
$4 \cdot vB \quad = \quad 36$

$vB \quad = \quad 12$ Substitute in equation 1 to solve for vA.

$2 \cdot vA - 12 \quad = \quad 0$

$vA \quad = \quad 6$ V. This is the voltage for vC2 at node A.

$vC2 \quad = \quad 6$ V. Answer.



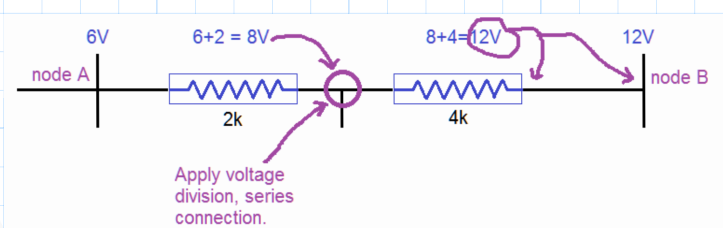
Total resistance = 2 + 4 = 6k

Voltage across 2K = $\left(\left(\frac{2 \cdot k}{6 \cdot k}\right)\right) \cdot (vB - vA)$

$\left(\left(\frac{2 \cdot k}{6 \cdot k}\right)\right) \cdot (12 - 6) = 2$ V

Voltage across 4K = $\left(\left(\frac{4 \cdot k}{6 \cdot k}\right)\right) \cdot (vB - vA)$

$\left(\left(\frac{4 \cdot k}{6 \cdot k}\right)\right) \cdot (12 - 6) = 4$ V



Voltage at node where capacitor C1 connects:

$vC1 \quad = \quad 8$ V Answer.

KCL at node A again:

$$3-\left(\frac{vA}{3}\right)-\left(\frac{vA-vB}{2+4}\right)+\left(\frac{vA-vB+18}{6}\right) = 0$$

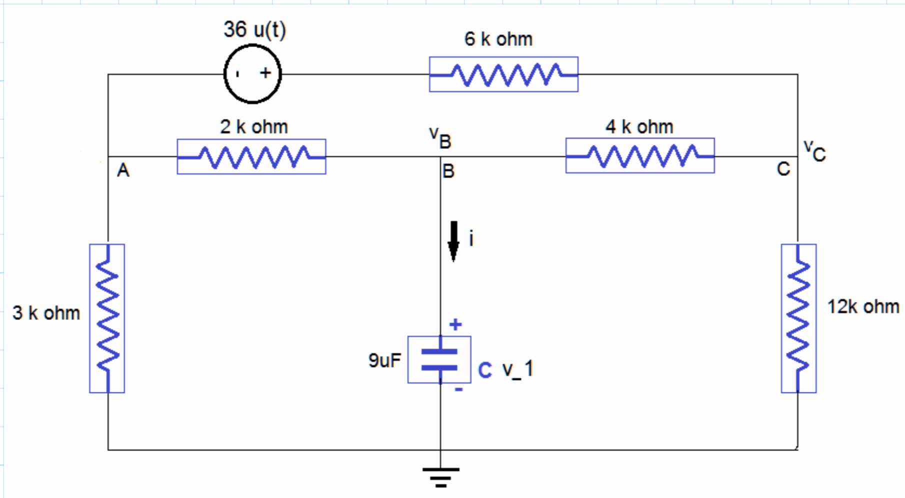
The last term on the LHS is the current in the inductor branch.

$$i_L = \frac{6-12+18}{6\cdot 1000} = 0.002$$ A. Current thru the inductor.

i_L = 2 mA Answer. *Enjoyed this example cleared my mind on that 'how could it be situations'.*

Example 7.8: DC Steady State In Inductors and Capacitors.

Find i and v in the circuit below:



Solution:
Similar to circuit in example 7.6 except for the addition of a voltage source 36 u(t), and we do not have a switch. No switch means the circuit is closed at time t = 0. It starts with a voltage of 36 V and stays constant.

We may say the capacitor was NOT charged prior to t<0 OR at t = 0. Why? Because we are not given such information and if we were there should be a switch at least. So we may say the voltage across the capacitor equal 0.

Current i is flowing thru the capacitor from the + to - terminal, opposite to example 7.6. Because the voltage source is supplying the current.

