
AP[®] Calculus AB

Sample Student Responses and Scoring Commentary

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Free Response Question 2

- Scoring Guideline**
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Part A (AB): Graphing calculator required**Question 2****9 points****General Scoring Notes**

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

Scoring guidelines and notes contain examples of the most common approaches seen in student responses. These guidelines can be applied to alternate approaches to ensure that these alternate approaches are scored appropriately.

A particle, P , is moving along the x -axis. The velocity of particle P at time t is given by $v_P(t) = \sin(t^{1.5})$ for $0 \leq t \leq \pi$. At time $t = 0$, particle P is at position $x = 5$.

A second particle, Q , also moves along the x -axis. The velocity of particle Q at time t is given by $v_Q(t) = (t - 1.8) \cdot 1.25^t$ for $0 \leq t \leq \pi$. At time $t = 0$, particle Q is at position $x = 10$.

Model Solution	Scoring	
(a) Find the positions of particles P and Q at time $t = 1$.		
$x_P(1) = 5 + \int_0^1 v_P(t) dt = 5.370660$	One definite integral	1 point
At time $t = 1$, the position of particle P is $x = 5.371$ (or 5.370).	One position	1 point
$x_Q(1) = 10 + \int_0^1 v_Q(t) dt = 8.564355$	The other position	1 point
At time $t = 1$, the position of particle Q is $x = 8.564$.		

Scoring notes:

- The first point is earned for the explicit presentation of at least one definite integral, either $\int_0^1 v_P(t) dt$ or $\int_0^1 v_Q(t) dt$.
- The first point must be earned to be eligible for the second and third points.
- The second point is earned for adding the initial condition to at least one of the definite integrals and finding the correct position.
- Writing $\int_0^1 v_P(t) + 5 = 5.370660$ does not earn a position point, because the missing dt makes this statement unclear or false. However, $5 + \int_0^1 v_P(t) = 5.370660$ does earn the position point because it is not ambiguous. Similarly, for the position of Q .
- Read unlabeled answers presented left to right, or top to bottom, as $x_P(1)$ and $x_Q(1)$, respectively.
- Special case 1: A response of $x_P(1) = 5 + \int_0^a v_P(t) dt = 5.370660$ AND $x_Q(1) = 10 + \int_0^a v_Q(t) dt = 8.564355$ for $a \neq 1$ earns one point.
- Special case 2: A response of $x_P(1) = 5 + \int v_P(t) dt = 5.370660$ AND $x_Q(1) = 10 + \int v_Q(t) dt = 8.564355$ or the equivalent, never providing the definite integrals, earns one point.
- Degree mode: A response that presents answers obtained by using a calculator in degree mode does not earn the first point it would have otherwise earned. The response is generally eligible for all subsequent points (unless no answer is possible in degree mode or the question is made simpler by using degree mode). In degree mode, $x_P(1)$ is 5.007 (or 5.006).

Total for part (a) 3 points

- (b) Are particles P and Q moving toward each other or away from each other at time $t = 1$? Explain your reasoning.

$v_P(1) = \sin(1^{1.5}) = 0.841471 > 0$ At time $t = 1$, particle P is moving to the right.	Direction of motion for one particle	1 point
$v_Q(1) = (1 - 1.8) \cdot 1.25^1 = -1 < 0$ At time $t = 1$, particle Q is moving to the left. At time $t = 1$, $x_P(1) < x_Q(1)$, so particle P is to the left of particle Q . Thus, at time $t = 1$, particles P and Q are moving toward each other.	Answer with explanation	1 point

Scoring notes:

- The first point is earned for using the sign of $v_P(1)$ or $v_Q(1)$ to determine the direction of motion for one of the particles. This point cannot be earned without reference to the sign of $v_P(1)$ or $v_Q(1)$.
- It is not necessary to present an explicit value for $v_P(1)$, or $v_Q(1)$, but if a value is presented, it must be correct as far as reported, up to three places after the decimal.
- Read with imported incorrect position values from part (a).
- If one or both position values were not found in part (a), but are found in part (b), the points for part (a) are not earned retroactively.
- To earn the second point the explanation must be based on the signs of $v_P(1)$ and $v_Q(1)$ and the relative positions of particle P and particle Q at $t = 1$. References to other values of time, such as $t = 0$, are not sufficient.
- Degree mode: $v_P(1) = 0.017$. (See degree mode statement in part (a).)

