11

Neutrino Oscillations

Recent experiments have shown that neutrinos can convert from one flavor to another (for instance, $\nu_e \leftrightarrow \nu_\mu$). This means that neutrinos have nonzero mass, and that the lepton numbers (electron, muon, and tau) are not separately conserved. Neutrino oscillations resolve the solar neutrino problem, and suggest modest changes in the Standard Model. The treatment here is largely self-contained, and could even be read immediately after Chapter 2.

11.1
The Solar Neutrino Problem

The story begins [1] in the middle of the nineteenth century, when Lord Rayleigh undertook to calculate the age of the sun. He assumed (as everyone did, at the time) that the source of the sun's energy was gravity – the energy accumulated when all that matter 'fell down' from infinity is liberated over time in the form of radiation. On the basis of the known rate of solar radiation (which he took to be constant), Rayleigh showed that the maximum possible age of the sun was substantially shorter than the age of the earth as estimated by geologists, and, more to the point, shorter than Darwin's theory of evolution required. This pleased Lord Rayleigh, who was opposed to evolution on quaint religious grounds, but it worried Darwin, who removed his own estimate from subsequent editions of his book.

In 1896, Becquerel discovered radioactivity. In subsequent studies he and the Curies noted that radioactive substances such as radium give off prodigious amounts of heat. This suggested that nuclear fission, not gravity, might be the source of the sun's energy, and this would allow for a much longer lifetime. The only trouble was that there didn't appear to *be* any radioactive stuff in the sun, which is made almost entirely of hydrogen (plus a small amount of light elements, but certainly not uranium or radium).

By 1920, Aston completed a series of meticulous measurements of atomic weights, and Eddington noticed that four hydrogen atoms weigh slightly more than one atom of helium-4. This implied (in view of Einstein's $E = mc^2$) that the fusion of four hydrogens would be energetically favorable, and would release a substantial

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amount of energy. Eddington suggested that this process (nuclear fusion) powers the sun, and in essence he was right. Of course, Eddington didn't know what the *mechanism* for binding the hydrogens together might be; this had to await the development of nuclear physics in the 1930s – in particular, Chadwick's discovery of the neutron and Pauli's invention of the neutrino.

In 1938, Hans Bethe worked out the details, which turn out to be quite complicated. In heavy stars the dominant mechanism is the CNO (Carbon–Nitrogen –Oxygen) cycle, in which the fusion process is 'catalyzed' by small amounts of those three elements. But in the sun (and other relatively light stars) the dominant route is the so-called *pp* chain (Figure 11.1). To begin with, a pair of protons (hydrogen nuclei) combine to make a deuteron, a positron, and a neutrino. (The deuteron is a proton and a neutron, so what really happened here is that a proton converted into a neutron, a positron, and a neutrino – the reverse of neutron decay.) Alternatively, the outgoing positron could be replaced by an incoming electron. Either way, we have produced deuterons (along with some neutrinos) from protons. The deuteron soon picks up another proton to make a helium-3 nucleus (two protons and a neutron), releasing energy in the form of a photon. Helium-3 has three options: it can join with another loose proton to make an alpha particle – the nucleus of helium-4 (two protons and two neutrons). Once again, a proton has been converted into a neutron (with emission of a positron and a neutrino). Or two helium-3s can get together to make an alpha particle and two leftover protons. Or the helium-3 can combine with an alpha particle (produced in one of the previous reactions) to make beryllium-7, with the emission of a photon. Finally, the beryllium can either

The pp Chain

Step 1: Two protons make a deuteron

$$p+p \rightarrow d+e^+ +\nu_e$$
$$p+p+e^- \rightarrow d+\nu_e$$

0.42 MeV

Step 2: Deuteron plus proton makes ^{3}He.

$$d+p \rightarrow {}^3\mathrm{He}+\gamma$$

Step 3: Helium-3 makes alpha particle or ^{7}Be.

$$^3\mathrm{He}+p \rightarrow \alpha+e^+ +\nu_e$$
$$^3\mathrm{He}+{}^3\mathrm{He} \rightarrow \alpha+p+p$$
$$^3\mathrm{He}+\alpha \rightarrow {}^7\mathrm{Be}+\gamma$$

Step 4: Berillium makes alpha particles.

$$^7\mathrm{Be}+e^- \rightarrow {}^7\mathrm{Li}+\nu_e$$
$$^7\mathrm{Li}+p \rightarrow \alpha+\alpha$$
$$^7\mathrm{Be}+p \rightarrow {}^8\mathrm{B}+\gamma$$
$$^8\mathrm{B} \rightarrow {}^8\mathrm{Be}^* +e^+ +\nu_e$$
$$^8\mathrm{Be}^* \rightarrow \alpha+\alpha$$

14.6 MeV

Fig. 11.1 The *pp* chain: how protons make alpha particles in the sun.

absorb an electron, making lithium, which picks up a proton, yielding two alpha particles, or else it absorbs a proton, making boron, which goes to an excited state of beryllium-8, and from there to two alpha particles.

The details are not so important; the *point* is that it all starts out as hydrogen (protons), and it all ends up as α particles (helium-4 nuclei) – precisely Eddington's reaction – plus some electrons, positrons, photons ... and neutrinos. But is this complicated story really *true*? How can we tell what is going on inside the sun? Photons take a thousand years to work their way out from the center to the surface, and what we see from earth doesn't tell us much about the interior. But neutrinos – because they interact so weakly, emerge virtually unscathed by passage through the sun. Neutrinos, therefore, are the perfect probes for studying the interior of the sun.

In the *pp* chain there are five reactions that yield neutrinos, and for each one the neutrinos come out with a characteristic energy spectrum, as shown in Figure 11.2 The overwhelming majority come from the initial reaction $p + p \to d + e + \nu_e$. Unfortunately, they carry relatively low energy, and most detectors are insensitive in this regime. For that reason, even though the boron-8 neutrinos are far less abundant, most experiments actually work with them.

There are certainly plenty of neutrinos coming from the sun. John Bahcall, who was responsible for most of the calculations of solar neutrino abundances, liked to say that 100 billion neutrinos pass through your thumbnail every second; and yet they are so ethereal that you can look forward to only one or two neutrino-induced

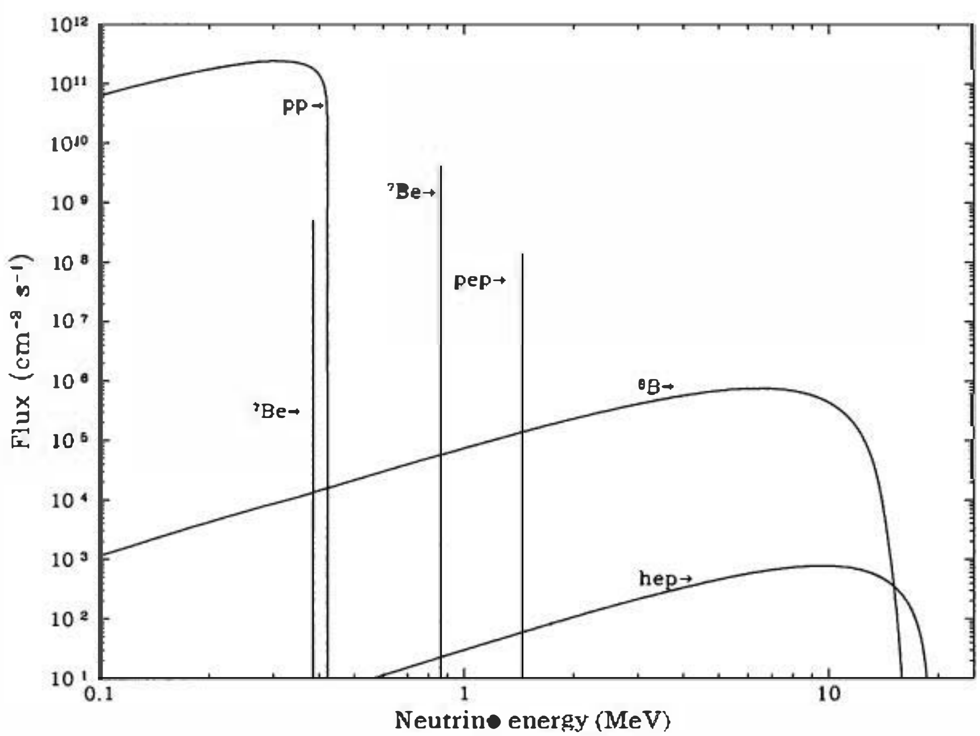


Fig. 11.2 The calculated energy spectra for solar neutrinos. (*Source:* J. N. Bahcall, A.M. Serenelli, and S. Basu, *Astrophysical Journal* **621**, L85 (2005).)

reactions in your body during your entire lifetime. In 1968, Ray Davis *et al.* [2] reported the first experiments to measure solar neutrinos, using a huge tank of chlorine (actually, cleaning fluid) in the Homestake mine in South Dakota (you have to do it deep underground to eliminate background from cosmic rays). Chlorine can absorb a neutrino and convert to argon by the reaction $\nu_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e$ (essentially again $\nu_e + n \rightarrow p + e$). The Davis experiment – for which he was finally awarded Nobel Prize in 2002 – collected argon atoms for several months (they were produced at a rate of about one atom every two *days*). The total accumulation was only about a third of what Bahcall predicted [3]. Thus was born the famous *solar neutrino problem*.

11.2 Oscillations

At the time, most physicists assumed the experiments were wrong. After all, Davis claimed to have flushed a total of 33 argon atoms out of a tank containing 615 metric tons of tetrachloroethylene – it was not hard to imagine that he might have missed a few. On the theory side, Bahcall's calculations required an audacious confidence in the so-called Standard Solar Model of the interior of the sun. But gradually the community came to take the solar neutrino problem seriously – especially when other experiments, using quite different detection methods, confirmed the deficit.

In 1968, Bruno Pontecorvo suggested a beautifully simple explanation for the solar neutrino problem. He proposed that the electron neutrinos produced by the sun are transformed in flight into a different species (muon neutrinos, say, or even antineutrinos), to which Davis' experiment was insensitive [4]. This is the mechanism we now call *neutrino oscillation*. The theory is quite simple – it is basically the quantum mechanics of mixed states, which itself is almost identical to the classical theory of coupled oscillators [5]. Consider the case of just *two* neutrino types – say, ν_e and ν_μ. If one can spontaneously convert into the other, it means that neither is an eigenfunction of the Hamiltonian.

