What does the number m in $y = mx + b$ measure?

To find out, suppose (x_1, y_1) and (x_2, y_2) are two points on the graph of $y = mx + b$.

Then $y_1 = mx_1 + b$ and $y_2 = mx_2 + b$.

Use algebra to simplify $\dfrac{y_2 - y_1}{x_2 - x_1}$

And give a geometric interpretation.

Try this!

Answer:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{(mx_2 + b) - (mx_1 + b)}{x_2 - x_1}$$

$$= \frac{mx_2 - mx_1 + b - b}{x_2 - x_1}$$

$$= \frac{mx_2 - mx_1}{x_2 - x_1}$$

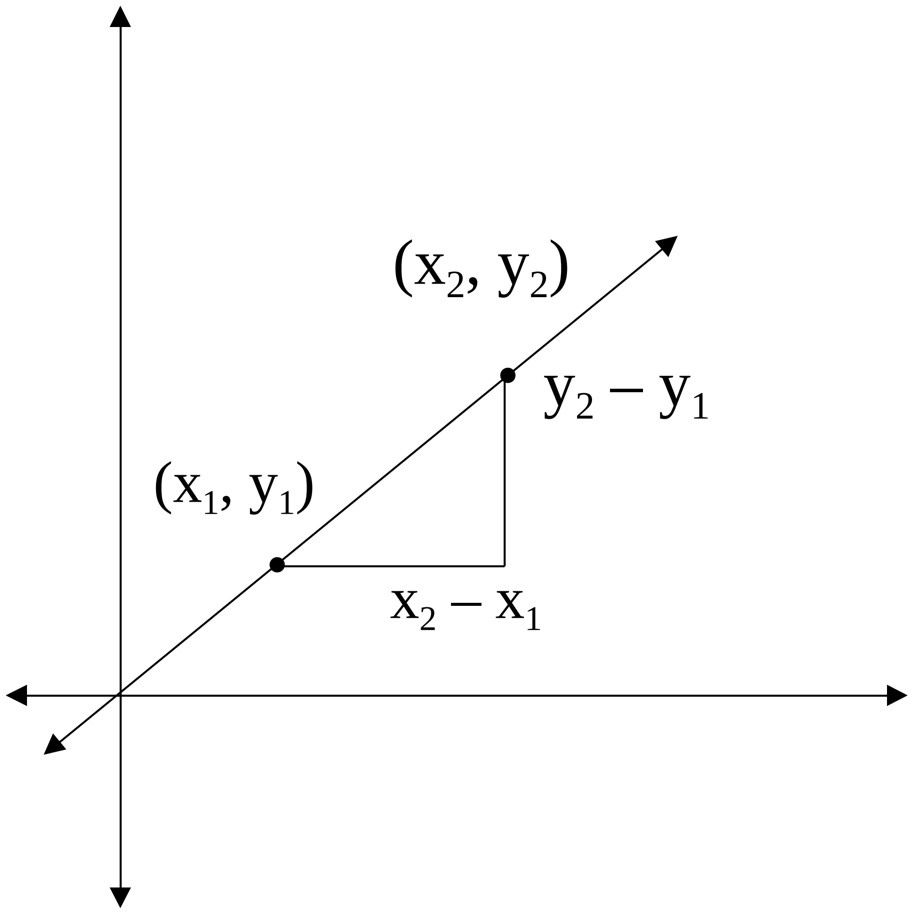
$$= \frac{m(x_2 - x_1)}{x_2 - x_1} \quad \text{distributive property}$$

$$= m$$

No matter which points (x_1,y_1) and (x_2, y_2) are chosen, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

But what does this mean?

Meaning of $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ in $y = mx + b$



$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ is the

“rise” (i.e. $y_2 - y_1$) over the

“run” (i.e. $x_2 - x_1$) and

m is called the <u>slope</u>.

Practice

Find the slope, m, of the line whose graph contains the points (1,2) and (2, 7).