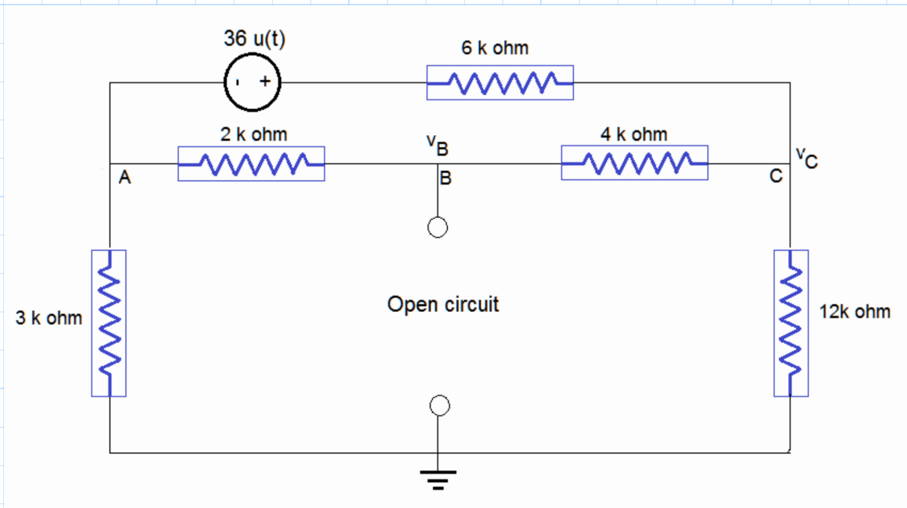
We can use some of the calculations from example 7.6.
Especially the Thevenin and Norton equivalents.

We may reuse the 3 simultaneous equations but need to update for the 36V at node A and B.

Under DC steady state conditions, the current across the capacitor would be constant, dv/dt, would result in 0. We have an open circuit

The circuit connection, resistor and capacitor values are the same, so the time constant will be the same.

$\tau_{RC} := 34$ after multiplying by 1000.



KCL at node A:

$$\left(\frac{vA - 0}{3 \cdot k}\right) + \left(\frac{vA - vB}{2 \cdot k}\right) + \left(\frac{vA - vC - 36}{6 \cdot k}\right) = 0$$

-36V in LHS because current from vA node enters DC voltage source thru -ve terminal. Then for vC-vA should be positive 36V.

$$(2 \cdot vA - 0) + (3 \cdot vA - 3 \cdot vB) + (vA - vC - 36) = 0$$ multiplied by 6

$$6 \cdot vA - 3 \cdot vB - vC = 36$$ Equation 1

KCL at node B:

$$\left(\frac{vB - vA}{2 \cdot k}\right) + \left(\frac{vB - vC}{4 \cdot k}\right) = 0$$

Example 7.6 equation 2 not exactly the same, capacitor is not here now since its a DC source at t=0 and t>0. Open circuit.

$$(2\ vB - 2\ vA) + (vB - vC) = 0$$

$$-2 \cdot vA + 3 \cdot vB - v \cdot C = 0$$

<---i? It not needed since the capacitor is not generating current, open circuit at t<0, or t=0. In reference to example 7.6

KCL at node C:

$$\left(\frac{vC - vB}{4 \cdot k}\right) + \left(\frac{vC - vA + 36}{6 \cdot k}\right) + \left(\frac{vC - 0}{12 \cdot k}\right) = 0$$

the other equation for - 36V.

$$(3 \cdot vC - 3 \cdot vB) + (2 \cdot vC - 2 \cdot vA + 72) + (vC) = 0$$

multiplied by 12

$$(6 \cdot vC - 3 \cdot vB) + (-2 \cdot vA) = -72$$

$$-2 \cdot vA - 3 \cdot vB + 6 \cdot vC = -72$$

Equation 3

The 3 equations:

$$6 \cdot vA - 3 \cdot vB - vC = 36$$ Equation 1.

$$-2 \cdot vA + 10\ vB - 6 \cdot vC = 0$$ Equation 2.

$$-2 \cdot vA - 3 \cdot vB + 6 \cdot vC = -72$$ Equation 3.

Resistance matrix:

$$v\_ovr\_R := \begin{bmatrix} 6 & -3 & -1 \\ -2 & 3 & -1 \\ -2 & -3 & 6 \end{bmatrix}$$

Inverse resistance matrix:

$$(v\_ovr\_R)^{-1} = \begin{bmatrix} 0.417 & 0.583 & 0.167 \\ 0.389 & 0.944 & 0.222 \\ 0.333 & 0.667 & 0.333 \end{bmatrix}$$

RHS of equations (1 -3), not current rather values to solve for voltages in v_m matrix.

$$v_M := \begin{bmatrix} 36 \\ 0 \\ -72 \end{bmatrix}$$

Solution for the coefficients:

$$v\_over\_R_{coeff} := (v\_ovr\_R)^{-1} \cdot v_M = \begin{bmatrix} 3 \\ -2 \\ -12 \end{bmatrix}$$

$$v\_over\_R_{coeff} = \begin{bmatrix} 3 \\ -2 \\ -12 \end{bmatrix}$$

$$vA := v\_over\_R_{coeff_0} = 3$$

$vB := v\_over\_R_{coeff_1} = -2$ V. <--- The voltage at node B from DC analysis. Answer.

