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Notices

of the American Mathematical Society

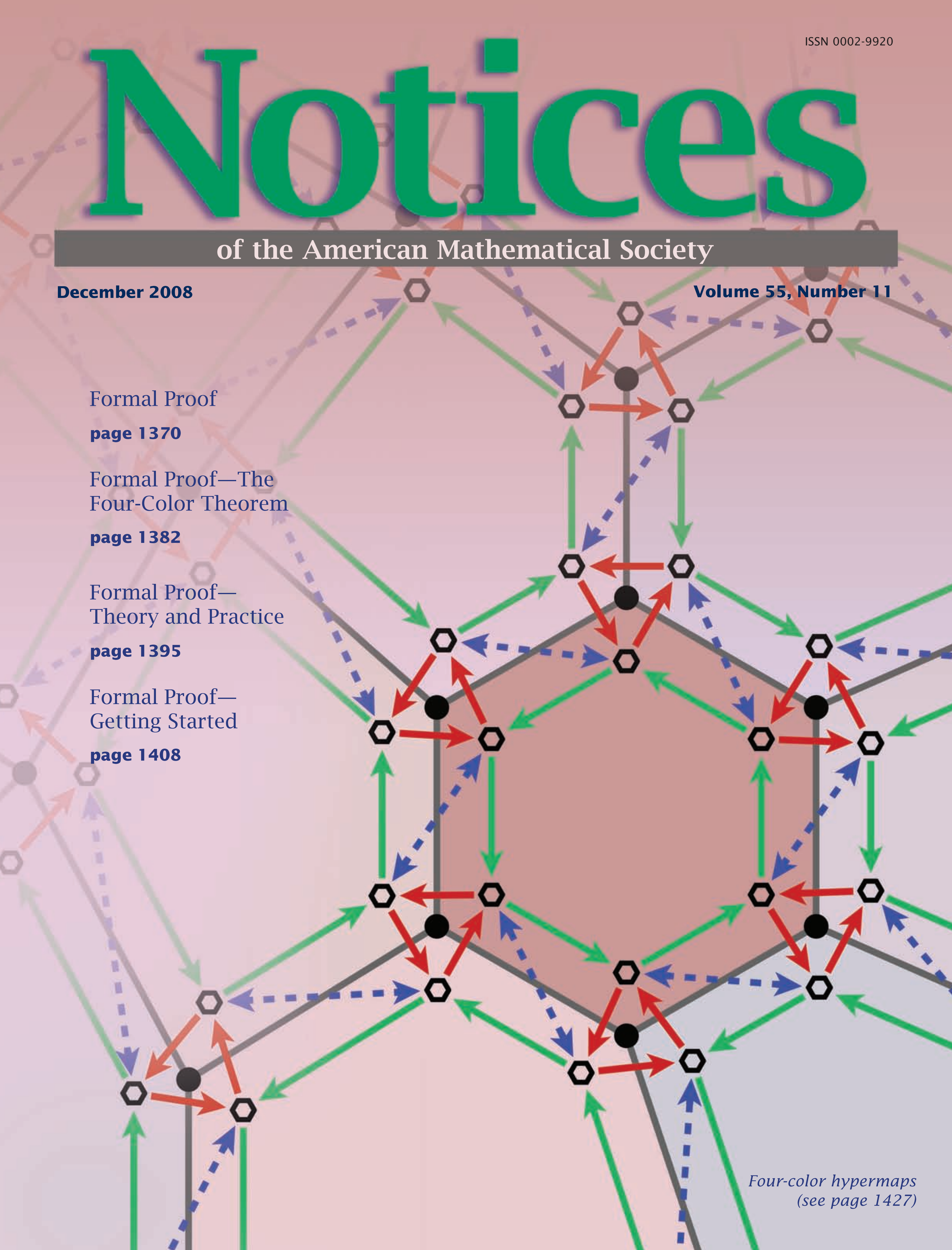
December 2008 **Volume 55, Number 11**

Formal Proof
page 1370

Formal Proof—The Four-Color Theorem
page 1382

Formal Proof—Theory and Practice
page 1395

Formal Proof—Getting Started
page 1408



Four-color hypermaps
(see page 1427)

Notices
of the American Mathematical Society

December 2008

Communications

1418 WHAT IS...a Period Domain?
James Carlson and Phillip Griffiths

1422 Adventures in Academic Year Undergraduate Research
Kathryn Leonard

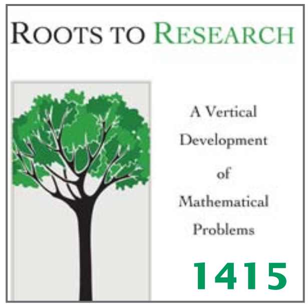
Commentary

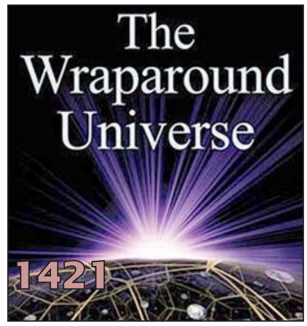
1367 Letter from the Editor: Job Talk
Andy Magid

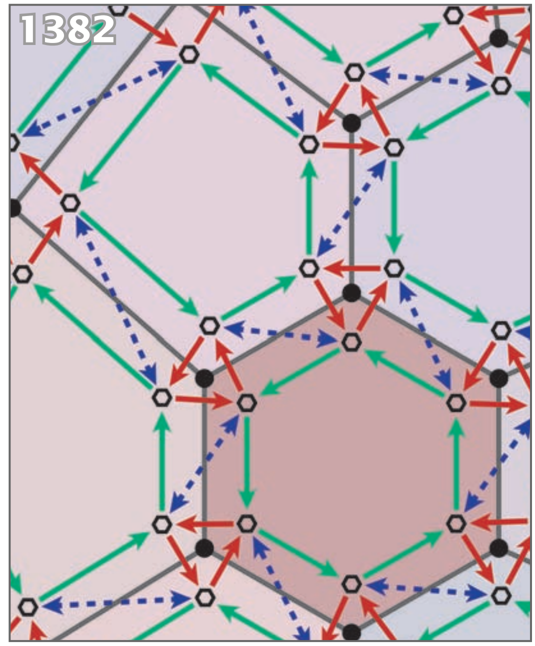
1368 Letters to the Editor

1415 *Mathematical Omnibus* and *Roots to Research*—A Book Review
Reviewed by Harriet Pollatsek

1421 *The Wraparound Universe*—A Book Review
Reviewed by George F. R. Ellis







A Special Issue on Formal Proof

Using computers in proofs both extends mathematics with new results and creates new mathematical questions about the nature and technique of such proofs. This special issue features a collection of articles by practioners and theorists of such formal proofs which explore both aspects.

1370 Formal Proof
Thomas C. Hales

1382 Formal Proof—The Four-Color Theorem
Georges Gonthier

1395 Formal Proof—Theory and Practice
John Harrison

1408 Formal Proof—Getting Started
Freek Wiedijk

Notices
of the American Mathematical Society

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Departments

About the Cover 1427

Mathematics People 1428

Kitaev Receives MacArthur Fellowship, Agrawal Receives Infosys Mathematics Prize, Bartels and Görtz Awarded von Kaven Prize, Alur and Dill Receive Computer-Aided Verification Award, MAA Awards, Petrosyan Awarded Emil Artin Junior Prize, Pi Mu Epsilon Student Paper Presentation Awards, B. H. Neumann Awards Given, Royal Society of Canada Elections.

Mathematics Opportunities 1433

NSF Computing Equipment and Instrumentation Programs, Jefferson Science Fellows Program, MfA Fellowship Program, CMI Liftoff Program for Summer 2009, National Academies Christine Mirzayan Graduate Fellowship Program, Call for Nominations for Waterman Award, Call for Nominations for Ostrowski Prize, News from IPAM.

Inside the AMS 1437

Trjitzinsky Memorial Awards Presented, Epsilon Scholarships Awarded for 2008, From the AMS Public Awareness Office, Deaths of AMS Members.

Reference and Book List 1439

Mathematics Calendar 1445

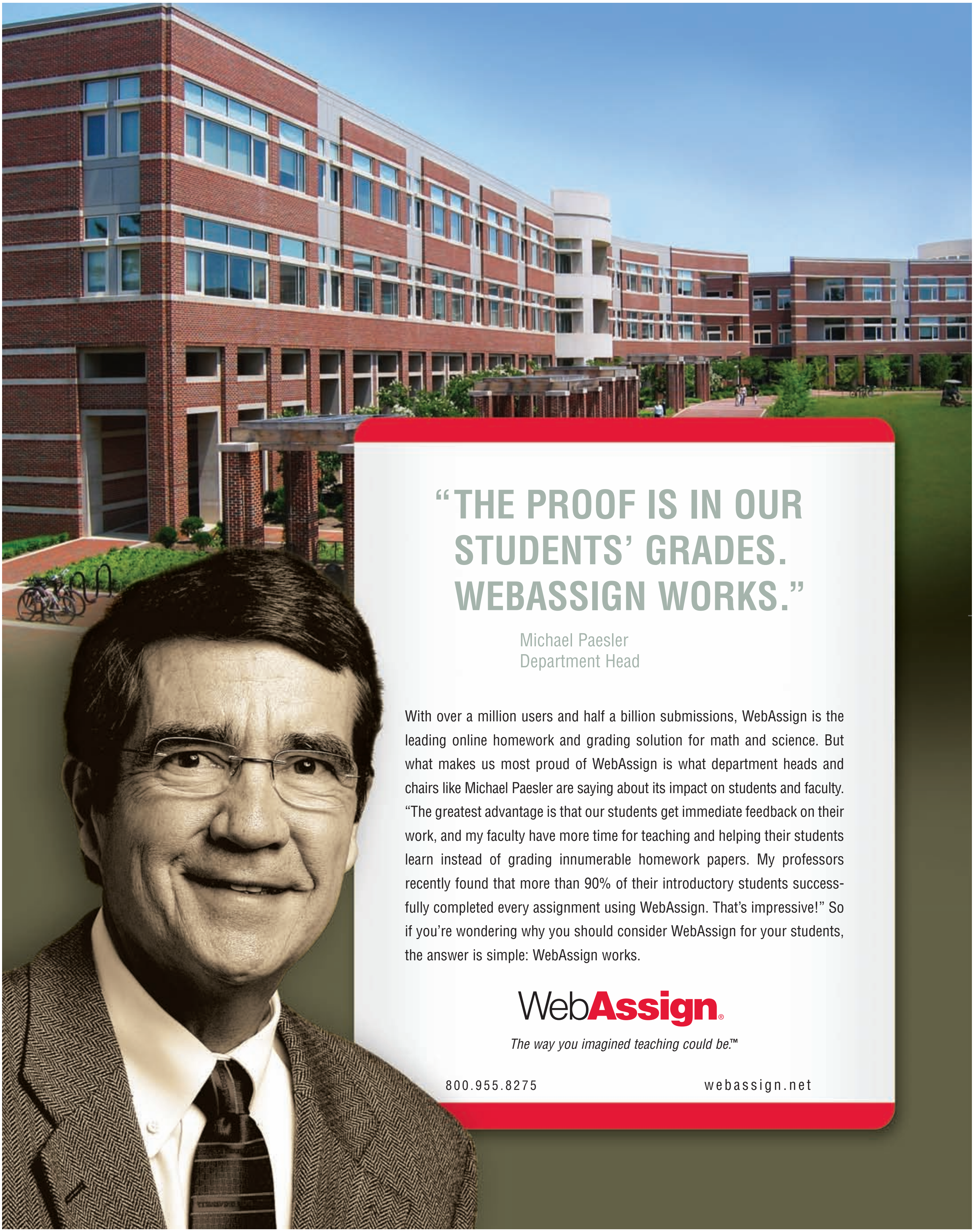
New Publications Offered by the AMS 1453

Classified Advertisements 1462

2008 *Notices* Index 1484

Meetings and Conferences Table of Contents 1502

Washington Meeting Registration Forms 1503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Job Talk

Recently, I attended a rather unusual lecture interview. The candidate's exposition was excellent: the general background of the subject was explained for nonspecialists, the history of the specific problem was covered in such a way that the importance of the candidate's own work was made clear, and finally there was enough detail in the statements of results and suggestions of proofs, that the audience could form a reasonable assessment of the candidate as a mathematician. Moreover, although the candidate's first language was not English, the presentation made it clear that the candidate should be an effective instructor. All in all, it was an exemplary lecture interview, better than many, but not differing in presentation from the several hundred or so others I've been to.

What made this one unusual was that the candidate was not an applicant for a job at my institution: this was an interview for a job at an institution ten time zones away. A video camera operator recorded the lecture on DVD for the hiring institution. We in the local audience were there for verisimilitude, or perhaps for moral support.

Maybe interviews via video recording are now routine, but this one was a novelty for me, and it got me to thinking about how such presentations could affect the job application process. Job candidates wouldn't need to wait for invitations to prepare their prerecorded lectures. Indeed, video files could be imbedded in online vitae or in electronic applications. Or perhaps not: with several hundred applicants for the typical job, a massive video file included with each application could overwhelm many department systems. There's also the issue of production values. The single amateur camera operator in the case I witnessed copied the raw footage on DVD, but in principle the candidate could have hired a professional video production company to record from multiple cameras with multiple takes. And then professional editing and special effects could be added. I'm afraid this escalation in production quality would quickly become the expected norm, just as projected TeX slideshows have replaced handwritten slides or chalkboard presentations.

In any event, from my observations of recent job searches, hiring institutions do not expect, and applicants are not submitting, video lecture interviews. I also believe that many job candidates, especially new and recent Ph.D.s, are still routinely contributing ten-minute talks on their work to the Society's annual meeting and inviting potential employers to attend. Such invitations are often included in job application cover letters (including emails). This is a useful practice. As a former department chair, and search committee chair or member, I often tried to attend, or have someone attend, such talks by applicants we were interested in. And one could judge the competition by the other department heads one saw in the audience.

I think it should be simple to make a video of a ten-minute talk—a few long shots to show the speaker, but mostly show the speaker's slides and voice—in fairly low resolution, and post it online at YouTube or a similar service. This is no substitute for actually hearing a ten-minute talk, and no excuse for new Ph.D.'s to skip presenting at AMS meetings, but it seems such a useful supplement that I'm surprised that messages from job applicants don't regularly point to such videos.

It was forty years ago this fall that, as a finishing doctoral student, I sent out my first job applications. In those post-Sputnik pre-Mansfield Amendment days, research opportunities were everyone's expectations. Most of us sought postdoctoral positions (or "named instructorships") at prestigious institutions. Then, as now, lecture interviews were not part of the process. In one case, I was invited to visit and give a talk after the position had been offered but before the acceptance deadline had arrived, a nice gesture and one that helped me decide to go there. If that practice is not typical, it should be.

Another feature shaping job searches then, at least for male U.S. citizens, was the Vietnam War and the draft. Students, including Ph.D. students under the age of 26, were eligible for draft deferments. But these ended upon receipt of the terminal degree (any terminal degree, which is why we didn't take Master's degrees). An accelerated doctoral program, therefore, produced draftable graduates. Teachers, including university teachers in some venues, also received deferments. So pure research postdocs, like those offered by the Institute for Advanced Study, were risky, but postdocs with some teaching, like most of the named instructorships, in the right places, were safe. Students, naturally, were well informed of the intricacies of draft deferments, although faculty usually weren't, so this was one aspect of the job application process where faculty couldn't help much. I believe the situation today regarding visa status for international students is analogous.

Fortunately, in most aspects of the job application process, advice is readily available. An article about the job application process in general appeared in the October 2006, *Notices*. An article specifically about how to give a lecture interview is in preparation and should appear in the *Notices* in early 2009.

—Andy Magid

Letters to the Editor

Varieties of Mathematical Truth

It is ironic that Melvyn Nathanson's essay on truth (*Notices*, August 2008) appeared in the same issue as Olle Häggström's review of *Irreligion*. The question is, what truth is Nathanson concerned with? My guess is that Brouwer or Heyting or someone of their schools of mathematics might find a lot more mistakes in papers than he does. After all, Brouwer spent a lot of years explaining why his celebrated theorem was not true. Presumably what Nathanson means by truth is playing mathematics by the standard rules of the game.

Meanwhile, Häggström shows us why the logic applied to some questions is not to be trusted. Question is, where is the line we can trust? The answer to that question divides a number of philosophies which have fallen from fashion in the latest century.

—*Jim Chaffee*
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(Received September 2, 2008)

On the Mumford Donation

In the September 2008 *Notices* David Mumford wrote an "Opinion" piece describing the aftermath of his decision to donate his share of the Wolf Prize money to Birzeit University and to an Israeli organization that advocates for Palestinian interests. In the piece Mumford touched on various issues that I believe require further explication.

Mumford relates that he received emails from Israelis expressing fear concerning radical campus activity on the West Bank, and notes that there is "no way to totally refute this fear". Indeed, there is much to fear. Hundreds of Israelis have been killed and maimed by suicide bombers who were students at Birzeit. Equally disturbing is the impact these events have had on the student body politic at Birzeit. In a campus debate in 2003, a Hamas candidate taunted his Fatah challenger by saying, "Hamas activists in this university killed 135 Zionists. How many did Fatah activists from Birzeit kill?" Only the fence and other security measures have reduced the number of suicide attacks and brought a relative calm in recent years.

Lest we think that radical student action only takes place on the West Bank, Mumford notes that "even Harvard had its unabomber." These cases could not be more different. The unabomber is regarded in the U.S. as a sociopath, and as soon as his family realized that Kaczynski might be the unabomber, they turned him in. By contrast, the suicide bombers from Birzeit were treated as heros by the populace and, with few exceptions, their families expressed pride at the deadly actions of their children and the murder of men, women, and children in Israel.

Mumford bases his support for Birzeit on general principles of academic freedom. However, when it comes to similar rights for Israelis, many faculty at Birzeit demur. For example, in 2006 a large number of faculty at Birzeit signed on to a call for a crippling boycott of Israeli academics. In the face of such actions, support for Birzeit on grounds of academic freedom rings hollow.

The true shining light in this story is the academic community in Israel. Although Mumford was a signatory to a particularly one-sided Harvard-MIT petition for divestment from Israel, the community in Israel nevertheless put politics aside and recommended Mumford for the Wolf Prize based on his mathematical achievements. This action is a model of the academic spirit that we should all try to emulate.

—*Percy Deift*
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(Received September 9, 2008)

David Mumford's generosity ("Opinion" *Notices*, September 2008) is marred by his one-sided blame of Israel for the situation in the West Bank. Israelis, both academics and others, would be delighted if checkpoints on the West Bank could be dismantled. They were instituted only after waves of suicide attacks, and continue to save lives.

It is unfortunate that Palestinian students are inconvenienced by Israeli security measures. What is forgotten is that Israeli students and academics have themselves been unable to travel to most Arab lands ever since 1948. They find themselves the subject of numerous boycotts by academic organizations in the Middle East. Those boycotts have been in force since the failed 1948 invasion of Israel, long before the 1967 war.

Mumford is himself a signatory to moves aimed at crippling Israeli universities. Such action would affect not only Jewish students and faculty, but also the many Israeli Arabs who study, and teach, in Israeli universities.

Mumford talks of Palestinians setting up Birzeit University after the 1967 "occupation". That "occupation" was the consequence of yet another attempt to destroy a small Jewish state, and would have ended had Palestinian leaders accepted President Clinton's Camp David plan. Moreover, Birzeit became a fully fledged university only during Israeli rule. During the preceding Jordanian rule, Birzeit could not offer four-year degrees.

It is counterproductive to continually target Israel, while ignoring the broader academic situation in the Middle East. There is less academic freedom outside Israel than within. There are numerous issues restricting academic freedom in the region that never receive attention.

We look forward to a time when Palestinians can, and are willing to, interact freely with their Israeli colleagues. Equally, we hope for a time when Israeli academics can travel freely across the Middle East and engage their colleagues in the region. One-sided censure of Israel will not hasten such an ideal. Rather it will encourage the many, both within and without academe, who have never given up hope of destroying a small Jewish state. And yes, Israel has a right to exist in the Middle East. Roughly half of its people are descendants of Jews who fled, or were expelled, from Arab countries, and who came to Israel penniless, but looked forward, and rebuilt their lives.

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(Received September 9, 2008)

Response to Deift, Lubinsky, and Nevai

I am writing to respond to criticisms of my "Opinion" piece in the letters above. First of all, my sincere belief is that assistance to and cooperation with the Palestinian universities is in the best interests of both the Palestinian people and the Israeli people. It saddens me that some people see this solely as a political matter and, in particular, a criticism of Israel. My actions are focused on what we, as mathematicians and educators, can do in a non-political way to help the situation. My personal view is that a long term guerilla-style conflict is going on and there have been times when I was sympathetic to Israel and times, especially when new settlements were made and old ones expanded, when I was not. It was in one of these latter times that I signed the Harvard-MIT divestment petition. But my plea now is to look beyond politics at the options which are open to us in our capacity as mathematicians.

Secondly, if we want to help, we certainly need to be clear what we are helping. Birzeit is and has been a center for intense political debate—a university in the midst of an occupation would not be serving its purpose if it were not. Yes, during the intifada, a handful of its students did succumb to the nihilistic lure of terrorism but it is my belief that the vast majority of its students are moderate and supporters of peace. For instance, in the latest student elections, Fatah won with a considerable majority. Birzeit does its utmost to counter the effect of a violent environment on its students through education, cultural activities and positive engagements. They require all their students to complete 120 hours of community service. Birzeit has graduated many of the top people in Palestine today and, above all, is a place that brings hope for thousands of Palestinian young people in the midst of a frustrating and violent environment. Once again, my plea is to look beyond the anger and bloodshed of the past and ask whether, through student and faculty exchanges, we can help in a positive way.

—*David Mumford*
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(Received September 12, 2008)

Julia and Raphael Robinson

I found Carol Wood's excellent review of George Csicsery's film *Julia Robinson and Hibert's Tenth Problem* in the May 2008 *Notices* warm and familial (in the sense of the "mathematical family"). I especially appreciate her paragraph on Raphael who, as Chern once said to me, was a greatly underrated mathematician. Julia felt that same way, and she told me in the last month of her life that she was planning to take his work as the subject of her Presidential Address.

—*Constance Reid*
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(Received September 15, 2008),

Noticed

This past summer *Science* magazine carried an article highlighting conclusions from a series of studies about mathematics education doctorates that have appeared in the *Notices*. The studies, by Robert Reys of the University of Missouri, present data about the imbalance between the relatively small number of mathematics education doctorates being produced and the large number of academic openings for people with such degrees. *Science* described the market for math education doctorates as "one of the hottest job markets in academia". The article goes on to say, "The reasons for the seller's market include a shortage of people entering the field, a growing demand by universities for their expertise as they become more involved in precollege education, a lack of consensus on how they should be trained, and a surfeit of other professional opportunities." The article, "Departments scramble to find math education faculty", by Jeffrey Mervis, appeared in the August 22, 2008, issue of *Science*. Reys's most recent article on the subject, "Jobs in mathematics education in institutions of higher education in the United States", is in the June/July 2008 *Notices*. (Due to an editing error, Robert Reys's title was truncated in the June/July *Notices*. He is Curators' Professor of Mathematics Education in the Department of Learning, Teaching and Curriculum at the University of Missouri.)

Formal Proof

Thomas C. Hales

> *There remains but one course for the recovery of a sound and healthy condition—namely, that the entire work of the understanding be commenced afresh, and the mind itself be from the very outset not left to take its own course, but guided at every step; and the business be done as if by machinery.*
>
> —F. Bacon, 1620
> *Novum Organum*

Daily, we confront the errors of computers. They crash, hang, succumb to viruses, run buggy software, and harbor spyware. Our tabloids report bizarre computer glitches: the library patron who is fined US$40 trillion for an overdue book, because a barcode is scanned as the size of the fine; or the dentist in San Diego who was delivered over 16,000 tax forms to his doorstep when he abbreviated "suite" in his address as "su".

On average, a programmer introduces 1.5 bugs per line while typing. Most are typing errors that are spotted at once. About one bug per hundred lines of computer code ships to market without detection. Bugs are an accepted part of programming culture. The book that describes itself as the "bestselling software testing book of all time" states that "testers shouldn't want to verify that a program runs correctly" [17]. Another book on software testing states "Don't insist that every bug be fixed ... When the programmer fixes a minor bug, he might create a more serious one." Corporations may keep critical bugs off the books to limit legal liability. Only those bugs should be corrected that affect profit. The tools designed to root out bugs are themselves full of bugs. "Indeed, test tools are often buggier than comparable (but cheaper) development tools" [18]. As for hardware reliability, former Intel President Andy Grove himself said "I have come to the conclusion that no microprocessor is ever perfect; they just come closer to perfection ..." [20, p. 221].

Bugs can be far-reaching. The bug causing the explosion of the Ariane 5 rocket cost hundreds of millions of dollars. As long ago as 1854, Thoreau wrote that "by the error of some calculator the vessel often splits upon a rock that should have reached a friendly pier." Last year, the *New York Times* reported Shamir's warning that even a small math error in a widely used computer chip could be exploited to defeat cryptography and would place "the security of the global electronic commerce system at risk ..." [27].

Mathematical Certainty

By contrast, philosophers tell us that mathematics consists of analytic truths, free of all imperfection. We prove that $1 + 1 = 2$ by recalling the definition of 1 as the successor of 0, 2 as the successor of 1, and then invoking twice the recursive definition of addition:

$$1 + 1 = 1 + S(0) = S(1 + 0) = S(1) = 2.$$

If only all proofs were so simple. Mathematical error is as old as mathematics itself. Euclid's very first proposition asks, "on a given straight line to construct an equilateral triangle." Euclid's construction makes the implicit assumption—not justified by the axioms—that two circles, each passing through the other's center, must intersect. We revere Euclid, not because he got everything right, but because he set us on the right path.

We have entered an era of proofs of extraordinary complexity. Take, for example, F. Almgren's masterpiece in geometric measure theory, called appropriately enough the "Big Paper". The preprint is 1728 pages long. Each line is a chore. He spent over a decade writing it in the 1970s and early 1980s. It was not published until 2000. Yet the theorem is fundamental. It establishes the regularity of minimizing rectifiable currents, up to codimension two; in basic terms, it shows that

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higher dimensional soap bubbles are smooth rather than jagged—just as one would naturally expect. How am I to develop enough confidence in the proof that I am willing to cite it in my own research? Do the stellar reputations of the author and editors suffice, or should I try to understand the details of the proof? I would consider myself very fortunate if I could work through the proof in a year.