Solution

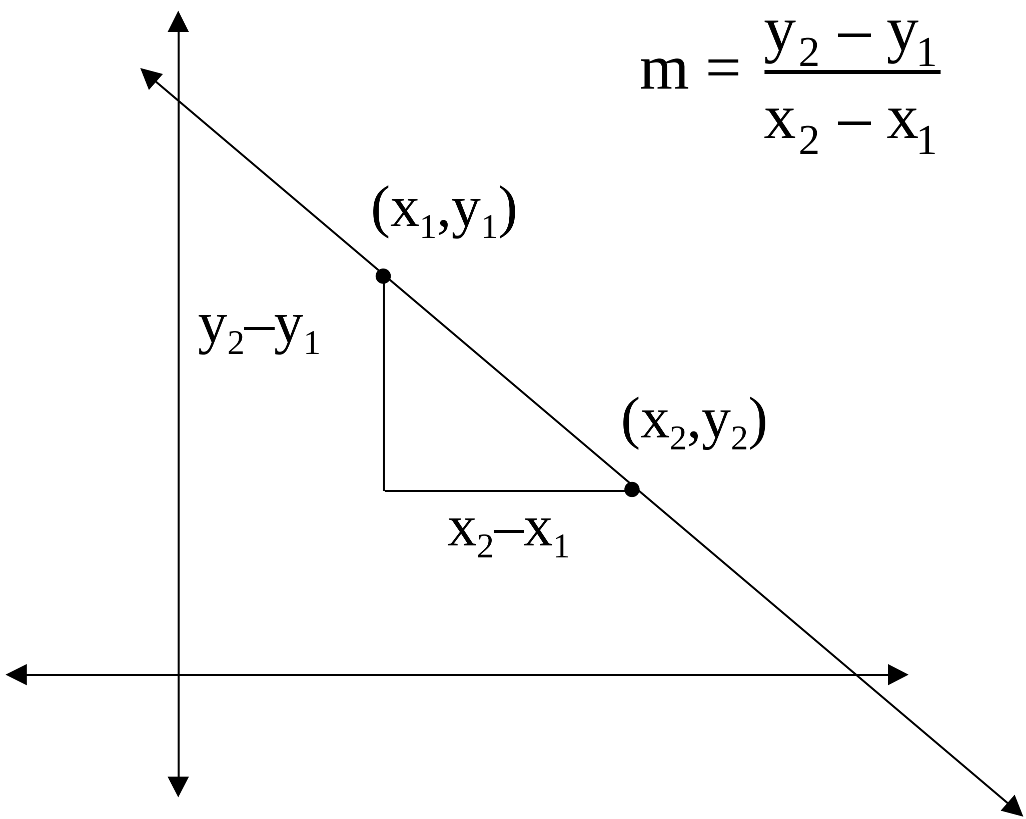
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-2}{2-1}$$

$$m = \frac{5}{1}$$

$$m = 5$$

The rise over the run, or slope, of the line whose graph includes the points (1,2) and (2,7) is 5.

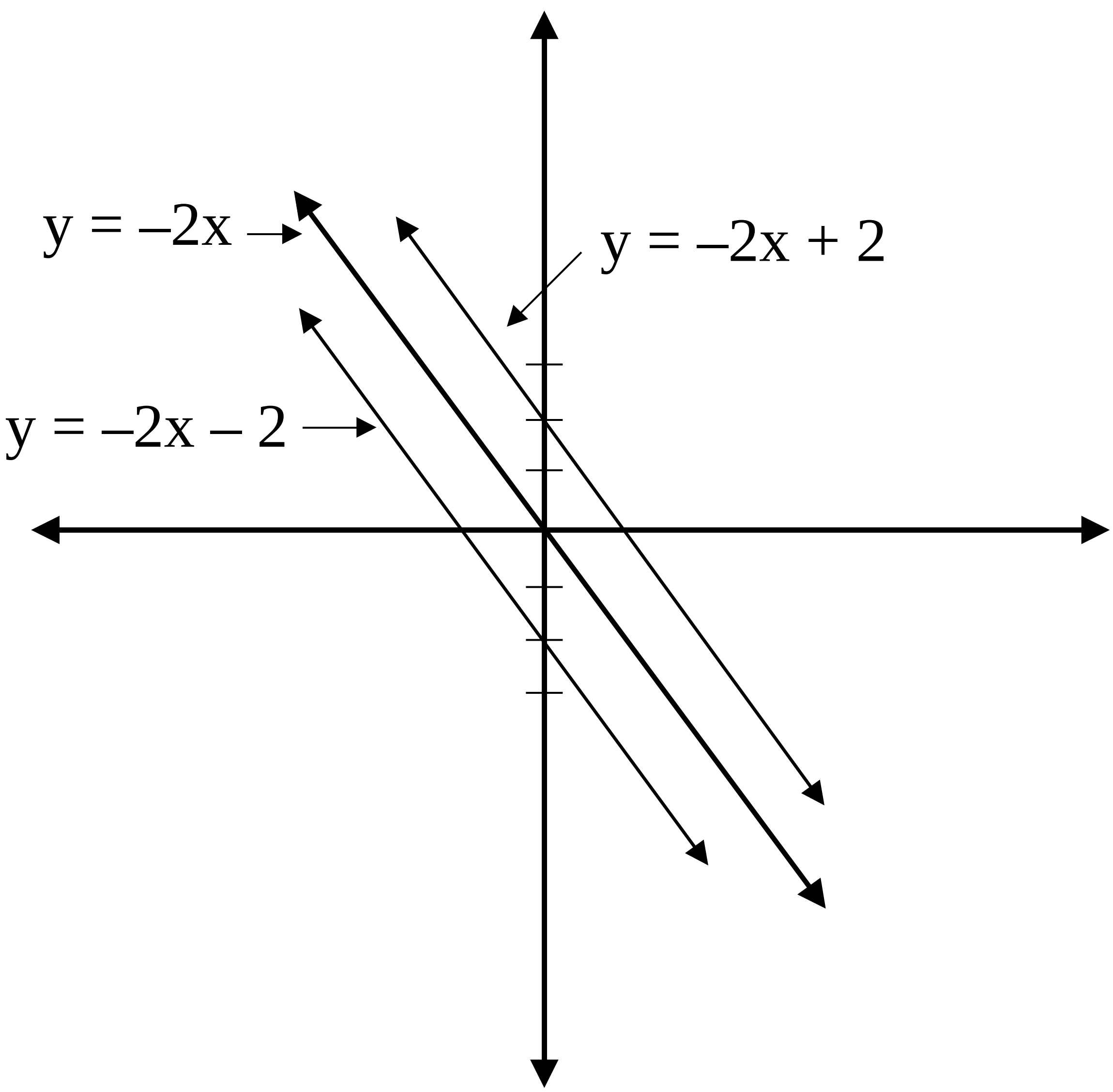
What does it mean if the slope, m, is negative in $y = mx + b$?