Total for part (b) 2 points

- (c) Find the acceleration of particle Q at time $t = 1$. Is the speed of particle Q increasing or decreasing at time $t = 1$? Explain your reasoning.

$a_Q(1) = v'_Q(1) = 1.026856$ The acceleration of particle Q is 1.027 (or 1.026) at time $t = 1$.	Setup and acceleration	1 point
$v_Q(1) = -1 < 0$ and $a_Q(1) > 0$ The speed of particle Q is decreasing at time $t = 1$ because the velocity and acceleration have opposite signs.	Speed decreasing with reason	1 point

Scoring notes:

- To earn the first point the acceleration must be explicitly connected to v'_Q (e.g., $v'_Q(1) = 1.026856$).
- The first point is not earned for an unsupported value of 1.027 (or 1.026). The setup, $v'_Q(1)$, must be shown. Presenting only $a_Q(1) = 1.027$ (or 1.026) without indication that $v'_Q = a_Q$ is not enough to earn the first point.
- A response does not need to present a value for $v_Q(1)$; the sign is sufficient.
- To earn the second point a response must compare the signs of a_Q and v_Q at $t = 1$. Considering only one sign is not sufficient.
- After the first point has been earned, a response declaring only “velocity and acceleration are of opposite signs at $t = 1$ so the particle is slowing down” (or equivalent) earns the second point.
- The second point may be earned without the first, as long as the response does not present an incorrect value or sign for $v_Q(1)$ and concludes the particle is slowing down because velocity and acceleration have opposite signs at $t = 1$.

Total for part (c) 2 points

- (d) Find the total distance traveled by particle P over the time interval $0 \leq t \leq \pi$.

$$\int_0^{\pi} |v_P(t)| dt = 1.93148$$

Definite integral

1 point

Over the time interval $0 \leq t \leq \pi$, the total distance traveled by particle P is 1.931.

Answer

1 point**Scoring notes:**

- The first point is earned for $\int_0^{\pi} |v_P(t)| dt$.
- The first point can also be earned for a sum (or difference) of definite integrals, such as $\int_0^{2.145029} v_P(t) dt - \int_{2.145029}^{\pi} v_P(t) dt$, provided the response has indicated $v_P(2.145029) = 0$.
- The second point can only be earned for the correct answer.
- The unsupported value 1.931 earns no points.
- A response reporting the distance traveled by particle Q as $\int_0^{\pi} |v_Q(t)| dt = 3.506$ earns the first point and is not eligible for the second point.
- In degree mode, the total distance traveled is 0.122. (See degree mode statement in part (a).) In the degree mode case, the response must present $\int_0^{\pi} |v_P(t)| dt$ in order to earn the first point because

$$\int_0^{\pi} |v_P(t)| dt = \int_0^{\pi} v_P(t) dt.$$

Total for part (d) 2 points**Total for question 2 9 points**

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$\text{position of } P = x_p = 5 + \int_0^1 \sin(t^{1.5}) dt = 5 + 0.37066$$

$$\text{position of } Q = x_Q = 10 + \int_0^1 v_Q(t) dt = 8.56435$$

At time $t=1$, the position of particle P is 5.37066 and the position of particle Q is 8.56435

Response for question 2(b)

$$v_Q(1) = -1$$

$$x_Q(1) = 5.37066$$

$$v_P(1) = 0.84147$$

$$x_P(1) = 8.56435$$

At time $t=1$ the 2 particles are moving closer to each other because $x_P(1) = 5.37066$ and $x_Q(1) = 8.56435$, which means that particle Q is to the right of particle P at $t=1$, and since particle Q has a negative velocity it is moving left and particle P has a positive velocity it is moving right, so the particles are ~~moving~~ moving toward each other at time $t=1$.

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

 $a_Q = \text{acceleration of particle } Q$

$$a_Q = v_Q'(t) \quad v_Q'(1) = 1.02686.$$

$v_Q(1) = -1$. Since the $a_Q(t)$ is 1.02686 and is positive and $v_Q(1)$ is negative, the speed of particle Q at time $t=1$ must be decreasing.

Response for question 2(d)

Total distance particle P travelled

$$\text{from } 0 \leq t \leq \pi = \int_0^{\pi} |v_P(t)| dt = 1.93148$$

The total distance particle P travelled from $0 \leq t \leq \pi$ is 1.93148.