Weak and mass ν eigenstates

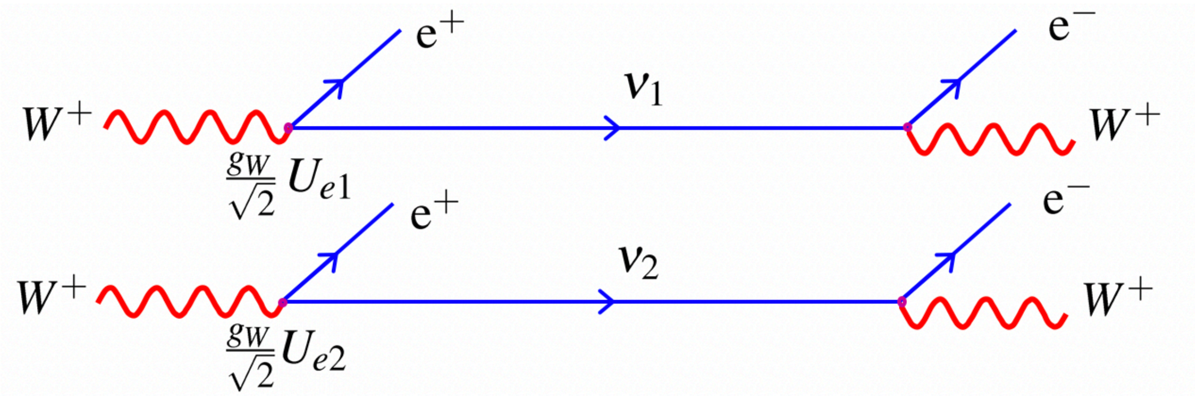
- Neutrinos are produced and interact as weak eigenstate

$$\nu_e, \nu_\mu, \nu_\tau$$ **= weak eigenstates**

they are produced with the corresponding lepton

- Let say that ν_e is a superposition of two mass eigenstates

$$\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2$$



- The weak and mass eigenstates are related by the unitary matrix. For two flavors

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \qquad \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

- The mass eigenstates are the free particle solutions to the wave-equation and propagate as plane waves

$$|\nu_1(t)\rangle = |\nu_1\rangle e^{i\vec{p}_1.\vec{x}-iE_1t} \qquad |\nu_2(t)\rangle = |\nu_2\rangle e^{i\vec{p}_2.\vec{x}-iE_2t}$$

- Suppose at time t=0 a neutrino is produced in a pure ν_e state

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

and evolves as the mass eigenstates before the neutrino interacts in a detector at a distance L and at a time T

$$|\psi(L,T)\rangle = \cos\theta|\nu_1\rangle e^{-i\phi_1} + \sin\theta|\nu_2\rangle e^{-i\phi_2}$$

Neutrino oscillations for two flavors

- Express the mass eigenstates in terms of weak eigenstates

$$|\psi(L,T)\rangle = \cos\theta(\cos\theta|\nu_e\rangle - \sin\theta|\nu_\mu\rangle)e^{-i\phi_1} + \sin\theta(\sin\theta|\nu_e\rangle + \cos\theta|\nu_\mu\rangle)e^{-i\phi_2}$$
$$|\psi(L,T)\rangle = |\nu_e\rangle(\cos^2\theta e^{-i\phi_1} + \sin^2\theta e^{-i\phi_2}) + |\nu_\mu\rangle \sin\theta\cos\theta(-e^{-i\phi_1} + e^{-i\phi_2})$$

- The mass eigenstates are in phase $\phi_1 = \phi_2$ if masses of $|\nu_1\rangle$ and $|\nu_2\rangle$ are the same and $\psi(L,T) = |\nu_e\rangle$
- If the masses are different the wave-function no longer remains a pure $|\nu_e\rangle$

  they are produced with the corresponding lepton
- Let say that ν_e is a superposition of two mass eigenstates

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= |\langle \nu_\mu|\psi(L,T)\rangle|^2 \\ &= \cos^2\theta\sin^2\theta(-e^{-i\phi_1} + e^{-i\phi_2})(-e^{+i\phi_1} + e^{+i\phi_2}) \\ &= \frac{1}{4}\sin^2 2\theta(2 - 2\cos(\phi_1 - \phi_2)) \\ &= \sin^2 2\theta \sin^2\left(\frac{\phi_1 - \phi_2}{2}\right) \end{aligned}$$

- Now need to evaluate the phase difference

$$\Delta\phi_{12} = \phi_1 - \phi_2 = (E_1 - E_2)T - (|p_1| - |p_2|)L$$

- Case 1: $|p_1| = |p_2| = \mathrm{p} \qquad L \approx (c)T$

  neglect the fact that different mass eigenstate propagate at different velocities and be observed at different time

$$\Delta\phi_{12} = (E_1 - E_2)T = \left[(\mathrm{p}^2 + m_1^2)^{1/2} - (\mathrm{p}^2 + m_2^2)^{1/2}\right]L$$
$$\Delta\phi_{12} = \mathrm{p}\left[\left(1 + \frac{m_1^2}{\mathrm{p}^2}\right)^{1/2} - \left(1 + \frac{m_2^2}{\mathrm{p}^2}\right)^{1/2}\right]L \approx \frac{m_1^2 - m_2^2}{2\mathrm{p}}L$$

- The full derivation requires a wave-packet treatment and gives the same result

Neutrino oscillations for two flavors

- Case 2:

$$\Delta\phi_{12} = (E_1 - E_2)T - \left(\frac{|p_1|^2 - |p_2|^2}{|p_1| + |p_2|}\right)L$$

$$\Delta\phi_{12} = (E_1 - E_2)\left[T - \left(\frac{E_1 + E_2}{|p_1| + |p_2|}\right)L\right] + \left(\frac{m_1^2 - m_2^2}{|p_1| + |p_2|}\right)L$$

- The first term vanishes either for $E_1 = E_2$ or the propagation velocity is the same. In all cases

$$\Delta\phi_{12} = \frac{m_1^2 - m_2^2}{2\mathrm{p}}L = \frac{\Delta m^2}{2E}L$$

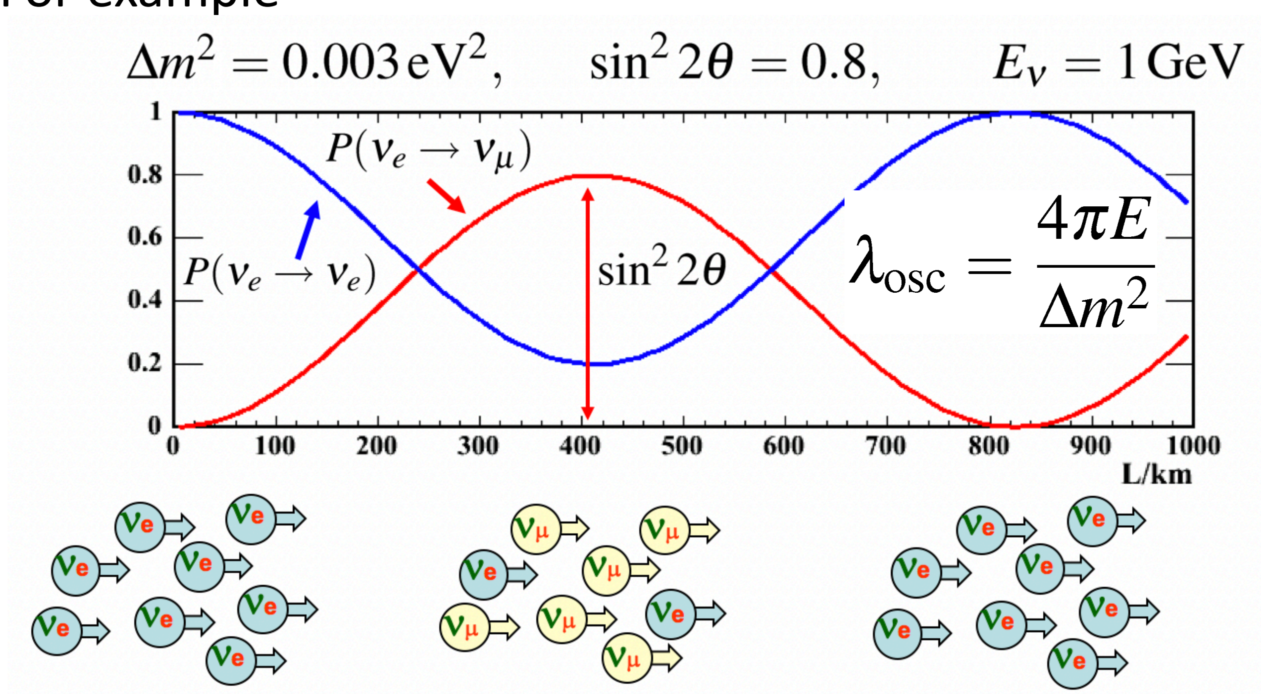
- For two flavors the oscillation probability is

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- The survival probability is

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- $\Delta m_{21}^2 = m_2^2 - m_1^2$ - can be negative
- For example



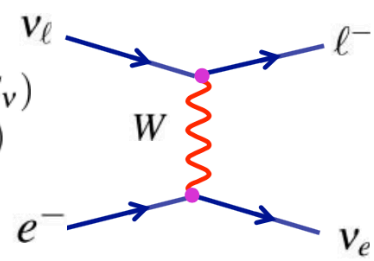
Experimental Studies of neutrino oscillations

detection method depends on the neutrino energy and flavor

X−section $\sigma_\nu \sim G_F^2\, s$, in the lab frame $\sigma_\nu \sim G_F^2\, m\, E_\nu$

1 Charged current interactions on atomic electrons (in laboratory frame)

$$p_\nu = (E_\nu, 0, 0, E_\nu)$$
$$p_e = (m_e, 0, 0, 0)$$



$$s = (p_\nu + p_e)^2 = (E_\nu + m_e)^2 - E_\nu^2$$

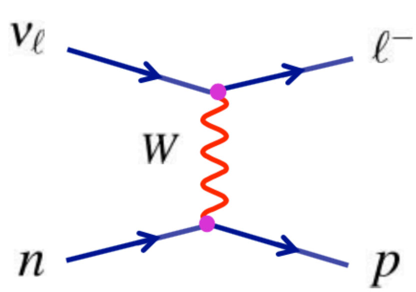
Require: $s > m_\ell^2$

$$\Rightarrow \quad E_\nu > \left[\left(\frac{m_\ell}{m_e}\right)^2 - 1\right]\frac{m_e}{2}$$

Putting in the numbers, for CC interactions with atomic electrons require

$$E_{\nu_e} > 0 \qquad E_{\nu_\mu} > 11\,\mathrm{GeV} \qquad E_{\nu_\tau} > 3090\,\mathrm{GeV}$$

2 charged current interactions on nucleons (in lab. frame)



$$s = (p_\nu + p_n)^2 = (E_\nu + m_n)^2 - E_\nu^2$$

Require: $s > (m_\ell + m_p)^2$

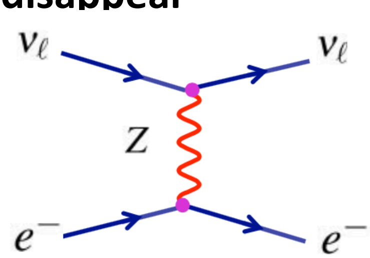
$$\Rightarrow \quad E_\nu > \frac{(m_p^2 - m_n^2) + m_\ell^2 + 2 m_p m_\ell}{2 m_n}$$

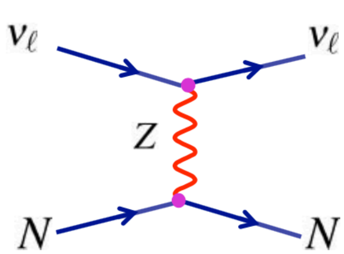
- For CC interactions from neutrons require

$$E_{\nu_e} > 0 \qquad E_{\nu_\mu} > 110\,\mathrm{MeV} \qquad E_{\nu_\tau} > 3.5\,\mathrm{GeV}$$

Low energy electron neutrinos from the sun and nuclear reactors which oscillate into muon or tau neutrinos cannot interact via charged current interactions – they effectively disappear

3 neutral current
observe scattered electron or nucleon





3 Nuclear reactions: $\nu_e + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar} + e^-$ $\qquad \nu_e + d \rightarrow p + n + \nu_e$

$$\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^-$$

Sources of Neutrinos and Experiments

Atmospheric/Beam Neutrinos $\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu : E_\nu > 1\,\mathrm{GeV}$

❶ **Water Čerenkov: e.g. Super Kamiokande**
❷ **Iron Calorimeters: e.g. MINOS, CDHS** (see handout 10)
- Produce high energy charged lepton – relatively easy to detect