$$vC := v\_over\_R_{coeff_2} = -12$$

vB has the particular solution v_p, here found to be -2V, and the homogeneous solution (or natural response) v_h, so the total of which is

$$v(t) = V_0 \cdot \left(1 - e^{\frac{-t}{R \cdot C}}\right) \cdot u(t) \qquad V_0 := 2$$

$v(t) = 2 \cdot \left(1 - e^{\frac{-1000\,t}{34}}\right) \cdot u(t)$ V. Voltage across capacitor. Answer.

$$i(t) = C \cdot \left(\frac{dv}{dt}\right)$$

$$\frac{d}{dt}\, 2 \cdot \left(1 - e^{\frac{-1000\,t}{34}}\right) = \left(\frac{-2000}{34}\right) \cdot e^{\frac{-1000 \cdot t}{34}}$$

$$i(t) = 9 \cdot 10^{-6} \cdot \left(\frac{-2000}{34}\right) \cdot e^{\frac{-1000 \cdot t}{34}}$$

$i(t) = -0.529 \cdot 10^{-3} \cdot \left(e^{\frac{-1000 \cdot t}{34}}\right) \cdot u(t)$ mA. Current through the capacitor. Answer.

Lets plot the graphs for the capacitor voltage and current.

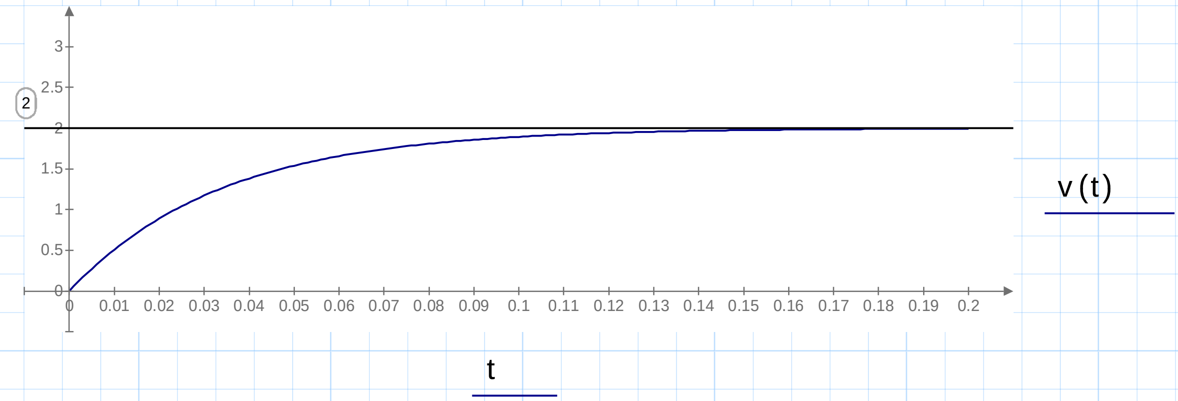
clear(t)

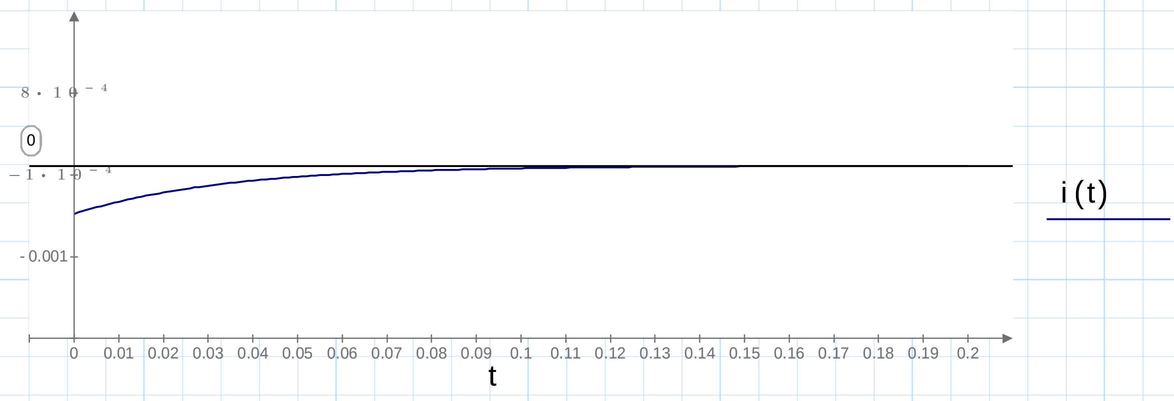
$\tau_{RC} := 0.034$ Plot between 0 and 5 tau or less.