Computer proofs, which are sprouting up in many fields, compound the complexity: the nonexistence of a projective plane of order 10, the proof that the Lorenz equations have a strange attractor, the double-bubble problem for minimizing soap bubbles enclosing two equal volumes, the optimality of the Leech lattice among 24-dimensional lattice packings, hyperbolic 3-manifolds, and the one that got it all started: the four-color theorem. What assurance of correctness do complex computer proofs provide?

Three Early Milestones

1954 – M. Davis programs the Presburger algorithm for additive arithmetic into the "Johniac" computer at the Institute for Advanced Study. Johniac proves that the sum of two even numbers is even, to usher in the era of computer proof.

1956 – The automation of Russell and Whitehead's *Principia Mathematica* begins [26]. By the end of 1959, Wang's procedure had generated proofs of every theorem of the *Principia* in the predicate calculus [30].

1968 – N.G. de Bruijn designs the first computer program to check the validity of general mathematical proofs. His program Automath eventually checked every proposition in a primer that Landau had written for his daughter on the construction of real numbers as Dedekind cuts.

N. G. de Bruijn

On April 24, 2008, F. Wiedijk and I visited N. G. de Bruijn at his home in Nuenen, shortly before his ninetieth birthday. (Nuenen is the Dutch town where Vincent van Gogh lived when he painted *The Potato Eaters*.) We discussed Automath, Brouwer, Heyting, and some of his coauthors (Knuth and Erdös). De Bruijn has contributed to many fields of mathematics, including analytic number theory, Penrose tilings, quasicrystals, and optimal control.

De Bruijn indices give a notation that eliminates all dummy variables from formulas with quantifiers: $\forall\, x.\ P(x)$ becomes $(\forall\ P\ 1)$. This notation solves the problem of free variable capture.

De Bruijn observed that the ratio of lengths of a formal proof to the corresponding conventional proof is remarkably stable across different proofs. The ratio, called the de Bruijn factor, has become the standard benchmark to measure the overhead of a formal proof.

Formal Proof

Traditional mathematical proofs are written in a way to make them easily understood by mathematicians. Routine logical steps are omitted. An enormous amount of context is assumed on the part of the reader. Proofs, especially in topology and geometry, rely on intuitive arguments in situations where a trained mathematician would be capable of translating those intuitive arguments into a more rigorous argument.

A formal proof is a proof in which every logical inference has been checked all the way back to the fundamental axioms of mathematics. All the intermediate logical steps are supplied, without exception. No appeal is made to intuition, even if the translation from intuition to logic is routine. Thus, a formal proof is less intuitive, and yet less susceptible to logical errors.

There is a wide gulf that separates traditional proof from formal proof. For example, Bourbaki's Theory of Sets was designed as a purely theoretical edifice that was never intended to be used in the proof of actual theorems. Indeed, Bourbaki declares that "formalized mathematics cannot in practice be written down in full" and calls such a project "absolutely unrealizable". The basic trouble with various foundational systems is that meta-mathematical arguments (for example, abbreviations that are external to the system or inductions over the syntactical form of an expression) are usually introduced early on, and without these simplifying meta-arguments, the vehicle stalls, never making it up the steep incline from primitive notions to high-level concepts. The gulf can be extreme: A. Matthias has calculated that to expand the definition of the number "1" fully in terms of Bourbaki primitives requires over 4 trillion symbols. In Bourbaki's view, the foundations of mathematics are roped-off museum pieces to be silently appreciated, but not handled directly.

There is an opposing view that regards the foundational enterprise as unfinished until it is realized in practice and written down in full. This article sketches the current state of this endeavor. It has been necessary to commence afresh, and to retool the foundations of mathematics for practical efficiency, while preserving its reliability and austere beauty. For anything beyond a trivial proof, the number of logical inferences is so large that a computer is used to ensure that no steps

Year	Theorem	Proof System	Formalizer	Traditional Proof
1986	First Incompleteness	Boyer-Moore	Shankar	Gödel
1990	Quadratic Reciprocity	Boyer-Moore	Russinoff	Eisenstein
1996	Fundamental - of Calculus	HOL Light	Harrison	Henstock
2000	Fundamental - of Algebra	Mizar	Milewski	Brynski
2000	Fundamental - of Algebra	Coq	Geuvers et al.	Kneser
2004	Four-Color	Coq	Gonthier	Robertson et al.
2004	Prime Number	Isabelle	Avigad et al.	Selberg-Erdös
2005	Jordan Curve	HOL Light	Hales	Thomassen
2005	Brouwer Fixed Point	HOL Light	Harrison	Kuhn
2006	Flyspeck I	Isabelle	Bauer-Nipkow	Hales
2007	Cauchy Residue	HOL Light	Harrison	classical
2008	Prime Number	HOL Light	Harrison	analytic proof

Table 1. Examples of formal proofs.

are omitted. This raises basic questions about trust in computers. This article also places formal proofs within a broader context of automating more general mathematical tasks.

As the art is currently practiced, each formal proof starts with a traditional mathematical proof, which is rewritten in a greatly expanded form, where all the assumptions are made explicit and all cases are treated in full. For example, a traditional mathematical proof might show that a graph is planar by drawing the graph on a sheet of paper. The expanded form of the proof replaces the picture by careful argument. From the expanded text, a computer script is prepared, which generates all the logical inferences of the proof. The transcription of a single traditional proof into a formal proof is a major undertaking.

Examples

Computer proof assistants have been under development for decades (see Box "Early Milestones"), but only recently has it become a practical matter to prove major theorems formally. The most spectacular example is Gonthier's formal proof of the four-color theorem. His starting point is the second-generation proof by Robertson et al. Although the traditional proof uses a computer and Gonthier uses a computer, the two computer processes differ from one another in the same way that a traditional proof differs from a formal proof. They differ in the same way that adding $1 + 1 = 2$ on a calculator differs from the mathematical justification of $1 + 1 = 2$ by definitions, recursion, and a rigorous construction of the natural numbers. In short, a large logical gulf separates them. As a result of Gonthier's formalization, the proof of the four-color theorem has become one of the most meticulously verified proofs in history.

In recent years, several other significant theorems have been formally verified. See Table 1. The table lists the theorems, which proof assistant was used (there are many to choose from), the person who produced a formal proof, and the mathematicians who produced the original proof. The Prime Number Theorem, asserting that the number of primes less than n is asymptotic to $n/\log n$, has two essentially different proofs: the elementary proof of Selberg and Erdös and the analytic proof of Hadamard and de la Vallée Poussin. Formal versions of both proofs have been produced. More ambitious projects are in store: Gonthier's team is now formalizing the Feit-Thompson odd order theorem, and the leading problem of the document *Ten Challenging Research Problems for Computer Science* is the formalization of the proof of Fermat's Last Theorem [3].

The Formal Jordan Curve Theorem

$$
\begin{aligned}
&\forall C.\ \textit{simple\_closed\_curve top2 } C \quad \Rightarrow \\
&\quad (\ \exists A\, B.\ \textit{top2 } A \wedge \textit{top2 } B \ \wedge \\
&\qquad \textit{connected top2 } A \wedge \textit{connected top2 } B \ \wedge \\
&\qquad A \neq \varnothing \wedge B \neq \varnothing \wedge \\
&\qquad A \cap B = \varnothing \wedge A \cap C = \varnothing \wedge B \cap C = \varnothing \wedge \\
&\qquad A \cup B \cup C = \textit{euclid } 2\ \ )
\end{aligned}
$$

The box above displays the statement of the Jordan Curve theorem, in computer readable form, as it appears in the formal proof. The complete specification of the theorem should also list all definitions, all the way back to the primitives. Without giving the detailed definitions here, we note that *top2* refers to the standard topology on the plane; *top2 A* indicates that A is an open set in the plane; *euclid* 2 is the Euclidean plane; and *connected top2 A* means that A is a connected set in the plane.

A large library is maintained of all previously established proofs in the system, and anyone may use any result that has been previously established. Although every step of every proof is always checked, as researchers contribute to the system, interaction with the system gradually moves away from the primitive foundations towards something more closely resembling the high-level practice of mathematicians. The hope is the system will eventually become sufficiently user-friendly to become a familiar part of the mathematical workplace, much as email, TEX, computer algebra systems, and Web browsers are today.

HOL Light

This section gives a brief introduction to one foundational system designed for doing mathematical proofs on a computer. The system is called HOL Light (an acronym for a lightweight implementation of Higher Order Logic). I have singled it out because of its simple design and because it is the system that I understand the best. Some understanding of the design of a simple system is helpful before turning to questions of soundness in the next section. HOL Light by itself is only a small part of the overall formal-theorem-proving landscape. There are several competing systems to choose from, built on various logical foundations, and with their own powerful features. People argue about the relative merits of the different systems much in the same way that people argue about the relative merits of operating systems, political loyalties, or programming languages. To some extent, preferences show a geographical bias: HOL in the UK, Mizar in Poland, Coq in France, and Isabelle in Germany and the UK.

The basic components of the HOL Light system are its types, terms, theorems, rules of inference, and axioms. Each is briefly described in turn. The HOL Light System box (next page) gives a summary of the entire system.

Types

Much day-to-day mathematics is written at a level of abstraction that is indifferent to its exact representation as sets. For example, it does not matter how an ordered pair is encoded as a set, as long as the ordered pair has the characteristic property

$$(x, y) = (x', y') \quad \Leftrightarrow \quad x = x' \text{ and } y = y'.$$

It is bad style to break the abstraction to write $2 \in (0, 1)$. This layer of abstraction is good news, because it allows us to shift from Zermelo-Fraenkel-Choice (ZFC) set theory to a different foundational system with equanimity and ease.

Many proof assistants are based on types. Types are familiar to computer programmers. In a typed computer language, 3 is an integer and $[1.0; 2.0; 3.0]$ is an array of floating point numbers. An attempt to add 3 to this array results in a type mismatch error, and the computer program will not compile. The type checking mechanism of programming languages conveniently detects many bugs at the time of compilation.

ZFC set theory has no such type checking mechanism. As de Bruijn puts it, "Theoretically, it seems perfectly legitimate to ask whether the union of the cosine function and the number e (the basis of natural logarithms) contains a finite geometry" [6]. Mathematicians have the good sense not to ask such questions. However, when moving mathematics to a computer, which is lacking in common sense, it is useful to introduce types into the foundations to prevent this kind of nonsense. By convention, a colon is written before the name of a type. For instance, we write the type of the real number e as $:\mathbb{R}$, or simply $e : \mathbb{R}$, to indicate that e is a real number. The cosine function has a different type $:\mathbb{R} \to \mathbb{R}$, or $\cos : \mathbb{R} \to \mathbb{R}$. The type of the union operator forces its two arguments to have the same type, so that an attempt to take the union of the cosine function with e is then flat out rejected.

HOL Light is a new axiomatic foundation with types, different from the usual ZFC. The types are presented in the HOL Light System box. There are only two primitive types, the boolean type *:bool* and an infinite type *:ind*. The rest are formed with type variables joined by arrows. A mechanism is also provided for creating a new type that is in bijection with a nonempty subset of an existing type, allowing the system to be extended with types for ordered pairs, integers, rational numbers, real numbers, and so forth.

Terms

Terms are the basic mathematical objects of the HOL Light system. The syntax is based on Church's λ-calculus, which uses the notation

$$\lambda x.\, f(x)$$

to represent the function that takes x to $f(x)$, what a mathematician would write as $f : \mathbb{N} \to \mathbb{N}$, $x \mapsto f(x)$. The name λ-calculus is derived from the use of the letter λ to mark function arguments. The HOL Light System box lays out the construction of terms.

In ZFC set theory, there is a bijection of sets

$$Z^{X\times Y} \simeq (Z^Y)^X.$$

In other words, a function $(x, y) \mapsto f(x, y)$ from the Cartesian product $X \times Y$ to Z can be viewed as a function on X that maps x to a function $f(x, \cdot) : Y \to Z$. The right-hand side of this bijection is called the curried form of the function (named after the logician Haskell Curry). In typed

The HOL Light System

HOL Light (Lightweight Higher Order Logic) is a foundational system designed for doing mathematical proofs on a computer. The notation is based on a typed λ-calculus.

1. Types: The collection of types is freely generated from *type variables* $:A, :B, \ldots$ and *type constants* $:bool$ (boolean), $:ind$ (infinite type), joined by *arrows* ($\to$). The colon is used as a notational device to indicate a type. For example, $:bool$, $:bool \to A$, and $:(bool \to A) \to (ind \to B)$ are types.

2. Terms: The collection of terms is freely generated from *variables* $x, y, \ldots$ and *constants* $0, \ldots$ using *abstraction* ($\lambda x.t$ where x is a variable and t a term) and *application* ($f(x)$ for compatibly typed terms x and f). Each term has a type. The notation $x{:}A$ indicates that the type of term x is $:A$. Variables and constants are assigned a type at the moment of creation; the types of abstractions and applications are defined recursively: the type of $\lambda x.t$ is $:A \to B$ when $x{:}A$ and $t{:}B$; the type of $f(x)$ is $:B$ if $f{:}A \to B$ and $x{:}A$.

3. Theorems: A theorem is a *sequent* $\{p_1, \ldots, p_k\} \vdash q$, where $p_1, \ldots, p_k, q$ are terms of type $:bool$. The terms $p_1, \ldots, p_k$ are called the assumptions and q is called the conclusion of the sequent. The design of the system prevents the construction of theorems except through inferences from existing theorems, new definitions, and axioms.

4. Inference Rules: The system has ten inference rules and a mechanism for defining new constants and types. Each inference rule is depicted as a fraction; the inputs to the rule are listed in the numerator, and the output in the denominator. The inputs to the rules may be terms or other theorems. In the following rules, we assume that p and p' are equal, up to a renaming of bound variables, and similarly for b and b'. (Such terms are called α-equivalent.)

On first reading, ignore the assumption lists Γ and Δ. They propagate silently through the inference rules, but are really not what the rules are about. When taking the union $\Gamma \cup \Delta$, α-equivalent assumptions should be considered as equal.

Equality is reflexive:

$$\frac{a}{\vdash a = a}$$

Equality is transitive:

$$\frac{\Gamma \vdash a = b;\ \Delta \vdash b' = c}{\Gamma \cup \Delta \vdash a = c}$$

Equal functions applied to equals are equal:

$$\frac{\Gamma \vdash f = g;\ \Delta \vdash a = b}{\Gamma \cup \Delta \vdash f\,a = g\,b}$$

The rule of abstraction holds. Equal terms give equal functions:

$$\frac{x;\ \Gamma \vdash a = b}{\Gamma \vdash \lambda x.\ a\ = \lambda x.\ b} \quad \text{(if } x \text{ is not free in } \Gamma\text{)}$$

The application of the function $x \mapsto a$ to x gives a:

$$\frac{(\lambda x.\ a)\,x}{\vdash (\lambda x.\ a)\,x = a}$$

Assume p, then conclude p:

$$\frac{p{:}bool}{p \vdash p}$$

An "equality-based" rule of modus ponens holds:

$$\frac{\Gamma \vdash p;\ \Delta \vdash p' = q}{\Gamma \cup \Delta \vdash q}$$

If the assumption q gives conclusion p and the assumption p gives q, then they are equivalent:

$$\frac{\Gamma \vdash p;\ \Delta \vdash q}{(\Gamma \setminus q) \cup (\Delta \setminus p) \vdash p = q}$$

Type variable substitution holds. If arbitrary types are substituted in parallel for type variables in a sequent, a theorem results. Term variable substitution holds. If arbitrary terms are substituted in parallel for term variables in a sequent, a theorem results.

5. Mathematical Axioms: There are only three mathematical axioms.

Axiom of Extensionality:	$\forall f.\quad (\lambda x. f\,x) = f.$
Axiom of Infinity:	$\exists f{:}ind \to ind.\ (\mathrm{ONE\_ONE}\,f) \wedge \neg(\mathrm{ONTO}\,f).$
Axiom of Choice:	$\forall P\,x.\quad P x \Rightarrow P(\varepsilon P).$

Extensionality asserts that every function is determined by its input-output relation. Dedekind's axiom of infinity asserts the existence of a function that is one-to-one but not onto. The Hilbert choice operator ε applied to a predicate P chooses a term that satisfies the predicate, provided the predicate is satisfiable.

systems, the curried form of multivariate functions is generally preferred. Treating X, Y, Z as types, we write the type of the curried function as $f : X \to (Y \to Z)$, or simply $f : X \to Y \to Z$.

The system has only two primitive constants. One of them[1] is the equality symbol $(=)$ of type $:A \to A \to bool$. That is, equality is a curried function that takes two arguments of the same type and returns the boolean type.

Axioms, Inference, and Theorems

There are three mathematical axioms: an axiom of extensionality that asserts that a function is determined by the values that it takes on all inputs, an axiom of infinity that asserts that the type $:ind$ is not finite, and an axiom of choice. The system has ten rules of inference, as described in the HOL Light System box. For example, the first two state that equality is reflexive and transitive. The final two rules of inference allow one to substitute new terms for the free variables in a theorem and allow one to substitute new types for the type variables in a theorem. Beyond these ten rules of inference are mechanisms for defining new constants and new types. A theorem is expressed in *sequent* form; that is, as a set of assumptions, followed by a conclusion.

Extending the Primitive System

This primitive system lacks the customary logical operators. There are no symbols for "and", "or", "not", and "implies." There are no universal or existential quantifiers. The set membership operator is absent. It is remarkable none of this is needed to express the rules of inference.

Logical operators are defined later. For example, the boolean constant T (true) can be defined as the conclusion of any theorem that has no assumptions. The most accessible yet jarringly iconoclast theorem comes from the reflexive law applied to equality itself:

$$\vdash (=)(=)(=).$$

Each new definition becomes a theorem. So then $\vdash T = ((=)(=)(=))$. Conjunction $(\wedge)$ is roundaboutly defined as the curried function that on boolean inputs p and q returns $(\lambda f.\ f\,p\,q) = (\lambda f.\ f\,T\,T)$; that is, conjunction yields true exactly when no curried function f is able to distinguish (p, q) from (T, T). The other logical operations are built with similar tricks.

[1] *The second constant is the Hilbert choice operator (ε). Recall that every term that is not a variable, a function application, or λ-abstraction is a constant. "Constancy" is thus a broader notion here than in first-order logic, and includes terms such as equality that take arguments. Parentheses are drawn around the equality symbol $(=)$ to denote the prefixed curried form, with $(=)\,x\,x$ an alternative syntax for $x = x$.*

The inference rules and axioms become bits of data that are processed by other computer procedures. For example, to give a formal proof that

$$2682440^4 + 15365639^4 + 18796760^4 = 20615673^4$$

a human is not required to type each primitive inference. An automated procedure takes any arithmetic identity as input, generates the inferences, and produces the theorem as output. A large number of such small decision procedures have been programmed into the system to handle routine tasks such as polynomial simplification, basic tautologies in logic, and decidable fragments of arithmetic. Procedures that automatically search for steps in a proof are also programmed into the computer. New procedures may be contributed by any user at any time to automate further tasks. The design of the kernel of the system prevents a rogue user from writing computer code that could compromise the soundness of the system.

All the basic theorems of mathematics up through the Fundamental Theorem of Calculus are proved from scratch on the user's laptop in about two minutes every time the system loads, so that the casual user does not need to be concerned with the low-level details. Basic facts of logic and elementary mathematics are simply there in the system to be used as needed.

Soundness

HOL Light is both an axiomatic system for doing mathematics and a computer program that implements the system. How trustworthy is it?

If the computer is set aside for a moment, and the axiomatic system alone analyzed, it is known to be consistent relative to ZFC. That is, an inconsistency in the HOL Light system would imply the inconsistency of ZFC.

Computer Implementation

> *You've got to prove the theorem-proving program correct. You're in a regression aren't you?*
>
> —*A. Robinson* [20, p. 288].

The more pressing question is the soundness and reliability of the computer program that implements the logic. An earlier section reported that a typical software program has approximately one bug per 100 lines of computer code. The most reliable software ever created, for example mission-critical software written for the space shuttle, has fewer than one bug per 10,000 lines of computer code. Various proof assistants vary widely in reliability, ranging from some of the world's most carefully crafted code at the upper end, to rubbish at the lower end. I confine my attention to the upper end.

The computer code that implements the axioms and rules of inference is referred to as the kernel of the system. It takes fewer than 500 lines of computer code to implement the kernel of HOL Light. (By contrast, a Linux distribution contains approximately 283 million lines of computer code.)

A bug anywhere in the kernel of this system might have fatal consequences. For example, if one of the axioms is incorrectly typed, it might lead to an inconsistent system.

Yes, it is a regress; but a rather manageable regress. The kernel is a tiny amount of computer code, but hundreds of thousands of lines of code are verified by the kernel. Eventually, there may be many millions of lines that are verified by this small kernel. The same kernel verifies everything from the prime number theorem to the correctness of hardware designs.

Since the kernel is so small, it can be checked on many different levels. The code has been written in a friendly programming style for the benefit of a human audience. The source code is available for public scrutiny. Indeed, the code has been studied by eminent logicians. By design, the mathematical system is spartan and clean. The computer code has these same attributes. A powerful type-checking mechanism within the programming language prevents a user from creating a theorem by any means except through this small fixed kernel. Through type-checking, soundness is ensured, even after a large community of users contributes further theorems and computer code. I wish to see a poster[2] of the lines of the kernel, to be taught in undergraduate courses, and published throughout the world, as the bedrock of mathematics. It is math commenced afresh as executable code.

Experience from other top-tier theorem-proving systems has been that about three to five bugs have been found in each system over a period of 15-20 years of use. After decades of use on many different systems, to my knowledge, only one proof has ever had to be retracted as a result of a bug in a theorem-proving system, and this in a system that I do not rank in the top-tier: in 1995 a heap overflow error led to the false claim that the theorem-prover REVEAL had solved the Robbins conjecture. We can assert with utmost confidence that the error rates of top-tier theorem-proving systems are orders of magnitude lower than error rates in the most prestigious mathematical journals. Indeed, since a formal proof starts with a traditional proof, then does strictly more checking even at the human level, it would be hard for the outcome to be otherwise.

As an extra check, J. Harrison gave what can almost be described as a formal proof in HOL Light of its own soundness [15]. To get around the self-referential limitations imposed by Gödel, he gave two separate proofs. In the first proof, a weakened version of HOL Light is created, without the axiom of infinity. The standard version is used to give a formal proof of the soundness of the weakened version. In the second proof, a strengthened version of HOL Light is created, with an additional axiom giving a large cardinal. The strengthened version then proves the standard version sound. These proofs go beyond traditional relative consistency proofs in logic in two respects. First of all, they are formal proofs, rather than conventional proofs. Second, the proofs establish not only the soundness of the logic, but also the underlying soundness of the computer code implementing the logic.[3]

Export

In the past few years, a number of programs have been written to automatically translate a proof written in one system into a proof in another system. If a proof in one system is incorrect because of an underlying flaw in the theorem-proving program itself, then the export to a different system fails, and the underlying flaw is exposed. (Except of course, unless the second theorem-proving program also has a bug that is perfectly aligned with the bug in the first system. Since these systems are largely independently designed and implemented, the events of failure in different systems are treated as nearly independent, so that the probability of a perfect alignment of failures across n systems, goes to zero roughly as p^n, where p is the individual failure rate.)

Consider what happens when the proof of the soundness of HOL Light is exported. (This has not happened yet, but should happen soon.) The exported proof is a formal proof within a second theorem-prover that the HOL Light logic and implementation are sound. It will soon be within reach for several systems to give proofs of one another's soundness. When this is achieved, the probability of a false certification of a pseudo-proof is pushed an order of magnitude closer to zero. With a computer—indeed with any physical artifact, whether a codex, transistor, or a flash drive made of proteins from salt-marsh bacteria—it is never a matter of achieving philosophical certainty. It is a scientific knowledge of the regularity of nature and human technology, akin to the scientific evidence that Planck's constant $\hbar$ lies reliably within its experimental range. Technology can push the probability of a false certification ever closer to zero: 10^{-6}, 10^{-9}, $10^{-12}, \ldots$. The intent is that one

[2] *A T-shirt has already been made!*

[3] *The soundness of the computer code is considered relative to a semantic model of the underlying programming language. This model may differ from the real-world behavior of the programming language, a reminder that the task of verification is never complete.*

day a system will store a million proofs without so much as a misplaced semicolon.

A bug in the compiler, operating system, or underlying hardware has the potential to compromise a formal proof. To minimize such bugs, formal proofs can be made about the correctness of the ambient computational environment. Indeed, verification of hardware design, compilers, and computer languages has long been one of the principal aims of formal methods. HOL itself was initially created for hardware verification. As early as 1989, a simple computer system from high-level language down to microprocessor was "formally specified and mechanically verified" [4]. Today, the semantics of various high-level programming languages have been defined with complete mathematical rigor [23]. In recent work that is nothing short of spectacular, X. Leroy has developed a formally verified compiler for the C programming language [19]. (When the target of a formal verification is a piece of computer code, rather than a standard mathematical text, the formalization checks that the computer code conforms to a precise specification of the algorithm; certifying that the computer code is bug free.)

Full Automation

Formal proofs are part of a larger project of automating all mechanizable mathematical tasks, from conjecture making to concept formation. This section touches on the problem of fully automated proofs—the discovery of proofs entirely by computer without any human intervention. The next section briefly describes the ultimate challenge of producing an automated mathematician. Progress has been gradual. Fifty years ago it was famously predicted that within a decade "a digital computer will discover and prove an important new mathematical theorem." This did not happen as scheduled.

Most success has been with the development of algorithms to solve special classes of problems. The WZ algorithm gives automated proofs of identities of hypergeometric sums. Gröbner basis methods solve ideal membership problems. Wu's geometry algorithm proves theorems such as Pappus' theorem and Pascal's theorem on the ellipse. Tarski's algorithm solves problems that can be formulated in the first-order language of the real numbers. The list of specialized algorithms is in fact enormous.

The most widely acclaimed example of a fully automated computer proof is the solution of the Robbins conjecture in 1996. The conjecture asserts that an alternative definition is equivalent to the usual definition of a Boolean algebra. It is remarkable because the solution does not involve any human assistance, specialized algorithms, or software designed with this particular problem in mind. Just type the problem into W. McCune's general purpose theorem prover *EQN*, hit return, and wait eight days for the solution to appear [21], [22].

Yet the story is only a qualified success. It has remained almost an isolated example, rather than the first in a torrent of results. The conjecture itself has the rather special form of a word problem in an abstractly defined algebraic system—a type of problem particularly suited for computer search. The proof that was found by computer can be expressed as a short yet non-obvious sequence of substitutions. (See box on next page.)