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Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$x_p(1) = 5 + \int_0^1 (\sin(t^2)) dx = 5 + 0.371 = 5.371 \text{ units}$$

$$x_q(1) = 10 + \int_0^1 (1.25^t (t - 1.8)) dx = 10 + (-1.436) = 8.564 \text{ units}$$

Response for question 2(b)

$$v_p(1) = 0.841 \quad a_p(1) = 0.810$$

$$v_q(1) = -1 \quad a_q(1) = 1.027$$

At $t=1$, the particles are moving away from each other as their velocities are going in different directions at $t=1$

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$a_Q(t) = v_Q'(t) = 1.25^t \ln 1.25(t - 1.8) + 1.25^t$$

$$a_Q(1) = 1.25^{(1)} \ln 1.25(1 - 1.8) + 1.25^1 = 1.027 \text{ units/t}^2$$

At $t=1$, the speed of Q is decreasing b/c $a_Q(t)$ which is $v_Q'(t)$ is positive while $v_Q(1)$ is negative.

Response for question 2(d)

$$\int_0^{\pi} |\sin(t^{1.5})| dx = 1.931$$

total distance traveled = 1.931

2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$$P: v(t) = \sin(t^{1.5})$$

$$Q: v(t) = (t - 1.8) \cdot 1.25^t$$

$$\int_0^1 v_P(t) dt = \underline{1.37066}$$

$$\int_0^1 v_Q(t) dt = \underline{-1.4356}$$

Response for question 2(b)

Particles P & Q are moving away from each other at time $t=1$ because one velocity (P) is positive and one velocity (Q) is negative.

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$$v(t) = \sin(t^{1.5})$$

~~at t~~
$$v'(t) = a(t)$$

$$a(1) = 1.0269$$

The speed of particle Q ~~is~~ is decreasing at time $t=1$ because the velocity is negative and the acceleration is positive, which are different signs. $v(1) < 0$ and $a(1) > 0$.

Response for question 2(d)

$$\int_0^{\pi} |v_p(t)| dt$$

$$\int_0^{\pi} |\sin(t^{1.5})| dt$$

1.9315 = total distance
on interval $0 \leq t \leq \pi$

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this problem particles P and Q move along the x -axis with velocities $v_P(t) = \sin(t^{1.5})$ and $v_Q(t) = (t - 1.8) \cdot 1.25^t$, respectively. The velocity of both particles applies for $0 \leq t \leq \pi$, and at time $t = 0$, particle P is at position $x = 5$, while particle Q is at position $x = 10$.

In part (a) students were asked to find the positions of both particles at time $t = 1$. A correct response should find the net change in each particle's position as the integral of their respective velocity across the interval $0 \leq t \leq 1$ and add this change to each particle's position at time $t = 1$.

In part (b) students were asked whether the particles were moving toward or away from each other at this time ($t = 1$). A correct response should evaluate the given velocity functions at $t = 1$ to determine the sign of each particle's velocity. This should lead to the conclusion that particle P is moving to the right while particle Q is moving to the left. In addition, a response should use the position functions found in part (a) to determine that at time $t = 1$ particle P is to the left of particle Q and, therefore, the particles are moving toward each other.

In part (c) students were asked to find the acceleration of particle Q at time $t = 1$ and whether the speed of particle Q was increasing or decreasing at time $t = 1$. A correct response should indicate that acceleration is the derivative of velocity and find the value of $a_Q = v'_Q$ at time $t = 1$ using a graphing calculator. The response should then indicate that the particle's speed is decreasing because the particle's acceleration and velocity (sign determined in part (b)) have opposite signs at this time.

Finally, in part (d) students were asked to find the total distance traveled by particle P over the entire time interval $0 \leq t \leq \pi$. A correct response would use a graphing calculator to determine the value of the definite integral of the speed, $\int_0^\pi |v_P(t)| dt$.

Sample: 2A

Score: 9

The response earned 9 points: 3 points in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response earned the first point for $\int_0^1 \sin(t^{1.5}) dt$ on line 1. The response earned the second point for $5 + \int_0^1 \sin(t^{1.5}) dt = 5 + 0.37066$ on line 1. Note that the numerical expression $5 + 0.37066$ need not be simplified.