$2 \cdot \tau_{RC} = 0.068$ $3 \cdot \tau_{RC} = 0.102$ $4 \cdot \tau_{RC} = 0.136$ $5 \cdot \tau_{RC} = 0.17$

$$v(t) := 2 \cdot \left(1 - e^{\frac{-1000\,t}{34}}\right) \qquad i(t) := -0.529 \cdot 10^{-3} \cdot \left(e^{\frac{-1000 \cdot t}{34}}\right)$$

$t := 0.0, 0.001 .. 0.2$





Observation:
Capacitor current gets closer to zero, the capacitor voltage gets closer to 2V.

u(t) is a unit step voltage source, it comes on, takes time to build up the voltage across the capacitor branch, while the current decreases to zero in this branch all over a time t>0. Current decreased because for capacitor its dependent on (dv/dt) which has become 0. At t>0, u(t) had come on it increased from 0 to 36u(t), during this time we had a (dv/dt), there was that current we show in the plot. Next it became constant at 36V.
The voltage on the capacitor branch of the circuit maintained a value of -2V....because its been charged up to it in the time t>0, while the current diminished to zero in the time t>0. The capacitor now is a? Energy storage device, stored charge which is potential. Well a small one. <--You disucss.

7.9 Transition at Switching Time.

1). Sudden switching of a source or a jump in its magnitude can translate into sudden jumps in voltages or currents in a circuit.

2). A jump in the capacitor voltage requires an impulse current.
3). A jump in the inductor current requires an impulse voltage.

4). If NO such impulses can be present, the capacitor voltages and the inductor currents remain continous.

5). Therefore, the post-switching conditions of L and C can be derived from their pre-switching conditions.

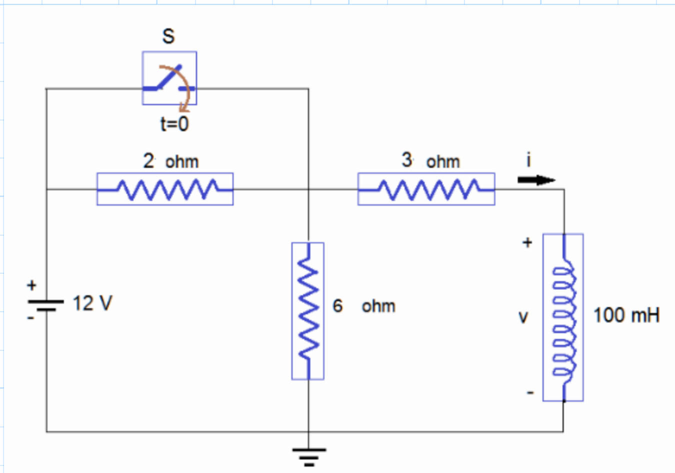
.....Schaum's page 153 chapter 7, 6th edition.

Point number 5 here is saying to determine the conditions at t>0, the conditions at t<0 and when appropriate t=0 can assist in determining those conditions when t>0.

The sudden switching condition results in a surge, spike, jump in magnitude. The cause of the sudden switching maybe lightning, a high current switch changed state from open to close or close to open.

Example 7.9 Transition at Switching Time.

In the circuit below the switch S is closed at t = 0.
Find i and v for all times.



The switch is closed at t = 0.

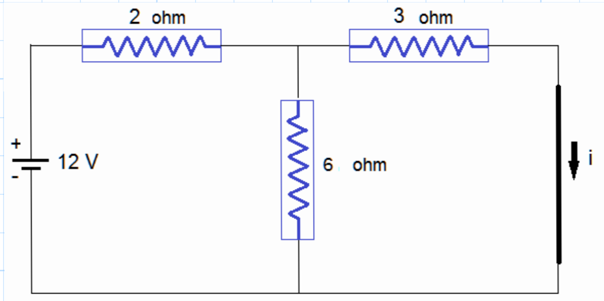
There is the pre-switching conditions at t<0.

At t>0 are the post switching conditions.

Solution:

Before the switch is closed the circuit is experiencing a STEADY STATE CONDITION, the voltage source supplies 12V dc. Thus the current is also dc current, which makes (di/dt) = 0. Hence the inductor is a short circuit, shown in heavy line in circuit below.

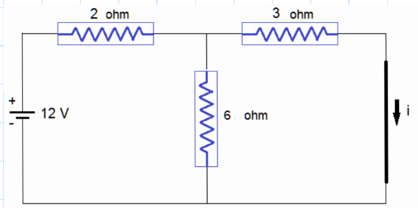
This circuit shown below with inductor shorted.



