6 Orbit Relative to the Sun. Crossing Times

We begin by studying the position of the orbit and ground track of an arbitrary satellite relative to the direction of the Sun. We then turn more specifically to Sun-synchronous satellites for which this relative position provides the very definition of their orbit.

6.1 Cycle with Respect to the Sun

6.1.1 Crossing Time

At a given time, it is useful to know the local time on the ground track, i.e., the LMT, deduced in a straightforward manner from the UT once the longitude of the place is given, using (4.49). The local mean time (LMT) on the ground track at this given time is called the crossing time or local crossing time.

To obtain the local apparent time (LAT), one must know the day of the year to specify the equation of time E_{T}. In all matters involving the position of the Sun (elevation and azimuth) relative to a local frame, this is the time that should be used.

The ground track of the satellite can be represented by giving the crossing time. We have chosen to represent the LMT using colour on Colour Plates IIb, IIIb and VII to XI.

For Sun-synchronous satellites such as MetOp-1, it is clear that the crossing time (in the ascending or descending direction) depends only on the latitude. At a given place (except near the poles), one crossing occurs during the day, the other at night. For the HEO of the Ellipso Borealis satellite, the stability of the crossing time also shows up clearly.

In the case of non-Sun-synchronous satellites such as Meteor-3-07, the time difference shows up through a shift in the time from one revolution to the next. For a low-inclination satellite such as Megha-Tropiques, we see that, if the ascending node crossing occurs at 06:00, the northern hemisphere will be viewed during the day, and the southern hemisphere during the night. After a few days the crossing time at the equator will have changed.

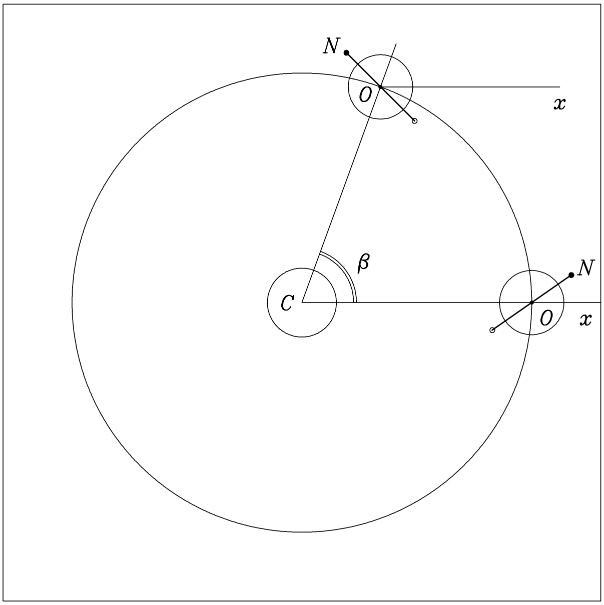

Figure 6.1. Nodal precession in the ecliptic. In the plane of the ecliptic, the centre of the Sun has been marked C, and the centre of the Earth O at two given times. The direction Ox is fixed relative to the stars. A satellite orbits the Earth. Its ascending node N is shown by a *small black dot*, whilst the descending node is shown by a *small circle*. The line segment joining the two nodes is the projection of the line of nodes on the ecliptic, not on the satellite orbit

6.1.2 Calculating the Cycle C_S

We consider the orbit of the Earth around the Sun, treating it as circular, since in this calculation of the cycle, we identify LAT and LMT. In Fig. 6.1, the centre of the Sun, and of the Earth's orbit, is denoted by C, whilst the centre of the Earth is O. The ascending node of a satellite in orbit around the Earth is denoted by N. The dihedral angle between the meridian plane of the Earth containing N and that containing C gives H, the hour angle of the ascending node. This angle is represented in Fig. 6.1 by $H = (\boldsymbol{OC}, \boldsymbol{ON})$. The diagram is schematic. To be precise, N should represent the projection of the ascending node on the plane of the ecliptic. However, this will not affect the following argument.

At the time $t = t_0$, the direction $\boldsymbol{CO}$ defines an axis $\boldsymbol{Ox}$, with fixed direction in the Galilean frame $\Re$. The hour angle of N is thus written $H(t_0) = H_0$. At another time $t = t_1$, the plane of the satellite orbit will have changed due to the phenomenon of nodal precession by an angle Ω relative to the frame $\Re$, i.e., relative to the direction $\boldsymbol{Ox}$. The hour angle of N is then given by

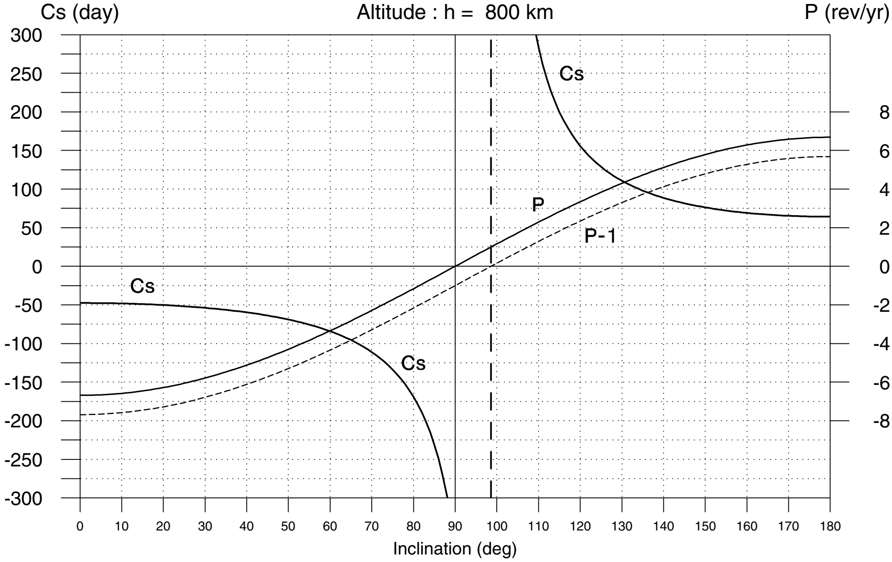

Figure 6.2. Variation of the cycle C_S relative to the Sun as a function of the inclination for a satellite at altitude $h = 800$ km. The cycle C_S is given in days on the *left ordinate*, and the nodal precession rate P is given in rev/yr on the *right ordinate*

$$H(t_1) = H_1 = H_0 + \Omega - \beta \ ,$$

where β is the angle through which the Earth has moved on its orbit around the Sun, viz.,

$$\beta = \big(\boldsymbol{CO}(t_0), \boldsymbol{CO}(t_1)\big) \ .$$

This angle β is equal to the difference in ecliptic longitude l of the Sun at the two given times. Hence,

$$\Delta H = H_1 - H_0 = \Omega - \beta \ ,$$

which represents the variation of the orbital plane relative to the direction of the Sun.

Setting $m = t_1 - t_0$ for the time interval, the angles can be written in terms of the angular speeds:

$$\Omega = m\dot{\Omega} \ , \quad \beta = m\dot{\Omega}_S \ ,$$

whence,

$$\Delta H = m(\dot{\Omega} - \dot{\Omega}_S) \ . \tag{6.1}$$

The time interval m_0 needed for the hour angle of the ascending node to vary by 24 hr, or one round trip, is called the cycle relative to the Sun. Hence,

$$H(t + m_0) = H(t) \quad [2\pi] \, ,$$

which implies that

$$m_0 = \frac{2\pi}{\dot{\Omega} - \dot{\Omega}_{\mathrm{S}}} \, .$$

Bringing in the nodal precession rate P in round trips per year as defined by (4.30) and using (4.28), m_0 becomes

$$m_0 = -J_{\mathrm{M}} \frac{N_{\mathrm{tro}}}{1 - P} \, .$$

The cycle relative to the Sun is usually given in days and we shall denote it by C_{S} (with C for 'cycle' and S for 'Sun'). Since m_0 is expressed in SI units, i.e., in seconds, we obtain C_{S} from the very simple expression

$$C_{\mathrm{S}} = \frac{N_{\mathrm{tro}}}{P - 1} \, . \tag{6.2}$$

The quantity P can be expressed in terms of the constant k_{h} defined by (4.63). This rate P is given by

$$P = -k_{\mathrm{h}} \left(\frac{R}{a} \right)^{7/2} \cos i \, . \tag{6.3}$$

One can check that for a Sun-synchronous satellite we do indeed have $P = 1$.

In this way we obtain the cycle relative to the Sun as a function of the orbital characteristics, taking care to note the signs:

$$C_{\mathrm{S}} = C_{\mathrm{S}}(a, i) = -\frac{N_{\mathrm{tro}}}{k_{\mathrm{h}} \left(\dfrac{R}{a} \right)^{7/2} \cos i + 1} \, , \tag{6.4}$$

or with approximate numerical values (C_{S} in days),

$$C_{\mathrm{S}} = -\frac{365.25}{10.11 \left(\dfrac{R}{a} \right)^{7/2} \cos i + 1} \, . \tag{6.5}$$

The cycle relative to the Sun, $C_{\mathrm{S}} = C_{\mathrm{S}}(a, i)$, is a very important feature of any satellite, but especially Earth-observation satellites.

6.1.3 Cycle C_{S} and Orbital Characteristics

Cycle C_{S} as a Function of Altitude and Inclination

The cycle C_{S} is a function of a and i. Figure 6.2 shows the variation $C_{\mathrm{S}}(i)$ for a fixed altitude $h = 800$ km. The cycle $C_{\mathrm{S}}(i)$ is given in days, with the sign indicating the direction of rotation. We have also plotted the nodal precession $P(i)$ in rev/yr, which is a sinusoidal curve, and $P-1$ which determines the vertical asymptote of $C_{\mathrm{S}}(i)$ by its intersection with the horizontal axis through the origin.