Overall, the level today of fully automated computer proof (lying outside special purpose algorithms) remains that of undergraduate homework exercises: a group in which every element has order two is necessarily abelian; Cantor's theorem asserting that a set is not in bijection with its powerset; if some iterate of a function has a unique fixed point, then the function has a fixed point; the base e for natural logarithms is irrational [1], [2]. Because of current limitations, fully automated proof tools generally serve to fill in intermediate steps of a larger formal proof. They are not ready to take on the Riemann hypothesis.

Automated Discovery

What happens if one sets aside rigor, and lets a computer explore? A groundbreaking project was D. Lenat's 1976 Stanford thesis. His computer program AM (for Automated Mathematician) was designed to discover new mathematical concepts. When AM was set loose to explore in the wild, it discovered the concepts of natural number, addition, multiplication, prime numbers, Pythagorean triples, and even the fundamental theorem of arithmetic. The thesis touched off a firestorm of criticism and praise.

To put AM in context, consider a hypothetical program that is instructed to discover new concepts by deleting conditions from the list of axioms defining a finite abelian group. The computer will then immediately discover the concepts of infinite group, nonabelian group, monoid, and so forth because these concepts all arise as subsets of the axioms. These discoveries could be sensationalized: *A program in Artificial Intelligence has made the ultimate leap from the finite to infinite, and from the abelian to the nonabelian, rediscovering fundamental concepts in seconds that mathematicians have grappled with for centuries.* There are nagging questions about the emptiness of AM's discoveries; a suggestive representation of the problem gives the answer away.

More recent projects stir the imagination, even if the field is still young. Computer programs have generated over one thousand conjectures in graph theory, expressing numerical relationships between different graph invariants. One

Full Automation of the Robbins Conjecture

Let S be a nonempty set with an associative commutative binary operation $(x, y) \mapsto xy$ and a unary operation $x \mapsto [x]$ (which, for convenience, we write synonymously as $x \mapsto \bar{x}$). The Robbins conjecture (in Winker form) asserts that the general Robbins identity

$$[[ab][a\bar{b}]] = a$$

implies the existence of $c, d \in S$ such that $[cd] = \bar{c}$. Here is the original proof that EQN discovered, as reconstructed in [10].

Proof. A solution is $c = x^3u$, $d = xu$, where $u = [x\bar{x}]$ and x is arbitrary. Abbreviate $j = [cd]$, $e = u[x^2]\bar{c}$. Over the equality sign, a prime indicates a direct application of the Robbins identity; a superscript indicates a substitution of the numbered line; no superscript indicates a rewriting of abbreviations c, d, e, j, u.

$$\begin{array}{ll}
0 : [u[x^2]] & = [[x\bar{x}][xx]] =' x. \\
1 : [xu[xu[x^2]\bar{c}]] & =' [[[xux^2][xu[x^2]]][xu[x^2]\bar{c}]] = [[\bar{c}[xu[x^2]]][\bar{c}xu[x^2]]] =' \bar{c}. \\
2 : [u\bar{c}] & = [u[x^2ux]] =^0 [u[x^2u[u[x^2]]]] =' [[[ux^2][u[x^2]]][x^2u[u[x^2]]]] \\
 & =' [u[x^2]] =^0 x. \\
3 : [ju] & = [[xcu]u] =' [[xcu][[uc][u\bar{c}]]] =^2 [[xcu][x[cu]]] =' x \\
4 : [x[x[x^2]u\bar{c}]] & =' [[[x[u\bar{c}]][xu\bar{c}]][x[x^2]u\bar{c}]] =^2 [[[x^2][xu\bar{c}]][[x^2]xu\bar{c}]] =' [x^2] \\
5 : [x\bar{c}] & =^1 [x[xu[xu[x^2]\bar{c}]]] =^0 [[u[x^2]][xu[xu[x^2]\bar{c}]]] \\
 & = [[u[x^2]][ux[xe]]] =^4 [[u[x[xe]]][ux[xe]]] =' u \\
6 : [jx] & =' [j[[xc][x\bar{c}]]] =^5 [j[[xc]u]] = [[uxc][u[xc]]] =' u \\
7 : [cd] & = j =' [[j[x\bar{c}]][jx\bar{c}]] =^5 [[ju][jx\bar{c}]] =^3 [x[jx\bar{c}]] =^2 [[\bar{c}u][\bar{c}jx]] \\
 & =^6 [[\bar{c}[jx]][\bar{c}jx]] =' \bar{c}.
\end{array}$$

□

open conjecture is described in the box "An Open Computer-Generated Conjecture". No technological barriers prevent us from unleashing conjecturing machines in all branches of mathematics, to see what moonshine they reveal.

Flyspeck

My interest in formal proofs grows out of a practical desire for a thorough verification of my own research that goes beyond what the traditional peer review process has been able to provide. A few years ago, I launched a project called *Flyspeck* to give a formal proof of the Kepler conjecture, asserting that no packing of congruent balls in three-dimensional Euclidean space can have density greater than the density of the face-centered cubic packing (also known as the cannonball arrangement). The name Flyspeck, which quite appropriately can mean to scrutinize, is derived from the acronym FPK, for the Formal Proof of the Kepler conjecture.

The original proof of this theorem was unusually difficult to check. In a letter of qualified acceptance for publication in the *Annals of Mathematics*, an editor described the process, "The referees put a level of energy into this that is, in my experience, unprecedented. They ran a seminar on it for a long time. A number of people were involved, and they worked hard. They checked many local statements in the proof, and each time they found that what you claimed was in fact correct. Some of these local checks were highly non-obvious at first, and required weeks to see that they worked out…They have not been able to certify the correctness of the proof, and will not be able to certify it in the future, because they have run out of energy to devote to the problem." In addition to a 300-page text, the proof relies on about forty thousand lines of custom computer code. To the best of my knowledge, the computer code was never carefully examined by the referees. The policy of the *Annals of Mathematics* states, "The human part of the proof, which reduces the original mathematical problem to one tractable by the computer, will be refereed for correctness in the traditional manner. The computer part may not be checked line-by-line, but will be examined for the methods by which the authors have eliminated or minimized possible sources of error…"

Ultimately, the mathematical corpus is no more reliable than the processes that assure its quality. A formal proof attains a much higher level of quality control than can be achieved by "local checks" and an "examination of methods".

Flyspeck may take as many as twenty work-years to complete. S. Obua and G. Bauer have already defended Ph.D. theses on the project. Together with the work of their advisor T. Nipkow (one of the principal architects of the Isabelle proof assistant), nearly half of the computer code used in the proof of the Kepler conjecture is now certified.

An Open Computer-Generated Conjecture

Let G be a finite graph with the following properties:

(1) It has at least two vertices.
(2) The graph is simple; that is, there are no loops or multiple joins.
(3) It is regular; that is, every vertex has the same degree.
(4) The graph is connected.

For example, the complete graph (the graph with an edge between every two vertices) on n vertices has these properties, when $n \geq 2$. Define the *total domination number* of G to be the size of the smallest subset of vertices such that every vertex of G is adjacent to some vertex in the subset. The *path covering number* is the size of the smallest partition of the vertices into subsets, such that there exists a path confined to each subset S that steps through each vertex of S exactly once (that is, the induced graph on S has a Hamiltonian path).

The computer program Graffiti.pc conjectures that *the total domination number of G is at least twice the path covering number of G*. For example, the complete graph on n vertices has path covering number one, because it has a Hamiltonian path. Its total domination number is two (take any two vertices). The conjecture is sharp in this case by these direct observations [9].

QED

The Flyspeck project is a minute speck in the overarching Q.E.D. project (an anonymous manifesto declaring that all significant mathematical results should be preserved in a vast library of formal proofs). The labor required to realize such a library would be staggering. In the *Notices* in 1991, de Bruijn proposed an assembly line to turn mathematical ideas into formally verified proofs [7]. The standard benchmark for the human labor to transcribe one printed page of textbook mathematics into machine-verified formal text is one week, or US$150 per page at an outsourced wage. To undertake the formalization of just $100,000$ pages of core mathematics would be one of the most ambitious collaborative projects ever undertaken in pure mathematics, the sequencing of a mathematical genome. One might imagine a massive wiki collaboration that settles the text of the most significant theorems in contemporary mathematics from Poincaré to Sato-Tate.

Outsourcing is the brute force solution to the Q.E.D. manifesto. Most researchers, however, prefer beauty over brute force; we may hope for advances in our understanding that will permit us someday to convert a printed page of textbook mathematics into machine-verified formal text in a matter of hours, rather than after a full week's labor. As long as transcription from traditional proof into formal proof is based on human labor rather than automation, formalization remains an art rather than a science. Until that day of automation, we fall short of a practical understanding of the foundations of mathematics.

Recommended Reading and Software

By far the best overview of the subject is the book *Mechanizing Proof,* winner of the 2003 Merton Book Award of the American Sociological Association [20]. The Q.E.D. Manifesto can be found at [29]. Historical surveys include [5], [13], [11], and [25]. For something more comprehensive, see [14].

Several theorem proving systems are extensively documented and are available for download, including HOL Light [12], Isabelle [16], Coq [8], Mizar [24], TPS, PVS, ACL2, NuPRL, and MetaPRL. A Web-browser version of Coq allows one to experiment with a proof assistant without downloading any software [28].

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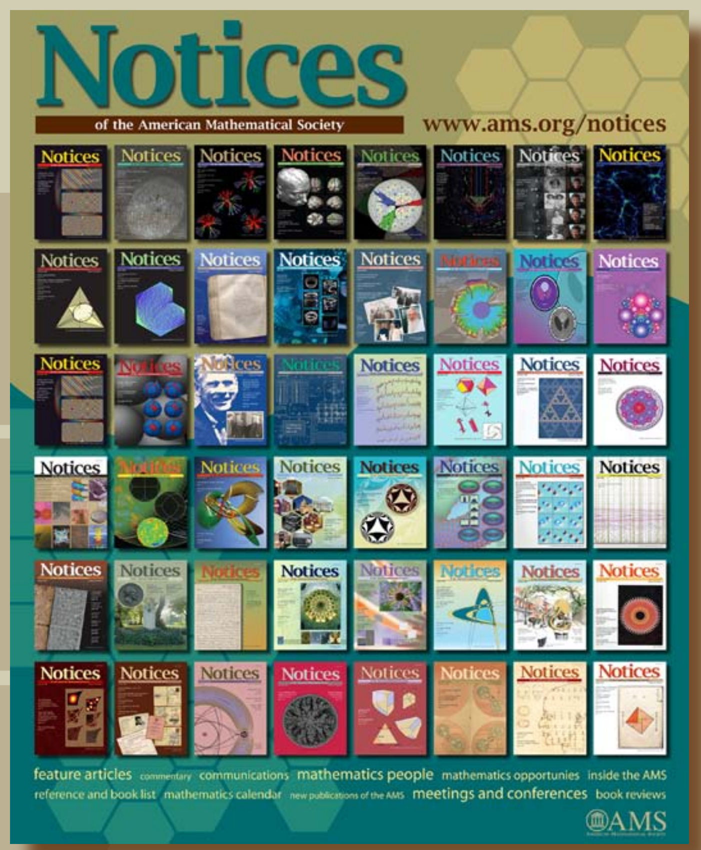

Formal Proof—The Four-Color Theorem

Georges Gonthier

The Tale of a Brainteaser

Francis Guthrie certainly did it, when he coined his innocent little coloring puzzle in 1852. He managed to embarrass successively his mathematician brother, his brother's professor, Augustus de Morgan, and all of de Morgan's visitors, who couldn't solve it; the Royal Society, who only realized ten years later that Alfred Kempe's 1879 solution was wrong; and the three following generations of mathematicians who couldn't fix it [19].

Even Appel and Haken's 1976 triumph [2] had a hint of defeat: they'd had a computer do the proof for them! Perhaps the mathematical controversy around the proof died down with their book [3] and with the elegant 1995 revision [13] by Robertson, Saunders, Seymour, and Thomas. However something was still amiss: both proofs combined a textual argument, which could reasonably be checked by inspection, with computer code that could not. Worse, the empirical evidence provided by running code several times with the *same* input is weak, as it is blind to the most common cause of "computer" error: programmer error.

For some thirty years, computer science has been working out a solution to this problem: formal program proofs. The idea is to write code that describes not only *what* the machine should do, but also *why* it should be doing it—a formal proof of correctness. The validity of the proof is an objective mathematical fact that can be checked by a *different* program, whose own validity can be ascertained empirically because it does run on *many* inputs. The main technical difficulty is that formal proofs are very difficult to produce, even with a language rich enough to express all mathematics.

In 2000 we tried to produce such a proof for part of code from [13], just to evaluate how the field had progressed. We succeeded, but now a new question emerged: was the statement of the correctness proof (the *specification*) itself correct? The only solution to that conundrum was to formalize the *entire* proof of the Four-Color Theorem, not just its code. This we finally achieved in 2005.

While we tackled this project mainly to explore the capabilities of a modern formal proof system—at first, to benchmark speed—we were pleasantly surprised to uncover new and rather elegant nuggets of mathematics in the process. In hindsight this might have been expected: to produce a formal proof one must make explicit every single logical step of a proof; this both provides new insight in the structure of the proof, and forces one to use this insight to discover every possible symmetry, simplification, and generalization, if only to cope with the sheer amount of imposed detail. This is actually how all of sections "Combinatorial Hypermaps" (p. 1385) and "The Formal Theorem" (p. 1388) came about. Perhaps this is the most promising aspect of formal proof: it is not merely a method to make absolutely sure we have not made a mistake in a proof, but also a tool that shows us and compels us to understand why a proof works.

In this article, the next two sections contain background material, describing the original proof and the Coq formal system we used. The following two sections describe the sometimes new mathematics involved in the formalization. Then the next two sections go into some detail into the two main parts of the formal proof: reducibility and

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unavoidability; more can be found in [8]. The Coq code (available at the same address) is the ultimate reference for the intrepid, who should bone up on Coq [4, 16, 9] beforehand.

The Puzzle and Its Solution

Part of the appeal of the four color problem is that its statement

Theorem 1. *The regions of any simple planar map can be colored with only four colors, in such a way that any two adjacent regions have different colors.*

can on the one hand be understood even by schoolchildren as *"four colors suffice to color any flat map"* and on the other hand be given a faithful, precise mathematical interpretation using only basic notions in topology, as we shall see in the section "The Formal Theorem".

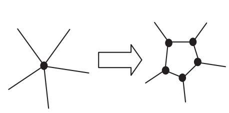
The first step in the proof of the Four-Color Theorem consists precisely in getting rid of the topology, reducing an infinite problem in analysis to a finite problem in combinatorics. This is usually done by constructing the dual graph of the map, and then appealing to the compactness theorem of propositional logic. However, as we shall see below, the graph construction is neither necessary nor sufficient to fully reduce the problem to combinatorics.

Therefore, we'll simply restrict the rest of this outline to connected finite maps whose regions are finite polygons and which are *bridgeless*: every edge belongs to exactly two polygons. Every such *polyhedral* map satisfies the Euler formula

$$N - E + F = 2$$

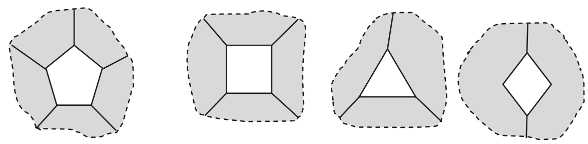
where N, E, and F are respectively the number of vertices (nodes), sides (edges), and regions (faces) in the map.

The next step consists in further reducing to *cubic* maps, where each node is incident to exactly three edges, by covering each node with a small polygon.



In a cubic map we have $3N = 2E$, which combined with the Euler formula gives us that the average number of sides (or *arity*) of a face is $2E/F = 6 - 12/F$.

The proof proceeds by induction on the size of the map; it is best explained as a refinement of Kempe's flawed 1879 proof [12]. Since its average arity is slightly less than 6, any cubic polyhedral map must contain an n-gon with $n < 6$, i.e., one of the following map fragments.

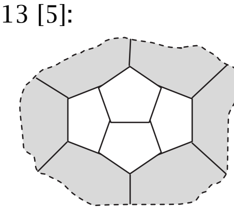


Each such *configuration* consists of a complete *kernel* face surrounded by a *ring* of partial faces.

Erasing an edge of a digon or triangle yields a smaller map, which is four-colorable by induction. This coloring uses at most three colors for the ring, leaving us a free color for the kernel face, so the original map is also four-colorable. Erasing an appropriate pair of opposite edges disposes of the square configuration similarly.

In the pentagon case, however, it is necessary to modify the inductive coloring to free a ring color for the kernel face. Kempe tried to do this by locally inverting the colors inside a two-toned maximal contiguous group of faces (a "Kempe chain"). By planarity, chains cannot cross, and Kempe enumerated their arrangements and showed that consecutive inversions freed a ring color. Alas, it is not always possible to do consecutive inversions, as inverting one chain can scramble other chains. It took ten years to spot this error and almost a century to fix it.

The correct proof gives up on pentagons and turns to larger *reducible* configurations for which Kempe's argument is sound. The first such configuration, which has ring-size 6, was discovered by Birkhoff in 1913 [5]:



Birkhoff also showed that all configurations with ring-size less than 6 are reducible *except* the pentagon; thus any minimal counter-example to the theorem must be *internally 6-connected* (we'll refer to this as the "Birkhoff lemma").

As we'll see below, showing that a given configuration is reducible is fairly straightforward, but very laborious: the number of cases to consider increases geometrically to about 20,000,000 for ring-size 14, and 137 of the 633 configurations used in the proof [13] are of that size.

The final part of the proof shows that reducible configurations are *unavoidable*, using a refinement of the average-arity argument published by Heesch in 1969 [11]. The idea is to look for reducible configurations near faces whose arity averaged over their 2-neighborhood is less than 6; the "averaging" is done by transfering (*discharging*) fractions of arities between adjacent faces according to a small set of local patterns: the "discharged" arity of a face a is

$$\overline{\delta}(a) = \delta(a) + \textstyle\sum_b (T_{ba} - T_{ab})$$

where $\delta(a)$ is the original arity of a, and T_{ba} is the arity fraction transfered from b. Thus the average discharged arity remains $6 - 12/F < 6$.

The proof enumerates all internally 6-connected 2-neighborhoods whose discharged arity is less than 6. This enumeration is fairly complex, but not as computationally intensive as the reducibility checks: the search is heavily constrained as the neighborhoods consist of two disjoint concentric rings of 5^+-gons. Indeed in [13] reducible configurations are always found inside the 2-neighborhoods, and the central face is a 7^+-gon.

Coq and the Calculus of Inductive Constructions

The Coq formal proof system (or *assistant*) [4, 16], which we used for our work is based on a version of higher-order logic, the Calculus of inductive Constructions (CiC) [6] whose specific features—propositions as types, dependent types, and reflection—all played an important part in the success of our project.

We have good reason to leave the familiar, dead-simple world of untyped first-order logic for the more exotic territory of Type Theory [10, 4]. In first-order logic, higher-level ("meta") arguments are second-class citizens: they are interpreted as informal procedures that should be expanded out to primitive inferences to achieve full rigor. This is fine in a non-formal proof, but rapidly becomes impractical in a formal one because of ramping complexity. Computer automation can mitigate this, but type theory supplies a much more satisfactory solution, levelling the playing field by providing a language that can express meta-arguments.

This can indeed be observed even with the simplest first-order type system. Consider the commutativity of integer addition,

$$\forall x, y \in \mathbb{N},\ x + y = y + x$$

There are two hidden premises, $x \in \mathbb{N}$ and $y \in \mathbb{N}$, that need to be verified separately every time the law is used. This seems innocuous enough, except x and y may be replaced by huge expressions for which the $x, y \in \mathbb{N}$ premises are not obvious, even for machine automation. By contrast, the typed version of commutativity

$$\forall x, y\ :\ \mathrm{Nat},\ x + y = y + x$$

can be applied to any expression $A + B$ without further checks, because the premises follow from the way A and B are written. We are simply not allowed to write drivel such as $1 + \mathrm{true}$, and consequently we don't need to worry about its existence, even in a fully formal proof—we have "proof by notation".

Our formal proof uses this, and much more: proof types, dependent types, and reflection, as we will now explain.

Proof types are types that encode logic (they're also called "propositions-as-types"). The encoding exploits a strong similarity between type and logic rules, which is most apparent when both are written in natural deduction style (see [10] in this issue), e.g., consider function application and modus ponens (MP):

$$\frac{f : A \to B \quad x : A}{f\ x : B} \qquad \frac{A \Rightarrow B \quad A}{B}$$

The rules are identical if one ignores the terms to the left of ":". However these terms can also be included in the correspondence, by interpreting $x : A$ and "x proves A" rather than "x is of type A". In the above we have that $f\ x$ proves B because x proves A and f proves $A \Rightarrow B$, so the application $f\ x$ on the left denotes the MP deduction on the right. This holds in general: proof types are inhabited by proof objects.

CiC is entirely based on this correspondence, which goes back to Curry and Howard. CiC is a formalism without a formal logic, a sensible simplification: as we've argued we need types anyway, so why add a redundant logic? The availability of proof objects has consequences both for robustness, as they provide a practical means of storing and thus independently checking proofs, and for expressiveness, as they let us describe and prove algorithms that create and process proofs—meta-arguments.

The correspondence in CiC is not limited to Herbrand term and minimal logic; it interprets most data and programming constructs common in computer science as useful logical connectives and deduction rules, e.g., pairs as "and"

$$\frac{x : A \quad y : B}{\langle x, y\rangle : A \times B} \qquad \frac{u : A \times B}{u.1 : A \quad u.2 : B}$$

$$\frac{A \quad B}{A \wedge B} \qquad \frac{A \wedge B}{A \quad B}$$

tagged unions as "or", conditional (`if-then-else`) as proof by cases, recursive definitions as proof by induction, and so on. The correspondence even works *backwards* for the logical rule of generalization: we have

$$\frac{\Gamma \vdash B[x] \quad x \text{ not free in } \Gamma}{\Gamma \vdash \forall x, B[x])}$$

$$\frac{\Gamma, x : A \vdash t[x] : B[x]}{\Gamma \vdash (\mathtt{fun}\ x : A \mapsto t[x]) : (\forall x : A, B[x])}$$

The proof/typing context Γ is explicit here because of the side condition. Generalization is interpreted by a new, stronger form of function definition that lets the *type* of the result *depend* on a formal parameter. Note that the nondependent case interprets the Deduction Theorem.

The combination of these *dependent types* with proof types leads to the last feature of CiC we wish to highlight in this section, computational reflection. Because of dependent types, data, functions and therefore (potential) computation can appear in types. The normal mathematical practice is to *interpret* such meta-data, replacing a constant by its definition, instantiating formal parameters, selecting a case, etc. CiC supports this through a typing rule that lets such computation happen transparently:

$$\frac{t : A \qquad A \equiv_{\beta\iota\delta\zeta} B}{t : B}$$

This rule states that the $\beta\iota\delta\zeta$-computation rules of CiC yield equivalent types. It is a *subsumption* rule: there is no record of its use in the proof term t. Aribitrary long computations can thus be elided from a proof, as CiC has an ι-rule for recursion.

This yields an entirely new way of proving results about specific calculations: computing! Henri Poincaré once pointed out that one does not "prove" $2 + 2 = 4$, one "checks" it. CiC can do just that: if `erefl` $: \forall x, x = x$ is the reflexivity axiom, and the constants +, 2, 4 denote a recursive function that computes integer addition, and the representation of the integers 2 and 4, respectively, then `erefl` 4 proves $2+2 = 4$ because $2+2 = 4$ and $4 = 4$ are just different denotations of the same proposition.

While the Poincaré example is trivial, we would probably not have completed our proof without computational reflection. At the heart of the reducibility proof, we define

```
Definition check_reducible cf : bool := ...
Definition cfreducible cf : Prop :=
  c_reducible (cfring cf) (cfcontract cf).
```

where `check_reducible cf` calls a complex reducibility decision procedure that works on a specific encoding of configurations, while `c_reducible r c` is a logical predicate that asserts the reducibility of a configuration map with ring `r` and deleted edges (contract) `c`; `cfring cf` denotes the ring of the map represented by `cf`. We then prove

```
Lemma check_reducible_valid :
  forall cf : config,
  check_reducible cf = true -> cfreducible cf.
```

This is the formal partial correctness proof of the decision procedure; it's large, but nowhere near the size of an explicit reducibility proof. With the groundwork done, all reducibility proofs become trivial, e.g.,

```
Lemma cfred232 : cfreducible (Config 11 33 37
   H 2 H 13 Y 5 H 10 H 1 H 1 Y 3 H 11 Y 4 H 9
   H 1 Y 3 H 9 Y 6 Y 1 Y 1 Y 3 Y 1 Y Y 1 Y).
Proof.
apply check_reducible_valid.
vm_compute; reflexivity.
Qed.
```

In CiC, this 20,000,000-cases proof, is almost as trivial as the Poincaré $2 + 2 = 4$: apply the correctness lemma, then reflexivity up to (a big!) computation. Note that we make essential use of dependent and proof types, as the cubic map computed by `cfring` is passed implicitly inside the type of the returned ring. `cfring` mediates between a string representation of configurations, well suited to algorithms, and a more mathematical one, better suited for abstract graph theory, which we shall describe in the next section.

Combinatorial Hypermaps

Although the Four-Color Theorem is superficially stated in analysis, it really is a result in combinatorics, so common sense suggests that the bulk of the proof should use solely combinatorial structure. Oddly, most accounts of the proof use graphs homeomorphically embedded in the plane or sphere, dragging analysis into the combinatorics. This does allow appealing to the Jordan Curve Theorem in a pinch, but this is hardly helpful if one does not already have the Jordan Curve Theorem in store.

Moreover, graphs lack the data to do geometric traversals, e.g., traversing the first neighborhood of a face in clockwise order; it is necessary to go back to the embedding to recover this information. This will not be easy in a fully formal proof, where one does not have the luxury of appealing to pictures or "folklore" when cornered.

The solution to this problem is simply to add the missing data. This yields an elegant and highly symmetrical structure, the combinatorial hypermap [7, 17].

Definition 1. *A* hypermap *is a triple of functions* $\langle e, n, f\rangle$ *on a finite set* d *of* darts *that satisfy the triangular identity* $e \circ n \circ f = 1$.

Note the circular symmetry of the identity: $\langle n, f, e\rangle$ and $\langle f, e, n\rangle$ are also hypermaps. Obviously, the condition forces all the functions to be permutations of d, and fixing any two will determine the third; indeed hypermaps are often defined this way. We choose to go with symmetry instead, because this lets us use our constructions and theorems three times over. The symmetry also clearly demonstrates that the dual graph construction plays no part in the proof.