Equivalent resistance:

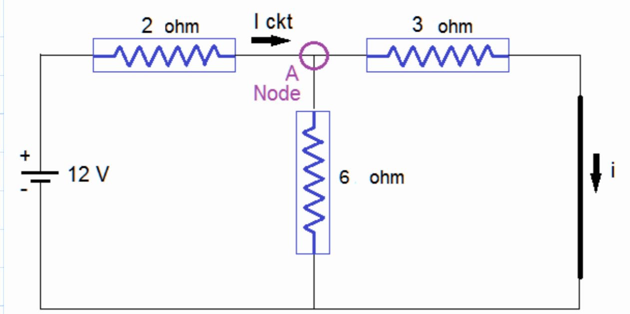
$$R3\_6 := \frac{3 \cdot 6}{3+6} = 2 \text{ ohm}$$

$$R_{eq} := 2 + 2 = 4 \quad \text{ohm}$$



Ickt = 12V/R_eq.

$$I_{ckt} := \frac{12}{4} = 3 \quad \text{A}$$



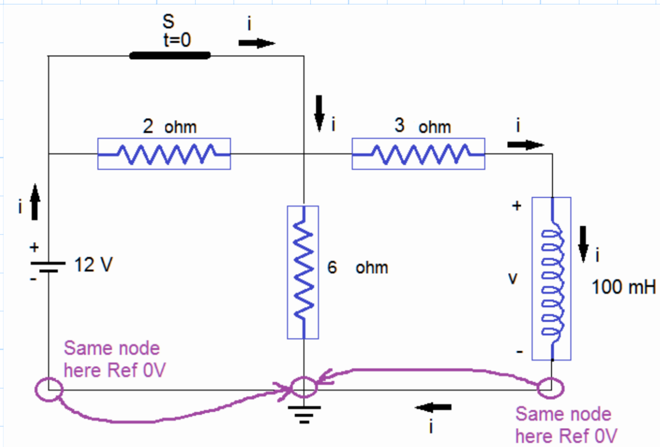
At node A current entering node A is Ickt = 3 A.

The 6 and 3 k ohm resistors are in parallel so we can apply current division, to find the current in the 3 ohm branch, inductor current, which is the branch with the inductor short circuited.

$$iL\left(0^{-}\right) \quad = \quad \left(\frac{6}{(6+3)}\right) \cdot 3 = 2 \quad \text{A.}$$

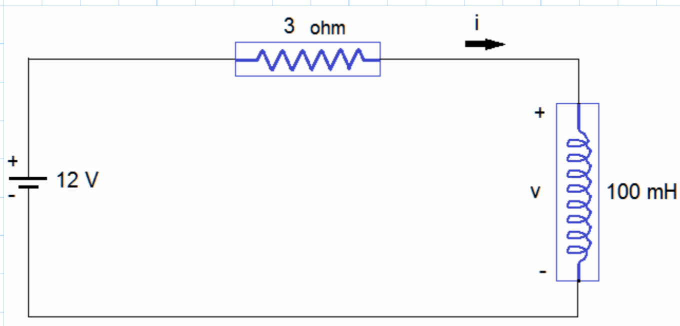
iL(0-) is iL(t<0) pre-switching condition at time t less than 0.

Next for the situation when the switch is closed at t = 0.
What conditions exist then.



The switch branch is the bypass branch current does not flow thru the 2 ohm resistor.

So the circuit under condition switch t is closed is shown below.



This is that circuit when switch is closed. Where is the 6 ohm? The current at the node between 3 and 6 ohm goes in the direction of the 3 ohm, because of the lower resistance. The inductor is shorted effectively, current passes but no voltage build up. There is no load.

We calculate (Vo/R) this is the final current.

In the above circuit, the inductor is experiencing a constant current from a constant voltage source so now under the switched condition the current thru the inductor (now shown short circuited) equals:

$$iL\,(t > 0) \quad = \quad \frac{12}{3} = 4 \quad \text{A.}$$

From the circuit above the time constant for the RL circuit is:

$$\tau_{RL} := \frac{100 \cdot 10^{-3}}{3} = 0.033 \quad \text{L/R.}$$

0.033 in fraction form is $\frac{1}{30} = 0.033$

(1/tau_RL) = L/R = 30/1 = 30 s. <---This is for in the exponential expression where its (1/tau).

Continued next page.

Inductor current and voltage t<0:

t<0 : 2 A 0 V...zero volt because L is short circuited,
both terminals are at the same point.

Inductor current and voltage at t>0:

t>0 : We apply the exponential
form for current and voltage.

$$\frac{1}{\tau_{RL}} = 30 \quad \text{s.}$$

$$i_L(t) := \left(\frac{V_0}{R}\right) \cdot \left(1 - e^{-\left(\frac{R}{L}\right) \cdot t}\right)$$

$$i_L(t) := \left(\frac{V_0}{R}\right) - \left(\frac{V_0}{R}\right) \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$$

$$i_L(t) = (\text{Final}) - (\text{Initial}) \cdot e^{-\left(\frac{R}{L}\right) \cdot t}$$

The LHS first term is final current 4A,
followed by the second term which is the intial current, 2A.