For the altitude represented here, typical of LEO satellites, we see that the cycle remains in the vicinity of two months ($C_{\mathrm{S}} \sim -60$ day) for inclinations below 45°. When i increases, the length of the cycle also increases. Above the Sun-synchronous inclination, the cycle decreases (but there are very few satellites in this configuration).

Specific Cases of the Cycle C_{S}

We note here certain specific values of the cycle C_{S} for different orbits.

- **Polar Satellites.** We see immediately from (6.4) or (6.5) that, if the satellite is strictly polar, $C_{\mathrm{S}} = -365.25$ day. The cycle is annual. One year goes by before we return to the same orbital configuration relative to the Sun, since the plane of the orbit does not rotate with respect to $\Re$. The negative value of C_{S} shows that the line of nodes moves in the retrograde direction relative to $\Re_{\mathrm{T}}$.
- **Sun-Synchronous Satellites.** Equation (6.2) shows that if $\dot{\Omega} = \dot{\Omega}_{\mathrm{S}}$, the cycle is infinite. This happens for Sun-synchronous satellites and we may indeed treat C_{S} as infinite, since after a very great number of days, the angle H will not have changed. For Sun-synchronous satellites, the hour angle of the ascending node, and hence the crossing time[1] of the satellite at the ascending node, is constant. For a given altitude, the cycle C_{S} is negative provided that i is less than the Sun-synchronous inclination given by (4.68). Beyond this value, C_{S} is positive.
- **Shortest Cycle.** The smallest value for the cycle is given by the minimum of $|C_{\mathrm{S}}(a, i)|$. According to (6.5), it is obtained for $i = 0$ and $a = R$ and the value is

$$|C_{\mathrm{S}}|_{\min} = \frac{N_{\mathrm{tro}}}{k_{\mathrm{h}} + 1} = \frac{365.25}{11.11} = 32.9 \text{ day} . \tag{6.6}$$

The cycle relative to the Sun C_{S} can never be less than 33 days.

[1] The time related to the hour angle is LAT. A Sun-synchronous satellite transits at the ascending node at the same LMT. If there is no difference between LAT and LMT here, it is because we have used a simplified scenario for the Earth orbit. However, for the calculation of the cycle C_{S}, this could not be otherwise: we only want to know how many days it will be before the next crossing (to within a few minutes), whatever time of year it is. To treat an elliptical Earth orbit, we would have to specify the day we choose to begin the cycle.

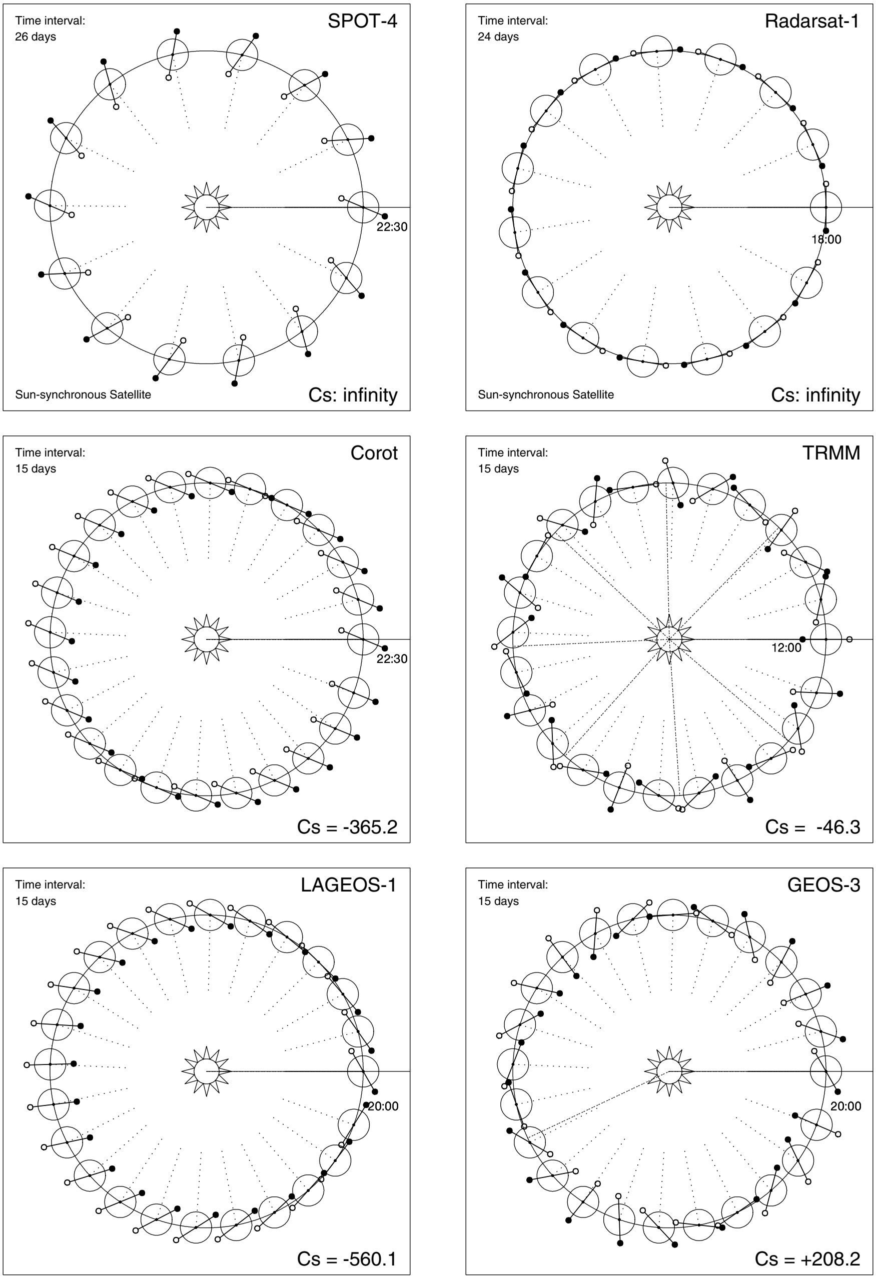

Figure 6.3. Cycle relative to the Sun for various satellites. The time given is the crossing time at the first ascending node

Example 6.1. *Calculate the cycle relative to the Sun for the satellites Meteor-3-07, TOPEX/Poseidon, ICESat, ERBS and UARS.*

These satellites have near-circular orbits. For Meteor-3-07, we have $h = 1\,194$ km and $i = 82.56°$. Using (6.5), we obtain

$$C_{\mathrm{S}} = -\frac{365.25}{10.11\left(\dfrac{6378}{7572}\right)^{7/2}\cos(82.56)+1} = -\frac{365.25}{10.11 \times 0.5477 \times 0.1295 + 1}$$

$$= -\frac{365.25}{0.7169+1} = -\frac{365.25}{1.7169} = -212.73\ ,$$

which gives a cycle of 213 days (advance of crossing time). In this case it is easier to use (6.2) because the value of P has already been calculated in Example 4.2:

$$P = -0.716 \quad \Longrightarrow \quad C_{\mathrm{S}} = \frac{365.25}{P-1} = -212.73\ .$$

For TOPEX/Poseidon, with $h = 1\,336$ km and $i = 66.04°$, we obtain $P = -2.107$, which gives a cycle $C_{\mathrm{S}} = -117.47$, or 117 days (advance of crossing time).
ICESat is at low altitude, $h = 592$ km, with inclination $i = 94°$ between the polar inclination for which the cycle is one year ($C_{\mathrm{S}} = -365.25$) and the Sun-synchronous inclination ($i_{\mathrm{HS}} = 97.8°$ at this altitude) for which the cycle is infinite. The calculation gives $P = 0.515$, whence $C_{\mathrm{S}} = -752.7$, which corresponds to a very long cycle of more than two years.
ERBS and UARS, both launched by the space shuttle, have the same inclination and the same altitude to within a few kilometres. The calculation gives $P = -3.986$ for ERBS, whence $C_{\mathrm{S}} = -73.2$, and $P = -4.090$ for UARS, whence $C_{\mathrm{S}} = -72.0$. One often reads for these satellites that their cycle relative to the Sun is 36 days. However, this is the half-cycle.

Nodal Precession and Cycle C_{S}

In order to visualise the nodal precession and bring out the significance of the cycle C_{S} as clearly as possible, let us return to the graph in Fig. 6.1 and apply it to a few satellites in the following example.

Example 6.2. *Visualising the cycle C_{S} for various satellites in prograde, polar, retrograde and Sun-synchronous orbit.*

Figure 6.3 shows the position of the Earth on its orbit around the Sun and the position of the nodes (ascending in black, descending in white) of the satellite orbit. For the two Sun-synchronous satellites, SPOT-4 and Radarsat-1, it is clear that the shift of the orbital plane compensates the Earth's annual motion. For Radarsat-1, the normal to the orbit lies in the meridian plane passing through the Sun.
For a strictly polar satellite like Nova-1 or Corot, the orbital plane is fixed in $\Re$.

For Corot, which has this inertial orbit, stars are observed perpendicularly to the orbit, six months in one direction, and six months in the opposite direction, in such a way as to avoid viewing the Sun.
Let us consider now several retrograde (negative) cycles, one very short, for TRMM (prograde orbit), one very long, for LAGEOS-1 (retrograde orbit). The satellite GEOS-3 (retrograde orbit) provides a rare case of precession in the prograde direction.