The negative slope means that y decreases as x increases.

Consider some examples.

x	y = -2x	y = -2x + 2	y = -2x – 2
0	-2 • 0 = 0	-2 • 0 + 2 = 2	-2 • 0 – 2 = -2
1	-2 • 1 = -2	-2 • 1 + 2 = 0	-2 • 1 – 2 = -4



DEFINITIONS

Definition 1

In the equation $y = mx + b$ for a straight line, the number m is called the slope of the line.

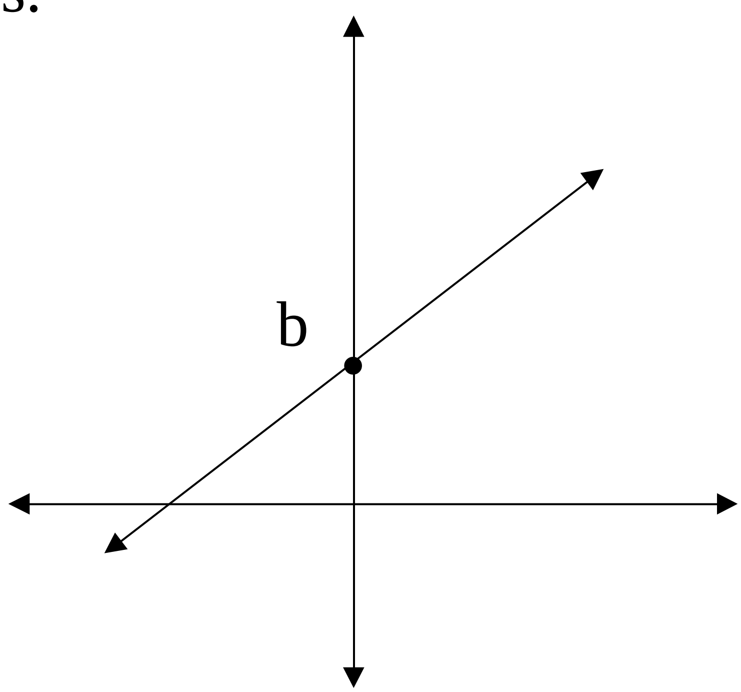
Definition 2

In the equation $y = mx + b$ for a straight line, the number b is called the y-intercept of the line.

Meaning of the y-intercept, b, in $y = mx + b$

Let $x = 0$, then $y = m \cdot 0 + b$, so $y = b$.

The number b is the coordinate on the y-axis where the graph crosses the y-axis.