$$i_L(t) := 4 - 2 \cdot e^{-30 \cdot t}$$

Here is where the forgetfulness seeps in how to find final V.
v_L = L (di/dt) we perform a differentiation on i(t). Not an initial condition.
For L differentiate i(t) for C its v(t).

d (i_L(t)) / dt = $60 \cdot e^{-30 \cdot t}$

$$v_L(t > 0) = L \cdot \left(\frac{di}{dt}\right) = 100 \cdot 10^{-3} \cdot 60 \cdot e^{-30 \cdot t}$$

$$v_L(t > 0) = 6 \cdot e^{-30 \cdot t}$$

Inductor current and voltage at t>0:

$$i_{L\_grt\_0}(t) := 4 - 2 \cdot e^{-30 \cdot t} \qquad v_{L\_grt\_0}(t) := 6 \cdot e^{-30 \cdot t}$$

Show the jump in magnitude in graphs:
Current and voltage graphs, at times t<0, t=0, and t>0.

clear (t , t1 , t2)

$\tau_{RL} = 0.033$ Plot between 0 and 5 tau or less.

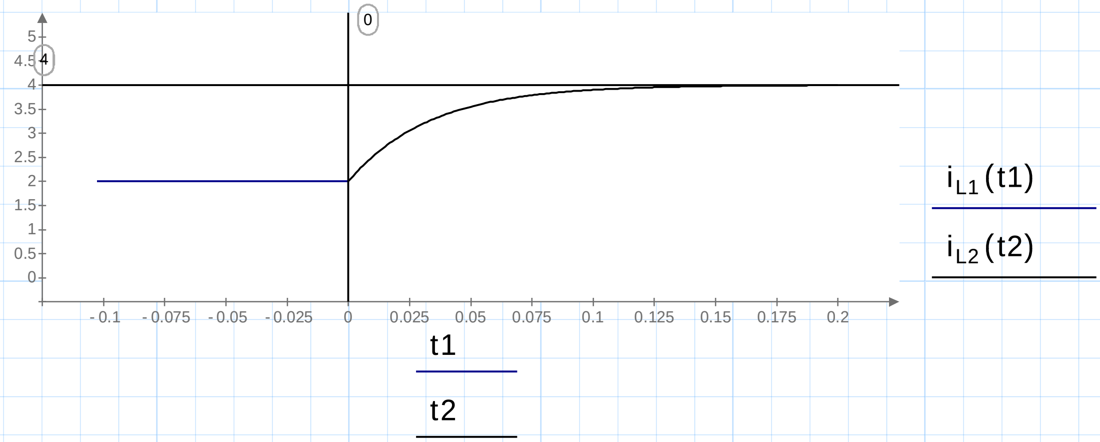
$2 \cdot \tau_{RL} = 0.067$ $3 \cdot \tau_{RL} = 0.1$ $4 \cdot \tau_{RL} = 0.133$ $5 \cdot \tau_{RL} = 0.167$

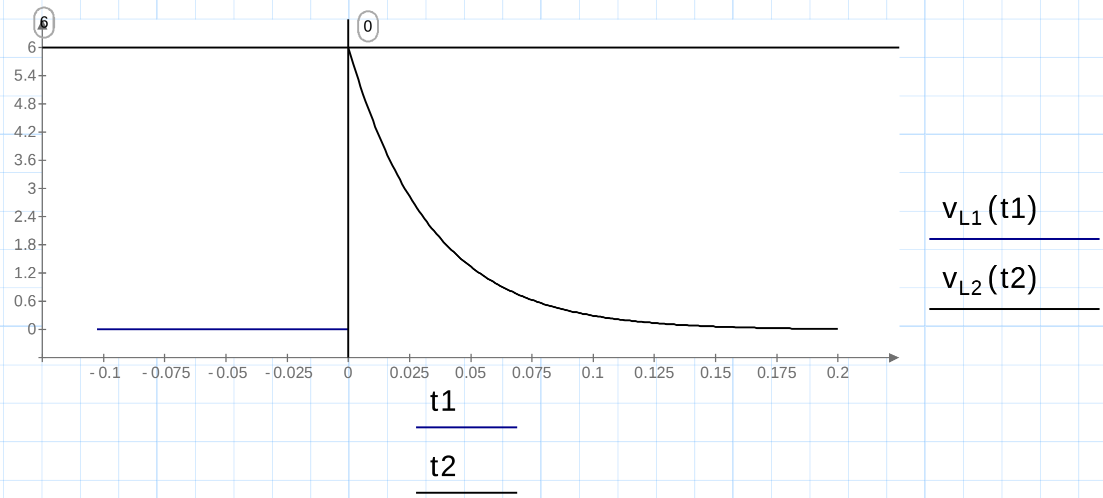
$t1 := -0.15, -0.125 .. 0.001$

$i_{L1}(t1) := 2$ $v_{L1}(t1) := 0$

$t2 := 0.0, 0.001 .. 0.2$

$i_{L2}(t2) := 4 - \left(2 \cdot e^{-(30 \cdot t2)}\right)$ $v_{L2}(t2) := 6 \cdot e^{-30 \cdot t2}$