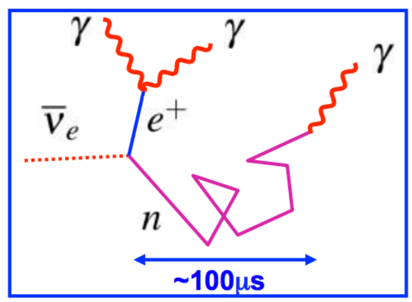
Solar Neutrinos $\nu_e \;:\; E_\nu < 20\,\mathrm{MeV}$

❶ **Water Čerenkov: e.g. Super Kamiokande**
- Detect Čerenkov light from electron produced in $\nu_e + e^- \rightarrow \nu_e + e^-$
- Because of background from natural radioactivity limited to $E_\nu > 5\,\mathrm{MeV}$
- Because Oxygen is a doubly magic nucleus don't get $\nu_e + n \rightarrow e^- + p$

❷ **Radio-Chemical: e.g. Homestake, SAGE, GALLEX**
- Use inverse beta decay process, e.g. $\nu_e + {}^{71}\mathrm{Ga} \rightarrow e^- + {}^{71}\mathrm{Ge}$
- Chemically extract produced isotope and count decays (only gives a rate)

Reactor Neutrinos $\overline{\nu}_e \;:\; E_{\overline{\nu}} < 5\,\mathrm{MeV}$

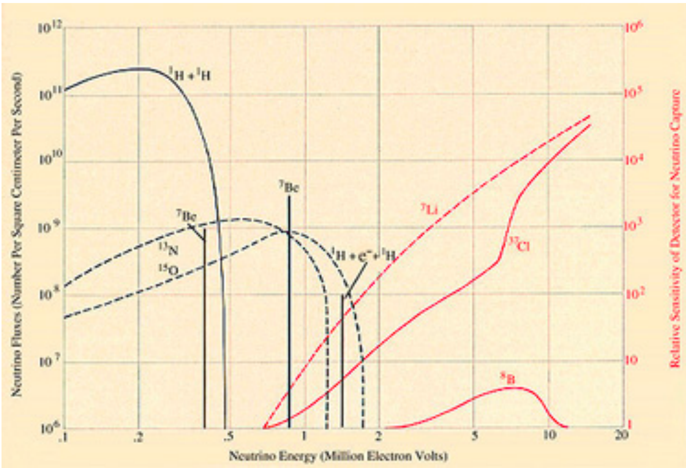
❶ **Liquid Scintillator: e.g. KamLAND, Borexino (solar ν)**
- Low energies → large radioactive background
- Dominant interaction: $\overline{\nu}_e + p \rightarrow e^+ + n$
- Prompt positron annihilation signal + delayed signal from n (space/time correlation reduces background)
- electrons produced by photons excite scintillator which produces light

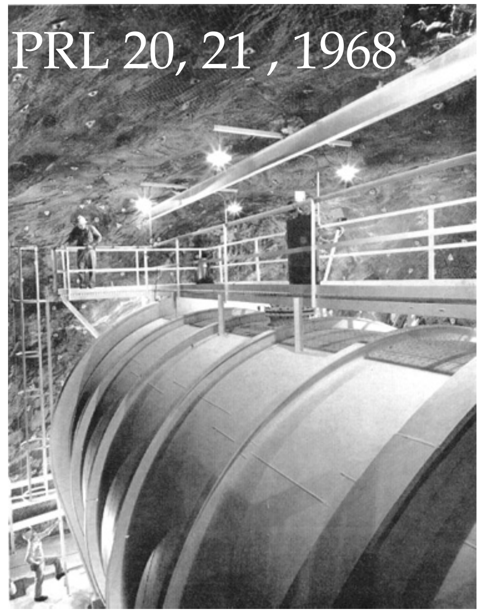


Low energy electron neutrinos from the sun and nuclear reactors which oscillate into muon or tau neutrinos cannot interact via charged current interactions – not visible

Solar neutrinos: Homestake

Ray Davis Chlorine experiment inside Homestake mine in Lead, South Dakota



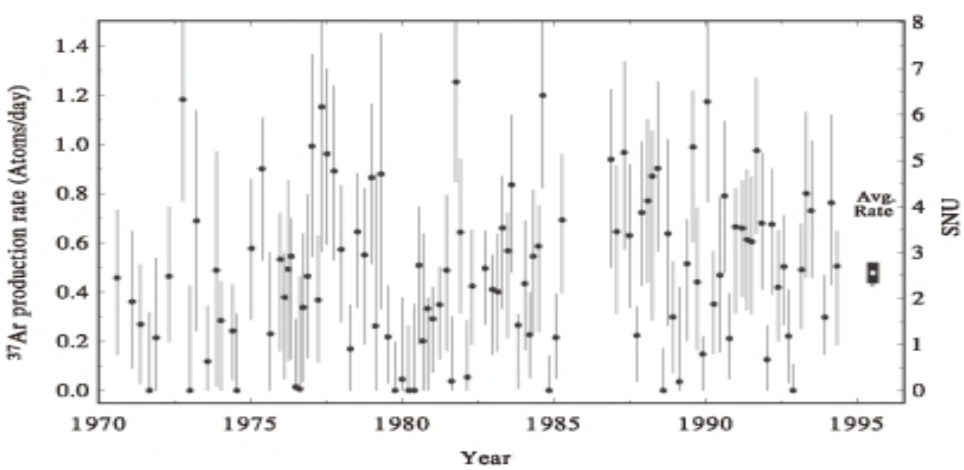


$$\nu_e + {}^{37}\mathrm{Cl} \longrightarrow {}^{37}\mathrm{Ar} + e^-$$

The reaction threshold is 0.814 MeV

Collect unstable Ar and measure the number of decays (electron capture resulted in emission of Auger electroin)

First experiment 110 days: expected 2-7 reactions per day, measured ~0.5 → first indication of Solar Neutrino Problem



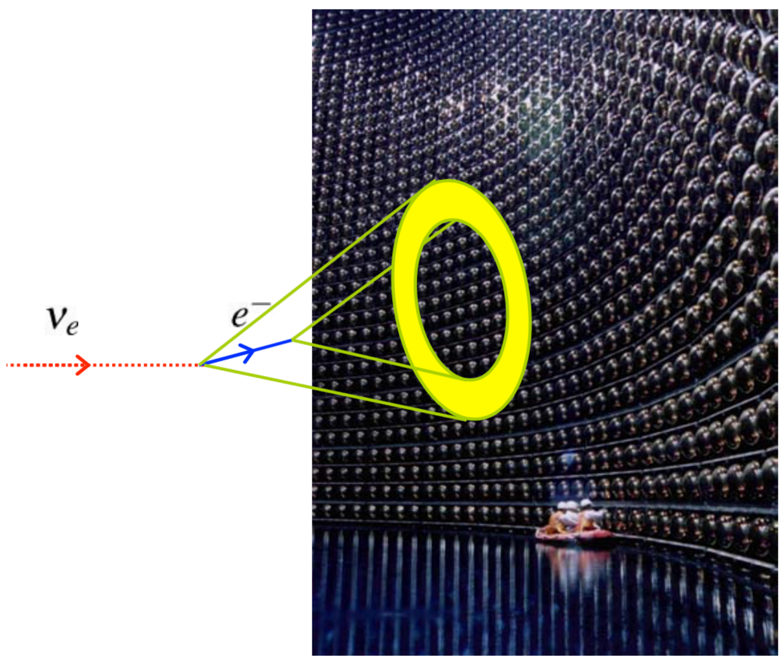
Radiochemical experiments SAGE and GALEX – more sensitive to low energy νe. Count extracted Ge $\nu_e + {}^{71}\mathrm{Ga} \rightarrow {}^{71}\mathrm{Ge} + e^-$

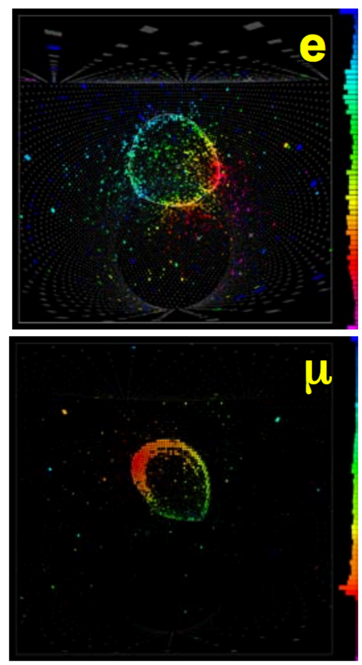
ratio detected/expected

Homestake: 0.27 ± 0.04
SAGE: 0.56 ± 0.08
GALLEX: 0.57 ± 0.08

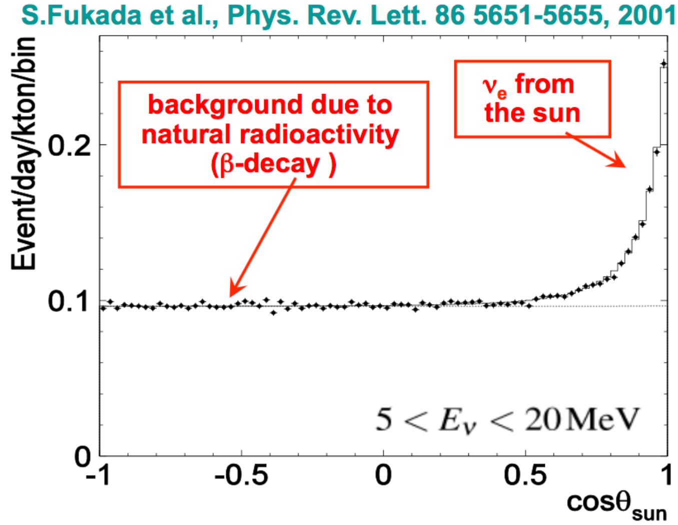
Solar neutrinos: SuperKamiokande

- 50000 ton water detector deep underground to filter out cosmic rays. Uses ~10000 photo-multiplier tubes
- Detect neutrinos by observing Cherenkov radiation from charged particles which travel faster than speed of light in water
- Can distinguish electrons from muons from pattern of light
  - muons produce clean rings
  - electrons produce more diffuse fuzzy rings due to EM showers
- Sensitive to solar neutrinos $E_\nu > 5MeV$
- LAB frame the electron is produced along the $\nu_e$direction





S.Fukada et al., Phys. Rev. Lett. 86 5651-5655, 2001

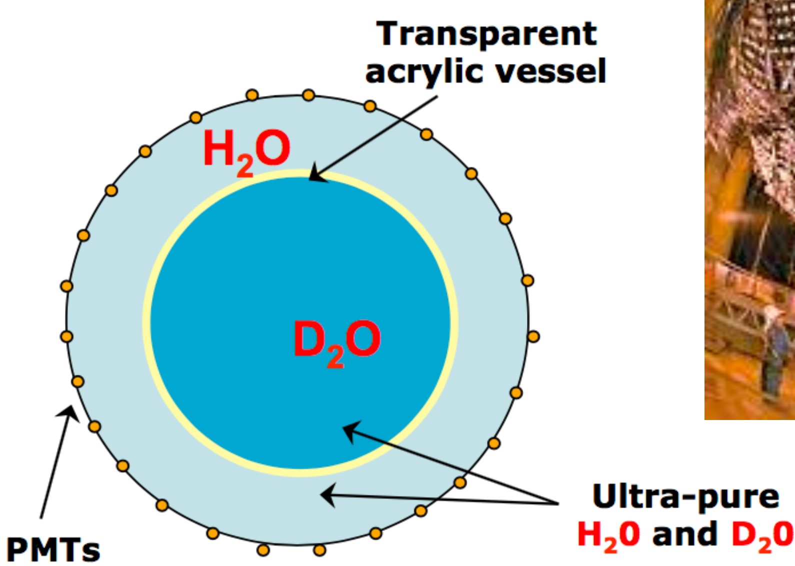


Homestake: 0.27 ± 0.04 SNU
SAGE: 0.56 ± 0.08 SNU
GALLEX: 0.57 ± 0.08 SNU
SuperK: 0.45 ± 0.02 SNU