We have found that the relation between hypermaps and "plain" polyhedral maps is best depicted by drawing darts as points placed at the corners of the polygonal faces, and using arrows for the three functions, with the cycles of the f function going counter-clockwise inside each face, and those of the n function around each node. On a plain map each edge has exactly two endpoints, and consequently each e cycle is a double-ended arrow cutting diagonally across the $n - f - n - f$ rectangle that straddles an edge (Figure 1).

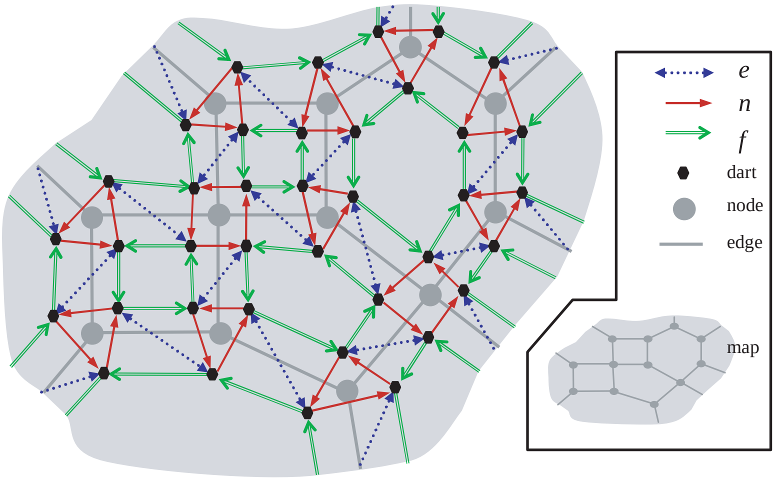


Figure 1. A hypermap.

The Euler formula takes a completely symmetrical form for hypermaps

$$E + N + F = D + 2C$$

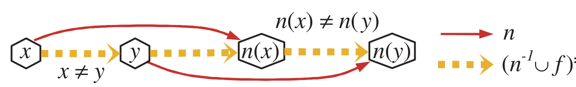
where E, N, and F are the number of cycles of the e, n, and f permutations, D and C are the number of darts and of connected components of $e \cup n \cup f$, respectively.

We define planar hypermaps as those that satisfy this generalized Euler formula, since this property is readily computable. Much of the proof can be carried out using only this formula. In particular Benjamin Werner found out that the proof of correctness of the reducibility part was most naturally carried out using the Euler formula only. As the other unavoidability part of the proof is explicitly based on the Euler formula, one could be misled into thinking that the whole theorem is a direct consequence of the Euler formula. This is not the case, however, because unavoidability also depends on the Birkhoff lemma. Part of the proof of the latter requires cutting out the submap inside an arbitrary simple ring of 2 to 5 faces. Identifying the inside of a ring is exactly what the Jordan Curve Theorem does, so we worked out a combinatorial analogue. We even show that our hypermap Jordan property is actually equivalent to the hypermap Euler formula.

The naïve transposition of the Jordan Curve Theorem from the continuous plane to discrete maps fails. Simply removing a ring from a hypermap, even a connected one, can leave behind any number of components: both the "inside" and the "outside" may turn out to be empty or disconnected. A possible solution, proposed by Stahl [14], is to consider paths (called chords below) that go from one face of the ring to another (loops are allowed). The Jordan Curve Theorem then tells us that such paths cannot start from the "inner half" of a ring face, and end at the "outer half" of a ring face.

Using the fixed local structure of hypermaps, "inner" and "outer" can be defined locally, by adhering to a certain traversal pattern. Specifically, we exclude the e function and fix *opposite* directions of travel on n and f: we define contour paths as dart paths for the $n^{-1} \cup f$ relation. A contour cycle follows the inside border of a face ring, clockwise, listing explicitly all the darts in this border. Note that n or n^{-1} steps from a contour cycle always go inside the contour, while f or f^{-1} steps always go outside. Therefore the Jordan property for hypermap contours is: "any chord must start and end with the same type of step." This can be further simplified by splicing the ring and cycle, yielding

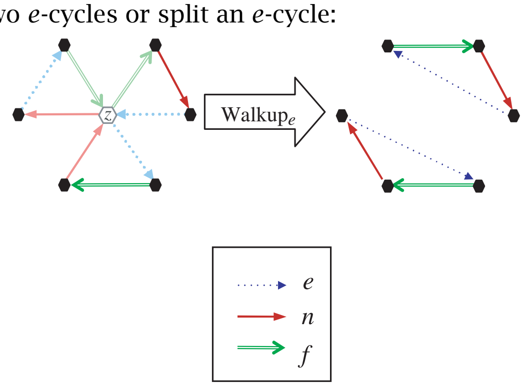
Theorem 2. *(the Jordan Curve Theorem for hypermaps): A hypermap is planar if and only if it has no duplicate-free "Möbius contours" of the form*



The $x \neq y$ condition rules out contour cycles; note however that we do allow $y = n(x)$.

As far as we know this is a new combinatorial definition of planarity. Perhaps it has escaped attention because a crucial detail, reversing one of the permutations, is obscured for plain maps (where $e^{-1} = e$), or when considering only cycles. Since this Jordan property is equivalent to the Euler identity, it is symmetrical with respect to the choice of the two permutations that define "contours", despite appearances. Oddly, we know no simple direct proof of this fact.

We show that our Jordan property is equivalent to the Euler identity by induction on the number of darts. At each induction step we remove some dart z from the hypermap structure, adjusting the permutations so that they avoid z. We can simply suppress z from two of the permutations (e.g., n and f), but then the triangular identity of hypermaps leaves us no choice for the third permutation (e here), and we have to either merge two e-cycles or split an e-cycle:



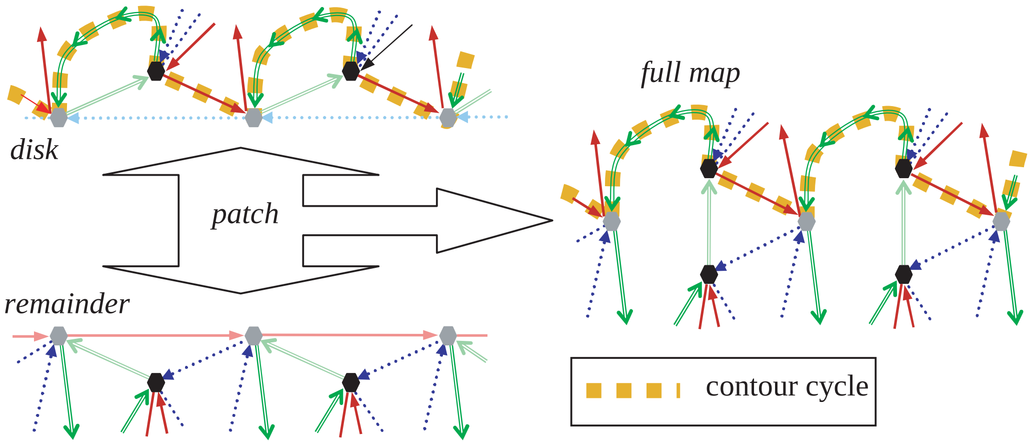


Figure 2. Patching hypermaps.

Following [14, 18], we call this operation the Walkup transformation. The figure on the previous page illustrates the Walkup_e transformation; by symmetry, we also have Walkup_n and Walkup_f transformations. In general, the three transformations yield different hypermaps, and all three prove to be useful. However, in the degenerate case where z is fixed by e, n, or f, all three variants coincide.

A Walkup transformation that is degenerate or that merges cycles does not affect the validity of the hypermap Euler equation $E+N+F=D+2C$. A splitting transformation preserves the equation if and only if it disconnects the hypermap; otherwise it increments the left hand side while decrementing the right hand side. Since the empty hypermap is planar, we see that planar hypermaps are those that maximize the sum $E+N+F$ for given C and D and that a splitting transformation always disconnects a planar hypermap.

To show that planar maps satisfy the Jordan property we simply exhibit a series of transformations that reduce the contour to a 3-dart cycle that violates the planarity condition. The converse is much more delicate, since we must apply *reverse* Walkup_e transformations that preserve both the existence of a contour and avoid splits (the latter involves a combinatorial analogue of the "flooding" proof of the original Euler formula).

We also use all three transformations in the main part of proof. Since at this point we are restricting ourselves to plain maps, we always perform two Walkup transformations in succession; the first one always has the merge form, the second one is always degenerate, and always yields a plain map. Each variant of this double Walkup transformation has a different geometric interpretation and is used in a different part of the proof:

- The double Walkup_f transformation erases an edge in the map, merging the two adjoining faces. It is used in the main proof to apply a contract.
- The double Walkup_e transformation concatenates two successive edges in the map; we apply it only at nodes that have only two incident edges, to remove edge subdivisions left over after erasing edges.
- The double Walkup_n transformation contracts an edge in the map, merging its endpoints. It is used to prove the correctness of the reducibility check.

Contours provide the basis for a precise definition of the patch operation, which pastes two maps along a border ring to create a larger map. This operation defines a three-way relation between a map, a configuration submap, and the remainder of that map. Surprisingly, the patch operation (Figure 2) is not symmetrical:

- For one of the submaps, which we shall call the disk map, the ring is an e cycle (a hyperedge). No two darts on this cycle can belong to the same face.
- For the other submap, the remainder map, the ring is an arbitrary n cycle.

Let us point out that although the darts on the border rings were linked by the e and n permutations in the disk and remainder map, respectively, they are not directly connected in the full map. However, because the e cycle is simple in the disk map, it is a subcycle of a contour that delineates the entire disk map. This contour is preserved by the construction, which is thus reversible: the disk map can be extracted, using the Jordan property, from this contour. The patch operation preserves most of the geometrical properties we are concerned with (planar, plane, cubic, 4-colorable; bridgeless requires a side condition).

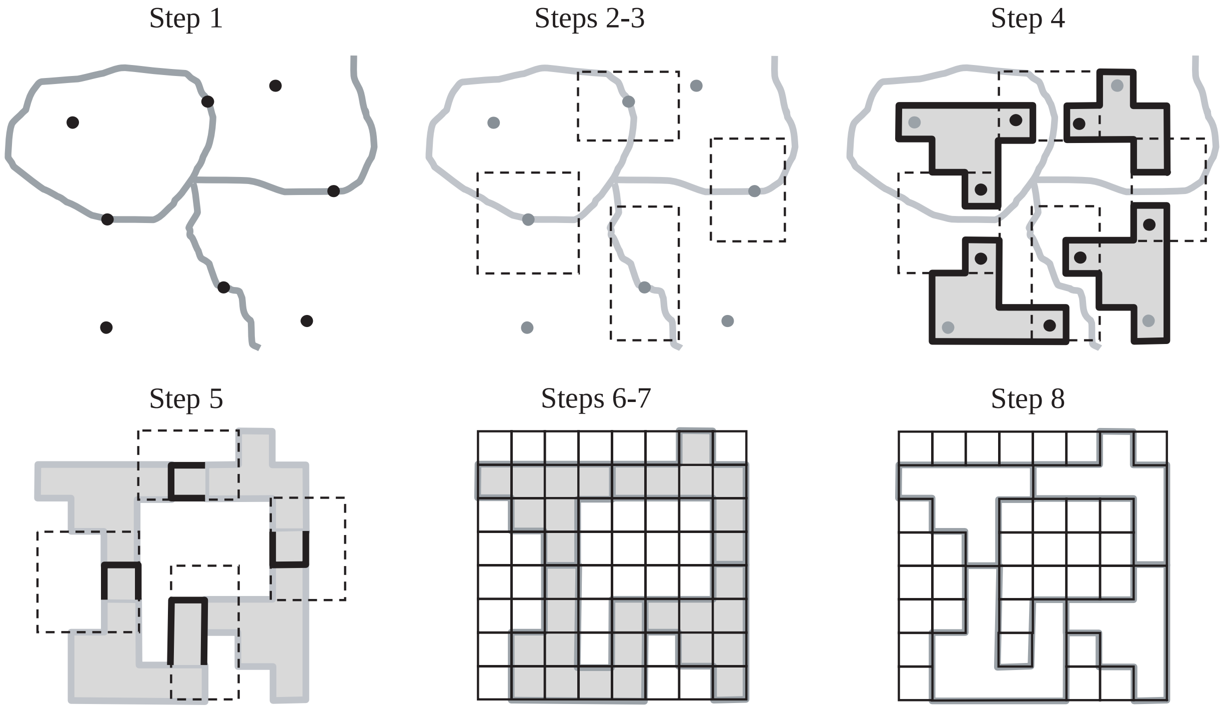


Figure 3. Digitizing the four color problem.

The Formal Theorem

Polishing off our formal proof by actually proving Theorem 1 came as an afterthought, after we had done the bulk of the work and proved

```
Theorem four_color_hypermap :
 forall g : hypermap, planar_bridgeless g ->
 four_colorable g.
```

We realized we weren't quite done, because the deceptively simple statement hides fairly technical definitions of hypermaps, cycle-counting, and planarity. While formal verification spares the skeptical from having to wade through the complete proof, he still needs to unravel all the definitions to convince himself that the result lives up to its name.

The final theorem looks superficially similar to its combinatorial counterpart

```
Variable R : real_model.
Theorem four_color : forall m : map R,
simple_map m -> map_colorable 4 m.
```

but it is actually quite different: it is based on about 40 lines of elementary topology, and about 100 lines axiomatizing real numbers, rather than 5,000 lines of sometimes arcane combinatorics. The 40 lines define simple point topology on $\mathbb{R} \times \mathbb{R}$, then simply drill down on the statement of Theorem 1:

Definition 2. *A planar map is a set of pairwise disjoint subsets of the plane, called regions. A simple map is one whose regions are connected open sets.*

Definition 3. *Two regions of a map are adjacent if their respective closures have a common point that is not a corner of the map.*

Definition 4. *A point is a corner of a map if and only if it belongs to the closures of at least three regions.*

The definition of "corner" allows for contiguous points, to allow for boundaries with accumulation points, such as the curve $\sin 1/x$.

The discretization construction (Figure 3) follows directly from the definitions: Pick a non-corner border point for each pair of adjacent regions (1); pick disjoint neighborhoods of these points (2), and snap them to a grid (3); pick a simple polyomino approximation of each region, that intersects all border rectangles (4), and extend them so they meet (5); pick a grid that covers all the polyominos (6) and construct the corresponding hypermap (7); construct the contour of each polyomino, and use the converse of the hypermap patch operation to cut out each polyomino (8).

It is interesting to note that the final hypermap is planar because the full grid hypermap of step 7 is, simply by arithmetic: the map for an $m \times n$ rectangle has $(m+1)(n+1)$ nodes, $mn+1$ faces, $m(n+1)+(m+1)n$ edges, hence $N+F-E = (m+1)(n+1)+(mn+1)-m(n+1)-(m+1)n = 2$ and the Euler formula holds. The Jordan Curve Theorem plays no part in this.

Checking Reducibility

Although reducibility is quite demanding computationally, it also turned out to be the easiest part of the proof to formalize. Even though we used more sophisticated algorithms, e.g., multiway decision diagrams (MDDs) [1], this part of the formal proof was completed in a few part-time months. By comparison, the graph theory in section "Combinatorial Hypermaps" (p. 1385) took a few years to sort out.

The reducibility computation consists in iterating a formalized version of the Kempe chain argument, in order to compute a lower bound for the set of colorings that can be "fitted" to match a coloring of the configuration border, using color swaps. Each configuration comes with a set of one to four edges, its *contract* [13], and the actual check verifies that the lower bound contains all the colorings of the *contract map* obtained by erasing the edges of contract.

The computation keeps track of both ring colorings and arrangement of Kempe chains. To cut down on symmetries, their representation uses the fact that four-coloring the faces of a cubic map is equivalent to three-coloring its edges; this result goes back Tait in 1880 [15]. Thus a coloring is represented by a word on the alphabet $\{1,2,3\}$.

Kempe inversions for edge colorings amount to inverting the edge colors along a two-toned path (a *chain*) incident to ring edges. Since the map is cubic the chains for any given pair of colors (say 2 and 3) are always disjoint and thus define an outerplanar graph, which is readily represented by a bracket (or Dyck) word on a 4-letter alphabet: traversing the ring edges counterclockwise, write down

- a $\bullet$ if the edge is not part of a chain because it has color 1
- a [if the edge starts a new chain
- a $]_0$ (resp. $]_1$) if the edge is the end of a chain of odd (resp. even) length

As the chain graph is outerplanar, brackets for the endpoints of any chain match. We call such a four-letter word a *chromogram*.

Since we can flip simultaneously any subset of the chains, a given chromogram will match any ring coloring that assigns color 1 to $\bullet$ edges and those only, and assigns different colors to edges with matching brackets if and only if the closing bracket is a $]_1$. We say that such a coloring is consistent with the chromogram.

Let us say that a ring coloring of the remainder map is suitable if it can be transformed, via a sequence alternating chain inversions and a cyclic permutation ρ of $\{1,2,3\}$, into a ring coloring of the configuration. A configuration will thus be reducible when all the ring colorings of its contract map are suitable. The reducibility check consists in computing an upper bound of the set of non-suitable colorings and checking that it is disjoint from the set of contract colorings.

The upper bound is the limit of a decreasing sequence $\Xi_0,\ldots,\Xi_i,\ldots$ starting with the set Ξ_0 of all valid colorings; we simultaneously compute upper bounds $\Lambda_0,\ldots,\Lambda_i,\ldots$ of the set of chromograms not consistent with any suitable coloring. Each iteration starts with a set $\Delta\Theta_i$ of suitable colorings, taking the set of ring colorings of the configuration for $\Delta\Theta_0$.

(1) We get Λ_{i+1} by removing all chromograms consistent with some $\theta \in \Delta\Theta_i$ from Λ_i
(2) We remove from Ξ_i all colorings ξ that are not consistent with any $\lambda \in \Lambda_{i+1}$; this is sound since any coloring that induces ξ will also induce a chromogram $\gamma \notin \Lambda_{i+1}$. As γ is consistent with both ξ and a suitable coloring θ, there exists a chain inversion that transforms ξ into θ.
(3) Finally we get $\Delta\Theta_{i+1}$ by applying ρ and ρ^{-1} to the colorings that were deleted from Ξ_i; we stop if there were none.

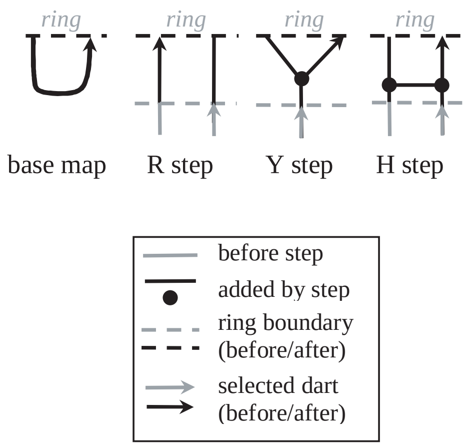
We use 3- and 4-way decision diagrams to represent the various sets: a $\{1,2,3\}$ edge-labeled tree represents a set of colorings, and a $\{\bullet,[,]_0,]_1\}$ edge-labeled tree represents a set of chromograms. In the MDD for Ξ_i, the leaf reached by following the branch labeled with some $\xi \in \Xi_i$ stores the number of matching $\lambda \in \Lambda_i$, so that step 2 is in amortized constant time (each consistent pair is considered at most twice); the MDD structures are optimized in several other ways.

The correctness proof consists in showing that the optimized MDD structure computes the right sets, mainly by stepping through the functions, and in showing the existence of the chromogram γ in step 2 above. Following a suggestion by B. Werner, we do this with a single induction on the remainder map, without developing any of the informal justification of the procedure: in this case, the formal proof turns out to be simpler than the informal proof that inspired it!

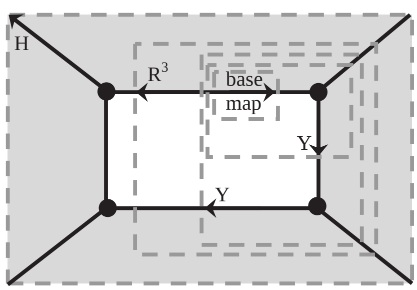
The 633 configuration maps are the only link between the reducibility check and the main proof. A little analysis reveals that each of these maps can be built inside-out from a simple construction program. We cast all the operations we need, such as computing the set of colorings, the contract map, or even compiling an occurrence check filter, as nonstandard interpretations of this program (the standard one being the map construction). This approach affords both efficient implementation and straightforward correctness proofs based on simulation relations between the standard and nonstandard interpretations.

The standard interpretation yields a pointed remainder map, i.e., a plain map with a distinguished dart x which is cubic except for the cycle

$n^*(x)$. The construction starts from a single edge and applies a sequence of steps to progressively build the map. It turns out we only need three types of steps.

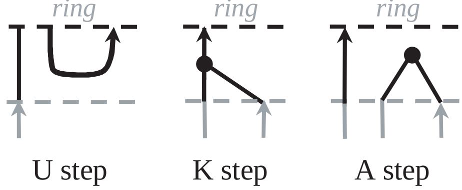


The text of a construction program always starts with an H and ends with a Y, but is executed *right to left*. Thus the program $\mathrm{H}\mathrm{R}^3\mathrm{Y}\mathrm{Y}$ constructs the square configuration:



The Ys produce the first two nodes; R^3 swings the reference point around so the H can close off the kernel.

We compile the configuration contract by locally rewriting the program text (Figure 4) into a program that produces a map with the ring same colorings as the contract map (Figure 5). The compilation uses three new kinds of steps:



These contract steps are more primitive than the H and Y steps; indeed, we can define the H, Y, and K steps in terms of U and a rotation variant N of K: we have $\mathrm{K} = \mathrm{R}^{-1} \circ \mathrm{N} \circ \mathrm{R}$, hence $\mathrm{Y} = \mathrm{N} \circ \mathrm{U}$ and $\mathrm{H} = \mathrm{N} \circ \mathrm{Y}$. Thus we only need to give precise hypermap constructions for the base map and the U, N, and A steps (Figure 7).

Proving Unavoidability

We complete the proof of the Four-Color Theorem proof by showing that in an internally 6-connected planar cubic bridgeless map, every 2-neighborhood either contains the kernel of one of the 633 reducible configurations from [13], or has averaged (discharged) arity at least 6.

Following [13], we do this by combinatorial search, making successive complementary assumptions about the arity of the various faces of the neighborhood, until the accumulated assumptions imply the desired conclusion. The search is partly guided by data extracted from [13] (from both the text and the machine-checked parts), partly automated via reflection (similarly to the reducibility check). We accumulate the assumptions in a data structure called a *part*, which can be interpreted as the set of 2-neighborhoods that satisfy all assumptions.

While we embrace the search structure of [13], we improve substantially its implementation. Indeed, this is the part of the proof where the extra precision of the formal proof pays off most handsomely. Specifically, we are able to show that if an internally 6-connected map contains a *homomorphic* copy of a configuration kernel, then the full configuration (comprising kernel and ring) is actually *embedded* in the map. The latter condition is required to apply the reducibility of the configuration but is substantially harder to check for. The code supplied with [13] constructs a candidate kernel isomorphism then rechecks its output, along with an additional technical "well-positioned" condition; its correctness relies on the structure theorem for 2-neighborhoods from [5], and on a "folklore" theorem ([13] theorem (3.3), p. 12) for extending the kernel isomorphism to an embedding of the entire configuration. In constrast, our reflected code merely checks that arities in the part and the configuration kernel are consistent along a face-spanning tree (called a *quiz*) of the edge graph of the configuration map. The quiz tree is traversed simultaneously in both maps, checking the consistency of the arity at each step, then using $n \circ f^{-1}$ and $f \circ n^{-1}$ to move to the left and right child. This suffices because

- The traversal yields a mapping ϕ from the kernel K to the map M matching the part, which respects f, and respects e on the spanning tree.
- Because M and K are both cubic, ϕ respects e on all of K.
- Because K has radius 2 and ring faces of the configuration C have arity at most 6, ϕ maps e arrows faithfully on K (hence is bijective).
- Because C and M are both cubic, and C is chordless (only contiguous ring faces are adjacent,

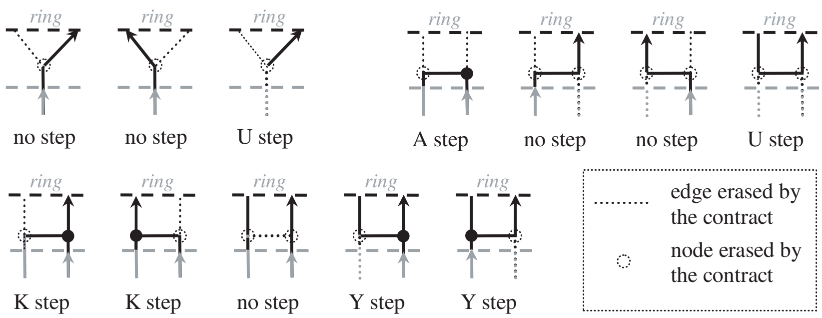


Figure 4. Contract interpretation.

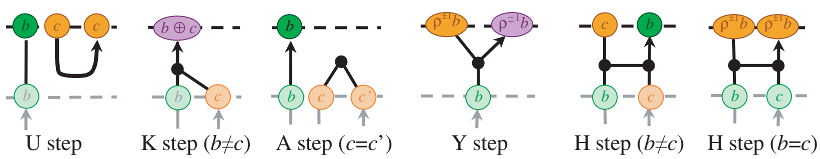


Figure 5. Coloring interpretation.

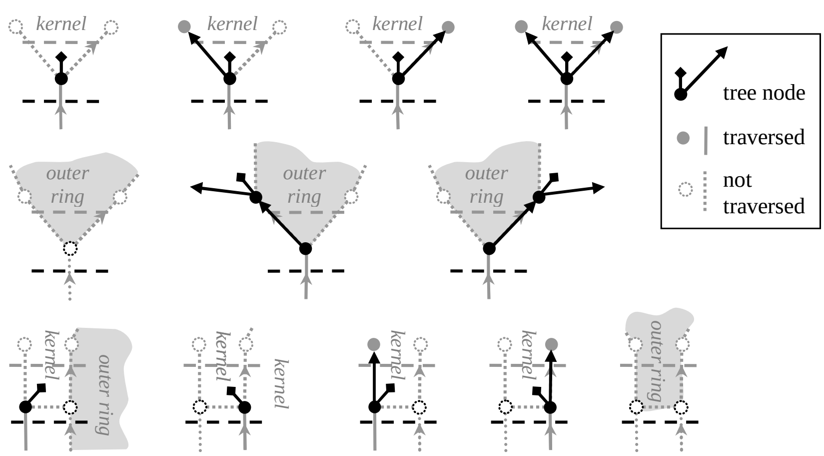


Figure 6. Quiz tree interpretation.