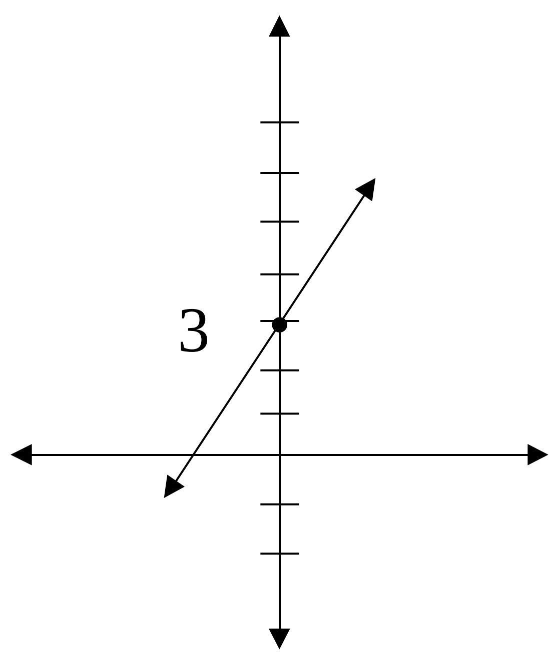


Example:

$$y = 2x + 3$$

What is the coordinate on the y-axis where the graph of $y = 2x + 3$ crosses y-axis?

Answer: 3



The Framework states…..

"… the following idea must be clearly understood before the student can progress further:

> A point lies on a line given by, for example, the equation $y = 7x + 3$, if and only if the coordinates of that point (a, b) satisfy the equation when x is replaced with a and y is replaced by b." (page 159)

Review this statement with the people at your table and discuss how you would present this to students in your classroom.

Verify whether the point (1,10) lies on the line

$y = 7x + 3$.

Verify whether the point (1,10) lies on the line $y = 7x + 3$.

Solution: If a point lies on the line, its x and y coordinates must satisfy the equation.

Substituting $x = 1$ and $y = 10$ in the equation $y = 7x + 3$, we have $10 = 7 \bullet 1 + 3$

$10 = 10$ which is true, therefore the point (1,10) lies on the line $y = 7x + 3$.

Practice

Tell which of the lines this point (2,5) lies on:

1. $y = 2x + 1$

2. $y = \frac{1}{2}x + 4$

3. $y = 3x + 1$

4. $y = -3x + 1$

5. $y = -4x + 13$

Example

Suppose we know that the graph of $y = -2x + b$ contains the point (1, 2).

What must the y-intercept be?

<u>Answer</u>: Substitute $x = 1$ and $y = 2$ in $y = -2x + b$, and then solve for b.