6.1.4 Cycle and Ascending Node Crossing Time

Knowing the initial conditions, it is a simple matter to obtain the crossing times at the ascending node at an arbitrary date, provided that we also know the cycle relative to the Sun C_{S}. Indeed, since the crossing time increases or decreases by 24 hours every C_{S} days, it is easy to calculate the increase or decrease per day. Here is an example of this calculation.

Example 6.3. *Calculate the dates during the year 1999 for which the LMT of the ascending node crossing is the same for the satellites TRMM and Resurs-O1-4.*

In order to study the Earth's radiation budget, TRMM and Resurs-O1-4 were equipped with the CERES and ScaRaB instruments, respectively. A joint measurement campaign was organised in January and February 1999. The aim was to compare the measurements obtained for the same region viewed by the two instruments at roughly the same times (with a leeway of ±15 min). The Sun-synchronous satellite Resurs-O1-4 crosses the ascending node at 22:20 LMT. The initial conditions for TRMM are given by an ascending node crossing (t_{AN} given in month day hr min s):

$$t_{\mathrm{AN}} = 1999\ 01\ 21\ 20{:}43{:}47\ (\mathrm{UT})\ , \quad \lambda = +5.157^\circ\ .$$

We calculate the value of τ_{AN}, LMT crossing time:

$$\tau_{\mathrm{AN}} = t_{\mathrm{AN}} + \frac{\lambda}{15} = 20{:}43{:}47 + 00{:}20{:}38 = 21{:}04{:}25\ .$$

In Example 4.1, we found $P = -6.89$, which gives the cycle

$$C_{\mathrm{S}} = -\frac{365.25}{7.89} = -46.29\ \mathrm{day}\ .$$

We thus obtain the daily drift as

$$\frac{1440}{C_{\mathrm{S}}} = -\frac{1440}{46.42} = -31.02\ \mathrm{min}\ .$$

The difference between $\tau_{\mathrm{AN}} = 21{:}04$ on 21 January 1999 ($J = 21$) and the chosen time of 22:20 is 76 min. The passage of TRMM at the chosen time thus occurs with a shift of $-76/31 = -2.45$ days, or 2 days earlier, i.e., on 19 January 1999 ($J = 19$). The ascending node crossing around 22:20 thus occurs on the days J_k given by

$$J_k = 19 + k|C_{\mathrm{S}}|\ ,$$

where the integer k takes 8 values over one year (since $|365/C_S| = 7.9$). Here, with $J_0 = 19$ and the values $k = 0, \dots, 7$, we obtain all the dates required for the year 1999. If we need to know the dates of passage of TRMM at 22:20 at the descending node, we merely add a half-cycle to the values of J_k, which gives dates shifted by 23 days with respect to the first series.

6.2 Crossing Time for a Sun-Synchronous Satellite

6.2.1 Passage at a Given Latitude

The time in LMT at which a Sun-synchronous satellite crosses the ascending node is constant in time (provided that the orbit is suitably maintained, of course), because in the frame $\Re$, the nodal precession balances the motion of the Earth's axis about the Sun. This is the defining feature of Sun-synchronous orbits, brought out in the next example.

Example 6.4. *Calculate the crossing time at two consecutive ascending nodes for a Sun-synchronous satellite.*

Consider the first crossing at the ascending node at longitude λ_1 and time $t = t_0$ in UT. Let τ_1 be the corresponding LMT, so that, according to (4.50),

$$\tau_1 = t_0 + \frac{\lambda_1}{15} ,$$

with time in hours and longitude in degrees.

The next passage (nodal period T) will occur at longitude λ_2 and at time $t = t_0 + T$. The corresponding LMT at the second crossing, denoted by τ_2, is therefore

$$\tau_2 = t_0 + T + \frac{\lambda_2}{15} .$$

The longitude λ_2 is obtained simply by considering the equatorial shift given by (5.22):

$$\lambda_2 = \lambda_1 + \Delta_E \lambda = \lambda_1 - 15T .$$

We thus have

$$\tau_2 = t_0 + T + \frac{\lambda_1 - 15T}{15} = t_0 + \frac{\lambda_1}{15} = \tau_1 ,$$

which shows that the LMT remains constant.

Since the mean motion is constant, the time taken to reach a given latitude from the equator will be the same for each revolution. We may thus say that, for a Sun-synchronous satellite:

- the LMT crossing time at a given latitude is constant,
- the LMT crossing time at a given meridian depends only on the latitude.

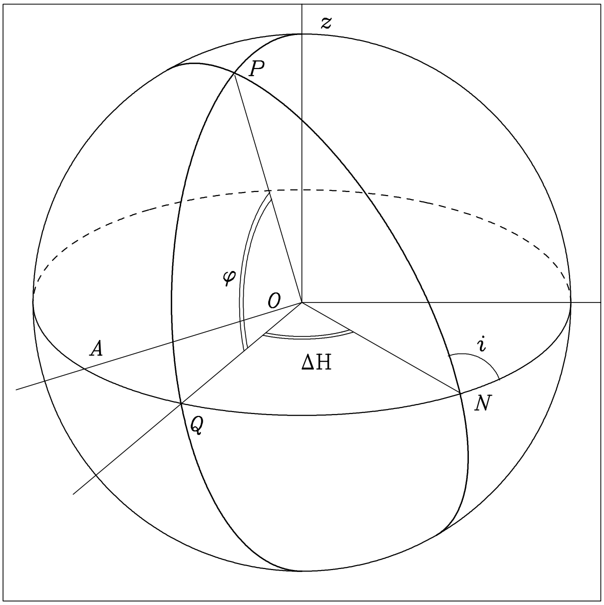

Figure 6.4. Intersection of the ground track of a Sun-synchronous satellite orbit (ascending node N) with a given meridian plane, defined by the point Q on the equator

Establishing the Relation Between ϕ and $\Delta\tau$

The relation between τ (the crossing time at the meridian in LMT) and ϕ (latitude) is found using the equations for the ground track and calculating the longitude corresponding to each latitude, whereupon the time can be found in LMT. But there is a simpler way to obtain this relation from geometric considerations.

Consider the Earth in the Galilean frame, as shown in Fig. 6.4. At a given time, let A be the intersection of the meridian plane of the direction of the Sun with the Earth's equator. We consider the orbital plane of a Sun-synchronous satellite. Its ground track cuts the equator at N, the projection of the ascending node on the Earth's surface. This plane makes an angle $i = i_{\mathrm{HS}}$ with the equatorial plane (this is indeed i since we are working in $\Re$, rather than the apparent inclination).

The angle $H_{\mathrm{AN}} = (\boldsymbol{OA}, \boldsymbol{ON})$ remains constant by the Sun-synchronicity condition, since H_{AN} measures the hour angle, and hence the time in LMT, of the ascending node.

Consider a meridian defined by a point Q on the equator. The ground track of the orbit cuts this meridian at a point P of latitude ϕ. The hour angle of P and of Q is $H = (\boldsymbol{OA}, \boldsymbol{OQ})$. We define