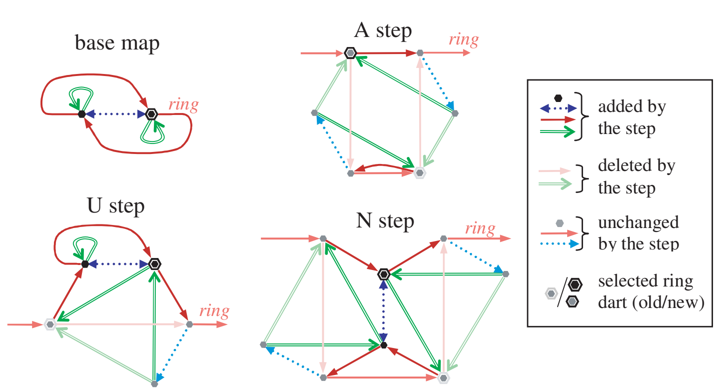


Figure 7. Standard (hypermap) interpretation.

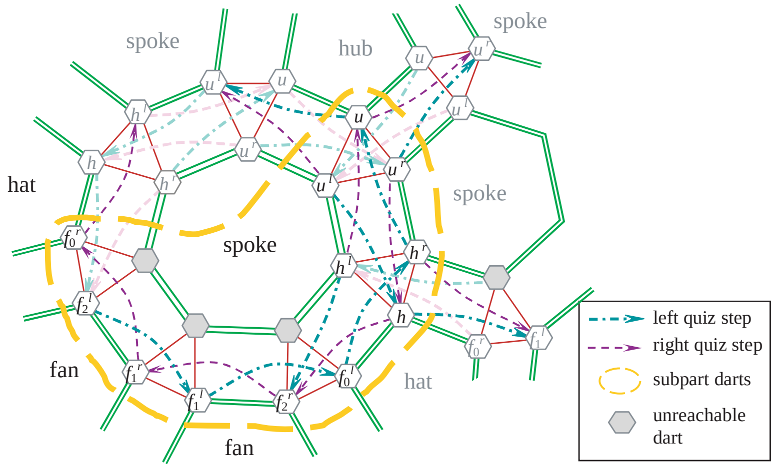


Figure 8. The hypermap of a subpart.

because each has arity at least 3), ϕ extends to an embedding of C in M.

The last three assertions are proved by induction over the size of the disk map of a contour containing a counter-example edge. For the third assertion, the contour is in M and spans at most 5 faces because it is the image of a simple non-cyclic path in K. Its interior must be empty for otherwise it would have to be a single pentagon (because M is internally 6-connected) that would be the image of a ring face adjacent to 5 kernel and 2 ring faces, contrary to the arity constraint.

We check the radius and arity conditions explicitly for each configuration, along with the sparsity and triad conditions on its contract [13]; the ring arity conditions are new to our proof. The checks and the quiz tree selection are formulated as non-standard interpretations; the quiz interpretation (Figure 6) runs programs left-to-right, *backwards*.

Because our proof depends only on the Birkhoff lemma (which incidentally we prove by reflection using a variant of the reducibility check), we do not need "cartwheels" as in [13] to represent 2-neighborhoods; the unavoidability check uses only "parts" and "quizzes". We also use parts to represent the arity discharging rules, using functions that intersect, rotate, and reflect (mirror) parts, e.g., a discharging rule applies to a part P if and only if its part subsumes P.

The search always starts by fixing the arity of the central "hub" face, so a part is really a counterclockwise list of subparts that each store a range of arities for each face of a sector of the neighborhood, comprising a single "spoke" face adjacent to the hub, a single "hat" face adjacent to the spoke and the preceding subpart, and 0 to 3 "fan" faces adjacent only to the spoke. There are only 15 possible ranges for arities, as there is no need to distinguish between arities greater than 9 (unconstrained faces range over $[5, +\infty)$). It is easy to simulate a quiz traversal on a part, because only 14 of the possible 27 darts of a subpart are reachable (Figure 8).

Finally, we translate the combinatorial search trees from [13] into an explicit proof script that refines a generic part until the lower bound on the discharged arity can be broken down into a sum of lower bounds (a *hubcap*) on the fractions of arity discharged from one or two spokes, which can be handled by a reflected decision procedure, or the part is a symmetry variant of a previous case. This *shallow embedding* allowed us to create our own scripts for pentagons and hexagons that were not covered by the computer proof in [13].

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Formal Proof—Theory and Practice

John Harrison

A *formal proof* is a proof written in a precise artificial language that admits only a fixed repertoire of stylized steps. This formal language is usually designed so that there is a purely mechanical process by which the correctness of a proof in the language can be verified. Nowadays, there are numerous computer programs known as *proof assistants* that can check, or even partially construct, formal proofs written in their preferred proof language. These can be considered as practical, computer-based realizations of the traditional systems of formal symbolic logic and set theory proposed as foundations for mathematics.

Why should we wish to create formal proofs? Of course, one may consider it just a harmless and satisfying intellectual activity like solving crosswords or doing Sudoku puzzles and not seek a deeper justification. But we can identify two more substantial reasons:

- To establish or refute a thesis about the nature of mathematics or related questions in philosophy.
- To improve the actual precision, explicitness, and reliability of mathematics.

Philosophical goals played an important role in the development of logic and indeed of computer science too [7]. But we're more interested in the actual use of formalization in mathematics, which we think is not such a radical departure from existing practice as it might appear. In some of its most fertile periods, mathematics has been developed in speculative and imaginative ways lacking obvious logical justification. Yet many great mathematicians like Newton and Euler *were* clearly self-conscious about a lack of rigor in their work [24]. Following the waves of innovation, there have always followed corresponding periods of retrenchment, analyzing foundations and increasingly adopting a strict axiomatic-deductive style, either to resolve apparent problems or just to make the material easier to teach convincingly [11]; the "ϵ-δ" explanation of limits in calculus is a classic example. Complete formalization is a natural further step in this process of evolution towards greater clarity and precision. To be more concrete, our own hopes for formalization are focused on two specific goals:

- Supplementing, or even partly replacing, the process of peer review for mainstream mathematical papers with an objective and mechanizable criterion for the correctness of proofs.
- Extending rigorous proof from pure mathematics to the verification of computer systems (programs, hardware systems, protocols, etc.), a process that presently relies largely on testing.

It is of course debatable whether, in either case, there is a serious problem with the existing status quo and whether formal proofs can really offer a solution if so. But we will argue in this paper that the answer is a resounding *yes* in both cases. Recent decades have seen substantial advances, with proof assistants becoming easier to use and more powerful and getting applied to ever more challenging problems.

A significant early milestone in formalization of mathematics was Jutting's 1970s formalization of Landau's very detailed proof of the complete

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ordered field axioms for the real numbers constructed by Dedekind cuts. Today we can point to formalizations starting from similarly basic foundations that reach nontrivial results in topology, analysis, and number theory such as the Jordan Curve Theorem, Cauchy's integral theorem, and the Prime Number Theorem. Perhaps most spectacularly, Gonthier has completely formalized the proof of the Four-Color Theorem, as described elsewhere in this issue.

Similar progress can be discerned in formal proofs of computer systems. The first proof of compiler correctness by McCarthy and Painter from 1967 was for a compiler from a simple expression language into an invented machine code with four instructions. Recently, Leroy has produced a machine-checked correctness proof for a compiler from a significant fragment of C to a real current microprocessor. In some parts of the computer industry, especially in critical areas such as avionics, formal methods are becoming an increasingly important part of the landscape.

The present author has been responsible for developing the HOL Light theorem prover, with its many special algorithms and decision procedures, and applying it to the formalization of mathematics, pure and applied. In his present role, he has been responsible at Intel for the formal verification of a number of algorithms implementing basic floating-point operations [13]. Work of this kind indicates that formalization of pure mathematics and verification applications are not separate activities, one undertaken for fun and the other for profit, but are intimately connected. For example, in order to prove quite concrete results about floating-point operations, we need nontrivial results from mainstream real analysis and number theory, even before we consider all the special properties of floating-point rounding.

Formal Symbolic Logic

The use of symbolic expressions denoting mathematical objects (numbers, sets, matrices, etc.) is well established. We normally write "$(x+y)(x-y)$" rather than "the product of, on the one hand the sum of the first unknown and the second unknown, and on the other hand the difference of the first and the second unknown". In ancient times such longwinded natural-language renderings were the norm, but over time more and more of mathematics has come to be expressed in symbolic notation. Symbolism is usually shorter, is generally clearer in complicated cases, and avoids some of the clumsiness and ambiguity inherent in natural language. Perhaps most importantly, a well-chosen notation can contribute to making mathematical reasoning itself easier, or even purely *mechanical*. The positional base-n representation of numbers is a good example: problems like finding sums and differences can then be performed using quite simple fixed procedures that require no mathematical insight or understanding and are therefore even amenable to automation in mechanical calculating machines or their modern electronic counterparts.

Symbolic logic extends the use of symbolism, featuring not only expressions called *terms* denoting mathematical *objects*, but also *formulas*, which are corresponding expressions denoting mathematical *propositions*. Just as there are operators like addition or set intersection on mathematical objects, symbolic logic uses *logical connectives* like "and" that can be considered as operators on propositions. The most important have corresponding symbolic forms; for example as we write "$x + y$" to denote the mathematical object "x plus y", we can use "$p \wedge q$" to denote the proposition "p and q". The basic logical connectives were already used by Boole, and modern symbolic logic also features the *universal quantifier* "for all" and the *existential quantifier* "there exists", whose introduction is usually credited independently to Frege, Peano, and Peirce. The following table summarizes one common notation for the logical constants, connectives and quantifiers:

English	Symbolic
false	$\bot$
true	$\top$
not p	$\neg p$
p and q	$p \wedge q$
p or q	$p \vee q$
p implies q	$p \Rightarrow q$
p iff q	$p \Leftrightarrow q$
for all x, p	$\forall x.\ p$
there exists x such that p	$\exists x.\ p$

For example, an assertion of continuity of a function $f : \mathbb{R} \to \mathbb{R}$ at a point x, which we might state in words as

> For all $\epsilon > 0$, there exists a $\delta > 0$ such that for all x' with $|x - x'| < \delta$, we also have $|f(x) - f(x')| < \epsilon$

could be written as a logical formula

$$\forall \epsilon.\ \epsilon > 0 \Rightarrow \exists \delta.\ \delta > 0 \wedge \forall x'.\ |x - x'| < \delta \Rightarrow |f(x) - f(x')| < \epsilon$$

The use of logical symbolism is already beneficial for its brevity and clarity when expressing complicated assertions. For example, we can make systematic use of bracketing, e.g., to distinguish between "$p \wedge (q \vee r)$" and "$(p \wedge q) \vee r$", while indicating precedences in English is more awkward. But logical symbolism really comes into its own in concert with *formal* rules of manipulation, i.e., symbolic transformations on formulas that can be applied mechanically without returning to the underlying meanings. For example, one sees at a glance that $x = 2y$ and $x/2 = y$ are equivalent, and applies corresponding manipulations without thinking about *why*. Logical notation creates a new vista of such mechanical transformations, e.g., from $(\exists x. P(x)) \Rightarrow q$ to $\forall x. (P(x) \Rightarrow q)$. Symbolism and formal rules of manipulation:

> […] have invariably been introduced to make things easy. […] by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain. […] Civilization advances by extending the number of important operations which can be performed without thinking about them. [27]

In modern *formal logic*, the emphasis on formal, mechanical manipulation is taken to its natural extreme. We not only make use of logical symbolism, but precisely circumscribe the permissible terms and formulas and define a precise counterpart to the informal notion of *proof* based purely on formal rules. We will see more details later, but first let us see how this idea arose in relation to foundational concerns, and how it may be useful in contemporary mathematics.

The Foundations of Mathematics

Arguably, the defining characteristic of mathematics is that it is a *deductive* discipline. Reasoning proceeds from *axioms* (or *postulates*), which are either accepted as evidently true or merely adopted as hypotheses, and reaches conclusions via chains of incontrovertible logical deductions. This contrasts with the natural sciences, whose theories, while strongly mathematical in nature, tend to become accepted because of empirical evidence. (In fact, it is more characteristic of physics to start from observations and seek, by *induction* or *abduction*, the simple axioms that can explain them.) A special joy of mathematics is that one can proceed from simple and entirely plausible axioms to striking and unobvious theorems, as Hobbes memorably discovered [2]:

> Being in a Gentleman's Library, Euclid's Elements lay open, and 'twas the 47 *El. libri* 1 [Pythagoras's Theorem]. He read the proposition. *By G—*, sayd he (he would now and then sweare an emphaticall Oath by way of emphasis) *this is impossible!* So he reads the Demonstration of it, which referred him back to such a Proposition; which proposition he read. That referred him back to another, which he also read. *Et sic deinceps* [and so on] that at last he was demonstratively convinced of that trueth. This made him in love with Geometry.

This idealized style of mathematical development was already established in Euclid's *Elements of Geometry*. However, its later critical examination raised numerous philosophical difficulties. If mathematics is a purely deductive discipline, what is its relationship with empirical reality? Are the axioms actually true of the real world? Can some axioms be deduced purely logically from others, or are they all independent? Would it make sense to use different axioms that contradict the usual ones? What *are* the incontrovertible logical steps admissible in a mathematical proof, and how are they to be distinguished from the substantial mathematical assumptions that we call axioms?

Foundational questions of this sort have preoccupied philosophers for millennia. Now and again, related worries have reached a broader community, often as a reaction to disquiet at certain mathematical developments, such as irrational numbers, infinitesimal calculus, and non-Euclidean geometry. Relatively recently, foundational concerns were heightened as the theory of infinite sets began to be generalized and pursued for its own sake by Cantor, Dedekind, and others.

It was precisely to clarify basic foundational questions that Frege in 1879 introduced his *Begriffsschrift* ("concept-script" or "ideography"), perhaps the first comprehensive formal system for logic and mathematics. Frege claimed that his formal rules codified acceptable logical inference steps. On that basis, he justified his "logicist" thesis that the basic axioms for numbers and geometry themselves are, properly understood, not extralogical assumptions at all, but are derivable from purely logical principles.

However, it was later observed that right at the heart of Frege's system was a logical inconsistency now known as *Russell's paradox*. Frege's system allowed the construction of (in modern parlance) the "set of all sets that are not members of themselves", $R = \{S \mid S \notin S\}$. This immediately leads to a contradiction because $R \in R$ if and only if $R \notin R$, by definition.

Later systems for the foundations of mathematics restricted the principles of set formation so that they were still able to talk about the sets needed in mathematics without, apparently, allowing such self-contradictory collections. Two somewhat different methods were adopted, and these streams of work have led to the development of modern "type theory" and "set theory" respectively.

- Russell's system, used in the monumental *Principia Mathematica*, shared many characteristics with Frege's formal system, but introduced an explicit notion of *type*, separating mathematical objects of different kinds (natural numbers, sets of natural numbers, etc.) The original system was subsequently refined and simplified, leading to the modern system of *simple type theory* or *higher-order logic* (HOL).

- Zermelo did not adopt a formal logic, but did specify explicit axioms for set construction. For example, Zermelo's axioms imply that whenever there is a set S, there is also a set of all its subsets $\wp(S)$, and that whenever sets S and T exist, so does the union $S \cup T$. With some later additions, this has become the modern foundational system of Zermelo-Fraenkel set theory (ZF, or ZFC when including the Axiom of Choice). It can be recast as a formal system by incorporating suitable rules for formal logic.

Type-based approaches look immediately appealing, because mathematicians generally *do* make type distinctions: between points and lines, or between numbers and sets of numbers, etc. A type discipline is also consonant with the majority of modern computer programming languages, which use types to distinguish different sorts of value, mainly for conceptual clarity and the avoidance of errors, but also because it sometimes reflects implementation differences (e.g., between machine integers and floating-point numbers). On the other hand, some consider the discipline imposed by types too inflexible, just as some programmers do in computer languages. For example, an algebraist might just want to expand a field F to an algebraic extension $F(a)$ without worrying about whether its construction as a subset or quotient of $F[x]$ would have a different type from F.

In fact, the distinction between type theory and set theory is not completely clear-cut. The universe of sets in ZF set theory can be thought of as being built in levels (the Zermelo or von Neumann hierarchy), giving a sort of type distinction, though the levels are cumulative (each level includes the previous one) and continued transfinitely. And the last few decades have seen the development of formal type theories with a wider repertoire of set construction principles. The development of many recent type theories has been inspired by the *Curry-Howard correspondence*, which suggests deep connections between propositions and types and between programs and (constructive) proofs.

Formalization or Social Process?

Much of the work that we have just described was motivated by genuine conceptual worries about the foundations of mathematics: how do we know which sets or other mathematical objects exist, or which axioms are logically self-consistent? For Frege and Russell, formalization was a means to an end, a way of precisely isolating the permissible proofs and making sure that all use of axioms was explicit. *Hilbert's program* caused renewed interest in formal logic, and Brouwer even derided Hilbert's approach to mathematics as *formalism*. But Hilbert too was not really interested in actually formalizing proofs, merely in using the theoretical possibility of doing so to establish results *about* mathematics ("metamathematics").

However, some logical pioneers envisaged a much more thoroughgoing *use* of formal proofs in everyday mathematical practice. Peano, independently of Frege, introduced many of the concepts of modern formal logic, and it is a modified form of Peano's notation that still survives today. Peano was largely motivated by the need to *teach* mathematics to students in a clear and precise way. Together with his colleagues and assistants, Peano published a substantial amount of formalized mathematics: his journal *Rivista di Matematica* was published from 1891 until 1906, and polished versions were collected in various editions of the *Formulaire de Mathématique*.

What of the situation today? The use of set-theoretic language is widespread, and books sometimes describe possible foundations for mathematical structures (e.g., the real numbers as Dedekind cuts or equivalences classes of Cauchy sequences). Quite often, lip service is paid to formal logical foundations:

> …the correctness of a mathematical text is verified by comparing it, more or less explicitly, with the rules of a formalized language. [4]

> A mathematical proof is rigorous when it is (or could be) written out in the first-order predicate language $L(\in)$ as a sequence of inferences from the axioms ZFC, each inference made according to one of the stated rules. [19]

Yet mathematicians seldom make set-theoretic axioms explicit in their work, except for those whose results depend on more "exotic" hypotheses. And there is little use of formal proof, or even formal logical notation, in everyday mathematics; Dijkstra has remarked that "as far as the mathematical community is concerned George Boole has lived in vain". Inasmuch as the logical symbols *are* used (and one does glimpse "$\Rightarrow$" and "$\forall$" here and there), they usually play the role of ad hoc abbreviations without an associated battery of manipulative techniques. In fact, the everyday use of logical symbols we see today closely resembles an intermediate "syncopation" stage in the development of existing mathematical notation, where the symbols were essentially used for their abbreviatory role alone [26].

Moreover, the correctness of mainstream mathematical proofs is almost never established by formal means, but rather by informal discussion between mathematicians and peer review of papers. The fallibility of such a "social process" is well-known, with published results sometimes containing unsubtle errors:

> Professor Offord and I recently committed ourselves to an odd mistake (*Annals of Mathematics* (2) 49, 923, 1.5). In formulating a proof a plus sign got omitted, becoming in effect a multiplication sign. The resulting false formula got accepted as a basis for the ensuing fallacious argument. (In defense, the final result was known to be true.) [18]

The inadequacy of traditional peer review is starkly illustrated by the case of the Four-Color Theorem. The first purported proof by Kempe in 1879 was accepted for a decade before it was found to be flawed. It was not until the 1970s that a proof was widely accepted [1], and even that relied on extensive computer checking which could not feasibly be verified by hand. (Gonthier's paper in this issue describes the complete formalization of this theorem and its proof.)

A book [17] written seventy years ago gave 130 pages of errors made by major mathematicians up to 1900. To bring this up to date, we would surely need a much larger volume or even a specialist journal. Mathematics is becoming increasingly specialized, and some papers are read by few if any people other than their authors. Many results are produced by those who are not by training mathematicians, but computer scientists or engineers. Perhaps because most "easy" proofs have long ago been found, many of the most impressive results of recent years are accompanied by huge proofs: for example the proof of the graph minor theorem by Robertson and Seymour was presented in a series of twenty papers covering about 500 pages. Others, such as Wiles's proof of Fermat's Last Theorem, are not only quite large and complex in themselves but rely heavily on a daunting amount of mathematical "machinery". Still others, like the Appel-Haken proof of the Four-Color Theorem and Hales's proof of the Kepler Conjecture, rely extensively on computer checking of cases. It's not clear how to bring them within the traditional process of peer review [16], even supposing one finds the status quo otherwise satisfying.

When considering the correctness of a conventional informal proof, it's a partly subjective question what *is* to be considered an oversight rather than a permissible neglect of degenerate cases, or a gap rather than an exposition taking widely understood background for granted. Proofs depend on their power to persuade individual mathematicians, and there is no objective standard for what is considered acceptable, merely a vague community consensus. There is frequently debate over whether "proofs" from the past can be considered acceptable today. For example, the Fundamental Theorem of Algebra was "proved" by, among others, d'Alembert and, in more than one way, by Gauss. Yet opinion is divided on which, if any, of these proofs should be considered as the first acceptable by present standards. (The result is usually referred to as d'Alembert's theorem in France.) The history of Euler's theorem $V - E + F = 2$, where the letters denote the number of vertices, edges, and faces of a polyhedron, reveals a succession of concerns over whether apparent problems are errors in a "proof" or indicate unstated assumption about the class of polyhedra considered [15].

Since mathematics is supposed to be an exact science and, at least in its modern incarnation, one with a formal foundation, this situation seems thoroughly lamentable. It is hard to resist the conclusion that we should be taking the idea of formal foundations at face value and *actually* formalizing our proofs. Yet is also easy to see why mathematicians have been reluctant to do so. Formal proof is regarded as far too tedious and painstaking. Arguably formalized mathematics may be more error-prone than the usual informal kind, as formal manipulations become more complicated and the underlying intuition begins to get lost. Russell in his autobiography remarks that his intellect "never quite recovered from the strain" of writing *Principia Mathematica*, and as Bourbaki [4] notes:

> If formalized mathematics were as simple as the game of chess, then once our chosen formalized language had been described there would remain only the task of writing out our proofs in this language, […] But the matter is far from being as simple as that, and no great experience is necessary to perceive that such a project is absolutely unrealizable: the tiniest proof at the beginning of the Theory of Sets would already require several hundreds of signs for its complete formalization. […] formalized mathematics cannot in practice be written down in full, […] We shall therefore very quickly abandon formalized mathematics, […]

However, we believe that the arrival of the computer changes the situation dramatically. While perfect accuracy in formal manipulations is problematic even for trained mathematicians, checking conformance to formal rules is one of the things computers are very good at. There is also the prospect that, besides merely checking the correctness of formal arguments, the computer may be able to help in their construction: the Bourbaki claim that the transition to a completely formal text is routine seems almost an open invitation to give the task to computers. Ideally, perhaps the computer may be able to find nontrivial proofs entirely automatically. We will examine in more detail later to what extent this is true.

Like Nidditch, who complained that "in the whole literature of mathematics there is not a single valid proof in the logical sense," we welcome the prospect of formalizing mathematics. In our view, the traditional social process is an anachronism to be swept away by formalization, just as empiricism replaced a similar "social process" used by the Greeks to decide scientific questions. But we should emphasize that we aren't trying to turn mathematics into drab symbol manipulation. Traditional informal proofs bear the dual burden of compelling belief *and* conveying understanding. These are not always mutually supportive and can be antagonistic, since the former pulls in the direction of low-level details, the latter in the direction of high-level concepts. Yet a result whose proof has been formalized can be presented to others in a high-level conceptual way, taking for granted that because of full formalization there is no reasonable doubt about correctness of the details nor uncertainty about precisely what has been proved and from what assumptions. And in principle, a computer program can offer views of the same proof at different levels of detail to suit the differing needs of readers.

Formal Verification

The woolly community process by which mathematical proofs become accepted seems all the worse when one considers the fact that mathematics is applied in the real world. That bridges do not collapse and aircraft do not fall out of the sky is a direct consequence of mathematical design principles. If the underlying mathematics is in doubt, then how can we trust these engineering artifacts? One may doubt the relevance of foundational concerns to the practice of applied mathematics [8], but everyday errors in mathematical procedures, like getting the sign wrong in an algebraic calculation, can have serious engineering consequences. Even so, such errors probably happen less in practice than other problems such as mechanical defects, the inaccurate modeling of the physical world, or the failure even to perform the appropriate mathematical analysis (e.g., checking for dangerous resonances in bridges or aircraft wings). For example, the failure that Frederick II of Prussia, in a 1778 letter to Voltaire, lays at the door of Euler (and of mathematics generally) was arguably caused instead by his contractors' failure to follow Euler's advice [9]:

> I wanted to have a water jet in my garden: Euler calculated the force of the wheels necessary to raise the water to a reservoir, from where it should fall back through channels, finally spurting out in Sanssouci. My mill was carried out mathematically and could not raise a mouthful of water closer than fifty paces to the reservoir. Vanity of vanities! Vanity of mathematics!

Nowadays, there is serious concern about the correctness of computer systems, given their ubiquity in everyday life, sometimes in safety-critical systems like fly-by-wire aircraft, antilock braking systems, nuclear reactor controllers, and radiation therapy machines. Yet most large computer programs or hardware systems contain "bugs", i.e., design errors that in certain situations can cause the system to behave in unintended ways. The consequences of bugs can be quite dramatic: the recall of some early Intel® Pentium® processors owing to a bug in the floating-point division instruction [25], and the explosion of the Ariane 5 rocket on its maiden voyage as the result of a software bug, were each estimated to have cost around US$500 million. At a more mundane level, many of us who use computers in daily life are depressingly familiar with strange glitches and crashes, even though they usually cause little more than minor annoyance.

The fundamental difficulty of writing correct programs, and delivering them on time, began to be recognized almost as soon as computers became popular. By the 1970s, the general situation was often referred to as the "Software Crisis". Brooks [5], drawing on the experience of managing the design of IBM's new operating system OS/360, recounted how adding more people to foundering projects often just made things worse, drawing a striking analogy with the struggles of prehistoric creatures trapped in a tar pit:

> In the mind's eye one sees dinosaurs, mammoths, and sabertoothed tigers struggling against the grip of the tar. The fiercer the struggle, the more entangling the tar, and no beast is so strong or so skillful but that he ultimately sinks. Large-system programming has over the past decade been such a tar pit.

Why should this be? Most engineering artifacts are unfailingly reliable: collapsing buildings or exploding cars are exceptionally rare and newsworthy events. Yet in the realm of computers, unreliability sometimes seems to be the norm.