The Solar Neutrino Problem

Solar neutrinos: SNO

- **Sudbury Neutrino Observatory located in a deep mine in Ontario, Canada**
- **The most model-independent demonstration that the neutrino deficit is due to oscillations**
- **1000 tons of heavy water D_2O inside the transparent acrylic vessel surrounded by 3000 tons of pure water H_2O**
- **The detector is viewed by 9546 photo-multiplier tubes to detect Cherenkov light from the product of neutrino interraction**
- **Main experimental challenge is to get very low background from radioactivity**



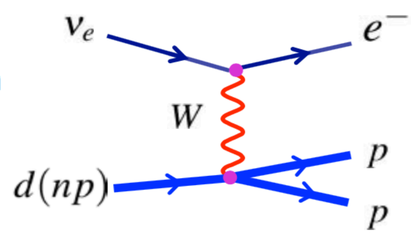
Solar neutrinos: SNO

Sensitive to three different reactions

CHARGE CURRENT

- **Detect Čerenkov light from electron**
- **Only sensitive to ν_e (thresholds)**
- **Gives a measure of ν_e flux**

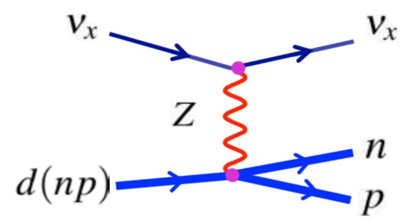
$$\text{CC Rate} \propto \phi(\nu_e)$$



NEUTRAL CURRENT

- **Neutron capture on a deuteron gives 6.25 MeV**
- **Detect Čerenkov light from electrons scattered by γ**
- **Measures total neutrino flux**

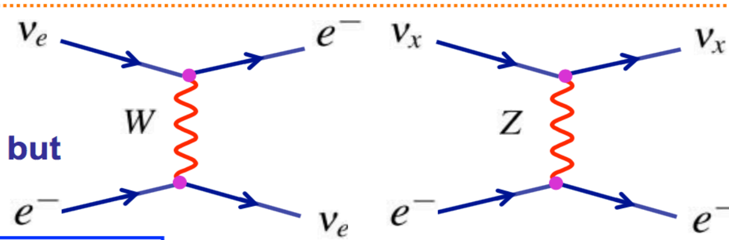
$$\text{NC Rate} \propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$$



ELASTIC SCATTERING

- **Detect Čerenkov light from electron**
- **Sensitive to all neutrinos (NC part) – but larger cross section for ν_e**

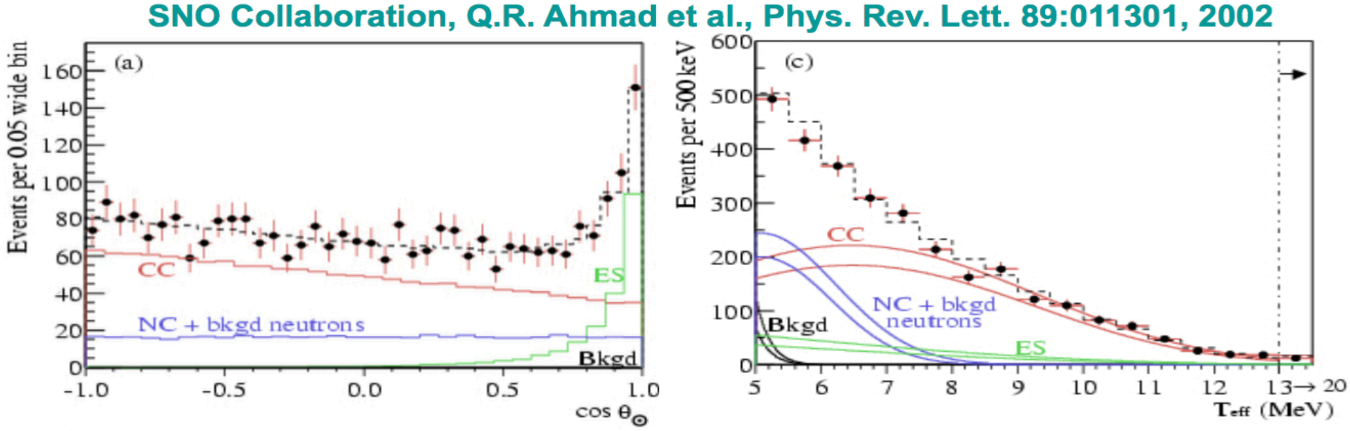
$$\text{ES Rate} \propto \phi(\nu_e) + 0.154(\phi(\nu_\mu) + \phi(\nu_\tau))$$



Measure rates from different reactions

- **Angle from the sun – electrons from neutral current point back to Sun**
- **Detected energy: neutral current events have lower energy – 6.25 MeV photon from neutron capture**
- **Use measured location of the interaction point for estimation of background**

SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89:011301, 2002

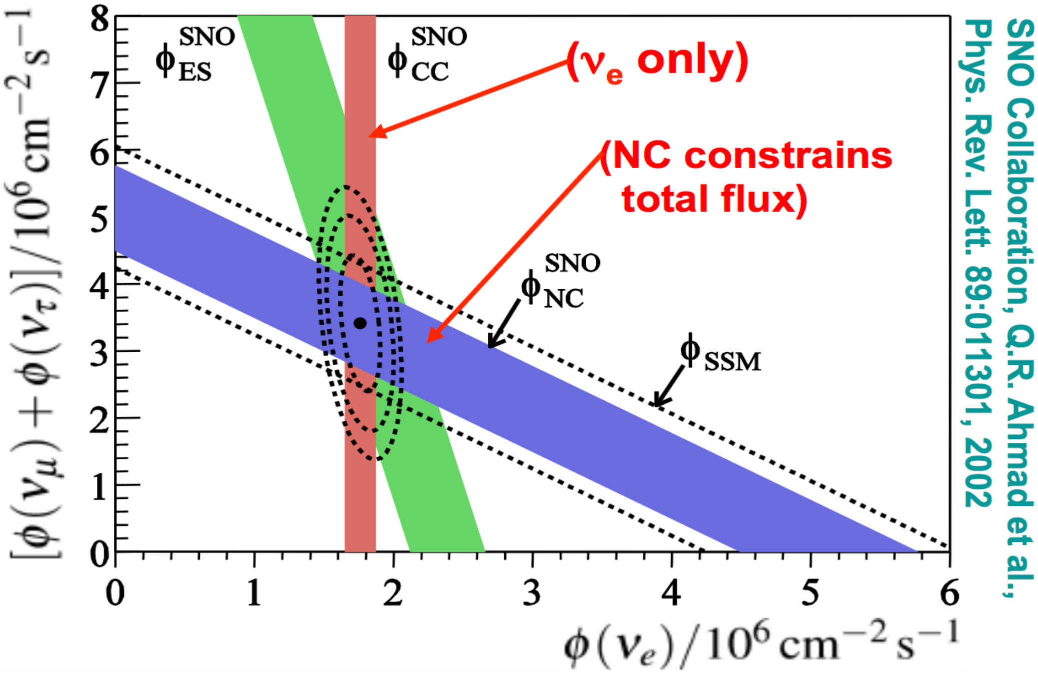


Solar neutrinos: SNO

- **From different distributions obtain a measure of numbers of events of each type**

CC : 1968 ± 61	$\propto \phi(\nu_e)$
ES : 264 ± 26	$\propto \phi(\nu_e) + 0.154[\phi(\nu_\mu) + \phi(\nu_\tau)]$
NC : 576 ± 50	$\propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$

- **Calculate flux of ν_e neutrinos and $\nu_\mu + \nu_\tau$ neutrinos**



- **Clear evidence of flux from $\nu_\mu + \nu_\tau$ neutrinos**

$$\phi(\nu_e) = (1.8 \pm 0.1) \times 10^{-6}\,\mathrm{cm^{-2}\,s^{-1}}$$
$$\phi(\nu_\mu) + \phi(\nu_\tau) = (3.4 \pm 0.6) \times 10^{-6}\,\mathrm{cm^{-2}\,s^{-1}}$$

- **Total flux in a good agreement with the SSM!**

$$\phi(\nu_e) = 5.1 \times 10^{-6}\,\mathrm{cm^{-2}\,s^{-1}}$$

- **Yet ν_e/ν_{total} < ½** → Mikheyev–Smirnov–Wolfenstein effect – neutrino oscillations are enhanced in the presence of matter - **for more details read Particle Physics in the LHC Era, p320-p322 – MSW fixes the sign of Δm_{12}^2**

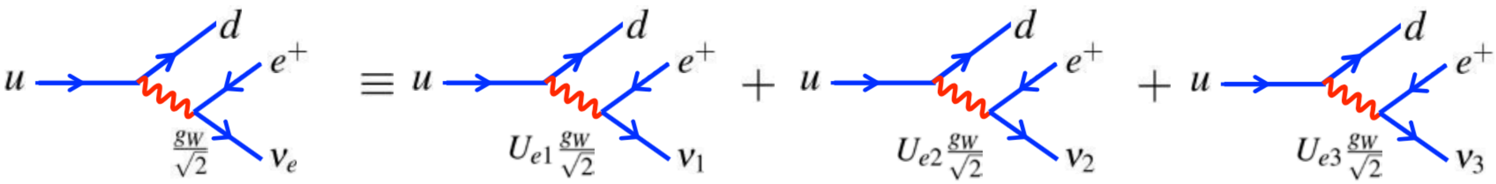
- **Combined analysis of all solar neutrino data**

$$\Delta m^2_{\text{solar}} \approx 8 \times 10^{-5}\,\mathrm{eV}^2,\ \sin^2 2\theta_{\text{solar}} \approx 0.85$$

Neutrino Oscillations for 3 flavors

- **Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS)**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



- **U is unitary (conserve probabilities) and Hermitian**

$$U^\dagger U = I,\ \ U^\dagger = (U^*)^T$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\tau 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

- **Consider a state which is produced at t=0 as $|\nu_e\rangle$**

$$|\psi(t=0)\rangle = |\nu_e\rangle = U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle$$

- **The wave function evolves, where $\phi_i = E_i t - |p_i| z$**

$$|\psi(L)\rangle = U_{e1}|\nu_1\rangle e^{-i\phi_1} + U_{e2}|\nu_2\rangle e^{-i\phi_2} + U_{e3}|\nu_3\rangle e^{-i\phi_3}$$

- **Expand wave function in terms of weak eigenstates**

$$\begin{aligned} |\psi(L)\rangle &= U_{e1}(U_{e1}^*|\nu_e\rangle + U_{\mu 1}^*|\nu_\mu\rangle + U_{\tau 1}^*|\nu_\tau\rangle)e^{-i\phi_1} \\ &+ U_{e2}(U_{e2}^*|\nu_e\rangle + U_{\mu 2}^*|\nu_\mu\rangle + U_{\tau 2}^*|\nu_\tau\rangle)e^{-i\phi_2} \\ &+ U_{e3}(U_{e3}^*|\nu_e\rangle + U_{\mu 3}^*|\nu_\mu\rangle + U_{\tau 3}^*|\nu_\tau\rangle)e^{-i\phi_3} \end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned} |\psi(L)\rangle &= (U_{e1}U_{e1}^* e^{-i\phi_1} + U_{e2}U_{e2}^* e^{-i\phi_2} + U_{e3}U_{e3}^* e^{-i\phi_3})|\nu_e\rangle \\ &+ (U_{e1}U_{\mu 1}^* e^{-i\phi_1} + U_{e2}U_{\mu 2}^* e^{-i\phi_2} + U_{e3}U_{\mu 3}^* e^{-i\phi_3})|\nu_\mu\rangle \\ &+ (U_{e1}U_{\tau 1}^* e^{-i\phi_1} + U_{e2}U_{\tau 2}^* e^{-i\phi_2} + U_{e3}U_{\tau 3}^* e^{-i\phi_3})|\nu_\tau\rangle \end{aligned}$$