$$\Delta H = H - H_{\mathrm{AN}} = (\boldsymbol{ON}, \boldsymbol{OQ}) \ .$$

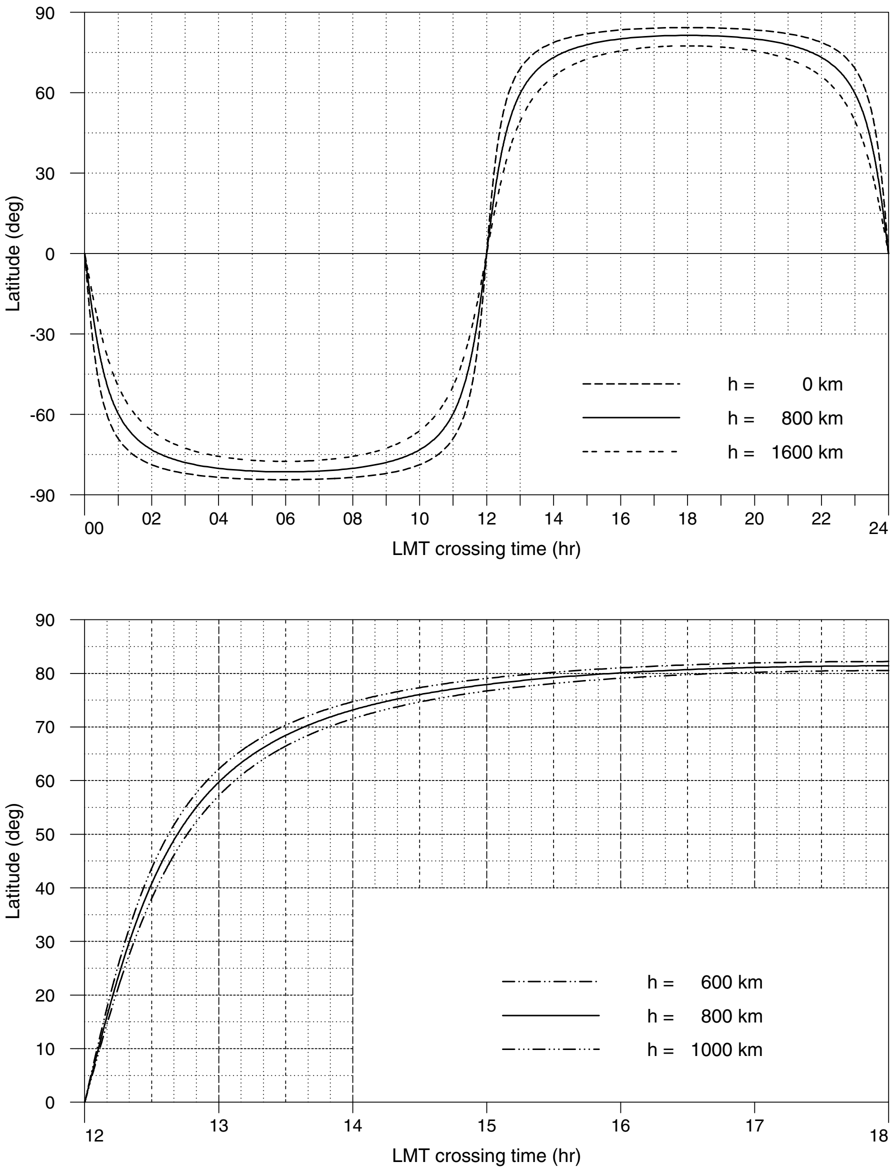

Figure 6.5. Graph of $\phi(\Delta\tau)$, the relation between the latitude of the point under consideration and the LMT time difference between transit at the ascending node and transit at this latitude, for a Sun-synchronous satellite. *Upper*: for three values of the altitude, $h = 800$ km and $h = (800 \pm 800)$ km. *Lower*: for three values of the altitude, $h = 800$ km and $h = (800 \pm 200)$ km. This is a magnified view of part of the upper figure

This angle thus measures the difference in hour angle between N and P (or Q).

In the spherical triangle PQN, with a right-angle at Q, we know the side PQ, $(\boldsymbol{OQ}, \boldsymbol{OP}) = \phi$ and the angle at N, representing the inclination of the orbital plane. We obtain ΔH from the standard relation of spherical trigonometry, corresponding to the relation (ST XII), identifying PQN with CAB:

$$\sin \Delta H = \frac{\tan \phi}{\tan i_{\mathrm{HS}}} \ . \tag{6.7}$$

Naturally, this formula is valid whether the satellite orbit is prograde or retrograde. In the prograde case, $\tan N$ and $\sin \Delta H$ are positive. In the retrograde case, as here, $\tan N = \tan(\pi - i_{\mathrm{HS}})$ and ΔH are negative.

Let τ_{AN} and τ be the local crossing times at the ascending node and P, respectively. Then,

$$\Delta \tau = \tau - \tau_{\mathrm{AN}} = \frac{1}{K} \Delta H \ , \tag{6.8}$$

where K is a constant depending on the units, so that if time is in hours and angles in degrees, then $K = 15$ (since 1 hr corresponds to 15°).

We thus have the following relations between the latitude ϕ and the difference in crossing times $\Delta \tau$:

$$\Delta \tau = \frac{1}{K} \arcsin \left(\frac{\tan \phi}{\tan i_{\mathrm{HS}}} \right) \ , \tag{6.9}$$

or

$$\phi = \arctan \left(\tan i_{\mathrm{HS}} \sin K \Delta \tau \right) \ . \tag{6.10}$$

Crossing Time at an Arbitrary Latitude

Let τ_{AN} and τ_{DN} be the crossing times at the ascending and descending nodes, respectively. Then,

$$\tau_{\mathrm{AN}} = 12 + \tau_{\mathrm{DN}} \quad [\mathrm{mod}\ 24] \ .$$

For $\Delta \tau$, we take the value defined by (6.9), i.e., between -6 hr and $+6$ hr. We thereby obtain the two daily crossing times $\tau_{(\mathrm{A})}$ and $\tau_{(\mathrm{D})}$ in the ascending and descending parts of the ground track, respectively:

$$\begin{cases} \tau_{(\mathrm{A})} = \tau_{\mathrm{AN}} + \Delta \tau \ , \\ \tau_{(\mathrm{D})} = \tau_{\mathrm{DN}} - \Delta \tau = \tau_{\mathrm{AN}} + 12 - \Delta \tau \ . \end{cases} \tag{6.11}$$

The time difference $\delta(\phi)$ between two crossings, one in the ascending part and the other in the descending part, at a given latitude is given by

$$\delta(\phi) = \tau_{(\mathrm{A})} - \tau_{(\mathrm{D})} = 12 + 2\Delta\tau \ . \tag{6.12}$$

We now give some examples of this calculation.

Example 6.5. *Calculate the LMT crossing time at latitude 15° N for a Sun-synchronous satellite at altitude h = 800 km, when the crossing time at the ascending node is 00:00 LMT.*

We have seen that the inclination of the satellite is $i = 98.6°$ for this altitude. Equation (6.9) yields

$$\Delta\tau = \frac{1}{K}\arcsin\left(\frac{\tan 15}{\tan 98.6}\right) = \frac{1}{15}\arcsin(-0.04052) = \frac{-2.32}{15}\ \mathrm{hr} = -9.3\ \mathrm{min}\ .$$

We thus take $\Delta\tau = -9$ min, and inserting $\tau_{\mathrm{AN}} = 00{:}00$ in (6.11), this implies that

$$\tau_{(\mathrm{A})} = \tau_{\mathrm{AN}} + \Delta\tau = 24\ \mathrm{h}\ 0\ \mathrm{min}\ -9\ \mathrm{min} = 23{:}51\ ,$$

$$\tau_{(\mathrm{D})} = \tau_{\mathrm{AN}} + 12 - \Delta\tau = 12\ \mathrm{h}\ 0\ \mathrm{min}\ +9\ \mathrm{min} = 12{:}09\ .$$

The two passages at this latitude thus occur at 23:51 LMT and 12:09 LMT, as can be checked on the upper part of Fig. 6.5.

Example 6.6. *Calculate the LMT crossing time at latitude 50° for the Sun-synchronous satellite SPOT-5, which transits the ascending node at 22:30 LMT.*

For this satellite and latitude 50°, (6.9) gives $\Delta\tau = -42$ min. With (6.11) and $\tau_{\mathrm{AN}} = 22{:}30$, we will thus have

$$\phi = 50°\mathrm{N} \quad \longrightarrow \quad 21{:}48 \text{ and } 11{:}12\ ,$$

$$\phi = 50°\mathrm{S} \quad \longrightarrow \quad 23{:}12 \text{ and } 09{:}48\ .$$

The daytime crossing will occur, in the northern hemisphere, well after 10:30, in fact, close to midday, with good solar lighting conditions. On the other hand, in the southern hemisphere, the crossing occurs rather early in the morning and the lighting conditions are not so good. The choice of node, e.g., descending at 10:30 rather than ascending) favours observation of the high latitudes of one hemisphere at the expense of the other. We shall return to this point.

6.2.2 Choice of Local Time at the Ascending Node

Restrictions on the Choice of Crossing Time

The local crossing time at the ascending node is determined by the aims of the mission. It is chosen as a compromise between various constraints which we shall number here from C1 to C6 (where C stands for ‘constraint’):

(C1) to obtain the best solar lighting conditions for the regions observed,
(C2) to reduce the risks of antisolar or specular reflection,[2]
(C3) to take meteorological factors into account, e.g., a certain region may be under cloud cover every day in the middle of the morning,
(C4) to take into account the crossing time of another Sun-synchronous satellite carrying out the same type of mission,
(C5) to limit periods of solar eclipse,
(C6) to limit thermal variations during each revolution.

We shall now discuss the various times chosen according to the type of mission.

Different Choices Depending on the Constraints

Satellites with High Energy Requirements. It is important to avoid long breaks in the power supply when satellites carry a radar or other instrument with high energy requirements. The solar panels must be almost continuously illuminated. To achieve this, the best-suited orbit has normal in the meridian plane (the normal at the centre of the orbit and the Earth–Sun direction are coplanar), because eclipses are then kept to a minimum (see Sect. 6.3). This Sun-synchronous orbit is such that $\tau_{\mathrm{AN}} = 06{:}00$ or 18:00 and it is called the dawn–dusk orbit.

Radarsat-1 is such a satellite ($\tau_{\mathrm{AN}} = 18{:}00$), as can be seen from Fig. 6.3: the constraint (C5) is given precedence. This orbit has been chosen for the future satellite Radarsat-2 ($\tau_{\mathrm{AN}} = 06{:}00$), for the Indian RISat-1 (Radar Imaging Satellite), and the Argentinian SAOCOM-1A. The same goes for oceanographic satellites using scatterometers, i.e., instruments measuring wind speeds at the sea surface, such as QuikScat ($\tau_{\mathrm{AN}} = 17{:}55$) and Coriolis ($\tau_{\mathrm{AN}} = 18{:}00$). It is also the orbit of the satellite Odin ($\tau_{\mathrm{AN}} = 18{:}00$).

This orbit is planned for the European projects GOCE, at very low altitude ($h \simeq 250$ km), Aeolus-ADM (Atmospheric Dynamics Mission) and WALES, at low altitude ($h \simeq 400$ km), and SMOS ($h = 755$ km). Other planned radar satellites will also be in dawn–dusk orbits: TerraSAR-X1 and TerraSAR-L1, and the COSMO-SkyMed constellation.

Satellites with Orbits Requiring a Specific Configuration Relative to the Sun. Solar observing satellites, if placed near the Earth, must gain

[2] Specular reflection occurs when the normal at the point P, viewed by the satellite, and the two directions P–satellite and P–Sun lie in the same plane to within a few degrees and, in addition, the normal is close to the bisector of these two directions. In this case, the satellite sensor may be blinded by the Sun, with the Earth's surface playing the role of mirror. This kind of reflection can be very efficient, in the case of a calm sea, for example, or quite imperceptible. All intermediate cases are possible, too. Antisolar reflection can occur when the Sun, the satellite and the point P being viewed are collinear. This can only happen between the two tropics.