Arguably, the fundamental reason is that computers are discrete and digital. In all but the most chaotic physical systems, there are certain continuity properties meaning that small changes in the system or environment are likely to have minimal consequences; indeed, physical parameters are only ever known approximately. In a discrete system like a computer, by contrast, the state is in general much more precise and well-defined. One might think that this would make computers easier to design and more reliable, because up to a point, one escapes the approximation and estimation that is necessary in most of the physical sciences and engineering. In principle, this should be so. Yet the flipside is that discrete systems are are also much more vulnerable to design errors since the smallest possible change, a single bit, may cause a completely different behavior, such as going the other way in an "if …then …else …".

Much of the appeal of using computers is that a single algorithm is supposed to work across a range of situations. For example, a traditional program that accepts some inputs, performs a computation and produces an output, is supposed to work for *any* set of inputs, or at least for a broad and clearly defined class. The number of possible inputs is often infinite ("any finite string of alphanumeric characters"), or at least immensely large ("any two 64-bit integers"). Testing on some relatively small set of inputs can often be an effective way of finding errors, particularly if the inputs are well-chosen, e.g., to exercise all paths through the program created by conditional statements. Yet we can seldom conclude with certainty, even after extensive and elaborate testing, that there are no remaining errors. In practice, however, while programs are written with clear intellectual ideas behind them, their correctness *is* usually checked by just this kind of testing.

An assertion that a program with k inputs is correct can be considered as a universally quantified proposition $\forall n_1, \ldots, n_k.\, P(n_1, \ldots, n_k)$: for all k-tuples of inputs $n_1, \ldots, n_k$ the program performs its intended function. For any *particular* tuple of inputs $n_1, \ldots, n_k$, e.g., $(0, 1, 42)$, we can usually test whether $P(n_1, \ldots, n_k)$ holds, i.e., whether the program works correctly on those inputs. Many nontrivial and/or open questions in pure mathematics can be expressed by formulas having the same characteristics. (These are roughly what logicians call Π_1^0 formulas.) For example, Goldbach's conjecture that every even integer > 2 is the sum of two primes can be expressed using quantification over natural numbers in the form $\forall n. \mathrm{even}(n) \wedge n > 2 \Rightarrow \exists p\ q. \mathrm{prime}(p) \wedge \mathrm{prime}(q) \wedge p + q = n$ (we could if we wish express the subsidiary concepts "even" and "prime" using just quantification over natural numbers and basic arithmetic). Once again, for a *specific* n, we know we can decide the body of the quantified formula, because we can restrict our search to $p, q \leq n$. Less obviously, the Riemann hypothesis about zeros of the complex ζ-function can also be expressed by a formula quantifying only over the natural numbers and with the same characteristics.

In typical programming practice, as we have noted, correctness claims of the form $\forall n. P(n)$ are usually justified by testing on particular values of n. In mathematics, by contrast, numerical evidence of that sort may suggest conjectures, and even be subjectively compelling, yet a result is not considered firmly established until it is rigorously *proved*. There are plenty of cautionary tales to justify this attitude. For example, Fermat conjectured that all integers $2^{2^n} + 1$ were prime because this was the case for all $n = 0, \ldots, 4$, yet it turned out later that even $2^{2^5} + 1$ was divisible by 641 and in fact *no* other primes of that form are currently known. Again, it is known by explicit calculations that $\pi(n) \leq \mathrm{li}(n)$ holds for $n \leq 10^{20}$, where $\pi(n)$ is the number of primes $\leq n$ and $\mathrm{li}(n) = \int_0^n du / \log u$, yet it is known that $\pi(n) - \mathrm{li}(n)$ changes sign infinitely often. The idea of *formal verification* is to adopt the same standard of evidence in programming as in mathematics: *prove* the correctness of a program in the manner of any other mathematical theorem, rather than relying on the evidence of particular test situations.

The idea of formal verification once aroused heated controversy [3]. One criticism is that we are ultimately interested in confirming that a physical computing system satisfies real-life requirements. What we produce instead is a mathematical proof connecting abstract mathematical models of each. We can represent this situation by the diagram in Figure 1. Formal verification aims to prove that the mathematical model satisfies the mathematical specification. But one must still be cognizant of the potential gaps at the top and the bottom. How do we know that the running of the actual system conforms to the idealized mathematical model?

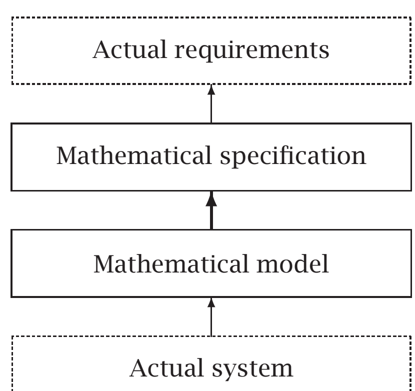

Figure 1. The role of models in formal verification.

And how do we know that our formal specification captures what we really intend?

A thorough discussion of these questions would take us too far afield, though we may note in passing that testing can suffer from analogous concerns. How do we know that the reference results we are testing against are correct? (The infamous FDIV bug was discovered at Intel by using the faulty device as a reference model against which to test the next generation.) And is our testing environment really an accurate model of the eventual deployed system? (This is particularly an issue in integrated circuit design, where the system is usually analyzed in simulation before being committed to silicon.)

We are more interested in another line of criticism, based on the claim that proofs of computer system correctness are often likely to be so long and tedious that humans cannot reasonably check them and discuss them. Even accepting that this claim is true, which some would not, we do not consider this an argument against formal verification. Rather, we think it further emphasizes the inadequacy of the traditional social process of proof and the need for a formal, computer-based replacement. Indeed McCarthy [22], one of the earliest proponents of program verification, emphasized the role of machine checking and generation of proofs. The subsequent evolution of automated reasoning has been closely intertwined with verification applications [20].

Automated Reasoning in Theory

The idea of reducing reasoning to mechanical calculation is an old dream [21]. Hobbes made explicit the analogy between reasoning and computation in his slogan "Reason […] is nothing but Reckoning". This connection was developed more explicitly by Leibniz, who emphasized that a system for reasoning by calculation must contain two essential components:

- A universal language (*characteristica universalis*) in which anything can be expressed
- A calculus of reasoning (*calculus ratiocinator*) for deciding the truth of assertions expressed in the *characteristica*.

Leibniz dreamed of a time when disputants unable to agree would not waste much time in futile argument, but would instead translate their disagreement into the *characteristica* and say to each other "*calculemus*" (let us calculate).

Leibniz was surely right to draw attention to the essential first step of developing an appropriate language. But he was far too ambitious in wanting to express all aspects of human thought. Eventual progress came rather by the gradual extension of the symbolic notations already used in mathematics, culminating in the systems of formal symbolic logic that we have already mentioned. In particular, a specific formal language called *first-order (predicate) logic* (FOL), is widely regarded as a good *characteristica*. Let us briefly sketch how we might define this precisely.

The permissible terms and formulas of FOL can be defined by *grammars* from formal language theory, similar to the BNF (Backus-Naur form) grammars often used to specify the syntax of computer programming languages. For example, given some previously-defined syntactic categories of *variables* and *functions* (more properly, variable names and function symbols), we can define the syntax of first-order terms as follows:

$$\begin{array}{rcl} \text{term} & \longrightarrow & \text{variable} \\ & | & \text{function}(\text{term}, \ldots, \text{term}) \end{array}$$

meaning that a term can be constructed from variables by applying functions to other terms as arguments (we consider constants as functions with zero arguments). The class of formulas is then built up using propositional connectives and quantifiers from *atomic formulas* that apply n-ary *relation symbol* to n terms:

$$\begin{array}{rcl} \text{formula} & \longrightarrow & \bot \\ & | & \top \\ & | & \text{relation}(\text{term}, \ldots, \text{term}) \\ & | & \neg\text{formula} \\ & | & \text{formula} \wedge \text{formula} \\ & | & \text{formula} \vee \text{formula} \\ & | & \text{formula} \Rightarrow \text{formula} \\ & | & \text{formula} \Leftrightarrow \text{formula} \\ & | & \forall\text{variable. formula} \\ & | & \exists\text{variable. formula} \end{array}$$

We will consider terms and formulas not as sequential strings, but as tree structures. In this tree-like *abstract syntax* we don't need bracketing to indicate precedences since the construction as a tree contains all this information. When actually writing down formulas, we may prefer a linear *concrete syntax* more like conventional notation. In this case we may again need bracketing to establish precedences, and we may prefer to use conventional infix notation for function and relation symbols, e.g., $x + y < 2$ instead of $<(+(x, y), 2())$. But we always keep in mind that the abstract syntax is what we are really talking about. In practical implementations, transforming from concrete to abstract syntax (*parsing*) and from abstract to concrete (*prettyprinting*) are well-understood tasks because of their role in compilers and other important applications.

Intuitive syntactic concepts like "the variables in a term" can be replaced by precise mathematical definitions, often by recursion on the grammar rules:

$$\begin{aligned} \mathrm{VARS}(v) &= \{v\} \\ \mathrm{VARS}(f(t_1,\ldots,t_n)) &= \bigcup_{i=1}^{n} \mathrm{VARS}(t_i) \end{aligned}$$

We say that a first-order formula ϕ is *logically valid*, and write $\models \phi$, if it *holds in any interpretation*, and *satisfiable* if it *holds in some interpretation*. An interpretation M consists of a nonempty domain D, for each n-ary function symbol f a function $f_M : D^n \to D$ and for each n-ary relation symbol a function $f_M : D^n \to \{\text{false}, \text{true}\}$. For reasons of space, we will not define precisely what it means to hold in an interpretation. But it is important to keep in mind how strong a requirement it is to hold in *all interpretations*. For example, $1 + 1 = 2$ does not hold in all interpretations, because it is perfectly permissible that an interpretation, even if it should happen to have $D = \mathbb{N}$ (which it need not), may choose to interpret the constant symbols "1" and "2" both as the number 7 and the "+" function symbol as multiplication. From abstract algebra we are already familiar with the way in which properties like $x \cdot y = y \cdot x$ may not hold in *all* groups, even if they hold in some familiar examples. For a first-order formula to be valid, it must essentially hold for *any* sensible way of interpreting the functions and relations, with only the logical constants, connectives, and quantifiers (and usually the equality relation) having a fixed meaning.

For first-order logic it is possible to define a notion of provability that is entirely formal in nature, and is *sound* (a provable formula is valid) and *complete* (a valid formula is provable). We write $\vdash \phi$ to indicate that ϕ is provable, so soundness and completeness means that for any formula ϕ we have $\vdash \phi$ if and only if $\models \phi$. There are various ways of defining a suitable notion of provability, but the usual choices are based on a set of formal *inference rules* that allow proof steps of a specific form. (To get started, we need at least one inference rule with no hypotheses, also known as an *axiom*.) For example, a typical inference rule is *modus ponens*, stating that if both p and $p \Rightarrow q$ have been proved, we can add a new step deducing q. The set of provable formulas is then generated inductively by these formal rules, and accordingly we write proof rules in the standard way for inductive definitions:

$$\frac{\vdash p \Rightarrow q \qquad \vdash p}{\vdash q}$$

It is now straightforward to define a corresponding notion of proof, such as a tree reflecting the patterns of inference, or simply a sequence of formulas with an indication of how it was derived from those earlier in the list.

For the following discussion, let us ignore the question of what a formal proof actually consists of, regarding both formulas and proofs as natural numbers and writing $\mathrm{Proves}(m, n)$ for "proof m is a valid proof of proposition n." (This is quite common when describing results of this nature, leaning on the trick of Gödel numbering, expressing a symbolic entity just as a large number. For example, one might express the symbolic expression as an ASCII string and regard the characters as base-256 digits.) Then soundness and completeness of the formal rules means that a proposition n is logically valid if and only if $\exists m.\, \mathrm{Proves}(m, n)$, i.e., if there exists a formal proof of n. From a suitably abstract point of view, the purely *formal* nature of the rules is manifested in the fact that there is a mechanical procedure, or computer program, that given any particular m and n as inputs will decide whether indeed $\mathrm{Proves}(m, n)$.

To return to a philosophical question that we raised early on, some might say that it is merely a question of terminology what we choose to call purely logical reasoning and what we consider as involving mathematical hypotheses with content going beyond pure logic. However, an important characteristic of a *proof* as traditionally understood is that even indifferent mathematicians should be able, with sufficient effort, to *check* that a long and difficult but clearly written proof really *is* a proof, even if they barely understand the subject matter and could not conceive of devising the proof themselves, just as Hobbes did for Pythagoras's theorem. The fact that for formal first-order logic, there is a proof-checking process that can be performed by machine is for many a solid reason for identifying "logical" reasoning with reasoning that is first-order valid.

As part of his foundational program, Hilbert raised further questions about logical reasoning, including the *Entscheidungsproblem* (decision problem) for first-order logic. If the binary "proof checking" relation $\mathrm{Proves}(m, n)$ is mechanically computable, what about the unary relation of provability, $\mathrm{Provable}(n) =_{def} \exists m.\, \mathrm{Proves}(m, n)$? Church and Turing showed that it is in fact uncomputable—a doubly significant step since they first needed to specify what it means to be computable. We can summarize this by saying that although formal proof checking is mechanizable, formal proof *finding* is not, even if we have an idealized digital computer without time or space limitations.

Having said that, proof finding is *semicomputable* (recursively enumerable) because we can systematically try $m = 0, 1, 2, \ldots$ in turn, testing in each case whether $\mathrm{Proves}(m, n)$. If indeed n is logically valid, we will eventually find an m that works and terminate our search with success.

However, if n turns out to be invalid, this process will continue indefinitely, and the Church-Turing result shows that the same must sometimes happen not just for this rather uninspired method, but for *any* other algorithm. Still, we might hope to come up with more intelligent programs that will find proofs of reasonably "simple" logically valid formulas relatively quickly.

We emphasized earlier the important distinction between holding in all interpretations and holding in some particular interpretation. The difficulty of the corresponding decision problems can also be very different. Consider first-order formulas built from constants 0 and 1, functions $+$, $-$ and $\cdot$ and relations $=$, $\leq$ and $<$, or as we say for brevity *arithmetic formulas*. By the results we have just described, whether a formula ϕ holds in all interpretations is *semicomputable* but not *computable* (the restriction to an arithmetic language does not affect either result). By contrast, it follows from famous undecidability results due to Gödel and Tarski that whether an arithmetic formula holds in $\mathbb{N}$ (i.e., the interpretation with domain $\mathbb{N}$ and the functions and relations interpreted in the obvious way) is *not even semicomputable*. This last result lends more support to the identification of *purely logical* with *first-order valid*, since it implies that validity for many natural extensions of first-order logic, e.g., to higher-order logic where quantification is permitted over functions and predicates, cease even to be semicomputable. This does not invalidate higher-order logic as a vehicle for formal mathematics. However it shows that for any sound formal proof system, we must reconcile ourselves to being able to prove only a proper subset of the higher-order valid formulas, or even of those first-order formulas that hold in $\mathbb{N}$.

Automated Reasoning in Practice

There are already well-established classes of computer programs that manipulate symbolic expressions, e.g., programming language compilers and computer algebra systems. The same techniques can be used to perform symbolic manipulations of the terms and formulas of formal logic. Using modern high-level languages, e.g., OCaml or Haskell, these manipulations can be expressed at a high level not far from their mathematical formulations on page 1402. For example, in OCaml we can define the first-order terms as a type of abstract syntax trees almost copying the abstract grammar:

```
type term = Var of string
          | Fn of string * term list;;
```

and express in a direct recursive way the function returning the set of variables in a term:

```
let rec vars tm =
  match tm with
    Var v -> [v]
  | Fn(f,ts) -> unions (map vars ts);;
```

The theoretical results in the last section suggest two contrasting approaches to the practical mechanization of proof. A *proof checker* expects the user to provide both the proposition n and the formal proof m, and simply checks that $\mathrm{Proves}(m, n)$. An *automated theorem prover*, by contrast, takes just the proposition n and attempts to find a suitable proof by itself. We use the broader term *proof assistant* or *interactive theorem prover* to cover the whole spectrum, including these extremes and various intermediate possibilities where the user provides hints or proof sketches to the program to direct the search.

We refer to programs that always terminate with a correct yes/no answer to a decision problem as *decision procedures*. From the Church-Turing result, we know that there is no decision procedure for first-order validity in general, but there are decision procedures for limited or modified forms of the same problem. For example, validity of purely *propositional* formulas (those without functions, variables or quantifiers) is computable, since the only predicates are nullary and therefore an interpretation simply assigns "true" or "false" to each of the relation symbols, and we can systematically try all combinations. This is the dual of the well-known propositional satisfiability problem SAT, and although it is NP-complete, there are tools that are surprisingly effective on many large problems. More generally, logical validity is computable for first-order formulas whose only quantifiers are universal and at the outside, e.g., $\forall x\ y.\ f(f(f(x)) = x \wedge f(f(f(f(f(x))))) = x \wedge f(f(x)) = y \Rightarrow x = y$ (which is valid); such methods have important applications in verification. Whether an arithmetic formula holds in $\mathbb{R}$ is also computable, albeit not very efficiently, and whether an arithmetic formula involving multiplication only by constants (a "Presburger formula") holds in $\mathbb{N}$ or $\mathbb{Z}$ is quite efficiently computable; even further restricted forms of this problem are useful in verification. There are also "combination" techniques for checking validity of formulas in languages including some symbols with a specific interpretation and others where all interpretations are permitted as in pure first-order validity. Modern SMT (satisfiability modulo theories) decision procedures are effective implementations of these methods.

In the early computer experiments in the late 1950s, most of the interest was in purely automated theorem proving. Perhaps the first theorem prover to be implemented on a computer was a decision procedure for Presburger arithmetic [6].

Subsequently, research was dominated by proof search algorithms for pure first-order logic. Decision procedures have proven useful in verification applications, while proof search has achieved some notable successes in mathematics, such as the solution by McCune [23], using the automated theorem prover EQP, of the longstanding "Robbins conjecture" concerning the axiomatization of Boolean algebra, which had resisted human mathematicians for some time. However, for some problems there seems to be no substitute for human involvement, and there was also considerable interest in proof checking at around the same time. The idea of a proof assistant that struck a balance between automation and human guidance appeared with the SAM (semi-automated mathematics) provers. Influential systems from the 1970s such as AUTOMATH, LCF, Mizar and NQTHM introduced most of the ideas that lie behind today's generation of proof assistants.

The purely automatic systems tend to adopt inference rules that are conducive to automated proof search, such as resolution, while the interactive systems adopt those considered more suitable for human beings such as natural deduction. However, in the SAM tradition, the leading interactive systems also tend to include an arsenal of decision procedures and proof search methods that can automate routine subproblems. Whatever the panoply of proof methods included, the system is *some* sort of computer program, and therefore its set of provable theorems is still at least semicomputable. However, since computer programs, especially large and complicated ones, are known to be prone to error, how can we be confident that such a system is sound, i.e., that the set of provable formulas is a subset of the set of valid ones?

Since we have been proposing theorem provers as an improvement on human fallibility and as a way of proving the correctness of other programs, this is a serious question: we seem to be in danger of an infinite regress. However, sound principles of design can provide a fairly satisfying answer. Some systems satisfy the *de Bruijn criterion*: they can output a proof that is checkable by a much simpler program. Others based on the LCF approach [10] generate all theorems internally using a small logical kernel: only this is allowed to create objects of the special type "theorem", just as only the kernel of an operating system is allowed to execute in privileged mode.

There is a fair degree of unanimity on the basic formal foundations adopted by the various proof assistants of today: most are based on either first-order logic plus set theory, or some version of simple type theory, or some constructive type theory. But the systems vary widely in other characteristics such as the level of automation and the style in which proof hints or sketches are provided [28]. One interesting dichotomy is between *procedural* and *declarative* proof styles [12]. Roughly, in a declarative proof one outlines *what* is to be proved, for example a series of intermediate assertions that act as waystations between the assumptions and conclusions. By contrast, a *procedural* proof explicitly states *how* to perform the proofs ("rewrite the second term with lemma 7 …"), and some procedural theorem provers such as those in the LCF tradition use a full programming language to choreograph the proof process.

Conclusions and Future Prospects

The use of formal proofs in mathematics is a natural continuation of existing trends towards greater rigor. Moreover, it may well be the *only* practical way of gaining confidence in proofs that are too long and complex to check in the traditional way (e.g., those in formal verification), or those that already involve ad hoc computer assistance. Hitherto, formalization has attracted little interest in the mathematical community at large because it seems too difficult. There is no escaping the fact that creating formal proofs *is* still difficult and painstaking. However, in barely fifty years, computer proof assistants have reached the stage where formalizing many nontrivial results is quite feasible, as the other papers in this issue illustrate. In our opinion, progress has come mainly through the following:

- The cumulative effects as libraries of formalized mathematics are developed and can be built upon by others without starting from scratch.
- The integration into proof assistants of more automated decision procedures, while maintaining high standards of logical rigor.
- More attention to the languages used to express proofs, and various interface questions that make the systems more convenient to use.

We believe that these trends will continue for some time, and perhaps other avenues for improvement will be more thoroughly explored. For example, computer algebra systems already feature many powerful algorithms for automating mainstream mathematics, which if incorporated in a logically principled way could be very valuable [14]. As proof assistant technology further improves, we can expect it to become increasingly accessible to mathematicians who would like to put the correctness of their proofs beyond reasonable doubt.

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Formal Proof—Getting Started

Freek Wiedijk

A List of 100 Theorems

Today highly nontrivial mathematics is routinely being encoded in the computer, ensuring a reliability that is orders of a magnitude larger than if one had just used human minds. Such an encoding is called a *formalization*, and a program that checks such a formalization for correctness is called a *proof assistant*.

Suppose you have proved a theorem and you want to make certain that there are no mistakes in the proof. Maybe already a couple of times a mistake has been found and you want to make sure that that will not happen again. Maybe you fear that your intuition is misleading you and want to make sure that this is not the case. Or maybe you just want to bring your proof into the most pure and complete form possible. We will explain in this article how to go about this.

Although formalization has become a routine activity, it still is labor intensive. Using current technology, a formalization will be roughly four times the size of a corresponding informal LaTeX proof (this ratio is called the *de Bruijn factor*), and it will take almost a full week to formalize a single page from an undergraduate mathematics textbook.

The first step towards a formalization of a proof consists of deciding which proof assistant to use. For this it is useful to know which proof assistants have been shown to be practical for formalization. On the webpage [1] there is a list that keeps track of the formalization status of a hundred well-known theorems. The first few entries on that list appear in Table 1.

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On the webpage [1] only eight entries are listed for the first theorem, but in [2] seventeen formalizations of the irrationality of $\sqrt{2}$ have been collected, each with a short description of the proof assistant.

When we analyze this list of theorems to see what systems occur most, it turns out that there are five proof assistants that have been significantly used for formalization of mathematics. These are:

proof assistant	*number of theorems formalized*
HOL Light	69
Mizar	45
ProofPower	42
Isabelle	40
Coq	39
all together	80

Currently in all systems together 80 theorems from this list have been formalized. We expect to get to 99 formalized theorems in the next few years, but Fermat's Last Theorem is the 33rd entry of the list and therefore it will be some time until we get to 100.

If we do not look for quantity but for quality, the most impressive formalizations up to now are:

Gödel's First Incompleteness Theorem: by Natarajan Shankar using the proof assistant nqthm in 1986, by Russell O'Connor using Coq in 2003, and by John Harrison using HOL Light in 2005.

Jordan Curve Theorem: by Tom Hales using HOL Light in 2005, and by Artur Korniłowicz using Mizar in 2005.

Prime Number Theorem: by Jeremy Avigad using Isabelle in 2004 (an elementary proof by Atle Selberg and Paul Erdös), and by

	theorem	*number of systems in which the theorem has been formalized*
1.	The Irrationality of $\sqrt{2}$	≥ 17
2.	Fundamental Theorem of Algebra	4
3.	The Denumerability of the Rational Numbers	6
4.	Pythagorean Theorem	6
5.	Prime Number Theorem	2
6.	Gödel's Incompleteness Theorem	3
7.	Law of Quadratic Reciprocity	4
8.	The Impossibility of Trisecting the Angle and Doubling the Cube	1
9.	The Area of a Circle	1
10.	Euler's Generalization of Fermat's Little Theorem	4
11.	The Infinitude of Primes	6
12.	The Independence of the Parallel Postulate	0
13.	Polyhedron Formula	1
...		...

Table 1. The start of the list of 100 theorems [1].

John Harrison using HOL Light in 2008 (a proof using the Riemann zeta function).

Four-Color Theorem: by Georges Gonthier using Coq in 2004.

All but one of the systems used for these four theorems are among the five systems that we listed. This again shows that currently these are the most interesting for formalization of mathematics.

Here are the *proof styles* that one finds in these systems:

proof assistant	*proof style of the system*
HOL Light	procedural
Mizar	declarative
ProofPower	procedural
Isabelle	both possible
Coq	procedural

A *declarative* system is one in which one writes a proof in the normal way, although in a highly stylized language and with very small steps. For this reason a declarative formalization resembles program source code more than ordinary mathematics. In a *procedural* system one does not write proofs at all. Instead one holds a dialogue with the computer. In that dialogue the computer presents the user with *proof obligations* or *goals*, and the user then executes *tactics*, which reduce a goal to zero or more new, and hopefully simpler, *subgoals*. Proof in a procedural system is an interactive game.

In this paper we will show HOL Light as the example of a procedural system, and Mizar as the example of a declarative system.

The main advantage of HOL Light is its elegant architecture, which makes it a very powerful and reliable system. A proof of the correctness of the 394 line HOL Light "logical core" even has been formalized. On the other hand HOL has the disadvantage that it sometimes cannot express abstract mathematics—mostly when it involves algebraic structures—in an attractive way. It *can* essentially express all abstract mathematics though. Another disadvantage of HOL is that the proof parts of the HOL scripts are unreadable. They can only be understood by executing them on the computer.

Mizar on the other hand allows one to write abstract mathematics very elegantly, and its scripts are almost readable like ordinary mathematics. Also Mizar has *by far* the largest library of already formalized mathematics (currently it is over 2 million lines). However, Mizar has the disadvantage that it is not possible for a user to automate recurring proof patterns, and the proof automation provided by the system itself is rather basic. Also, in Mizar it is difficult to express the formulas of calculus in a recognizable style. It is not possible to "bind" variables, which causes expressions for constructions like sums, limits, derivatives, and integrals to look unnatural.

The Example: Quadratic Reciprocity

In this article we will look at two formalizations of a specific theorem. For this we will take the *Law of Quadratic Reciprocity*, the seventh theorem from the list of a hundred theorems. This theorem has thus far been formalized in four systems: by David Russinoff using nqthm in 1990, by Jeremy Avigad using Isabelle in 2004, by John Harrison using HOL Light in 2006, and by Li Yan, Xiquan Liang, and Junjie Zhao using Mizar in 2007.