$$2 = -2 \cdot 1 + b$$

$$2 = -2 + b$$

$$4 = b \qquad b = 4$$

Practice

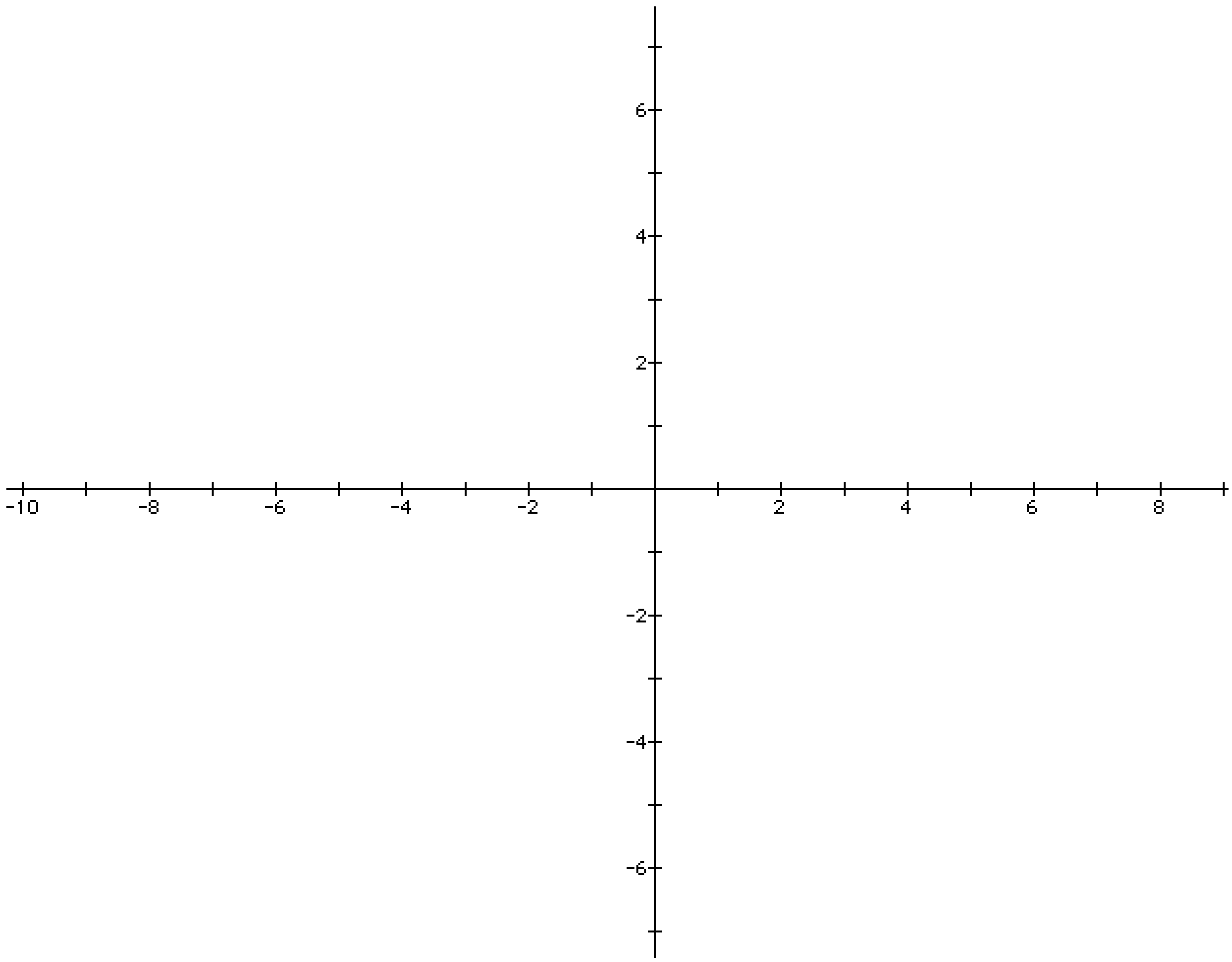
Find b for the given lines and points on each line.

1. $y = 3x + b$; (2,7)

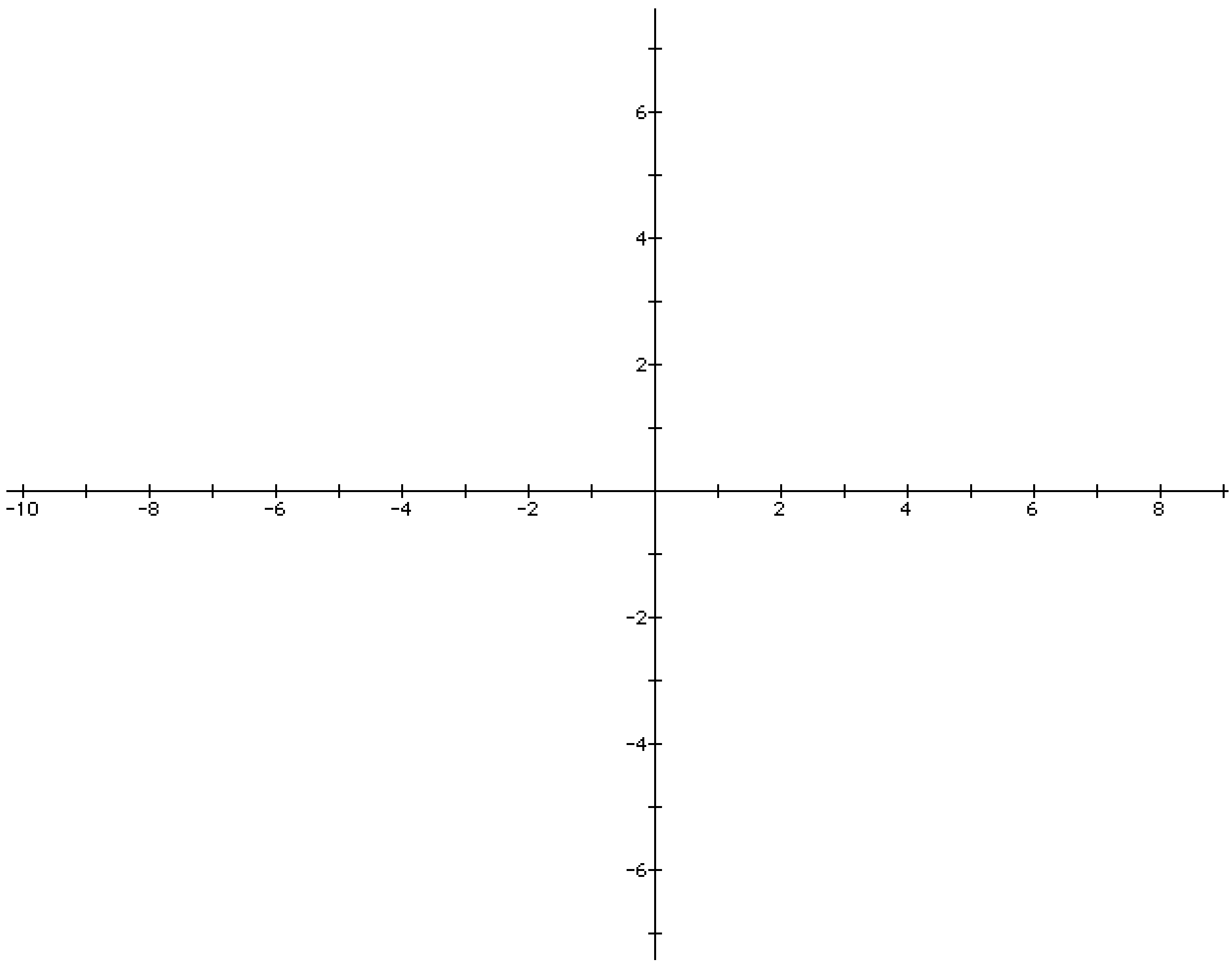
2. $y = –5x + b$; (–1,–3)