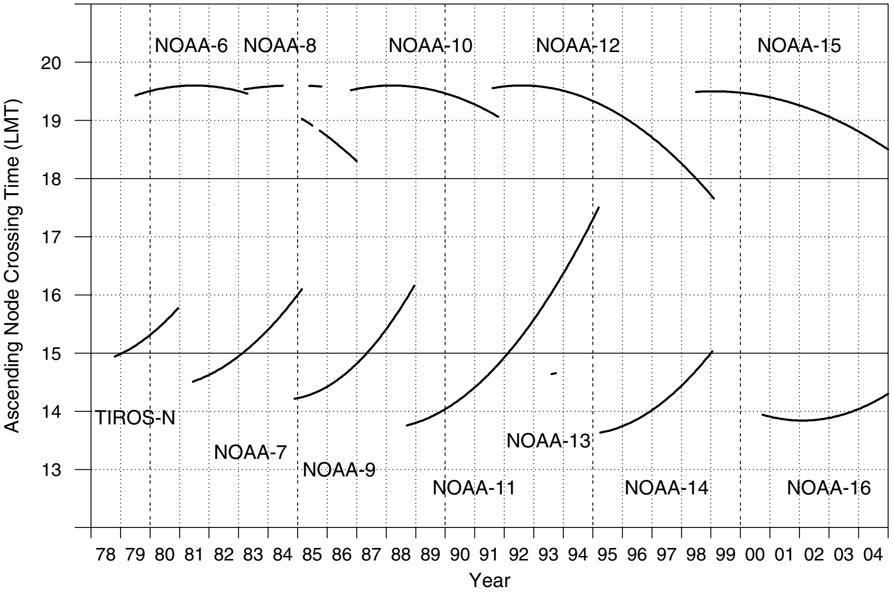

Figure 6.6. Drift of the ascending node crossing time τ_{AN} for Sun-synchronous meteorological satellites in the POES programme. The time τ_{AN} is given for the operating period of each satellite. From NOAA data

maximum advantage of their view of the day star. In its response to the constraint (C5), only the dawn–dusk orbit can allow such continuous observation. The satellite TRACE (τ_{AN} = 06:00) is on this type of orbit, also expected for Picard.

Satellites Subject to Limited Temperature Variation. It is of the utmost importance for satellites carrying out fundamental physics experiments on the equivalence principle that temperature variations should be kept to a minimum. The dawn–dusk orbit satisfies constraint (C6). This will be the orbit for µSCOPE and STEP.

Oceanographic Satellites. When they are not specialised in altimetry, oceanographic satellites are Sun-synchronous. If they do not carry scatterometers, the equatorial crossing is often chosen around midday and midnight, to satisfy constraint (C1): τ_{AN} = 00:00 for Oceansat-1, τ_{AN} = 00:20 for SeaStar, and τ_{N} around midday for Ocean-1 and -2 (also called HY-1 and -2).

Meteorological Satellites. For these satellites which observe meteorological phenomena, the crossing time is not critical. The ascending node crossing times of the various satellites are therefore rather varied, as can be seen from Table 6.1. Moreover, in most cases, these satellites are not kept at their station, the crossing time being allowed to drift. This drift is quadratic in time, as shown by (4.77). For the NOAA satellites, the drift, which can become

Table 6.1. Ascending node crossing time τ_{AN} for various Sun-synchronous satellites. The value of τ_{AN} is that of the first orbits for satellites actually launched or the planned value for satellites still under development. Meteorological satellites. For NOAA satellites, see also Fig. 6.6

Sun-synchronous satellite	τ_{AN}	Sun-synchronous satellite	τ_{AN}
NOAA-2	20:30	Nimbus-6	11:45
NOAA-3	20:30	Nimbus-7	23:50
NOAA-4	20:30	HCMM	14:00
NOAA-5	20:30	DMSP-5D2 F-8	06:15
NOAA-17	22:20	DMSP-5D2 F-10	19:30
FY-1A	15:30	DMSP-5D2 F-11	18:11
FY-1B	19:50	DMSP-5D2 F-12	21:22
FY-1C	18:20	DMSP-5D2 F-13	17:42
FY-1D	20:15	DMSP-5D2 F-14	20:29
FY-3A	21:30	DMSP-5D3 F-15	21:15
FY-3B	21:30	DMSP-5D3 F-16	19:58
Meteor-3M-1	09:15	NPP	22:30
MetOp-1	21:30	NPOESS-1, -4	21:30
MetOp-2	21:30	NPOESS-2, -5	13:30
MetOp-3	21:30	NPOESS-3, -6	17:30

quite significant, is shown in Fig. 6.6. The same goes for the DMSP satellites. For example, for the satellite DMSP-5D2 F-10, the drift was 47 min during 1991.

For the NOAA satellites from TIROS-N and NOAA-6 onwards, the constraint (C4) has been taken into account: for a given region, and with solar illumination, one satellite overflies in the morning and the other in the afternoon.

Satellites for Remote-Sensing of Earth Resources. A satellite may carry instruments pertaining to different types of mission. For example, the Russian satellite Resurs-O1-4 carries the Russian imaging device MSU for remote-sensing and the French instrument ScaRaB to study the Earth radiation budget (which can be classified as meteorological). But it is the remote-sensing aspect that determined the choice of crossing time.

As already mentioned, satellites of this type are Sun-synchronous, with very few exceptions. For satellites devoted to remote sensing of Earth resources, constraints (C1) and (C2) are given priority. The local crossing time at the node must be close to midday for (C1), but not too close because of (C2). Moreover, considering the curve $\phi(\Delta\tau)$, a shift away from midday yields good solar lighting conditions for high latitudes. Mission designers generally consider that the optimal time slot for viewing lasts for three hours centered on noon, i.e., from 10:30 to 13:30 LMT for the crossing at the relevant place, although these limits do not have to be strictly observed.

Table 6.2. Ascending node crossing time τ_{AN} for various Sun-synchronous satellites. The value of τ_{AN} is that of the first orbits for satellites actually launched or the planned value for satellites still under development. Remote-sensing and resource management satellites

Sun-synchronous satellite	τ_{AN}	Sun-synchronous satellite	τ_{AN}
Landsat-1	21:30	IRS-1A	22:25
Landsat-2	21:30	IRS-1B	22:25
Landsat-3	21:30	IRS-1C	22:30
Landsat-4	21:45	IRS-1D	22:30
Landsat-5	21:45	IRS-P2	22:40
Landsat-7	22:00	IRS-P3	22:30
EO-1	22:01	IRS-P6	22:30
SAC-C	22:15	TES	22:30
SPOT-1	22:30	Cartosat-1	22:30
SPOT-2	22:30	Cartosat-2	22:30
SPOT-3	22:15	CBERS-1	22:30
SPOT-4	22:30	CBERS-2	22:30
SPOT-5	22:30	TMSat	22:20
Hélios-1A	13:17	OrbView-3	22:30
Hélios-1B	13:16	OrbView-4	22:30
Pléiades-1	22:15	QuickBird-2	22:20
Pléiades-2	22:15	Ikonos-2	22:30
ERS-1	22:15	EarlyBird-1	22:30
ERS-2	22:30	QuikTOMS	22:30
Envisat	22:00	BIRD	22:30
EarthCARE	22:30	Terra (EOS-AM-1)	22:30
MOS-1	22:25	Aqua (EOS-PM-1)	13:30
MOS-1B	22:30	CloudSat	13:31
JERS-1	22:30	Calipso	13:31
ADEOS-1	22:30	PARASOL	13:33
ADEOS-2	22:30	Aura (EOS-Chem-1)	13:38
ALOS	22:30	OCO	13:15
EROS-A1	21:45	Rocsat-2	21:45
Kompsat-1	22:50	Tan Suo-1	23:00
Resource21-01	22:30	Diamant-1	23:30
Resource21-02	22:30	Diamant-2	23:30
Resurs-O1-4	22:15	RapidEye-1	12:00
TechSat-1B	22:15	NEMO	10:30
FaSat-2	22:20	SSR-1/ss	09:30

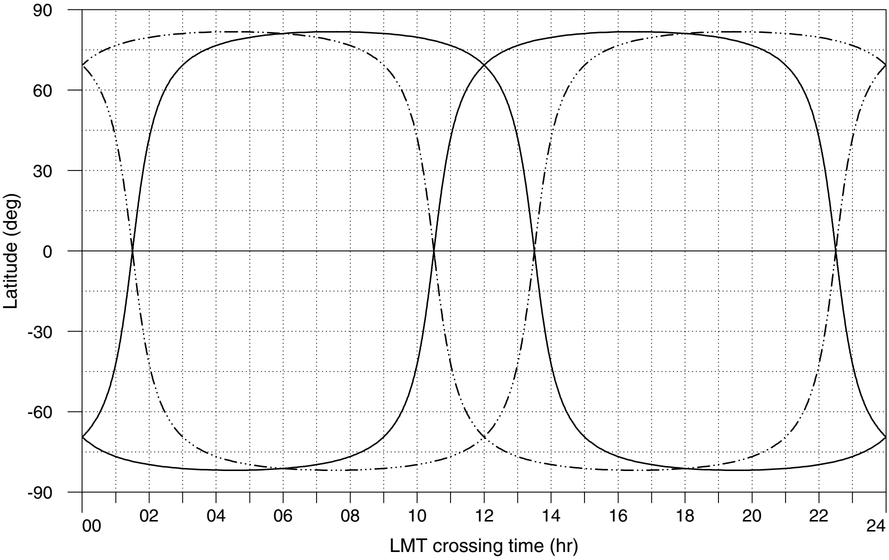

Figure 6.7. Complementarity of Terra and Aqua. LMT crossing time as a function of latitude for the Sun-synchronous EOS satellites. For various values of the LMT ascending node crossing time: 10:30 and 22:30 for EOS-AM-1, 01:30 and 13:30 for EOS-PM-1. The *continuous curve* shows the graph for values corresponding to the crossing time retained in the final project, i.e., 22:30 for EOS-AM-1 and 13:30 for EOS-PM-1 (Terra and Aqua, respectively)