When I was a student, my algebra professor Hendrik Lenstra always used to say that the Law of Quadratic Reciprocity is the first nontrivial theorem that a student encounters in the mathematics curriculum. Before this theorem, most proofs can be found without too much trouble by expanding the definitions and thinking hard. In contrast the Law of Quadratic Reciprocity is the first theorem that is totally unexpected. It was already conjectured by Euler and Legendre, but was proved only by the "Prince of Mathematicians", Gauss, who called it the *Golden Theorem* and during his lifetime gave eight different proofs of it.

The Law of Quadratic Reciprocity relates whether an odd prime p is a square modulo an odd prime q, to whether q is a square modulo p. The theorem says that these are equivalent unless both p and q are 3 modulo 4, in which case they have opposite truth values. There also are two *supplements* to the Law of Quadratic Reciprocity, which say that -1 is a square modulo an odd prime p if and only if $p \equiv 1 \pmod 4$, and that 2 is a square modulo an odd prime p if and only if $p \equiv \pm 1 \pmod 8$.

The Law of Quadratic Reciprocity is usually phrased using the *Legendre symbol*. A number a is called a *quadratic residue* modulo p if there exists an x such that $x^2 \equiv a \pmod p$. The Legendre symbol $\left(\frac{a}{p}\right)$ for a coprime to p then is defined by

$$\left(\frac{a}{p}\right) = \begin{cases} 1 & \text{if } a \text{ is a quadratic residue modulo } p \\ -1 & \text{otherwise.} \end{cases}$$

Using the Legendre symbol, the Law of Quadratic Reciprocity can be written as:

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\frac{q-1}{2}}$$

The right hand side will be -1 if and only if both p and q are $3 \pmod 4$.

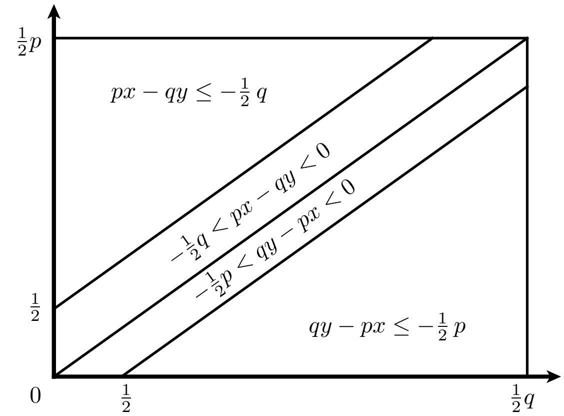
There are many proofs of the Law of Quadratic Reciprocity [3]. The formalizations shown here both formalize elementary counting proofs that go back to Gauss' third proof. We will not give details of these proofs here, but will just outline the main steps, following [4]. First by using that only half of the residues modulo p can be a square, one proves *Euler's criterion*:

$$\left(\frac{a}{p}\right) \equiv a^{\frac{1}{2}(p-1)} \pmod p$$

Using this criterion, by calculating the product of $a, 2a, \ldots, \frac{1}{2}(p-1)a$ in two different ways, one then proves *Gauss' lemma*, which says that

$$\left(\frac{a}{p}\right) = (-1)^l,$$

in which l is the number of $1 \le j \le \frac{1}{2}(p-1)$ for which there is an $-\frac{1}{2}p < a' < 0$ such that $aj \equiv a' \pmod p$. Finally one uses Gauss' lemma to derive the Law of Quadratic Reciprocity by counting lattice points in the four regions of the following figure:



HOL Light

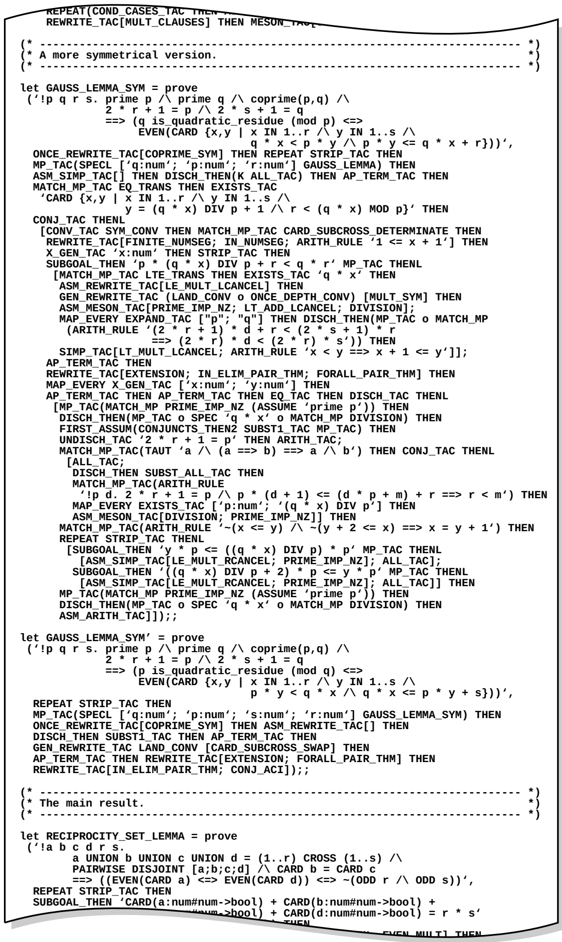
Suppose that we select HOL Light as our proof assistant. The second step will be to download and install the system. This does not take long. First download the ocaml compiler from [5] and install it. Next download the `tar.gz` file with the current version of the HOL Light sources from [6] and unpack it. Then follow the installation instructions in the `README` file. If you use Linux or Mac OS X, all you will need to do is type "`make`". Under Windows, installation is a bit more involved: you will have to copy the "`pa_j_....ml`" file that corresponds to your version of ocaml as given by "`ocamlc -version`" to a file called "`pa_j.ml`", and then compile that copy using one of the two "`ocamlc -c`" commands that are in the `Makefile`.

When you have installed the system, start the ocaml interpreter by typing "`ocaml`", and then enter the command "`#use "hol.ml";;`" This checks and loads the basic library of HOL Light, which takes a few minutes. After that you can load HOL files by typing for example "`loadt "100/reciprocity.ml";;`".

The third step will be to write a formalization of the proof. For this you will have to learn the HOL proof language. To do this it is best to study two documents: the HOL Light manual [7] and the HOL Light tutorial [8].

Instead of taking this third step and describing how one writes a formalization, here we will just look at a formalization that already exists. It can be found in the file "`100/reciprocity.ml`" (see Figure 1) and formalizes the proof from [4]. This file can also be found on the Web by itself as [9]. It consists of 753 lines of HOL Light code and proves 41 lemmas on top of the already existing HOL Light mathematical libraries. The statement of the final lemma is:

```
!p q. prime p /\ prime q /\
  ODD p /\ ODD q /\ ~(p = q)
  ==> legendre(p,q) * legendre(q,p) =
      --(&1) pow ((p - 1) DIV 2 * (q - 1) DIV 2)
```



```
    REWRITE_TAC[MULT_CLAUSES] THEN MESON_TAC[

(* ------------------------------------------------------------------------- *)
(* A more symmetrical version.                                               *)
(* ------------------------------------------------------------------------- *)

let GAUSS_LEMMA_SYM = prove
 (`!p q r s. prime p /\ prime q /\ coprime(p,q) /\
             2 * r + 1 = p /\ 2 * s + 1 = q
             ==> (q is_quadratic_residue (mod p) <=>
                    EVEN(CARD {x,y | x IN 1..r /\ y IN 1..s /\
                                     q * x < p * y /\ p * y <= q * x + r}))`,
  ONCE_REWRITE_TAC[COPRIME_SYM] THEN REPEAT STRIP_TAC THEN
  MP_TAC(SPECL [`q:num`; `p:num`; `r:num`] GAUSS_LEMMA) THEN
  ASM_SIMP_TAC[] THEN DISCH_THEN(K ALL_TAC) THEN AP_TERM_TAC THEN
  MATCH_MP_TAC EQ_TRANS THEN EXISTS_TAC
   `CARD {x,y | x IN 1..r /\ y IN 1..s /\
               y = (q * x) DIV p + 1 /\ r < (q * x) MOD p}` THEN
  CONJ_TAC THENL
   [CONV_TAC SYM_CONV THEN MATCH_MP_TAC CARD_SUBCROSS_DETERMINATE THEN
    REWRITE_TAC[FINITE_NUMSEG; IN_NUMSEG; ARITH_RULE `1 <= x + 1`] THEN
    X_GEN_TAC `x:num` THEN STRIP_TAC THEN
    SUBGOAL_THEN `p * (q * x) DIV p + r < q * r` MP_TAC THENL
     [MATCH_MP_TAC LTE_TRANS THEN EXISTS_TAC `q * x` THEN
      ASM_REWRITE_TAC[LE_MULT_LCANCEL] THEN
      GEN_REWRITE_TAC (LAND_CONV o ONCE_DEPTH_CONV) [MULT_SYM] THEN
      ASM_MESON_TAC[PRIME_IMP_NZ; LT_ADD_LCANCEL; DIVISION];
      MAP_EVERY EXPAND_TAC ["p"; "q"] THEN DISCH_THEN(MP_TAC o MATCH_MP
       (ARITH_RULE `(2 * r + 1) * d + r < (2 * s + 1) * r
                    ==> (2 * r) * d < (2 * r) * s`)) THEN
      SIMP_TAC[LT_MULT_LCANCEL; ARITH_RULE `x < y ==> x + 1 <= y`]];
    AP_TERM_TAC THEN
    REWRITE_TAC[EXTENSION; IN_ELIM_PAIR_THM; FORALL_PAIR_THM] THEN
    MAP_EVERY X_GEN_TAC [`x:num`; `y:num`] THEN
    AP_TERM_TAC THEN AP_TERM_TAC THEN EQ_TAC THEN DISCH_TAC THENL
     [MP_TAC(MATCH_MP PRIME_IMP_NZ (ASSUME `prime p`)) THEN
      DISCH_THEN(MP_TAC o SPEC `q * x` o MATCH_MP DIVISION) THEN
      FIRST_ASSUM(CONJUNCTS_THEN2 SUBST1_TAC MP_TAC) THEN
      UNDISCH_TAC `2 * r + 1 = p` THEN ARITH_TAC;
      MATCH_MP_TAC(TAUT `a /\ (a ==> b) ==> a /\ b`) THEN CONJ_TAC THENL
       [ALL_TAC;
        DISCH_THEN SUBST_ALL_TAC THEN
        MATCH_MP_TAC(ARITH_RULE
         `!p d. 2 * r + 1 = p /\ p * (d + 1) <= (d * p + m) + r ==> r < m`) THEN
        MAP_EVERY EXISTS_TAC [`p:num`; `(q * x) DIV p`] THEN
        ASM_MESON_TAC[DIVISION; PRIME_IMP_NZ]] THEN
      MATCH_MP_TAC(ARITH_RULE `~(x <= y) /\ ~(y + 2 <= x) ==> x = y + 1`) THEN
      REPEAT STRIP_TAC THENL
       [SUBGOAL_THEN `y * p <= ((q * x) DIV p) * p` MP_TAC THENL
         [ASM_SIMP_TAC[LE_MULT_RCANCEL; PRIME_IMP_NZ]; ALL_TAC];
        SUBGOAL_THEN `((q * x) DIV p + 2) * p <= y * p` MP_TAC THENL
         [ASM_SIMP_TAC[LE_MULT_RCANCEL; PRIME_IMP_NZ]; ALL_TAC]] THEN
      MP_TAC(MATCH_MP PRIME_IMP_NZ (ASSUME `prime p`)) THEN
      DISCH_THEN(MP_TAC o SPEC `q * x` o MATCH_MP DIVISION) THEN
      ASM_ARITH_TAC]]);;

let GAUSS_LEMMA_SYM' = prove
 (`!p q r s. prime p /\ prime q /\ coprime(p,q) /\
             2 * r + 1 = p /\ 2 * s + 1 = q
             ==> (p is_quadratic_residue (mod q) <=>
                    EVEN(CARD {x,y | x IN 1..r /\ y IN 1..s /\
                                     p * y < q * x /\ q * x <= p * y + s}))`,
  REPEAT STRIP_TAC THEN
  MP_TAC(SPECL [`q:num`; `p:num`; `s:num`; `r:num`] GAUSS_LEMMA_SYM) THEN
  ONCE_REWRITE_TAC[COPRIME_SYM] THEN ASM_REWRITE_TAC[] THEN
  DISCH_THEN SUBST1_TAC THEN AP_TERM_TAC THEN
  GEN_REWRITE_TAC LAND_CONV [CARD_SUBCROSS_SWAP] THEN
  AP_TERM_TAC THEN REWRITE_TAC[EXTENSION; FORALL_PAIR_THM] THEN
  REWRITE_TAC[IN_ELIM_PAIR_THM; CONJ_ACI]);;

(* ------------------------------------------------------------------------- *)
(* The main result.                                                          *)
(* ------------------------------------------------------------------------- *)

let RECIPROCITY_SET_LEMMA = prove
 (`!a b c d r s.
        a UNION b UNION c UNION d = (1..r) CROSS (1..s) /\
        PAIRWISE DISJOINT [a;b;c;d] /\ CARD b = CARD c
        ==> ((EVEN(CARD a) <=> EVEN(CARD d)) <=> ~(ODD r /\ ODD s))`,
  REPEAT STRIP_TAC THEN
  SUBGOAL_THEN `CARD(a:num#num->bool) + CARD(b:num#num->bool) +
```

Figure 1. Fragment of the HOL Light formalization of the Law of Quadratic Reciprocity.

This is the Law of Quadratic Reciprocity in HOL syntax. In this expression the exclamation mark is ASCII syntax for the universal quantifier, the combination of slash and backslash is supposed to resemble the $\wedge$ sign and represents conjunction, the tilde is logical negation, and the ampersand operator maps the natural numbers into the real numbers. The functions `pow` and `DIV` represent exponentiation and division, and the term `legendre(p,q)` represents the Legendre symbol $\left(\frac{p}{q}\right)$. This last function is defined in the formalization by the HOL syntax:

```
let legendre = new_definition
 `(legendre:num#num->int)(a,p) =
     if ~(coprime(a,p)) then &0
     else if a is_quadratic_residue (mod p)
          then &1 else --(&1)`;;
```

In this the expression `num#num->int` corresponds to functions of type $\mathbb{N} \times \mathbb{N} \to \mathbb{Z}$. That is, `num` and `int` are the natural numbers and the integers, the hash symbol represents the Cartesian product, and the combination of a minus and a greater than sign is supposed to look like an arrow.

A formalization, in any proof assistant, mainly consists of a long chain of lemmas, where each lemma consists of a label, a statement, and a proof. In between these lemmas there occasionally are a few definitions. In `reciprocity.ml` there are three: a definition of the notion of quadratic residue, a definition of the Legendre symbol, and a definition of the notion of iterated product.

The formalization of the Law of Quadratic Reciprocity is too large to explain in full here. Therefore we will now zoom in on one of its smallest lemmas, the third lemma in the file (see Figure 2 below).

```
let CONG_MINUS1_SQUARE = prove
 (`2 <= p ==> ((p - 1) * (p - 1) == 1) (mod p)`,
  SIMP_TAC[LE_EXISTS; LEFT_IMP_EXISTS_THM] THEN
  REPEAT STRIP_TAC THEN
  REWRITE_TAC[cong; nat_mod;
    ARITH_RULE `(2 + x) - 1 = x + 1`] THEN
  MAP_EVERY EXISTS_TAC [`0`; `d:num`] THEN
  ARITH_TAC);;
```

Figure 2. Small lemma from the HOL Light formalization of the Law of Quadratic Reciprocity.

This ASCII text consists of the three parts mentioned above: the first line gives the label under which the result will be referred to later, the second line states the statement of the lemma, and the last three lines encode the proof.

In this proof, there are references to four earlier lemmas from the HOL Light library:

```
LE_EXISTS            !m n. m <= n <=>
                            (?d. n = m + d)
LEFT_IMP_EXISTS_THM  !P Q. (?x. P x) ==> Q <=>
                            (!x. P x ==> Q)
cong                 !rel x y. (x == y) rel <=>
                          rel x y
nat_mod              !x y n. mod n x y <=>
                               (?q1 q2. x + n * q1 =
                                        y + n * q2)
```

The proof of this lemma encodes a dialog with the system. We can execute the proof all at once (this happens when we load the file as a whole), but we can also process the proof step by step, in an interactive fashion. This is the way in which an HOL Light proof is developed. To do this, we enter the following command (where the # character is the prompt of the system):

```
# g `2 <= p ==> ((p - 1) * (p - 1) == 1) (mod p)`;;
```

The g command asks the system to set the "goal" to the statement between the backquotes. The system then replies with:

```
Warning: Free variables in goal: p
val it : goalstack = 1 subgoal (1 total)

`2 <= p ==> ((p - 1) * (p - 1) == 1) (mod p)`
```

indicating that it understood us and that this statement now is the current goal. Next we execute the first "tactic" (a command to the system to reduce the goal) by using the e command:

```
# e (SIMP_TAC[LE_EXISTS; LEFT_IMP_EXISTS_THM]);;
```

Note that this corresponds to the initial part of the third line of the lemma in the way that it is written in the file. The tactic SIMP_TAC uses the theorems given in its argument to simplify the goal. It transforms the goal to:

```
`!d. p = 2 + d ==> (((2 + d) - 1) * ((2 + d)
 - 1) == 1) (mod (2 + d))`
```

As already noted, the "!" symbol is the universal quantifier, which means that the statement that now is the goal is universally quantified over all natural numbers d. The existential quantifier, which will occur below, is written as "?".

We now display the rest of the interactive session without further explanations between the commands. This is a dialogue between the human user executing tactics and the computer presenting the resulting proof obligations ("goals"). In square brackets are the *assumptions* that may be used when proving the goal.

```
# e (REPEAT STRIP_TAC);;
val it : goalstack = 1 subgoal (1 total)

0 [`p = 2 + d`]

`(((2 + d) - 1) * ((2 + d) - 1) == 1) (mod (2
 + d))`

# e (REWRITE_TAC[cong; nat_mod;
 ARITH_RULE `(2 + x) - 1 = x + 1`]);;
val it : goalstack = 1 subgoal (1 total)

  0 [`p = 2 + d`]

`?q1 q2. (d + 1) * (d + 1) + (2 + d) * q1 = 1
 + (2 + d) * q2`

# e (MAP_EVERY EXISTS_TAC [`0`; `d:num`]);;
val it : goalstack = 1 subgoal (1 total)

 0 [`p = 2 + d`]

`(d + 1) * (d + 1) + (2 + d) * 0 = 1 + (2 + d
) * d`

# e ARITH_TAC;;
val it : goalstack = No subgoals
```

At this point the proof is finished, as there are no unproved subgoals left.

To effectively use HOL Light you will need to learn the dozens of tactics available in the system. This example uses the following tactics:

SIMP_TAC	Simplify the goal by theorems
REPEAT	Apply a tactic repeatedly until it fails
STRIP_TAC	Break down the goal
REWRITE_TAC	Rewrite conclusion of goal with equational theorems
ARITH_RULE	Linear arithmetic prover over $\mathbb{N}$
MAP_EVERY	Map tactic over a list of arguments
EXISTS_TAC	Provide a witness to an existential statement
ARITH_TAC	Tactic to solve linear arithmetic over $\mathbb{N}$

A final note about this example. The lemma talks about natural numbers, which means that for $p = 0$ the difference $p - 1$ is defined to be 0. This is called "truncated" subtraction. This complicates the proof, and also explains the need for the condition `2 <= p` in the statement of the lemma. If the lemma had been stated using integers, it would have been provable without human input by the automated prover INTEGER_RULE:

```
# INTEGER_RULE `!p:int.
  ((p - &1) * (p - &1) == &1) (mod p)`;;
1 basis elements and 0 critical pairs
val it : thm = |- !p. ((p - &1) * (p - &1) ==
&1) (mod p)
```

In that case no explicit proof script would have been necessary.

Mizar

If instead of HOL Light we choose Mizar as our proof assistant, again the second step consists of downloading and installing the system. Download the system from the Mizar website [10], unpack the tar or exe file, and follow the instructions in the README. Mizar is distributed as compiled binaries, which means that we do not need to install anything else first.

The third step then again is to write a formalization of the proof. The best way to learn the Mizar language is to work through the Mizar tutorial [11].

The Law of Quadratic Reciprocity is formalized in the file "mml/int_5.miz" (part of this file is shown in Figure 2; it also is on the Web by itself as [12]). This file again primarily consists of a long chain of lemmas. It consists of 4701 lines proving 51 lemmas. It also has 3 definitions: a definition of integer polynomials, a definition of quadratic residues, and a definition of the Legendre symbol.

To have Mizar check this file for correctness, copy the file "mml/int_5.miz" inside a fresh directory called "text", and *outside* this directory type

```
  for n be Element of NAT holds
  hence thesis;
end;

reserve X for finite set,
  F for FinSequence of bool X;

definition
  let X, F;
  redefine func Card F -> Cardinal-yielding FinSequence of NAT;
  coherence
  proof
    rng Card F c= NAT
    proof
      let y be set;
      assume y in rng Card F;
      then consider x being set such that
A1:   x in dom Card F & y = (Card F).x by FUNCT_1:def 5;
A2:   x in dom F by A1,CARD_3:def 2;
      then F.x in rng F by FUNCT_1:12;
      then reconsider Fx = F.x as finite set;
      y = card Fx by A1,A2,CARD_3:def 2;
      hence thesis;
    end;
    hence thesis by FINSEQ_1:def 4;
  end;
end;

theorem Th48:
  for f be FinSequence of bool X st len f = n &
  (for d,e st d in dom f & e in dom f & d<>e holds f.d misses f.e) holds
  Card union rng f = Sum Card f
proof
  defpred P[Nat] means for f be FinSequence of bool X st
  len f = $1 & (for d,e st d in dom f & e in dom f & d<>e
  holds f.d misses f.e) holds Card union rng f = Sum Card f;
A1: P[0]
  proof
    let f be FinSequence of bool X;
    assume len f = 0 & (for d,e st d in dom f & e in dom f & d<>e
    holds f.d misses f.e);
    then f = {};
    hence thesis by CARD_1:47,CARD_3:9,RVSUM_1:102,ZFMISC_1:2;
  end;
A2: for n be Element of NAT st P[n] holds P[n+1]
  proof
    let n be Element of NAT;
    assume
A3: P[n];
    P[n+1]
    proof
      let f be FinSequence of bool X;
      assume
A4:   len f = n+1 &
      (for d,e st d in dom f & e in dom f & d<>e holds f.d misses f.e);
      then
A5:   f <> {};
      then consider f1 be FinSequence of bool X,Y be Element of bool X
      such that
A6:   f = f1^<*Y*> by HILBERT2:4;
A7:   n+1 = len f1 +1 by A4,A6,FINSEQ_2:19;
      for d,e st d in dom f1 & e in dom f1 & d<>e holds f1.d misses f1.e
      proof
        let d,e;
        assume
A8:     d in dom f1 & e in dom f1 & d<>e;
        then
A9:     f.d = f1.d & f.e = f1.e by A6,FINSEQ_1:def 7;
        d in dom f & e in dom f by A6,A8,FINSEQ_2:18;
        hence thesis by A4,A8,A9;
      end;
      then
A10:  Card union rng f1 = Sum Card f1 by A3,A7;
      Union f1 is finite;
      then reconsider F1 = union(rng f1) as finite set;
      F1 misses Y
      proof
        assume F1 meets Y;
        then consider x be set such that
A11:    x in F1 /\ Y by XBOOLE_0:4;
        x in F1 by A11,XBOOLE_0:def 3;
        then consider Z be set such that
A12:                    f1 by TARSKI:def 4;
```

Figure 3. Fragment of the Mizar formalization of the Law of Quadratic Reciprocity.

```
mizf text/int_5.miz
```

This will print something like:

```
Processing: text/int_5.miz

Parser   [4701]   0:02
Analyzer [4700]   0:13
Checker  [4700]   1:14
Time of mizaring: 1:29
```

which means that the file was checked without errors. Try modifying `int_5.miz` and see whether the checker will notice that the file now no longer is correct.

The Law of Quadratic Reciprocity is the 49th lemma from the file. It reads:

```
p>2 & q>2 & p<>q
implies Lege(p,q)*Lege(q,p)=
  (-1)|^(((p-'1) div 2)*((q-'1) div 2))
```

The proof of this statement takes 1268 lines of Mizar code! Here is a smaller example of a Mizar proof. This is the 11th lemma from the file. See Figure 4 below.

```
theorem Th11:
  i gcd m = 1 & i is_quadratic_residue_mod m &
    i,j are_congruent_mod m
  implies j is_quadratic_residue_mod m
proof
  assume
A1: i gcd m = 1 &
    i is_quadratic_residue_mod m &
    i,j are_congruent_mod m;
  then consider x being Integer such that
A2: (x^2 - i) mod m = 0 by Def2;
  m divides (i - j) by A1,INT_2:19;
  then
A3: (i - j) mod m = 0 by Lm1;
  (x^2 - j) mod m
     = ((x^2 - i) + (i - j)) mod m
    .= (((x^2 - i) mod m) + ((i - j) mod m))
            mod m by INT_3:14
    .= 0 by A2,A3,INT_3:13;
  hence thesis by Def2;
end;
```

Figure 4. Small lemma from the Mizar formalization of the Law of Quadratic Reciprocity.

The lemmas from the Mizar library to which this proof refers are:

```
INT_2:19   a,b are_congruent_mod c iff
             c divides (a-b)
INT_3:13   (a mod n) mod n = a mod n
INT_3:14   (a + b) mod n =
             ((a mod n) + (b mod n)) mod n
Lm1        (x divides y implies y mod x = 0) &
           (x<>0 & y mod x = 0 implies
             x divides y)
Def2       a is_quadratic_residue_mod m iff
             ex x st (x^2 - a) mod m = 0
```

If you think that the condition "`i gcd m = 1`" is not used in this proof, you can try removing it, both from the statement and from the "`assume`" step, and see what happens when you check the file again.

The Future of Formal Mathematics

In mathematics there have been three main revolutions:

- The introduction of *proof* by the Greeks in the fourth century BC, culminating in Euclid's *Elements*.
- The introduction of *rigor* in mathematics in the nineteenth century. During this time

the nonrigorous calculus was made rigorous by Cauchy and others. This time also saw the development of mathematical logic by Frege and the development of set theory by Cantor.

- The introduction of *formal mathematics* in the late twentieth and early twenty-first centuries.