- **Calculate probability $\nu_e \to \nu_\mu$**

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(L)\rangle|^2 = |U_{e1}U_{\mu 1}^* e^{-i\phi_1} + U_{e2}U_{\mu 2}^* e^{-i\phi_2} + U_{e3}U_{\mu 3}^* e^{-i\phi_3}|^2$$

- **As follows from unitarity condition** $U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0$ **the probability is zero unless the phases are different**
- **Similar derivation for other flavors**

Neutrino Oscillations for 3 flavors

- **Using other unitarity conditions evaluate**

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}[e^{-i(\phi_1-\phi_2)}-1]\} \\ &+ 2\Re\{U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}[e^{-i(\phi_1-\phi_3)}-1]\} \\ &+ 2\Re\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}[e^{-i(\phi_2-\phi_3)}-1]\} \end{aligned}$$

$$\begin{aligned} P(\nu_\mu \to \nu_e) &= 2\Re\{U_{\mu 1}U^*_{e1}U^*_{\mu 2}U_{e2}[e^{-i(\phi_1-\phi_2)}-1]\} \\ &+ 2\Re\{U_{\mu 1}U^*_{e1}U^*_{\mu 3}U_{e3}[e^{-i(\phi_1-\phi_3)}-1]\} \\ &+ 2\Re\{U_{\mu 2}U^*_{e2}U^*_{\mu 3}U_{e3}[e^{-i(\phi_2-\phi_3)}-1]\} \end{aligned}$$

- **Note, $P(\nu_e \to \nu_\mu) = P(\nu_\mu \to \nu_e)$ only if PMNS is real. If not - neutrino oscillations are not invariant under time reversal t → -t**
- **The electron neutrino survival probability is obtained from**

$$P(\nu_e \to \nu_e) = |\langle \nu_e|\psi(L)\rangle|^2 = |U_{e1}U^*_{e1}e^{-i\phi_1} + U_{e2}U^*_{e2}e^{-i\phi_2} + U_{e3}U^*_{e3}e^{-i\phi_3}|^2$$

Using unitarity condition
$|U_{e1}U^*_{e1} + U_{e2}U^*_{e2} + U_{e3}U^*_{e3}|^2 = 1$ →

$$\begin{aligned} P(\nu_e \to \nu_e) = 1 &+ 2|U_{e1}|^2|U_{e2}|^2\Re\{[e^{-i(\phi_1-\phi_2)}-1]\} \\ &+ 2|U_{e1}|^2|U_{e3}|^2\Re\{[e^{-i(\phi_1-\phi_3)}-1]\} \\ &+ 2|U_{e2}|^2|U_{e3}|^2\Re\{[e^{-i(\phi_2-\phi_3)}-1]\} \end{aligned}$$

- **Phase of mass eigenstate *i* at z=L**

$$\phi_i \approx \frac{m_i^2}{2E}L \quad \rightarrow \quad \Re\{e^{-i(\phi_1-\phi_2)}-1\} = -2\sin^2\left(\frac{(m_2^2-m_1^2)L}{4E}\right)$$

$$\Delta_{21} = \frac{(m_2^2-m_1^2)L}{4E} = \frac{\Delta m_{21}^2 L}{4E} \qquad \Delta m_{21}^2 = m_2^2 - m_1^2$$

$$P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2\sin^2\Delta_{21} - 4|U_{e1}|^2|U_{e3}|^2\sin^2\Delta_{31} - 4|U_{e2}|^2|U_{e3}|^2\sin^2\Delta_{32}$$

- **Only 2 independent Δ:**

$$m_3^2 - m_1^2 = (m_3^2 - m_2^2) + (m_2^2 - m_1^2)$$

- **Oscillation wavelength** $\lambda_{21} = \dfrac{4\pi E}{\Delta m_{21}^2}$

$$\lambda_{\rm osc}({\rm km}) = 2.47\frac{E({\rm GeV})}{\Delta m^2({\rm eV}^2)}$$ **for solar neutrinos ~30 km**

Neutrino Oscillations for 3 flavors

- **The PMNS matrix is usually expressed in terms of three rotation angles $\theta_{12}, \theta_{13}, \theta_{22}$ and a complex phase δ**

$$c_{ij} = cos(\theta_{ij}), \qquad s_{ij} = sin(\theta_{ij})$$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

- **dominates by** "High Energy" "solar"

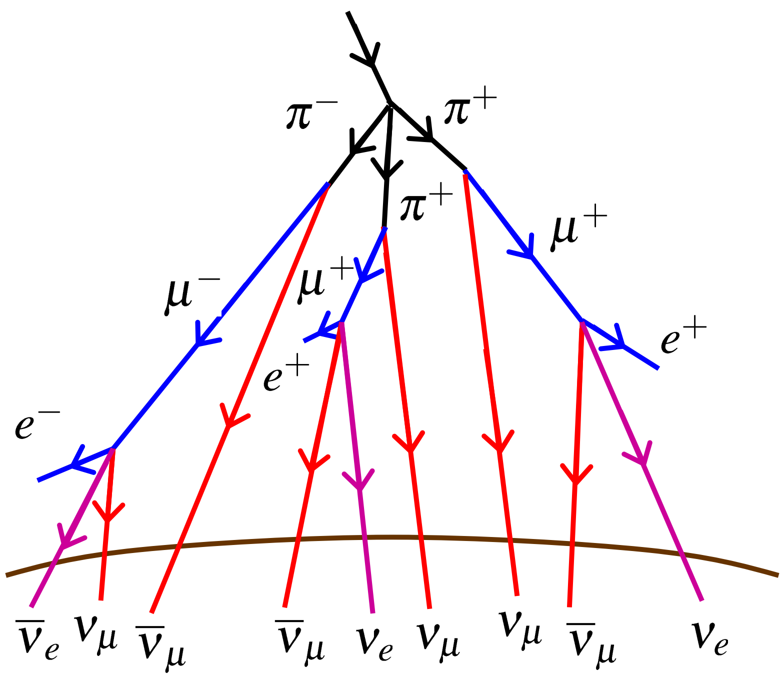
- **To measure six SM parameters need low and high neutrino oscillation experiments**

$\|\Delta m_{21}\|^2 = \|m_2^2 - m_1^2\|$	θ_{12}	Solar and reactor neutrino experiments
$\|\Delta m_{32}\|^2 = \|m_3^2 - m_2^2\|$	θ_{23}	Atmospheric and beam neutrino experiments
	θ_{13}	Reactor neutrino experiments + future beam
	δ	Future beam experiments

- **Non-zero δ will be an indication of CP violation – this is discussed later**

High Energy Atmospheric Neutrinos

- **Need high energy to measure oscillations Δ_{23} of μ and τ neutrinos. → long baseline experiments**
- **Atmospheric neutrinos are produced by cosmic rays (protons and heavier nuclei) colliding with air nuclei in the atmosphere.**
- **Produce charged pions, which weakly decay to muons and neutrinos – typical energy** $E_\nu \sim 1\,\mathrm{GeV}$



$$\pi^+ \to \mu^+ \nu_\mu$$
$$\hookrightarrow e^+ \nu_e \overline{\nu}_\mu$$

$$\pi^- \to \mu^- \overline{\nu}_\mu$$
$$\hookrightarrow e^- \overline{\nu}_e \nu_\mu$$

- **Roughly half of neutrinos are $\overline{\nu}_e, \nu_e$ - detect them with SuperK detector**

$$\frac{N(\nu_\mu + \overline{\nu}_\mu)}{N(\nu_e + \overline{\nu}_e)} \approx 2$$

- **Identify electron and muon neutrinos by shape of Cherenkov ring**
- **Observe a lower ratio with deficit of muon neutrinos coming from below the horizon, i.e. large distance from production point on other side of the Earth**

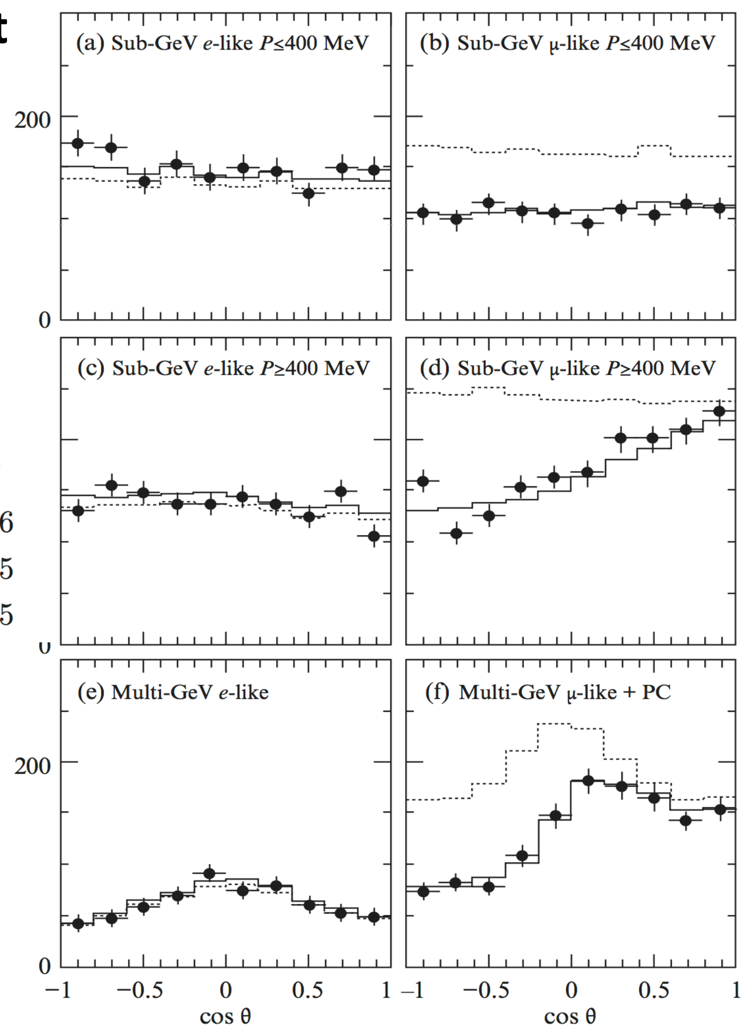
High Energy Atmospheric Neutrinos

- **Measure rate as a function of angle cos(θ) with respect to local vertical**
- **Neutrinos coming from above travel ~20 km: cos(θ)=1**
- **Neutrinos coming from below (i.e. other side of the Earth) travel ~12800 km: cos(θ)= -1**
- **To reduce systematic error due to absolute flux uncertainty use the ratio of flux R = Flux(ν_m)/Flux(ν_e)**
- **Measure asymmetry A = $R_{experiment}/R_{predicted}$**

- **Strong evidence for disappearance of ν_μ for large distances → Consistent with $\nu_\mu \rightarrow \nu_\tau$ oscillations**
- **Don't detect the oscillated ν_τ as typically below interaction threshold of 3.5 GeV**

$\lambda_e \sim 30000km$
no oscillations

$\lambda_\mu \sim 1/\Delta m^2_{23}$
oscillations



Kamiokande-s	$0.60^{+0.06}_{-0.05} \pm 0.05$
Kamiokande-m	$0.57^{+0.08}_{-0.07} \pm 0.07$
Soudan-2 (iron)	$0.66 \pm 0.11 \pm 0.06$
Super-Kamiokande-s	$0.64 \pm 0.02 \pm 0.05$
Super-Kamiokande-m	$0.68 \pm 0.03 \pm 0.05$