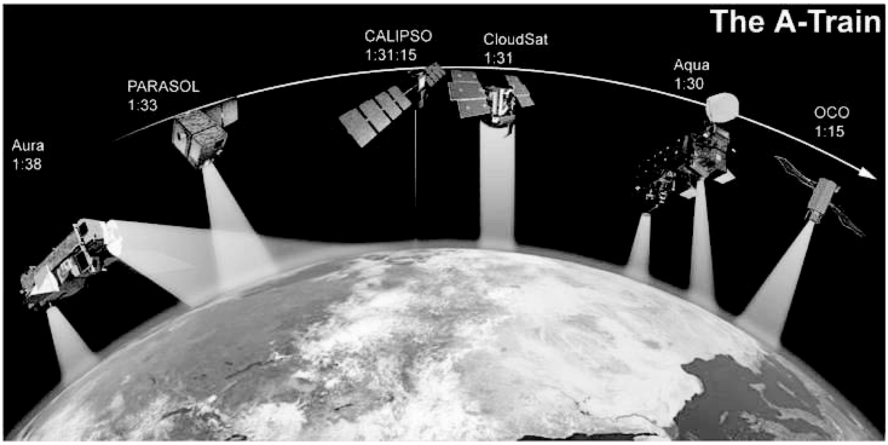

Figure 6.8. A-Train mission spacing (with notation of descending node crossing time). Credit: NASA, ESMO Project

One can thus envisage the following cases, calculated for a satellite at altitude $h = 800$ km:

- Equatorial crossing at the lower time limit. If the ascending node is at 10:30, $\tau_{\mathrm{AN}} = 10{:}30$, latitudes viewed between $\Delta\tau = 0$ and $\Delta\tau = 03{:}00$ are obtained using (6.10). With $K = 15$, the calculation for $\Delta\tau = 3$ gives

$$\phi = \arctan\left[(\tan 98.6) \times (\tan 45)\right] = -78° ,$$

  which corresponds to latitudes lying between 0° (at 10:30) and 78°S (at 13:30). If the descending node occurs at 10:30, $\tau_{\mathrm{AN}} = 22{:}30$, latitudes viewed during this time interval lie between 0° (at 10:30) and 78°N (at 13:30).
- Equatorial crossing at the upper time limit. If the ascending node is at 13:30, $\tau_{\mathrm{AN}} = 13{:}30$, latitudes viewed between $\Delta\tau = 0$ and $\Delta\tau = -3{:}00$ then lie between 78°N (at 10:30) and 0° (at 13:30). If the descending node is at 13:30, $\tau_{\mathrm{AN}} = 01{:}30$, latitudes lie between 78°S (at 10:30) and 0° (at 13:30).
- Equatorial crossing at midday. If the ascending node occurs at 12:00, $\tau_{\mathrm{AN}} = 12{:}00$, latitudes are viewed between $\Delta\tau = -1{:}30$ and $\Delta\tau = 1{:}30$. The calculation for $\Delta\tau = 1.5$ gives

$$\phi = \arctan\left[(\tan 98.6) \times (\tan 22.5)\right] = -68° ,$$

  which corresponds to latitudes lying between 68°N (at 10:30) and 68°S (at 13:30). If the descending node is at 12:00, $\tau_{\mathrm{AN}} = 00{:}00$, latitudes lie between 68°S (at 10:30) and 68°N (at 13:30).
- Choice of time. As the midday crossing time at the node is not chosen, to avoid specular reflection, the choice of the equatorial crossing time at 10:30 or 13:30 is guided by the choice between the northern and southern hemispheres. Naturally, the northern hemisphere is generally favoured, since it encompasses more visible land mass than the other hemisphere, but also because it comprises more nations financing satellite launches.

For satellites observing Earth resources, the choice is between the two equatorial crossing times:

$$\tau_{\mathrm{AN}} = 22{:}30 \quad \Longrightarrow \quad \text{descending node } 10{:}30 ,$$

$$\tau_{\mathrm{AN}} = 13{:}30 \quad \Longrightarrow \quad \text{ascending node } 13{:}30 .$$

The graphs in Fig. 6.7 clearly explain these choices for the satellites EOS-AM-1 and EOS-PM-1 (renamed Terra and Aqua, respectively).

The A-Train refers to the constellation of satellites that plan to fly together with EOS Aqua to enable coordinated science observation. These satellites have an afternoon crossing time close to the local mean time of the lead satellite, Aqua, which is 1:30 p.m. This explains the name: A is short for 'afternoon' and 'Train' is self-explanatory (see Fig. 6.8).

The EROS satellites should form a constellation of six satellites for which the choice of crossing times corresponds to the same strategy. The crossing times retained for this project are $\tau_{\mathrm{AN}} = 22{:}00$, 22:30, and 23:00 for EROS-B1, -B2 and -B3, and $\tau_{\mathrm{AN}} = 13{:}00$, 13:30, and 14:00 for EROS-B4, B-5 and -B6.

The choice between the two possibilities $\tau_{\mathrm{AN}} = 22{:}30$ or $\tau_{\mathrm{AN}} = 13{:}30$ is generally decided in response to the constraint (C3). In this way, one avoids the rather systematic formation of cloud cover at certain times of the day in certain well-defined regions. For example, the descending node was chosen at the end of the morning for the seven satellites in the Landsat series and the five SPOT satellites.

Table 6.2 shows the supremacy of the 22:30 crossing time for the ascending node with this type of satellite.

We may lay stakes that, if Australia sends up a satellite to study Earth resources across its territory, the ascending node will be at 10:30! Remaining for a moment in the southern hemisphere, note that Brazil had a project for a Sun-synchronous satellite, SSR-1 (here called SSR-1/ss), with ascending node at 09:30. This project has been transformed into another, for surveillance of the Amazon, requiring an equatorial orbit, although the satellite will still be called SSR-1.

The crossing times of remote-sensing satellites are generally maintained quite accurately, to within a few minutes.

Other Types of Mission. Other types of mission not mentioned above use Sun-synchronous orbits. Here are a few examples of ascending node crossing times: $\tau_{\mathrm{AN}} = 12{:}00$ for TOMS-EP, $\tau_{\mathrm{AN}} = 14{:}00$ for ARGOS, $\tau_{\mathrm{AN}} = 22{:}50$ for ACRIMSAT. Note that $\tau_{\mathrm{AN}} = 08{:}40$ was planned for TERRIERS.

6.3 Appendix: Duration of Solar Eclipse

The satellite undergoes solar eclipse when the Sun is hidden from it by the Earth. During the eclipse, the satellite cools down and its solar panels no longer produce electricity. For some satellites, an eclipse is a critical phenomenon, and in this case, the Earth–Sun–satellite geometry is examined in detail. We shall discuss here two types of orbit: dawn–dusk LEO and GEO.

6.3.1 Dawn–Dusk LEO Orbit

Consider a Sun-synchronous satellite in low circular orbit. If the LMT crossing time at the equator is around midday and midnight, the satellite is illuminated by the Sun for roughly a little more than half the period. The rest of the time, it moves in the shadow of the Earth.

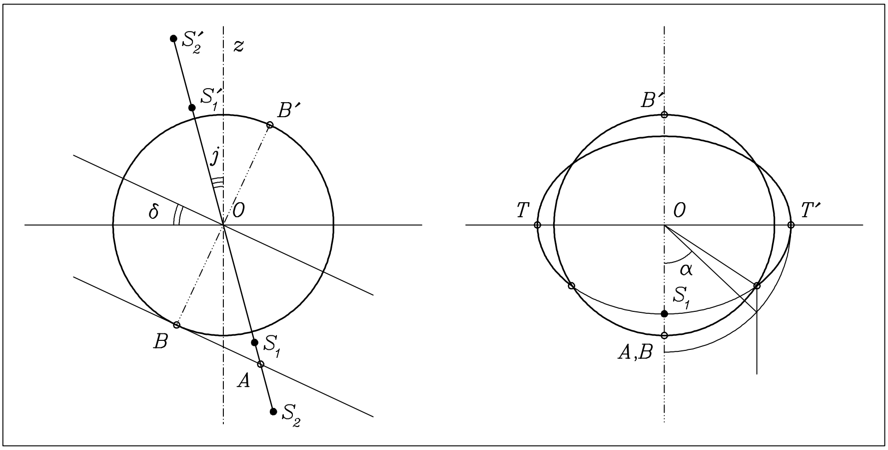

Figure 6.9. Schematic diagram of the Earth and the orbit of a Sun-synchronous satellite in a dawn–dusk configuration. *Left*: Meridian plane. Intersection of orbit with this plane: S_i and S_i'. *Right*: Plane perpendicular to the meridian plane and perpendicular to the direction of the Sun. The projection of the circular orbit on this plane is an ellipse

On the other hand, if the equatorial crossing times are around 06:00 and 18:00, the satellite is rarely in the Earth's shadow. This Sun-synchronous LEO orbit, with τ_{AN} = 06:00 or 18:00 is called a dawn–dusk orbit, as we have seen. In this configuration, which limits the length of the eclipse, one finds satellites that cannot tolerate long breaks in their power supply, or are sensitive to the sudden temperature change between day and night.