The response earned the third point for $10 + \int_0^1 v_Q(t) dt = 8.56435$ on line 2. Lines 3-5 summarize the results and contain correct information. Note the presented decimals are accurate to three decimal places, rounded or truncated. In part (b) the response earned the first point by stating that "since particle Q has a negative velocity it is moving left" on lines 7 and 8. The response earned the second point by stating that " Q is to the right of particle P at $t = 1$," "particle Q has a negative velocity it is moving left," and "particle P has a positive velocity it is moving right, so the particles are moving toward each other at time $t = 1$ " on lines 6-10. In part (c) the response earned the first point on line 2 for $a_Q = v'_Q(t)$ and $v'_Q(1) = 1.02686$. Note that without $a_Q = v'_Q(t)$, the response would still have earned the first point for $v'_Q(1) = 1.02686$. The response earned the second point for comparing the signs of $a_Q(1)$ (positive) and $v_Q(1)$ (negative) and concluding that the speed of particle Q at time $t = 1$ must be decreasing on

Question 2 (continued)

lines 3-6. In part (d) the response earned the first point for $\int_0^\pi |v_P(t)| dt$ on line 2. The response earned the second point for $\int_0^\pi |v_P(t)| dt = 1.93148$ on line 2. Lines 3 and 4 summarize the result and contain correct information. Note the presented decimal is accurate to three decimal places, rounded or truncated.

Sample: 2B**Score: 7**

The response earned 7 points: 3 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response earned the first point for $\int_0^1 \sin(t^{1.5}) dx$ on line 1. The second point was earned for

$5 + \int_0^1 \sin(t^{1.5}) dx = 5 + 0.371$ on line 1. Note the correct position of particle P need not be simplified; however, the response simplifies correctly to obtain 5.371. The third point was earned for

$10 + \int_0^1 (1.25^t (t - 1.8)) dx = 10 + (-1.436) = 8.564$ on line 2. The response was not penalized for the use of dx in

place of dt . In part (b) no points were earned because the response fails to connect the direction of motion of each particle with the correct signs of the respective velocities at $t = 1$. Also, the response fails to reference the relative positions of P and Q at $t = 1$. In part (c) the first point was earned on lines 1 and 2 of the response. Note that the correct expression for $v_Q'(t)$ is given on line 1; however, this was not required. On line 1 the connection between

$a_Q(t)$ and $v_Q'(t)$ is made. Note the required connection is also made if the response begins the statement on line 1 with $v_Q'(t)$. On line 2 the correct value of $a_Q(1)$ is given. The second point was earned on lines 3 and 4 by

comparing the signs of v_Q and a_Q at $t = 1$ and concluding that the speed of Q is decreasing. In part (d) the

response earned the first point for $\int_0^\pi (|\sin(t^{1.5})|) dx$ on line 1. The response was not penalized for the use of dx in

place of dt . The second point was earned for the correct total distance traveled, $\int_0^\pi (|\sin(t^{1.5})|) dx = 1.931$, on line

1. The response goes on to summarize the result, which is unnecessary but correct, so the response earned the second point on line 2.

Sample: 2C**Score: 5**

The response earned 5 points: 1 point in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In part (a) the response earned the first point for $\int_0^1 v_P(t) dt$ on line 3. The second and third points were not earned

because both positions are incorrect. In part (b) the response earned no points because there is no connection made between the direction of motion of either particle and the sign of its velocity. Also, there is no reference to the relative positions of the particles at $t = 1$, and the response incorrectly concludes that the particles are moving away from each other. In part (c) the first point was earned on lines 2 and 3 where the connection between $v'(t)$ and $a(t)$

is made explicit, and the correct value of $a(1) = 1.0269$ is stated. The second point was earned for “The speed of particle Q is decreasing at time $t = 1$ because the velocity is negative and the acceleration is positive, which are different signs.” The response goes on to state the signs of $v(1) < 0$ and $a(1) > 0$ on lines 7 and 8. In part (d) the

Question 2 (continued)

response earned the first point for the definite integral $\int_0^{\pi} |v_P(t)| dt$ on line 1. The second point was earned for $\int_0^{\pi} |v_P(t)| dt$ and the correct total distance traveled, 1.9315, stated on line 3. Note that the stated answer of 1.9315, which is given to four decimal places, is correct when truncated to three decimal places.