3. $y = \frac{1}{2}x + b$; (4,5)

Graph $y = 3x + 1$ by plotting two points and connecting with a straight edge.



Example: $y = 2x - 5$. Use the properties of the y-intercept and slope to draw a graph.

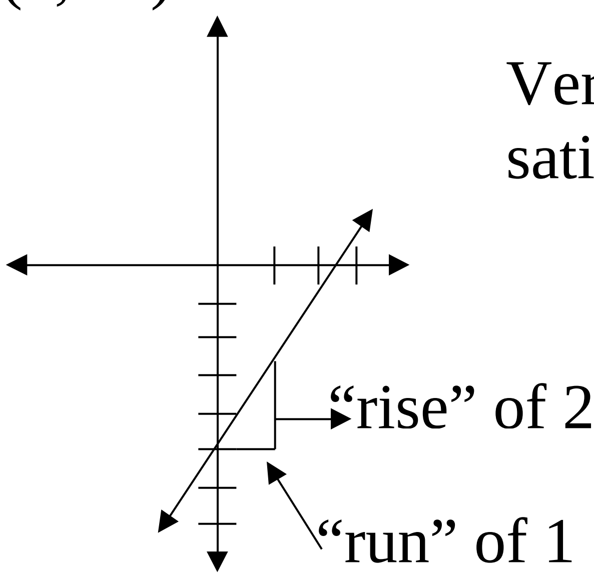


Solution:

Use b. In the equation y = 2x – 5, the y-intercept, b, is –5. This means the line crosses the y-axis at –5. What is the x coordinate for this point?

The coordinates of one point on the line are (0,–5), but we need two points to graph a line. We'll use the slope to locate a second point. From the equation, we see that m = 2 = $\frac{2}{1}$. This tells us the "rise" over the "run". We will move over 1 and up 2 from our first point. The new point is (1, –3).

Verify that (1, –3) satisfies the equation.



Standard 7

Algebra I, Grade 8 Standards

Students verify that a point lies on a line given an equation of a line. Students are able to derive linear equations using the point-slope formula.

Look at the Framework and see how this relates to the algebra and function standards for your grade.

Determine the equation of the line that passes through the points (1, 3) and (3, 7).

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 1: Use the formula above to determine the slope.

$$m = \frac{7 - 3}{3 - 1} = \frac{4}{2} = 2$$

Writing an equation of a line continued:

Step 2: Use the formula $y = mx + b$ to determine the y-intercept, b.

Replace x and y in the formula with the coordinates of one of the given points, and replace m with the calculated value, (2).

$$y = mx + b$$

If we use (1,3) and $m = 2$, we have

$$3 = 2 \bullet 1 + b$$
$$3 = 2 + b$$
$$1 = b \quad \text{or} \quad b = 1$$

If we use the other point (3,7) and $m = 2$, will we obtain the same solution for b?

$$7 = 2 \bullet 3 + b$$
$$7 = 6 + b$$
$$1 = b \quad \text{or} \quad b = 1$$

So, substituting $m = 2$ and $b = 1$ into $y = mx + b$ the equation of the line is $y = 2x + 1$ or $y = 2x + 1$.