Eclipse Conditions

A Sun-synchronous satellite at altitude h (reduced distance η) has inclination $i = i_{\mathrm{HS}}$ given by (4.69). We set

$$j = i_{\mathrm{HS}} - \frac{\pi}{2} \,. \tag{6.13}$$

Figure 6.9 (left) shows the Earth (polar axis Oz, radius R) in the meridian plane containing the Sun (hour angle zero). Light rays from the Sun make an angle δ (declination) with the equatorial plane. The satellite orbit, which is perpendicular to the meridian plane because it is a dawn–dusk orbit, cuts this plane at S_i and S_i'. One of the two points is illuminated, e.g., S_i', and so is the other if it is not in the Earth's shadow, i.e., if $OS_i > OA$, where the point A is the intersection of the edge of the Earth's shadow with the plane of the orbit, in the meridian plane (the plane of the figure). In the example given in the figure, if the satellite is in position S_1, it undergoes solar eclipse, whereas if it is at S_2, there is no eclipse.

We have immediately

$$OS = R + h = a \; , \quad OA = \frac{R}{\cos(\delta + j)} \; .$$

The condition for there to be no eclipse is therefore

$$K > H \quad \text{(for a given declination)} \tag{6.14}$$

$$\text{with:} \quad H = 1/\eta \; , \quad K = \cos(\delta + j) \; .$$

The strongest constraint obtains at the two solstices, with $|\delta| = \varepsilon = 23.44°$. In these conditions, when η is varied between 1 and 1.9367, the maximal value for a Sun-synchronous satellite, given by (4.72), the condition $(K > H)$ is satisfied when η lies between 1.2181 and 1.5221. Using the altitude, we obtain

$$\text{no eclipse} \quad \Longleftrightarrow \quad 1\,391 < h < 3\,330 \text{ km} \; .$$

If the altitude of the satellite is less than 1 391 km, there is eclipse, because the satellite is not high enough to escape from the Earth's shadow (at least, at the solstice). If the altitude is greater than 3 330 km, the orbit is close enough to the equatorial plane (i tends to 180°) and the ecliptic to mean that, despite its high altitude, the satellite moves into the shadow.

These observations are rather theoretical. In practice, most satellites in dawn–dusk orbit are equipped with radar – with the constraint (C5) considered earlier – and an altitude less than 800 km is thus the norm. The eclipse phenomenon is then inevitable at some point during the year.

Calculating the Duration of Eclipse

We calculate the duration of eclipse when the satellite has altitude less than the limiting value $h = 1\,391$ km. Looking along the direction of the Sun's rays, the Earth appears as a circle $(\mathcal{C}_1)$ of radius R and the dawn–dusk orbit appears as an ellipse $(\mathcal{C}_2)$ with semi-major axis a, the actual radius of the circular orbit, and semi-minor axis b, the projection of a on a plane perpendicular to the direction of the Sun. Figure 6.9 (right) shows, for $(\mathcal{C}_1)$, $R = OB$, and for $(\mathcal{C}_2)$, $a = OT$, $b = OS_1$, so that

$$a = R + h \; , \quad b = a \cos(\delta + j) = aK \; .$$

With the axes (O, x, y), the equations defining curves $(\mathcal{C}_1)$ and $(\mathcal{C}_2)$ can be written

$$(\mathcal{C}_1) : \quad x^2 + y^2 = R^2 \; , \quad (\mathcal{C}_2) : \quad x^2 + \frac{y^2}{K^2} = a^2 \; .$$

We calculate the intersection (x_1, y_1) of these two curves, which yields

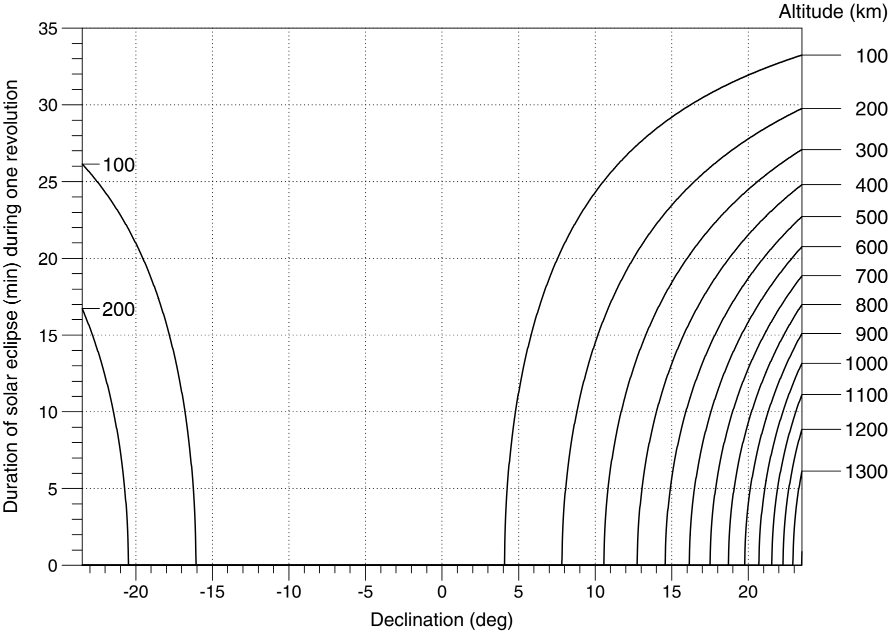

Figure 6.10. Sun-synchronous satellite in dawn–dusk orbit. Duration of solar eclipse in minutes during one revolution, for the altitude indicated, as a function of the declination. Graphs are drawn for $\tau_{\mathrm{AN}} = 18{:}00$. For $\tau_{\mathrm{AN}} = 06{:}00$, take the opposite value of the declination

$$x_1^2 = \frac{1 - \eta^2 K^2}{1 - K^2} R^2 \,.$$

Rotating the orbital plane onto the plane of the figure, we obtain the actual value of the angle α which determines the duration of the eclipse (see Fig. 6.9, right). Hence,

$$\sin\alpha = \frac{x_1}{a} = \sqrt{\frac{H^2 - K^2}{1 - K^2}} \,. \tag{6.15}$$

The duration Δt_{e} of the eclipse is

$$\Delta t_{\mathrm{e}} = \frac{\alpha}{\pi} T_0 \,, \tag{6.16}$$

since the orbit is circular, with uniform motion of period T_0.

In the case $K > H$, there is no eclipse, as explained above, and we put $\alpha = 0$.

Using the altitude h, the angle i_{HS} and the value of the period $T_{0(h=0)}$ defined by (2.17), we have (time in minutes, angles in radians), for declination δ,

$$\Delta t_{\mathrm{e}}\ (\mathrm{min}) = 84.5 \left(1 + \frac{h}{R}\right)^{3/2} \frac{1}{\pi} \arcsin \frac{\sqrt{[R/(R+h)]^2 - \sin^2(\delta + i_{\mathrm{HS}})}}{|\cos(\delta + i_{\mathrm{HS}})|} \,. \tag{6.17}$$

Figure 6.10 plots representative graphs of the duration of solar eclipse over one revolution for various altitudes, as a function of the declination. For easier understanding, Fig. 6.11 (upper) shows the same as a function of the day of the year.

Ascending Node Crossing Time and Dates of Eclipse

Figure 6.9 (left) shows the situation in a northern summer ($\delta > 0$, Sun at the zenith in the northern hemisphere) with a satellite orbit crossing the ascending node at 18:00 (taking into account the direction of rotation of the Earth). The maximal eclipse occurs at the summer solstice when the satellite passes close to the South Pole. The most favorable situation with regard to the question of eclipse, even at very low orbit, occurs for $\delta = -j$, i.e., during the northern winter, when the direction of the Sun is exactly perpendicular to the orbit.

For a satellite crossing the ascending node at 06:00, the straight line $S_i S_i'$ occupies a symmetric position with respect to the polar axis Oz. All the above calculations remain the same, provided that we take the opposite value of the declination. For example, in this case, the value $\delta = -23.44°$ corresponds to the northern summer solstice.