- **s – sub-GeV neutrinos**
- **m – multi-GeV neutrinos**

- **Data is consistent with**

$$\Delta m^2_{23} \sim 0.0025 eV^2$$

$$sin^2 2\theta_{23} \sim 1$$

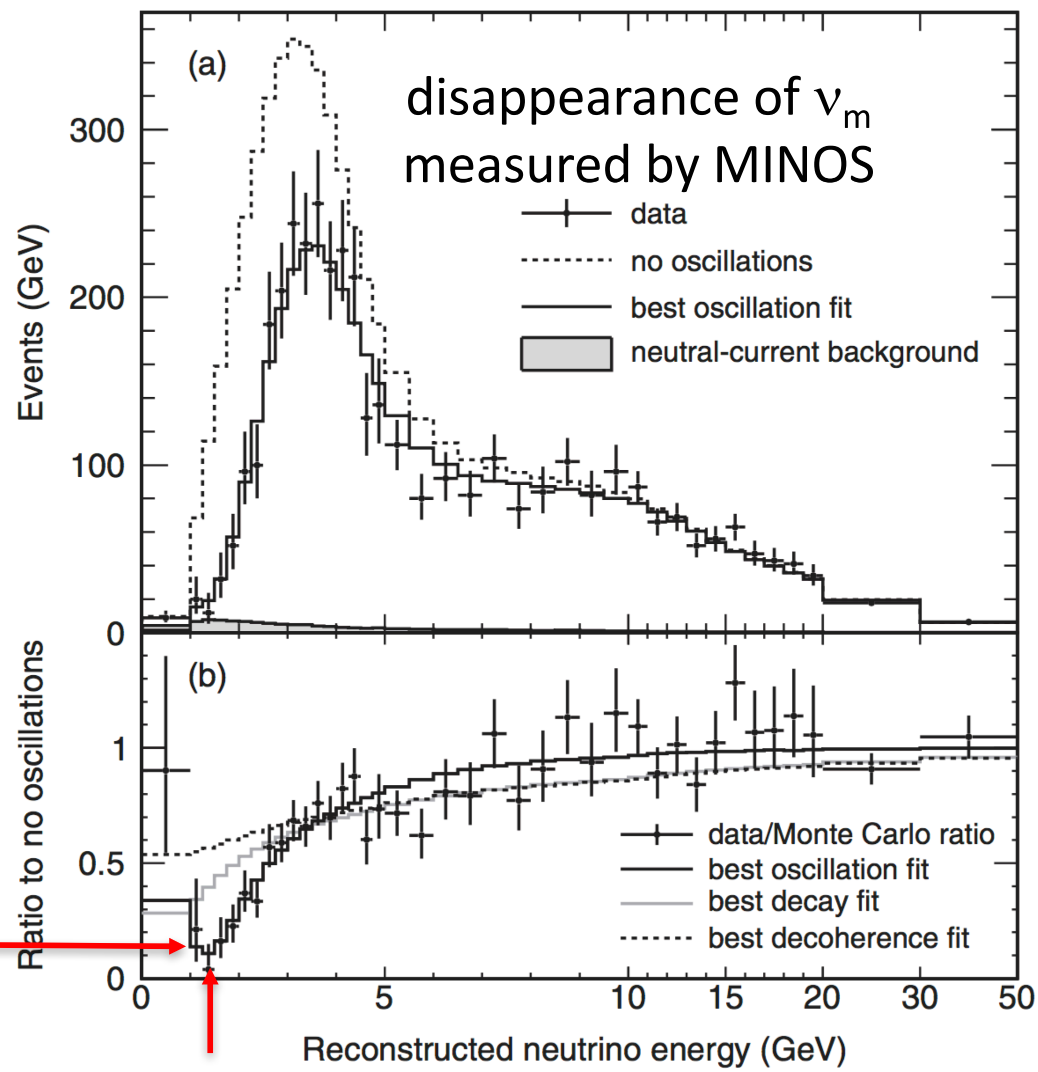
High Energy Beam Neutrinos

- **neutrino research is shifting to beam experiments. design experiment with specific goals**
- **neutrino experiments: K2K, MINOS, CNGS, T2K**
  - **Intense neutrino beam**
  - **Measure ratio of the neutrino energy spectrum in far detector (oscillated) to that in the near detector →less systematic err.**

near detector

far detector (oscillated)

$sin^2\theta_{23}$

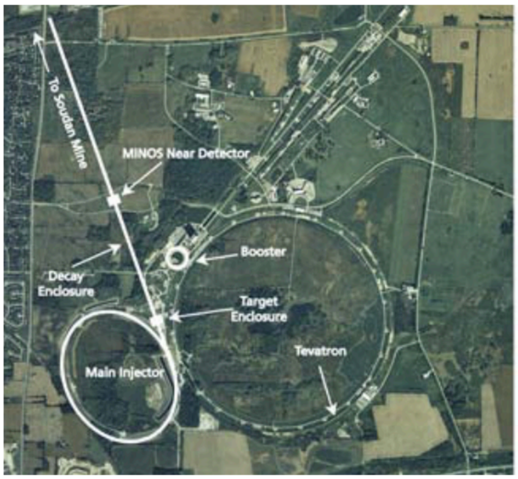


Δm^2

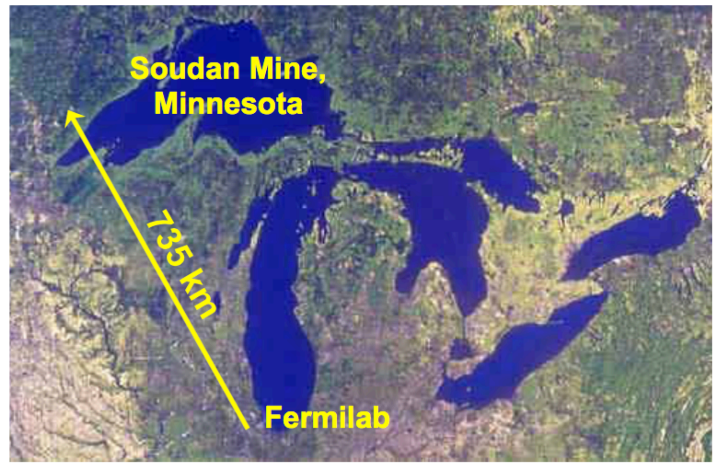
Infer oscillation parameters from the ratio

MINOS

- **120 GeV protons from the MAIN INJECTOR at Fermilab**
- **2.5×10^{13} protons per pulse hit target → very intense beam - 0.3 MW on target**

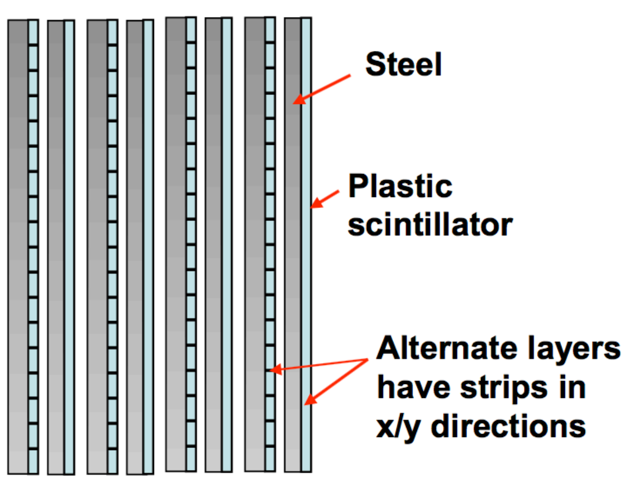


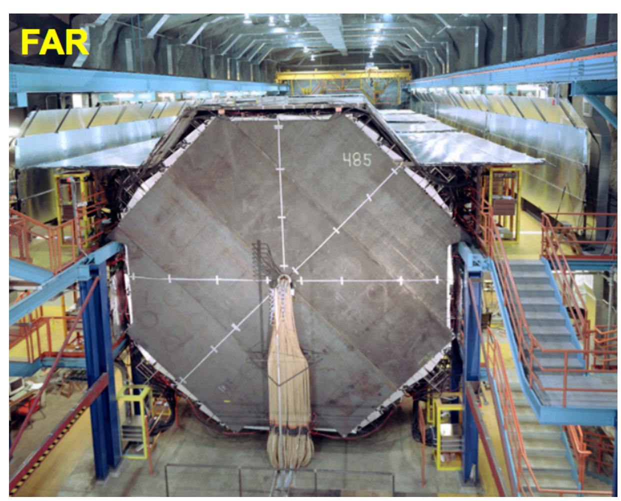
1000 ton, NEAR Detector at Fermilab



5400 ton FAR Detector, 720m underground in Soudan mine, N. Minnesota: 735 km away

- **Detect high energy hadrons & μ from neutrino interactions via CC interactions on nucleon:** $\nu_\mu + N \rightarrow \mu^- + X \qquad E_\nu = E_\mu + E_X$
- **Steel-Scintillator sampling calorimeter: each plane: 2.54 cm steel +1 cm scintillator charged particles crossing the scintillator produce light – detect with photomultipliers.**
- **E_μ from range/curvatire in B field, E_X – from hadronic shovers**
- **Do not see long-range Δ_{12} oscillations because Eν>50MeV**





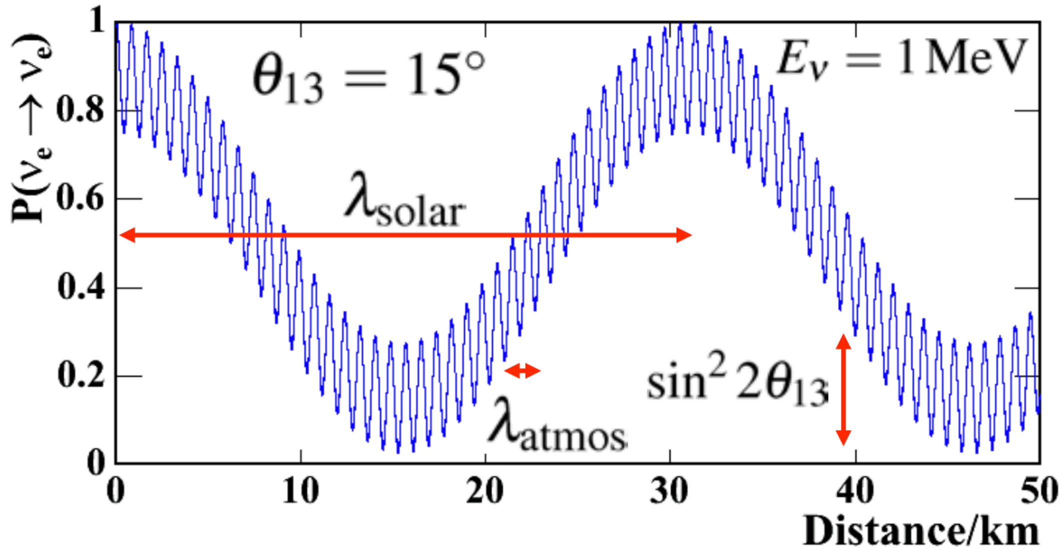
$$|\Delta m^2_{32}| = (2.43 \pm 0.12) \times 10^{-3}\,\text{eV}^2$$

Reactor Experiments

- **Reactors produce intense flux of electron anti-neutrinos with typical energies of few MeV → measurement of sin(θ_{13})**