Guided Practice

Find the equation of the line whose graph contains the points (1,–2) and (6,5).

The answer will look like

$$y = mx + b.$$

Step 1: Find m

Step 2: Find b

Step 3: Write the equation of the line by writing your answers from Steps 1 and 2 for m and b in the equation $y = mx + b$.

Try this!

Solution:

Find the equation of the line whose graph contains the points (1,–2) and (6,5).

Step 1: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-(-2)}{6-1} = \dfrac{7}{5}$

Step 2: $y = \dfrac{7}{5}x + b$

Substitute x = 1 and y = –2 into the equation above.

$$-2 = \frac{7}{5}(1) + b$$

$$-2 = \frac{7}{5} + b$$

$$-2 - \frac{7}{5} = b$$

$$b = -\frac{17}{5}$$

Step 3: $y = \dfrac{7}{5}x - \dfrac{17}{5}$

Practice

Find the equation of the line containing the given points:

1. (1,4) and (2,7)

   Step 1:

   Step 2:

   Step 3:

2. (3,2) and (–3,4)

   Step 1:

   Step 2:

   Step 3:

Point-Slope Formula

The equation of the line of slope, m, whose graph contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Example: Find the equation of the line whose graph contains the point (2,3) and whose slope is 4.

$$\begin{aligned} y - 3 &= 4(x - 2) \\ y - 3 &= 4x - 8 \\ y &= 4x - 5 \end{aligned}$$

Practice with point-slope formula

$$y - y_1 = m(x - x_1)$$

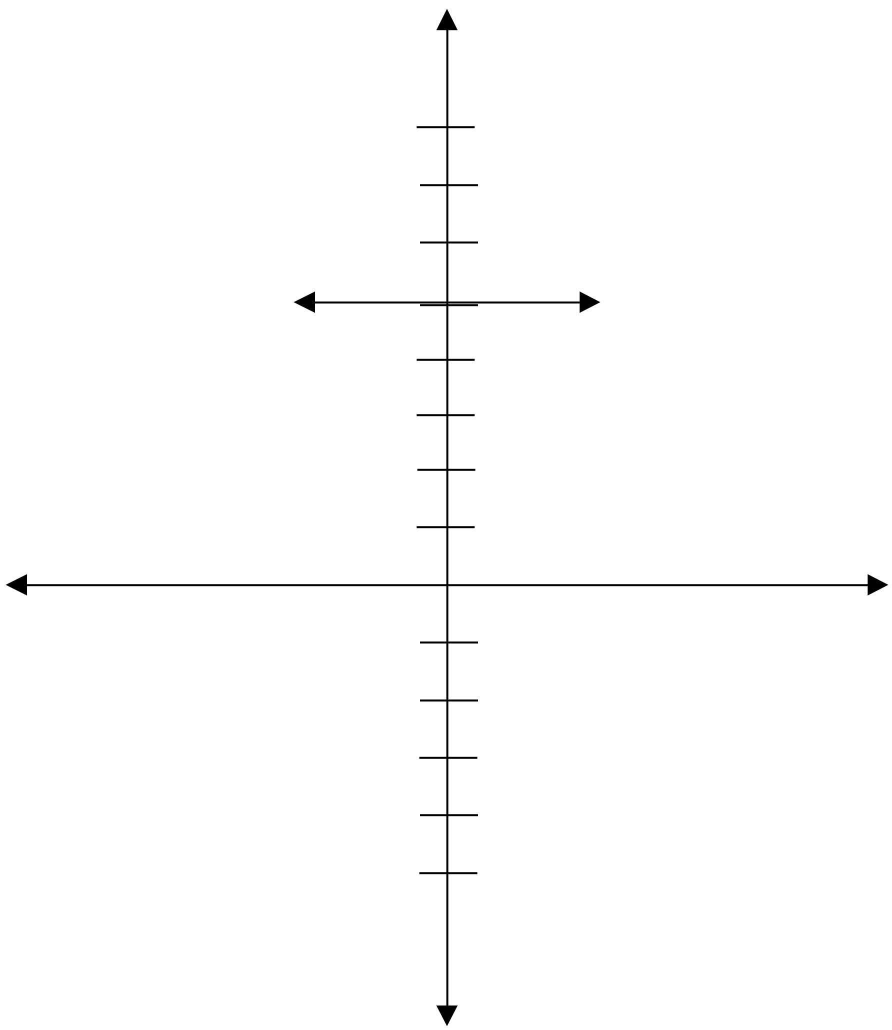
1. Find the equation of the line with a slope of 2 and containing the point (5,7)

2. Find the equation of the line through (2,7) and (1,3). (Hint: Find m first.)

Horizontal Lines

If m = 0, then the equation y = mx + b becomes y = b and is the equation of a horizontal line.