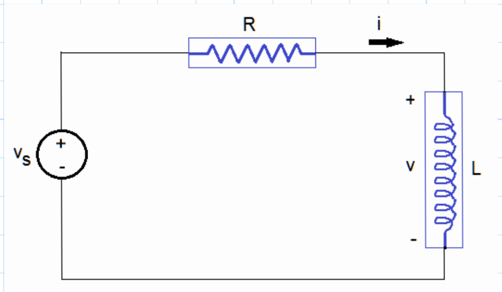




At time t =0 see the jump in magnitude of current and voltage.
Shown above same as Schaums Outline solution.

Example 7.10 Transition at Switching Time.

Find i and v for t=0 and t>0 in the circuit below.
Given R = 5 ohm.
L = 10 mH.
and
v_s = 5V for t <0.
v_s = 5 sin wt V for t >0.



Solution:

This is that problem where the explanation is founded on "you should know what i meant, of course it wasn't that'. So at the end the plots will help solve those questions. After having done this example already, post-mortem, the concern is, is there a surge higher than that could be caused by the voltage source in its two conditions before t =0 and after t = 0. The jump in voltage is what we are looking for not the change in voltage wave form. This is where I failed in the understanding of the question-solution. Should be looking for how the waveform impact the circuit rather than a change in amplitude which was greater than V = 5V and this in both the dc and sinusoidal case.

Time t less than zero (t < 0):

We have voltage equal 5V, continous 5V, as given above.
From t<0 to t =0.

$V_{less0} := 5$ $R := 5$

$i_{less\_0} := \frac{V_{less0}}{R} = 1$ A.

Current thru the inductor:
Since the voltage source is constant the current would be constant at 1 A.
What is the voltage across the inductor at (t<0):

v_L(t<0) = L (di/dt)
di/dt = 0
v_L(t<0) = 0.

Time t = 0, transition from -t to 0:
The inductor current is the same at t =0 either from the dc or sinusoidal waveform.
i(t<0) = i(t=0). *See this later NOT the waveform rather the magnitude.*

Time t>0, from 0 to t>0:

Here the voltage is sinusoidal with an amplitude of 5. No change in magnitude from V = 5 V dc. We see no impulse because its at 5V.
5 sin (wt) V, amplitude 5.

$V_{grt\_0}(t) \quad = \quad 5 \cdot \sin(\omega \cdot t) \qquad V_{amp} := 5$

$i_{grt\_0} := \frac{V_{amp}}{R} = 1 \qquad$ A.

i(t<0) to i(t>0) = 1A. Answer.

What is the voltage across the inducor at t>0:
Since the voltage is sinusoidal, a change in voltage will be seen oscillating over the y=0 axis. But the amplitude of the voltage is 5V. The sine wave does travel from peak to peak over the period T. *w = 2 Pi f, f = w/2 Pi, 1/f =T = 2 Pi/w.*

Tricky solution. How do we solve from here?
Schaums Outline instructs us to form a KVL equation for the circuit under the condition t>0.

$v_s(t>0) \quad = \quad R \cdot i(t>0) + v\_L(t>0)$

$v_s(t>0) \quad = \quad$ vs(t=0) $= 5 \cdot \sin(\omega\, t)$ at t =0 $= 5 \cdot \sin(\omega \cdot 0)$ at t =0 $= 5 \cdot \sin(0) = 0$

Why would the voltage across the inductor at time t>0 be equal to t=0 and t<0?
Lets look at the transition from t<0 to t>0, it passes thru t =0.

Voltage here at t=0 is 0V.
As time passes the sine wave goes from 0 to a peak of 5V and -ve 5V later as it oscillates about 2 Pi. Sin(0)=0, Sin(90)=1, Sine(180)=0 and sin(270)=-1.
From 0 to 90 degs or 0 to Pi/2 the voltage rises from 0 to 5V.
This time period is t>0. It passes thru t=0. Capacitor voltage at t =0 is sin(0) = 0.

Since the voltage source is not generating an impulse the current 1A will not result with any impulse either. The voltage is continous. But it goes from a DC to a SINUSOIDAL. The Authors-Engineers do not make a case on this. Why? They said the current is continous, though the sinusoidal may oscillate between 5V and -5V, the peak amplitude is the same. So their reasoning is the current for the best part is continous between 0 and 1A. At t=0 current is 0, but just immediately before and after its 1A. See plots.