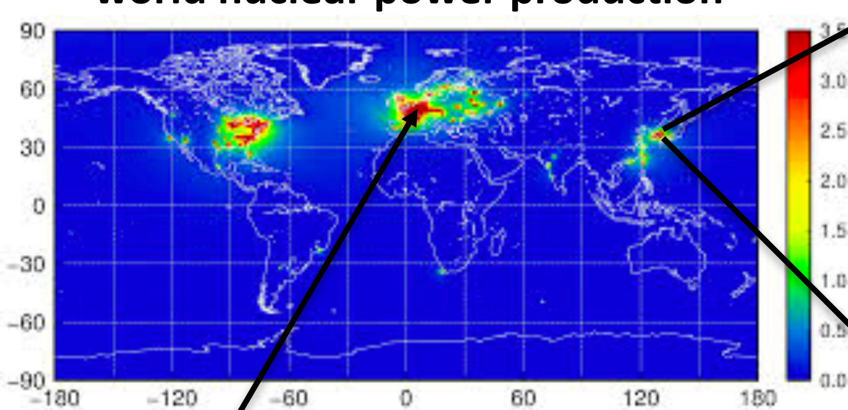
$$P(\nu_e \to \nu_e) \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32}$$

- $\boldsymbol{\lambda_{solar} \sim 30\ kn}$
- $\boldsymbol{\lambda_{atmos} \sim 1\ km}$



- **Amplitude of short wavelength oscillations given by sin(θ_{13})**

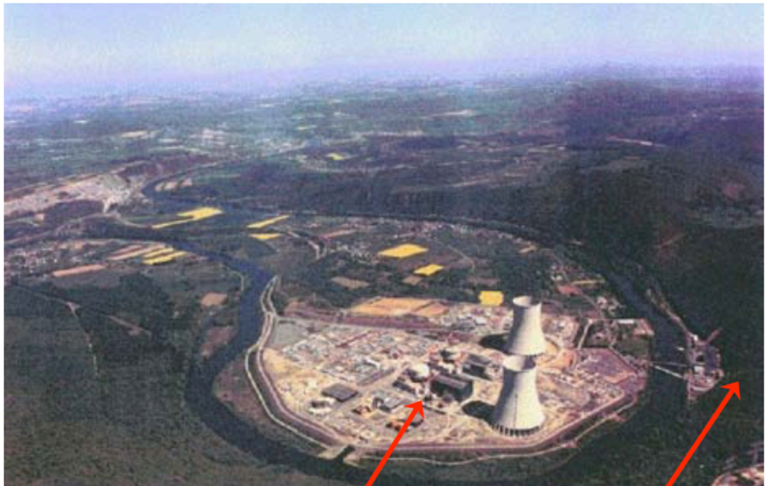
world nuclear power production



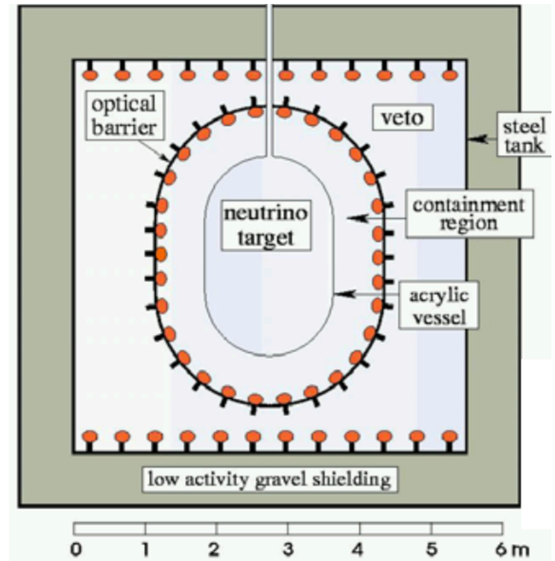
- **GHOOZ reactor, France, Daya Bay(China) – 1km short baseline experiment**
- **Japan + neighbors ~ 70 GW from nuclear power from reactors within 130-240 km – long baseline experiment KamLAND**

CHOOZ

- **Two nuclear reactors, each producing 4.2 GW**
- **detector 1 km away – liquid scintillator doped with Gd**



reactors **Detector 150m underground**



- **Detection via inverse beta decay** $e^+ + e^- \rightarrow \gamma + \gamma$

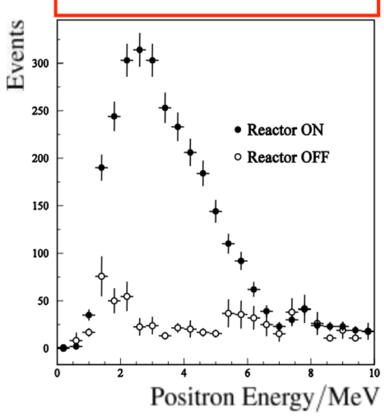
  prompt 2γ $\nu_e + p \rightarrow e^+ + n$

  delayed γ $n + \text{Gd} \rightarrow \text{Gd}^* \rightarrow \text{Gd} + \gamma + \gamma + \ldots$

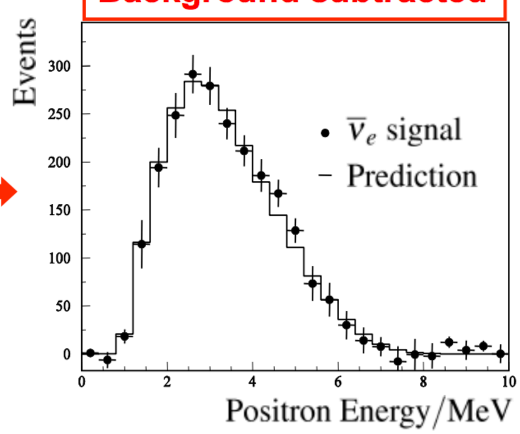
- **Oscillations due to Δm^2_{12} are very rapid → $\boldsymbol{sin^2(\Delta_{12}) = 0}$**

$$P(\nu_e \rightarrow \nu_e) \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32}$$

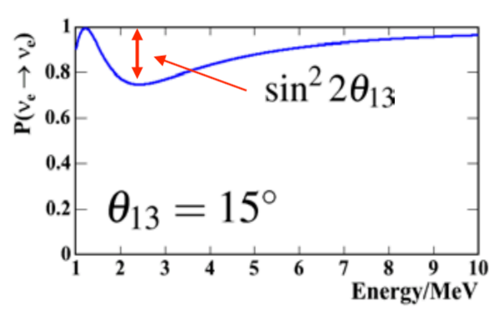
CHOOZ Raw Data



Background subtracted



Compare to effect of oscillations



PRL B420, 397-404, 1998

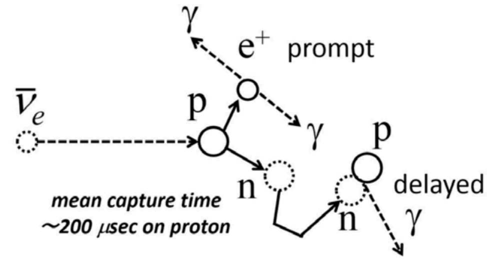
- $P(\nu_e \rightarrow \nu_e) \approx 1$ $\quad \sin^2 2\theta_{13} < 0.12 - 0.2$ $\quad$ exact limit depends on Δm^2_{23}

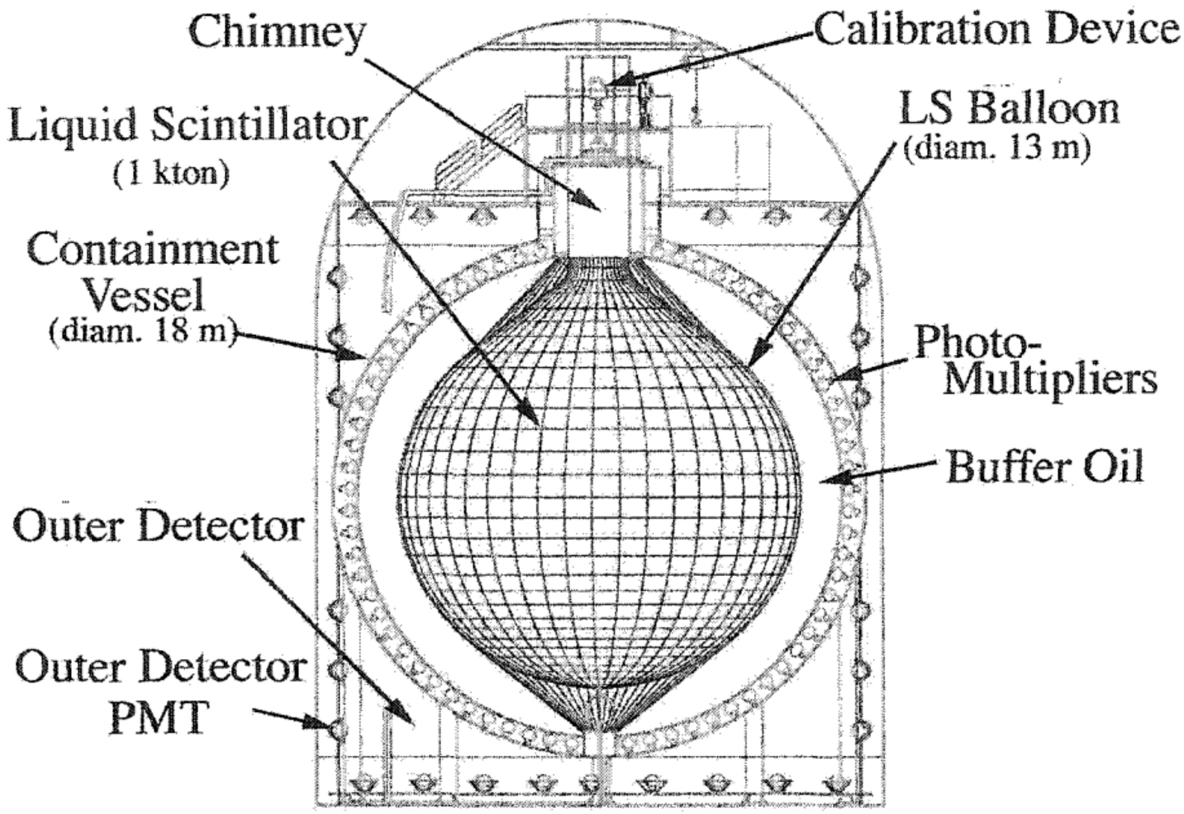
KamLAND

- **Liquid scintillator detector, 1789 PMTs, located in same mine as the Super Kamiokande detector**
- **Detection via inverse beta decay**

  prompt 2γ

  delayed γ





- **Oscillations due to Δm^2_{23} are very rapid → $\langle sin^2(\Delta_{23})\rangle$=1/2**

$$
\begin{aligned}
P(\nu_e \to \nu_e) &= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32} \\
&\approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13} \\
&= \cos^4\theta_{13} + \sin^4\theta_{13} - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\
&\approx \cos^4\theta_{13}(1 - \sin^2 2\theta_{12}\sin^2\Delta_{21})
\end{aligned}
$$