Example: y = 5



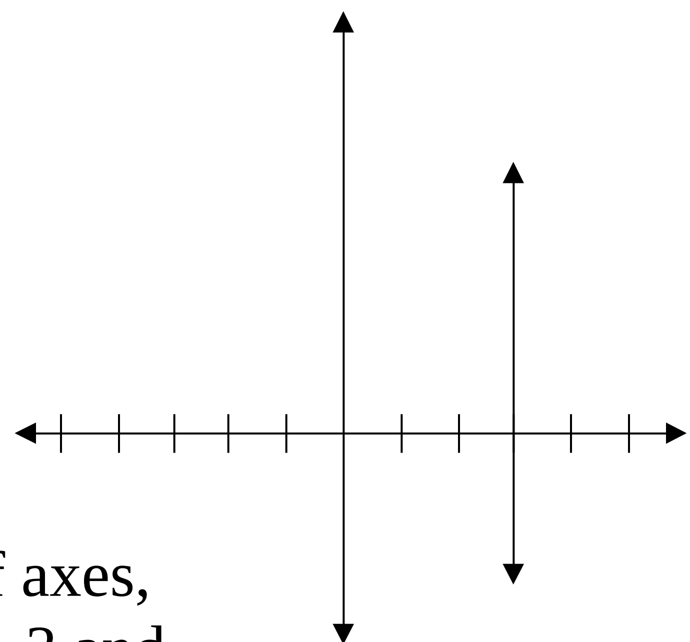
On the same pair of axes, graph the lines y = 2 and y = –3.

What about vertical lines?

A vertical line consists of all points of the form (x,y) where x = a constant.

This means x = a constant and y can take any value.

Example: $x = 3$



On the same pair of axes, graph the lines $x = -3$ and $x = 5$.

What about the slope of a vertical line? Let's use two points on the line $x = 3$, namely (3,5) and (3,8), then $m = \frac{8-5}{3-3} = \frac{3}{0}$. Division by 0 is undefined. The slope of a vertical line is undefined.

Standard Form for Linear Equations

The equation $Ax + By = C$ is called the general linear equation. Any equation whose graph is a line can be expressed in this form, whether the line is vertical or nonvertical.

Why?

Any non-vertical line is the graph of an equation of the form $y = mx + b$. This may be rewritten as $-mx + y = b$.

Now if $A = -m$, $B = 1$, and $C = b$, we get

$$Ax + By = C.$$

So, the equation $y = mx + b$ may be expressed in the form $Ax + By = C$.

Example:

Express $y = -3x + 4$ in the general linear form $Ax + By = C$.

$$\begin{aligned} y &= -3x + 4 \\ 3x + y &= 3x - 3x + 4 \\ 3x + y &= 0 + 4 \\ 3x + y &= 4 \end{aligned}$$

Here $A = 3$, $B = 1$, and $C = 4$.

What about vertical lines?

Any vertical line has an equation of the form $x = k$ where k is a constant.

$$x = k$$

can be rewritten as

$$Ax + By = C$$

where $A = 1$, $B = 0$, and $C = k$.

For example, $x = 2$ can be rewritten as

$$1 \bullet x + 0 \bullet y = 2.$$

The general linear equation

$$Ax + By = C$$

Can also be expressed in the form

$$y = mx + b$$

provided $B \neq 0$.

Reason:

$$\begin{aligned} Ax + By &= C \\ By &= -Ax + C \\ y &= \frac{1}{B}(-Ax + C) \\ y &= -\frac{A}{B}x + \frac{C}{B} \end{aligned}$$

Algebra Practice

Rewrite the equation $-2x + 3y = 4$ in the form

$$y = mx + b.$$

Solution:

$$\begin{aligned} -2x + 3y &= 4 \\ 3y &= 2x + 4 \\ y &= \frac{1}{3}(2x + 4) \\ y &= \frac{2}{3}x + \frac{4}{3} \end{aligned}$$

Here $m = \frac{2}{3}$ and $b = \frac{4}{3}$.