So they said it will be a continous uninterupted current thru the inductor.
$R \cdot i(t>0) \quad = \quad 5 \cdot 1 = 5 \qquad$ V.
Re-arranging the equation: $\quad v\_L(t>0) = v_s(t>0) - R \cdot i(t>0)$ $= 0 - 5$ V <---This equation shown above this page.

$v\_L(t>0) = -5V$ Answer. Lets plot to satisfy the answer.

clear $(t, t1)$

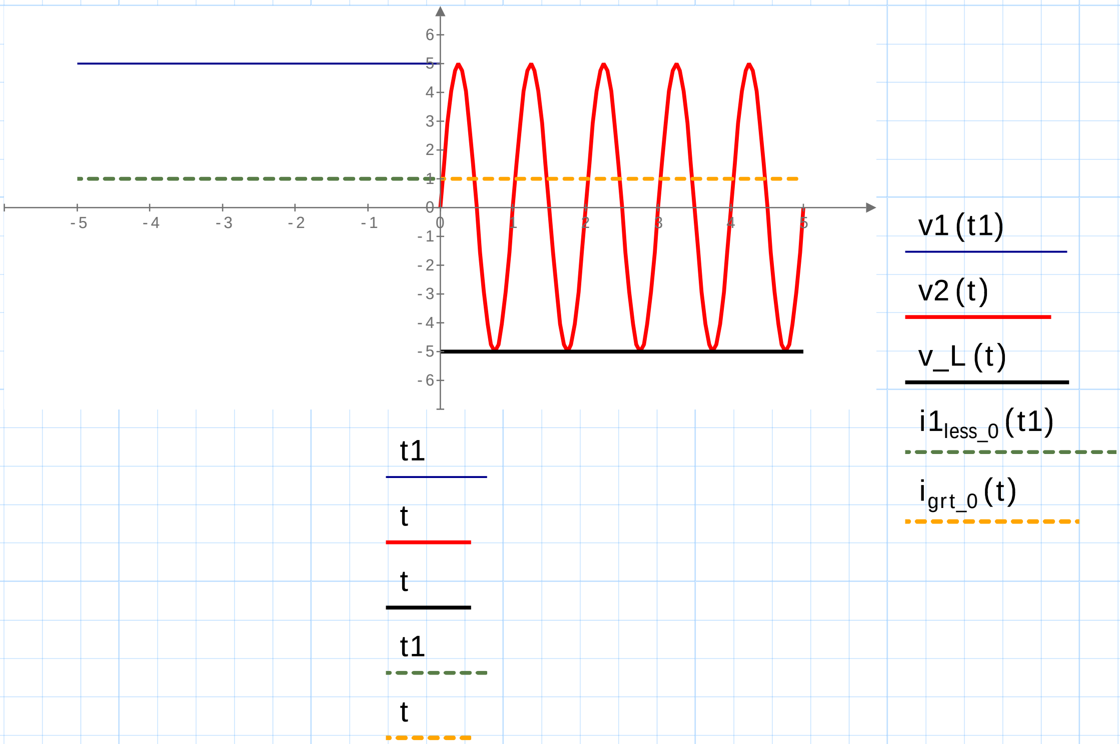
$t1 := -5, -4 .. 0$ $\quad t := 0, 0.05 .. 5$

$\omega := 2 \cdot \pi = 6.283$

$v1(t1) := 5$ $\quad v2(t) := 5 \cdot \sin(\omega \cdot t)$

$v\_L(t) := -5$

$i1_{less\_0}(t1) := 1$ $\quad i_{grt\_0}(t) := 1$



Discussion: The reasoning may not be satisfactory at first, but the key point I think/ sense is there is no voltage impulse. From 5V dc to '+5V to -5V sinusoidal' here the amplitude is 5 V for the sinusoidal it does NOT rise higher than 5V dc. So this is saying its normal and no impulse is present. If it was 20V somewhere then that would be a jump. So we do not see a jump in voltage maximum magnitude, the current should be the same as a maximum at 1A. For for t<0, t=0, and t>0, current maximum is 1A and voltage maximum is -5V and 5V.

*NEXT SECTIONS TAKE ON A NEW AREA IT DEALS WITH THE INPUT TYPE TO THE CIRCUIT AND THE RESPONSE OF THE CIRCUIT THE OUTPUT TO EXPECT. NOT A GOOD FEELING MAYBE, THIS MAYBE WHERE STUDENTS DECIDE TO QUIT EE. THIS CHAPTER IS THAT DECISION MAKING POINT WHETHER TO **CONTINUE/HOLD-ON/DROP-OUT**. THE R L AND C ANALOG COMPONENTS RUN THRU THE WHOLE ELECTRICAL ENGINEERING DURATION OF STUDIES. PATIENCE? OR A PATIENT LECTURER MAY GET THE AVERAGE STUDENT THRU. Suggest go at it slowly there is no need to rush.*