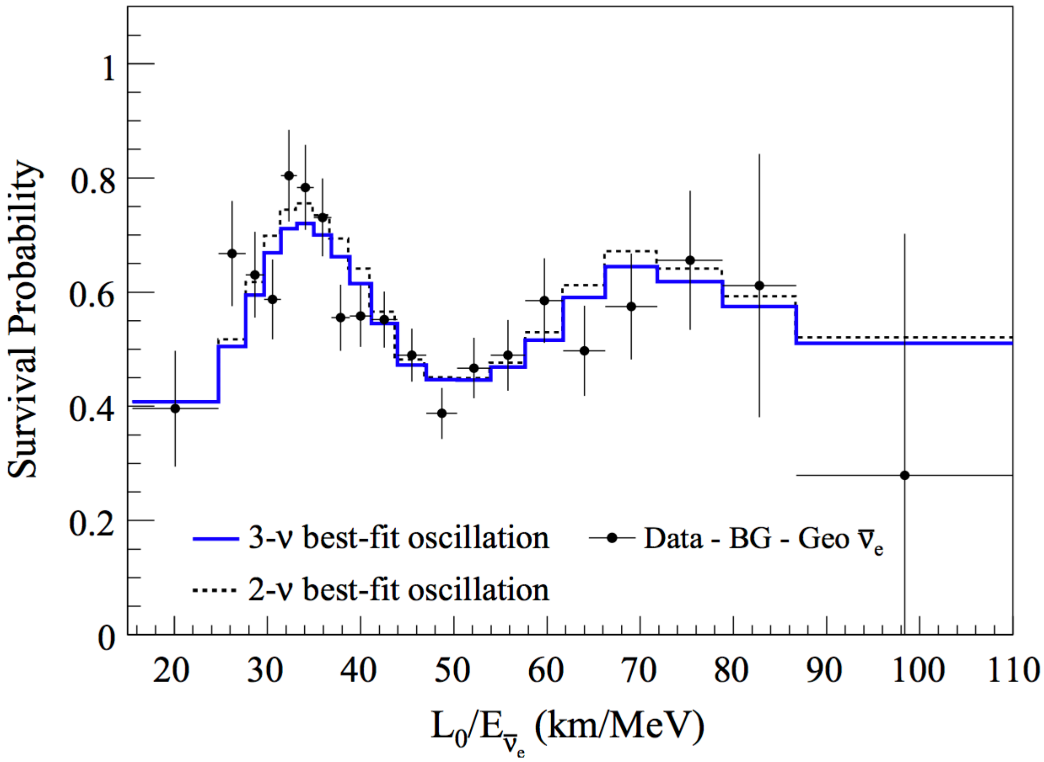
KamLAND

- **Survival probability**

$$P(\nu_e \to \nu_e) \quad \approx \quad \cos^4\theta_{13}(1 - \sin^2 2\theta_{12}\sin^2\Delta_{21})$$

from CHOOZ experiment neglect $\boldsymbol{sin^4\theta_{13}}$ **and** $\boldsymbol{cos^4\theta_{13} > 0.9}$

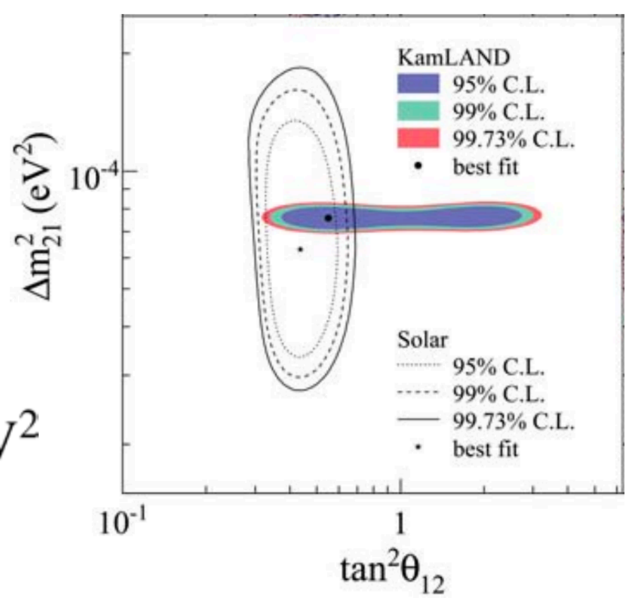
KamLAND Collaboration, Phys. Rev. Lett., 221803, 2008



- **first measurement of the oscillation behavior**
- **Compare data with expectations and perform fit to extract measurement**
- **KamLAND data provides strong constraints on** Δm^2_{12}
- **SNO constraints** $sin^2\theta_{12}$
- **Combined measurement**

$$|\Delta m^2_{21}| = (7.59 \pm 0.21) \times 10^{-5}\,\mathrm{eV}^2$$

$$\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$$

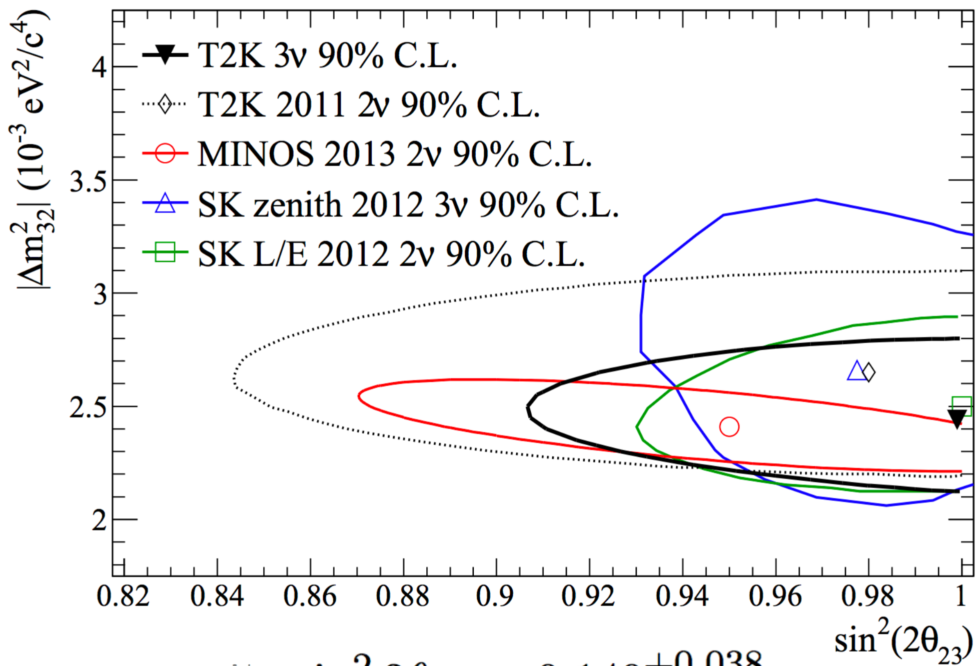


Summary of Current Knowledge

- **Solar experiments + KamLAND**

$$|\Delta m^2_{21}| \approx (7.6 \pm 0.2) \times 10^{-5}\,\mathrm{eV}^2$$
$$\tan^2\theta_{12} \approx 0.47 \pm 0.05$$

- **Long Baseline/Beam Experiments**



T2K $\sin^2 2\theta_{13} = 0.140^{+0.038}_{-0.032}$

- **Nuclear reactor experiments (topic for final exam)**

Daya Bay (China) $\sin^2 2\theta_{13} = 0.090^{+0.008}_{-0.009}$

RENO (S.Korea) $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$

Double CHOOZ $\sin^2 2\theta_{13} = 0.109 \pm 0.030 \pm 0.025$

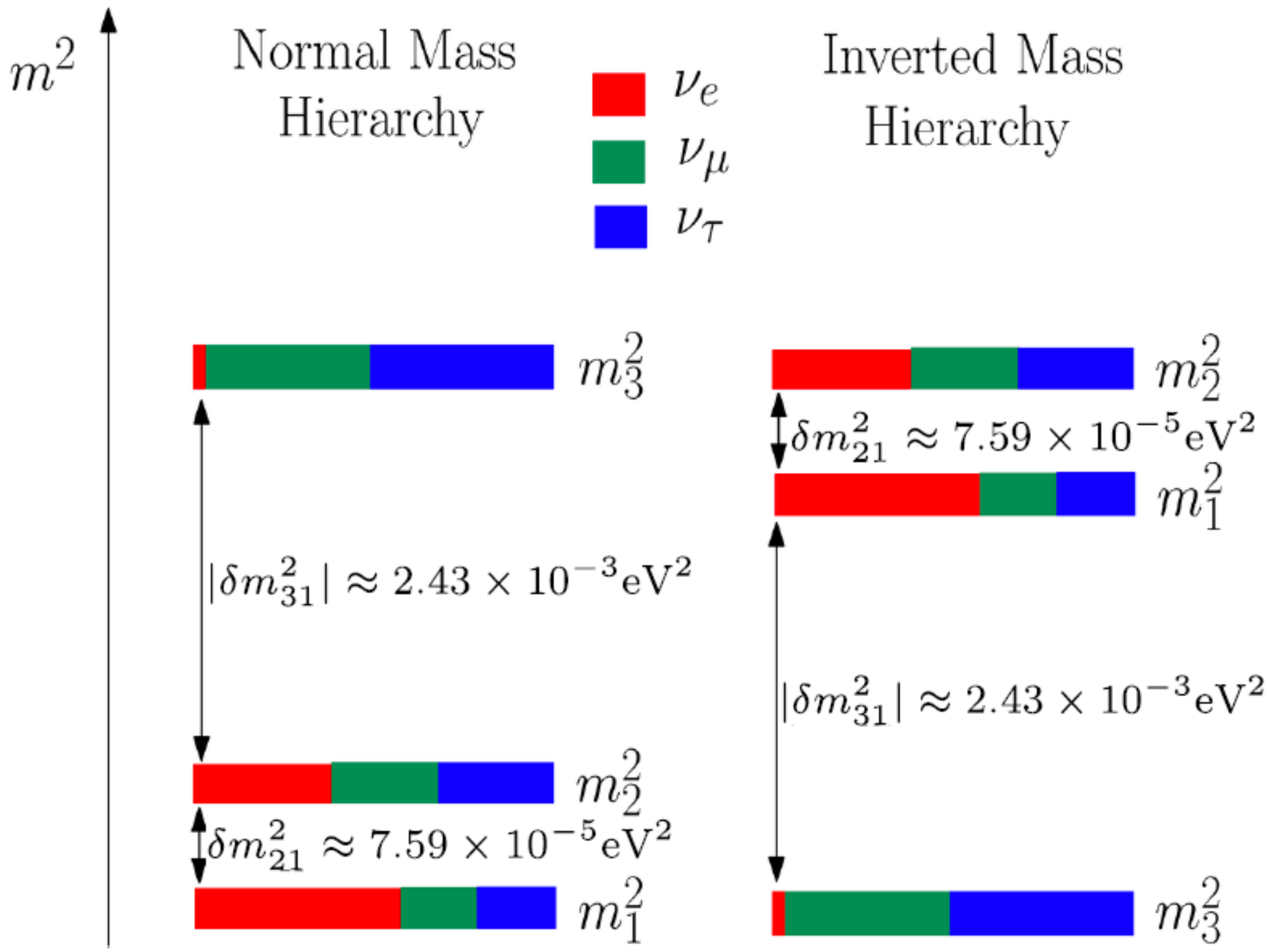
- **No evidence for non-zero δ – assume PMNS matrix is real**
- **Combiner result for measurement of PMNS matrix**

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix} 0.82 & 0.56 & \sim 0.15 \\ 0.31\text{–}0.43 & 0.51\text{–}0.59 & 0.75 \\ 0.37\text{–}0.47 & 0.59\text{–}0.66 & 0.66 \end{pmatrix} \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

Mass Hierarchy

$$|\Delta m_{13}^2| \approx |\Delta m_{23}^2| \approx 2\times10^{-3} eV^2$$

- **What is the hierarchy of mass eigenstates?**



- **Neutrino masses**

$$m_\nu(e) < 2\,\mathrm{eV}; \quad m_\nu(\mu) < 0.17\,\mathrm{MeV}; \quad m_\nu(\tau) < 18.2\,\mathrm{MeV}$$

- **The most stringent constraint from the tritium experiments**

$$m < 2\ eV$$

See Martin & Shaw chapter 2.3.3