Example 6.7. *Calculate the eclipse dates and duration of eclipse for Radarsat-1 and SMOS.*

Radarsat-1 has a near-circular Sun-synchronous dawn–dusk orbit with characteristics: $a = 7\,167.064$ km, $i_{\mathrm{HS}} = 98.58°$, $T_{\mathrm{d}} = 100.76$ min, $\tau_{\mathrm{AN}} = 18{:}00$. Using the above notation, we obtain

$$\eta = 1.1237\ , \quad H = 0.8899\ , \quad j = 8.58°\ .$$

To find the date of eclipse, we apply (6.14). With $\arccos(0.8899) = 27.14°$, we obtain

$$\delta + j = \pm 27.14\ , \quad \text{hence} \quad \delta = \pm 27.14 - 8.58\ .$$

The solution $\delta = 18.56°$ is the only possible value, since for the other, $|\delta| > \varepsilon$. As we have $\tau_{\mathrm{AN}} = 18{:}00$, the dates are given directly by the values of δ. In the northern summer, there is eclipse for days when the declination is greater than $18.56°$, i.e., in the interval from 15 May to 20 July. To calculate the duration of the longest eclipse, at the summer solstice, we use (6.15). With $K = \cos(\varepsilon + j) = \cos(32.02) = 0.8479$, we obtain $\sin\alpha = 0.5096$, whence $\alpha = 30.6°$. Then, with (6.16),

$$\Delta t_{\mathrm{e}} = 0.170 T_0 \approx 17\ \mathrm{min}\ .$$

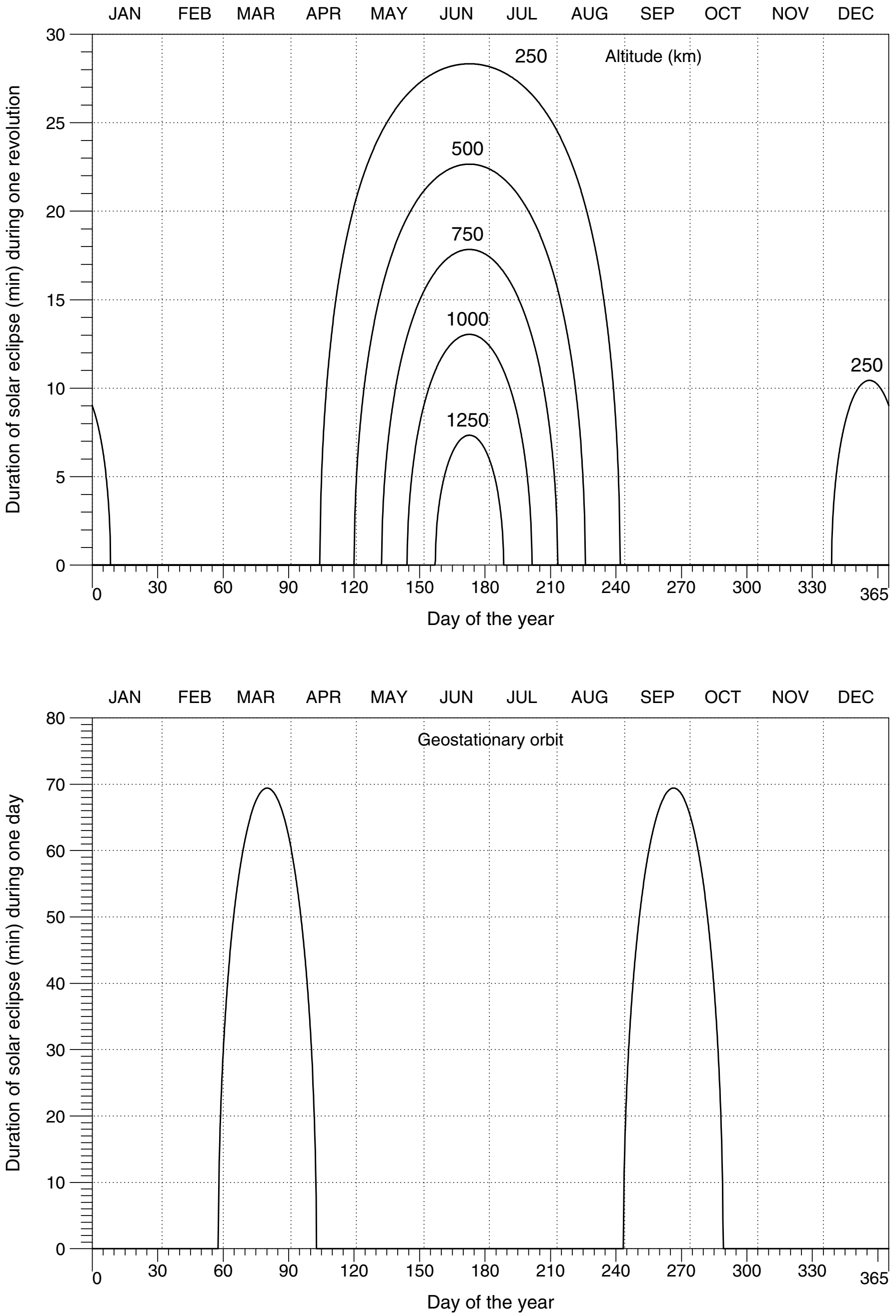

Figure 6.11. Duration of solar eclipse in minutes as a function of the day of the year. *Upper*: Sun-synchronous satellite in dawn–dusk orbit. Duration of the eclipse during one revolution at the given altitude. Graphs are drawn for $\tau_{AN} = 18{:}00$. For $\tau_{AN} = 06{:}00$, take the opposite value of the declination. *Lower*: geostationary satellite. Duration of eclipse over one day

We can also obtain these results directly using (6.17).

The satellite SMOS should fly at an altitude of 755 km, so that $i_{\mathrm{HS}} = 98.44°$. We deduce that $H = 0.8942$ and $j = 8.44°$. For the eclipse dates, we find once again that there is only one possible value for the declination, viz., $\delta = 26.60 - 8.44 = 18.16$. If we choose $\tau_{\mathrm{AN}} = $ 18:00, eclipse will occur for declinations greater than $+18.16°$, or in the interval between 13 May and 31 July, around the summer solstice. If we choose $\tau_{\mathrm{AN}} = $ 06:00, eclipse will occur for declinations less than $-18.16°$, that is, in the interval from 15 November to 28 January, around the winter solstice. Concerning the duration of eclipse at the solstice, the calculation gives $\Delta t_{\mathrm{e}} = 18$ min per revolution.

Example 6.8. *Constraints imposed by eclipse on the satellites STEP and GOCE, in very low orbit.*

For STEP, the requirements of temperature stability forbid any period of solar eclipse during its operating time. Moreover, the orbit must be low, the altitude being fixed at 400 km. A Sun-synchronous dawn–dusk orbit is the only suitable orbit. With $i_{\mathrm{HS}} = 97.05°$ and $\eta = 1.0627$, we obtain $j = 7.05°$ and $H = 0.9410$ and hence, $\delta + j = \pm 19.78$. We thus have just one value for the declination, viz., $\delta = 19.78 - 7.05 = 12.73°$. Depending on the value of τ_{AN}, this corresponds to the interval 25 April to 21 August or the interval 28 October to 15 February. Note that, in the first case, the eclipse lasts for 118 days, whilst in the second case, it lasts for 110 days, a consequence of the eccentricity of the Earth's orbit. Finally, with this orbit, there is a period of 8 months without eclipse. The accelerometers of the STEP experiment are maintained at a temperature of 2 K, using a superfluid helium cryostat, which limits the time over which the experiment can operate to around 6 months. The satellite must be launched shortly after 21 August, if $\tau_{\mathrm{AN}} = $ 18:00 is chosen, or shortly after 15 February, if $\tau_{\mathrm{AN}} = $ 06:00 is chosen.

GOCE flies at the very low altitude $h = 250$ km. With $i_{\mathrm{HS}} = 96.52°$ and $\eta = 1.0392$, we obtain $j = 6.52°$ and $H = 0.9623$, whence $\delta + j = \pm 15.79$ and $\delta = \pm 15.79 - 6.52$. There are now two values of the declination:

$$\delta = -22.31° \ , \quad \delta = +9.27° \ .$$

We can say that there are two eclipse 'seasons', one short, with $|\delta|$ close to ε, the other long, as can be seen very clearly in Fig. 6.11 (upper). The mission is planned to last 20 months (limited by the amount of fuel needed to compensate for atmospheric drag) and the satellite should be launched at the end of the long eclipse season. There are therefore only two possible launch windows: either in July, taking $\tau_{\mathrm{AN}} = $ 18:00, or in January, taking $\tau_{\mathrm{AN}} = $ 06:00.

6.3.2 GEO Orbit

For a geostationary satellite, no shadow is cast by the Earth on the circular orbit as long as the direction of the Sun has an inclination (declination δ), with respect to the equatorial plane, greater than the angle with which the satellite views the Earth. Let f_0 be this angle, which is the half-angle at the apex of the observation cone with which the satellite views the Earth, and to which we shall return in Chap. 8 [see (8.24)]. With η_{GS} defined by (4.58), the relation $\sin f_0 = 1/\eta_{\mathrm{GS}}$ gives $f_0 = 8.7°$.

There is therefore an eclipse if $|\delta| < f_0$. This happens twice a year, around the equinoxes:

$$\text{eclipse for GEO} \quad \Longleftrightarrow \quad [\text{27 Feb–12 Apr}] \; , \quad [\text{01 Sep–16 Oct}] \; .$$

During these periods, each lasting 45 days (from $J = 58$ to $J = 102$ and from $J = 244$ to $J = 289$, although dates may vary by one day from year to year), the longest eclipse occurs at the equinox itself. On this day, it lasts $\Delta t_{\mathrm{e}0}$ given by

$$\Delta t_{\mathrm{e}0} = \frac{f_0}{\pi} T_0 = \frac{8.7}{180} J_{\mathrm{sid}} = 69.5 \text{ min} \approx 1 \text{ hr } 10 \text{ min} \; . \tag{6.18}$$

On the other days, the duration of the eclipse is found by considering the Earth's disk, viewed by the satellite, occulting the Sun. The 'ground track' of the Sun cuts the disk along parallel chords, passing through the centre of the disk for $\delta = f_0$. This gives, for the duration Δt_{e} of the eclipse as a function of δ,

$$\Delta t_{\mathrm{e}} = \sqrt{1 - \left(\frac{\delta}{f_0} \right)^2} \Delta t_{\mathrm{e}0} \; . \tag{6.19}$$

The value of Δt_{e} as a function of the day of the year is shown in Fig. 6.11 (lower).