Most mathematicians are not aware that this third revolution already has happened, and many probably will disagree that this revolution even is needed. However, in a few centuries mathematicians will look back at our time as the time of this revolution. In that future most mathematicians will not consider mathematics to be definitive unless it has been fully formalized.

Although the revolution of formal mathematics already has happened and formalization of mathematics has become a routine activity, it is not yet ready for widespread use by all mathematicians. For this it will have to be improved in two ways:

- First of all, formalization is *not close enough to existing mathematical practice* yet to be attractive to most mathematicians. For instance, both HOL Light and Mizar define

  $$\frac{1}{0} = 0$$

  because they do not have the possibility to have functions be undefined for some values of the arguments. This is just a trivial example, but in many other places the statements of formalized mathematics are not close to their counterpart in everyday mathematics. Here there exists room for significant progress.

  However, it is *not* important to have proof assistants be able to process existing mathematical texts. Writing text in a stylized formal language is easy. The fact that proof assistants are not able to understand natural language will not be a barrier to having formalization be adopted by the working mathematician.

- The second improvement that will be needed is on the side of *automation*. With this I do not mean that the computer should take steps that a mathematician would need to think about. Formalization of mathematics is about checking, and not about discovery.

  However, currently steps in a proof that even a high school student can easily take without much thought often take many minutes to formalize. This lack of automation of "high school mathematics" is the most important reason why formalization currently still is a subject for a small group of computer scientists, instead of it having been discovered by all mathematicians.

Still, there are no fundamental problems that block these improvements from happening. It is just a matter of good engineering. In a few decades it will no longer take one week to formalize a page from an undergraduate textbook. Then that time will have dropped to a few hours. Also then the formalization will be quite close to what one finds in such a textbook.

When this happens we will see a quantum leap, and suddenly all mathematicians will start using formalization for their proofs. When the part of refereeing a mathematical article that consists of checking its correctness takes more time than formalizing the contents of the paper would take, referees will insist on getting a formalized version before they want to look at a paper.

However, having mathematics become utterly reliable might not be the primary reason that eventually formal mathematics will be used by most mathematicians. Formalization of mathematics can be a very rewarding activity in its own right. It combines the pleasure of computer programming (craftsmanship, and the computer doing things for you), with that of mathematics (pure mind, and absolute certainty.) People who do not like programming or who do not like mathematics probably will not like formalization. However, for people who like both, formalization is the best thing there is.

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[9] http://www.cs.ru.nl/~freek/notices/reciprocity.ml

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Mathematical Omnibus: Thirty Lectures on Classic Mathematics *and* Roots to Research: A Vertical Development of Mathematical Problems

Reviewed by Harriet Pollatsek

Mathematical Omnibus: Thirty Lectures on Classic Mathematics
Dmitry Fuchs and *Serge Tabachnikov*
American Mathematical Society, 2007
US$59.00, 463 pages
ISBN-13: 978-0821843161

Roots to Research: A Vertical Development of Mathematical Problems
Judith D. Sally and *Paul J. Sally Jr.*
American Mathematical Society, 2007
US$49.00, 338 pages
ISBN-13: 978-0821844038

These are dangerous books! Perched on your desk, they will lure you away from duty with their lovely mathematics and engaging exposition. Seductiveness isn't all the books have in common. Both aspire to be of interest to an extraordinarily wide range of readers, from high school students to researchers "curious about results in fields other than their own". And both aim to traverse topics from school mathematics to the current research frontier. The inclusion of contemporary (since 1990) mathematics is especially notable, and in this respect these books differ from some others aimed at a similarly broad audience.

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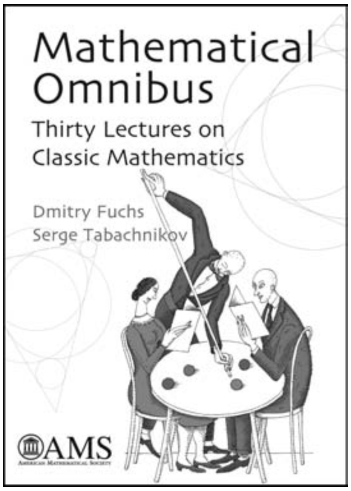
How would you use such a book? The Sallys write, "Our book . . . can be used by teachers at all of the above-mentioned levels [high school through graduate school] for the enhancement of standard curriculum materials or extra-curricular projects." Fuchs and Tabachnikov write that their "book may be used for an undergraduate honors mathematics seminar (there is more than enough material for a full academic year), various topics courses, mathematics clubs at high school or college, or simply as a coffee table book to browse through at one's leisure."

These authors set lofty goals for their books (which we refer to as *Roots* and *Omnibus* for short). Do they achieve them? Yes they do, but using somewhat different strategies and with somewhat different strengths and weaknesses.

What do the books cover? The thirty lectures in *Omnibus* are organized into eight chapters (number of lectures in parentheses): Arithmetic and Combinatorics (3), Equations (5), Envelopes and Singularities (4), Developable Surfaces (3), Straight Lines (4), Polyhedra (6), Surprising Topological Constructions (2), and Ellipses and Ellipsoids (3). The over-arching themes are algebra and arithmetics for the first two chapters and geometry and topology for the rest. The authors say they have followed their eyes for beauty and have not attempted systematic development of ideas nor uniformity of length or difficulty in different lectures. They "do not assume much by way of preliminary knowledge: a standard calculus course will do in most cases, and quite often even calculus is not required."

The five problems in *Roots* are treated in five chapters, each divided into roughly a dozen sections: The Four Numbers Problem (the finiteness of an elementary arithmetic game—the only chapter that doesn't lead all the way to current mathematics research), Rational Right Triangles and the Congruent Number Problem, Lattice Point Geometry, Rational Approximation, and Dissection. The over-arching themes are similar to those in *Omnibus*. The prerequisites for different sections vary from school mathematics to topics in the upper-level undergraduate curriculum, and the beginning of each chapter specifies the mathematical background required for each section. The authors' goal is to "provide a source for the mathematics (from beginning to advanced) needed to understand the emergence and evolution" of these problems.



The chapter on Lattice Point Geometry provides an illustrative sample of *Roots*. It begins with simple observations about lengths, angle measure, and areas of polygons with vertices in the lattice $\mathbb{Z}^2$. A reminder of the Two Squares Theorem in the preceding chapter leads to theorems characterizing integers that occur as areas of lattice squares and on the numbers of lattice points in and on particular circles. Dissection of lattice polygons into primitive lattice triangles leads to the algebraic structure of the lattice $\mathbb{Z}^2$ and its group of isometries. Then come two lovely proofs of Pick's Theorem, one independent of the preceding development and the other via Euler's formula. Applying Pick's Theorem leads to Farey sequences and to extension to bounded convex regions and to Minkowski's theorem, first in the plane and then in $\mathbb{R}^k$. The grand finale comes via the attempt to extend Pick's theorem to $\mathbb{R}^k$ for $k > 2$, leading to Ehrhart's theorem on the number of lattice points in a convex polytope in $\mathbb{Z}^k$.

The flavor of *Omnibus* shows, for example, in the titles of the lectures in the chapter on polyhedra: Curvature and Polyhedra, Non-inscribable Polyhedra, Can One Make a Tetrahedron Out of a Cube?, Impossible Tilings, Rigidity of Polyhedra, and Flexible Polyhedra. The lecture on making a tetrahedron from a cube opens with the statement of Hilbert's Third Problem as solved by Dehn and notes the (unique) omission of the Third Problem from the 1976 AMS collection *Mathematical Developments Arising from Hilbert Problems*—"no developments, no influence on mathematics, nothing to discuss", the authors say. Then they go on, "How strange it seemed just a couple of years later! Dehn's Theorem, Dehn's theory, Dehn's invariant became one of the hottest subjects in geometry." The lecture then shifts to a similar problem in the plane, moves on to a different planar problem with a similar solution (engagingly commenting on the roles of geometric and algebraic methods in the two planar problems), and culminates in Dehn's proof. The lecture ends with a brief discussion of the origin of the initial statement of Hilbert's Third Problem in the foundations of geometry and a statement of Sydler's 1965 result on polyhedra with equal volumes and equal Dehn invariants.

Where is the post-1990 mathematics, you may ask? Admittedly, not in this particular lecture, but somewhat similar themes recur in the lecture on flexible polyhedra, with a lovely description of Connelly's "courageous" search for and "breathtaking discovery" of a flexible, non-intersecting polyhedron, the improvement by Steffen, and the 1995 proof by Sabitov of the "bellows conjecture" that the volume inside a flexible polyhedron does not vary in the process of deformation.

The lectures in *Omnibus* have the enthusiasm and verve of a dynamic live presentation. The authors frequently editorialize about the beauty of the ideas or the cleverness of the arguments or their rationale for including one thing versus another. For example, they write on page 75, "This proof is convincing but it does not reveal the reasons for the existence of an expression for a, b, c via p, q, r. Let us try to explain these reasons." On page 225, they write "What is a surface? We would prefer to avoid answering this question honestly, but to prove theorems, we need precise definitions." The value of the book, especially for young readers, is enhanced by the authors' many side remarks, as on page 148, where they write, "Most mathematicians are brought up to believe that things like non-differentiable functions do not appear in 'real life' ... [but] Proposition 10.3 provides a perfectly natural example of such a situation."

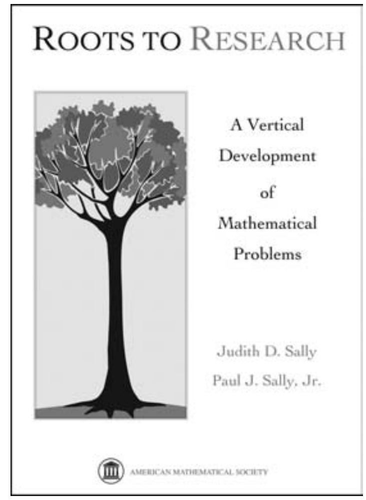
While the mathematical prerequisites for *Omnibus* are low, a high degree of mathematical sophistication and cleverness is often expected of the reader. Indeed, the authors warn in their preface that "it will take considerable effort from the reader to follow the details of the arguments." That was true for this reader. Some of the difficulties are avoidable, as in the confusing notation in section 8.4 or the failure to make clear at the start that much of Lecture 10 depends on a careful reading of Lecture 8. Also, I noticed more typos and careless errors in *Omnibus* than in *Roots*. Examples in the first few lectures include the omission of the exception $\alpha = 1/2 + n$ for n an integer on pages 9 and 10, the reversal of the interior and boundary points in Pick's formula on page 24, the reversal of p and q on page 39, and the incorrect arithmetic in the calculation of $\boldsymbol{p}(7)$ on page 51—none serious, but perhaps troubling for an inexperienced reader.

The tone of *Roots* is somewhat more formal and restrained than that of *Omnibus*, but it is never dull. Although there is less editorializing, the ideas are carefully motivated, often with natural questions arising from examples and previous results. Difficulties are not dodged, but the arguments are laid out with great care, so that the reading is challenging, but never discouraging. I found *Roots* much easier to read than *Omnibus*. For example, maybe I'm just more comfortable with Farey sequences than continued fractions, but I found the proof of Hurwitz's theorem in *Roots* as clear as glass, while I had to struggle with the one in *Omnibus*. I could more easily imagine handing *Roots* to a student to read on his or her own, than *Omnibus*. Of course, in a seminar or other group setting, this difference would be less significant.

As noted above, when background beyond calculus is required in *Roots*, this is made explicit. Sometimes lucid summaries are provided of the prerequisite mathematics, as in the description of basic facts from field theory in the section on Liouville's theorem in Chapter 4. The handling of Roth's Theorem, on pages 227 and following, is a particularly nice example of making something beyond the reader's preparation comprehensible in a general way, but without hiding the difficulties. The authors write, "The proof…is immensely more complex than those of the theorems of Liouville and of Thue, but the framework is, in essence, the same … [however] at every step, there are constants that are dependent only on α and δ, but are not specified until later stages … The delicate balancing of these constants is the point of the proof, but is not even considered here." There then follows a discussion of the steps of the proof of Roth's Theorem modeled on the preceding proof of Thue's. Roth's result is also mentioned near the end of Lecture 1 in *Omnibus*. This brief description includes the fact that Roth was awarded a Fields Medal for it in 1958, an interesting tidbit not appearing in *Roots*.

Both books are rich in exercises, many of them challenging. This is a particular strength of *Omnibus*, where the exercises are more abundant. In *Roots* there are some missed opportunities for exercises to test a reader's understanding of a complex argument. Both books include hints (more in *Omnibus*) and references for some exercises. Only *Omnibus* includes selected solutions; the solutions are numerous and they are written out in full—a great strength. The absence of any solutions in *Roots* is a disadvantage for a solitary reader. Both books have generous bibliographies and extensive indexes.

Illustrations are also more abundant in *Omnibus* than in *Roots*. There are, the authors say, about four hundred figures in the book, and they are mathematically precise and invariably illuminating. The authors also include photographs (or drawings) of almost every mathematician they mention, more than eighty portraits, including more than twenty of mathematicians still living. Regrettably, none is a woman. Every lecture includes a drawing by Sergey Ivanov, formerly artist-in-chief of the magazine *Kvant* and now of its cousin *Quantum*. Many of these illustrations are witty, mathematically apt, and attractive. Unlike the pictures of mathematicians, many of Ivanov's drawings include women. Unfortunately, the women pictured are sometimes movie-star bosomy and revealingly clad; in particular, the use of female nudity (especially on pages 45 and 123) seems inappropriate. The authors make clear in their preface that they want to encourage young women as well as men (even using "(s)he" as a pronoun), but the absence of real women mathematicians and the presence of women who seem mainly decorative works against their goal.



To summarize, I loved both books. Both are filled with beautiful mathematics and sometimes surprising connections. Both show the authors' love for their subject and their eagerness to share their enthusiasm. Neither book occupies a standard niche, but each has many possible uses. I am already thinking about ways to sneak bits into my courses, students to whom I might give parts to read, and talks and projects for our Math/Stat Club that I might draw from them.

WHAT IS...

a Period Domain?

James Carlson and Phillip Griffiths

The notion of period domain goes back to the very beginnings of algebraic geometry, to the study of elliptic curves. These are compact Riemann surfaces of genus one, defined as the complex solutions of $y^2 = x^3 + ax + b$, plus one point at infinity. Such a surface E is a compact torus and so has a homology basis $\{\delta, \gamma\}$, where the intersection number of the two cycles is $\delta \cdot \gamma = 1$. Consider the differential one-form $\omega = dx/y$, which is holomorphic in local coordinates on E. The *period matrix* of E is given by the integrals

$$(A, B) = \left(\int_\delta \omega, \int_\gamma \omega \right). \tag{1}$$

Multiplying ω by a suitable nonzero scalar, we may assume that its A period is one. Then a calculation, based on the fact that

$$\sqrt{-1} \int_S \omega \wedge \bar{\omega} > 0, \tag{2}$$

shows that its B period has positive imaginary part. Consequently, the upper half plane $\mathcal{H} = \{ z = x + iy \in \mathbb{C} \mid y > 0 \}$ parametrizes the set of so-called normalized B periods. The upper half plane is the first example of a period domain.

An elliptic curve plus a homology marking, i.e., a choice of integer homology basis such that $\delta \cdot \gamma = 1$, determines a point in the period domain $\mathcal{H}$. Two normalized homology bases are related by an element of the group Γ of unimodular matrices with integer entries, and the two normalized B periods are related by the corresponding fractional linear transformation. If one has a family of elliptic curves E_t that depends holomorphically on t, then $B(t)$ is locally defined and varies holomorphically. The map $t \mapsto B(t)$ is the *period map*. Since $\mathcal{H}$ is biholomorphic to the unit disk, one finds, as a consequence of the uniformization theorem, that any nonconstant family of elliptic curves parametrized by the Riemann sphere must have at least three singular fibers. The equivalence class of the normalized period modulo the action of the group Γ is intrinsically defined and lies in the quotient $\Gamma \backslash \mathcal{H}$. Thus, if E_t is a family of elliptic curves parametrized by a base S, one has a globally defined period map $S \to \Gamma \backslash \mathcal{H}$.

The notion of period domain is easily generalized to Riemann surfaces of higher genus, in which case the period matrix (A, B) has size g by $2g$. The role of the upper half plane is played by the Hermitian symmetric space of $Sp(2g, \mathbb{R})$: the Siegel upper half space of genus g, given by $g \times g$ complex symmetric matrices with positive definite imaginary part. Written $\mathcal{H}_g$, this is the space of normalized B periods. The group acting on it is $\Gamma = Sp(2g, \mathbb{Z})$.

To make the transition to algebraic manifolds of higher dimension, we think in terms of Hodge structures: the decomposition of the complex cohomology into the spaces $H^{p,q}$ spanned by closed differential forms expressible locally as a sum of terms $f dz_{i_1} \wedge \cdots \wedge dz_{i_p} \wedge d\bar{z}_{j_1} \wedge \cdots \wedge d\bar{z}_{j_q}$, where $z_1, \cdots, z_n$ are holomorphic local coordinates. For a projective algebraic manifold one has $H^k(X, \mathbb{C}) = \oplus_{p+q=k} H^{p,q}$ where $\overline{H^{p,q}} = H^{q,p}$. Such a decomposition, together with the lattice given by the integer cohomology modulo torsion, is a *Hodge structure of weight* k. For a Riemann surface the Hodge structure $H^1(X, \mathbb{C}) = H^{1,0} \oplus H^{0,1}$ is of weight one, and $H^{1,0}$ is identified with the row space of the period matrix. This space is subject to two important relations. One comes from the fact that for holomorphic differentials $\phi = f dz$ and $\psi = g dz$, the product $\phi \wedge \psi$ vanishes. The other comes from the fact that $i\phi \wedge \bar{\phi}$ is a positive multiple of the volume form. These are the *first and second Riemann bilinear relations*. A Hodge structure satisfying these relations is *polarized* (by cup product). In terms of normalized B periods, (1) B is symmetric, and (2) B has positive definite imaginary part. The Siegel upper half space parametrizes polarized Hodge structures of weight one.

General period domains are parameter spaces for polarized Hodge structures of weight k. The model is the subspace of the k-th cohomology of a complex projective algebraic manifold of dimension k which is annihilated by cup-product with the hyperplane class. Polarization is the generalization

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of the first and second Riemann bilinear relations. The resulting parameter space D can be represented, just as in the case of $\mathcal{H}_g = Sp(2g, \mathbb{R})/U(g)$, as a complex homogeneous space G/V, where G is a Lie group and V is a compact subgroup. However, V is rarely a maximal compact subgroup, and so D is rarely hermitian symmetric. Important special cases in which D is of weight $k > 1$ but is nonetheless hermitian symmetric are the period domains of K3 surfaces and of the cyclic cubic threefolds associated to cubic surfaces.

For period domains of weight $k > 1$, there is a differential relation not seen in the weight one case. To explain it, consider the subspaces $F^p = H^{k,0} \oplus \cdots \oplus H^{p,k-p}$. They define the *Hodge filtration* $F^k \subset F^{k-1} \subset \cdots$, denoted by $F^\bullet$. To give a Hodge decomposition is to give a Hodge filtration and *vice versa*. The Hodge filtration of a family of algebraic varieties that varies holomorphically with parameters also varies holomorphically. However, more is true. If t is a parameter on which $F^p(t)$ depends holomorphically, then the derivative satisfies

$$\dot{F}^p(t) \subset F^{p-1}(t). \tag{3}$$

This relation is now known as *Griffiths transversality*.

More formally, let TD be the holomorphic tangent bundle of D. The relation (3) defines a holomorphic subbundle I to which period mappings coming from geometry are tangent. Mappings satisfying this differential relation are called *horizontal*. A general period map is just a horizontal holomorphic map. An immediate consequence of horizontality is that most Hodge structures do not come from geometry.

Curvature computations along the horizontal distribution imply that period maps defined on the unit disk are distance decreasing with respect to the Poincaré metric on the disk and the G-invariant metric on D. The distance-decreasing property of period maps from the punctured disk Δ^* to D forces them to extend across the origin. Thus a version of the Riemann removable singularity theorem holds. Period domains act, with respect to horizontal holomorphic mappings, as if they were bounded domains.

On the n-th cohomology of a family of nonsingular algebraic varieties over Δ^* is defined a *monodromy transformation* T. It controls the analytic continuation of the period map along a loop around the origin. The period mapping associated to the family over the punctured disk takes the form $\tau : \Delta^* \to \{ T^i \}\backslash D$. Using the fact that T is an integral matrix and τ is distance-decreasing, one finds that the eigenvalues of T are m-th roots of unity. Passing to a finite covering of Δ^* we may assume that T is unipotent with logarithm N.

The distance decreasing properties of maps tangent to I make it possible to take limits of Hodge structures, just as one takes limits in calculus. The starting point is the asymptotic formula for a period map on the punctured disk,

$$\phi(t) \sim \exp\left(\left(\frac{\log t}{2\pi\sqrt{-1}}\right) N\right) F_0^\bullet, \tag{4}$$

where the "limit filtration" $F_0^\bullet$, which lies in $\check{D}$, defines a so-called *mixed Hodge structure*. The previous relation, due to Schmid, is the starting point for the result that the index of unipotency of T^m is $n+1$, i.e., $(T^m - I)^{n+1} = 0$.

The boundary points for the limit Hodge filtration lie in the *compact dual* $\hat{D}$ of D, obtained by ignoring the positivity condition in the definition of polarization. For elliptic curves, $\hat{D}$ is just $\mathbb{P}^1$, and the limiting mixed Hodge structures added to compactify $\Gamma\backslash\mathcal{H}$ correspond to cusps of the fundamental domain of $SL_2(\mathbb{Z})$. It is a remarkable fact, encoded in the Clemens-Schmid exact sequence, that the limit mixed Hodge structure can largely be read from the geometry of the singular fiber.

The subbundle I usually generates TD under Lie bracket, as in the case of the contact distribution on the three-sphere or its holomorphic analogue, given in local coordinates by the null space of $\omega = dz - xdy$. As with the contact distribution, the dimension of integral submanifolds of I is smaller than the dimension of I, indeed, often much smaller. One may suspect that a nontrivial period mapping defined on a quasi-projective variety "comes from geometry". However, with the exception of weight one (abelian varieties) and K3 surfaces, almost nothing is known about this question.

We close with some observations of a more arithmetic character. First, the projective variety $\hat{D}$ is defined over $\mathbb{Q}$. Thus it makes sense to ask for the field of definition of $F^\bullet(t)$. If $F^\bullet$ is simple, then $End(F^\bullet) \otimes \mathbb{Q}$ is a division algebra whose center is a field k with $[k : \mathbb{Q}] \le \dim H$. We say that the Hodge structure has CM type when the division algebra is commutative and equality holds. Equivalently, the *Mumford-Tate group* $M(F^\bullet)$ is an algebraic torus. The Mumford-Tate group is the $\mathbb{Q}$-subgroup of $Aut(H, \mathbb{Q})$ that fixes all the rational (p, p) classes ("Hodge classes") in the tensor algebra on H and its dual. The nature of Hodge structures of CM type, which have played an essential role in the weight one case, is just beginning to be explored in higher weight. The interface between Hodge theory, period domains, and arithmetic is one of the deepest and most promising areas for future work.

Further reading. Some foundational articles are those by P. A. Griffiths and W. Schmid in *Acta Math.* (1968), by P. Deligne in *Pub. Math. IHES* (1971), and the article on the singularities of the period mapping by W. Schmid in *Invent. Math.* (1973). Recent expository works include *Hodge Theory and Complex Algebraic Geometry I and II*, by C. Voisin, and *Period Domains and Period Mappings*, by J. Carlson, C. Peters, and S. Müller-Stach. The notes by B. Moonen on Mumford-Tate groups (1999) illuminate the material of the last paragraph.

AMERICAN MATHEMATICAL SOCIETY

D. R. FULKERSON PRIZE

Call for NOMINATIONS

The Fulkerson Prize Committee invites nominations for the Delbert Ray Fulkerson Prize, sponsored jointly by the Mathematical Programming Society and the American Mathematical Society. Up to three awards are presented at each (triennial) International Symposium of the MPS. The Fulkerson Prize is for outstanding papers in the area of discrete mathematics. The Prize will be awarded at the 20th International Symposium on Mathematical Programming to be held in Chicago, August 23-29, 2009.

Eligible papers should represent the final publication of the main result(s) and should have been published in a recognized journal, or in a comparable, well-refereed volume intended to publish final publications only, during the six calendar years preceding the year of the Symposium (thus, from January 2003 through December 2008). The prizes will be given for single papers, not series of papers or books, and in the event of joint authorship the prize will be divided.

The term "discrete mathematics" is interpreted broadly and is intended to include graph theory, networks, mathematical programming, applied combinatorics, applications of discrete mathematics to computer science, and related subjects. While research work in these areas is usually not far removed from practical applications, the judging of papers will be based only on their mathematical quality and significance.

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The Fulkerson Prize Committee consists of Bill Cook (Georgia Tech), chair, Michel Goemans (MIT) and Danny Kleitman (MIT).

Please send your nominations (including reference to the nominated article and an evaluation of the work) by January 15th, 2009, to the chair of the committee. Electronic submissions to bico@isye.gatech.edu are preferred.

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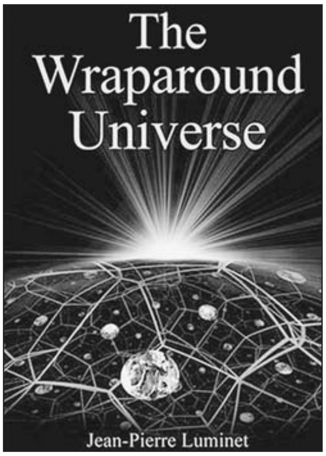
Book Review



The Wraparound Universe

Reviewed by George F. R. Ellis

The Wraparound Universe
Jean-Pierre Luminet
A K Peters, Ltd., 2008
US$39.00, 400 pages
ISBN-13: 978-1568813090

The standard models of relativistic cosmology have preferred 3-dimensional spatial sections of constant curvature; these are surfaces of spatial homogeneity in the expanding universe. It has been known since the earliest days of relativistic cosmology that they need not have the obvious simply connected topology, and indeed it was already known in the 1930s that their topology could be extremely complex (with an infinite number of possibilities in the negatively curved case).

What is new in the last few decades is that observational investigation of this spatial topology has become an active field of research in cosmology. While the simply connected topologies (S^3 in the case of positive curvature, R^3 in the cases of zero and negative curvature) are usually assumed in most cosmological studies, there is in fact no known physical mechanism that determines this spatial topology (the Einstein field equations are differential equations that determine the space-time curvature locally but do not directly specify global connectivity; and no mechanism related to quantum gravity has so far been proposed that will determine this topology). Furthermore since the advent of string theory, the idea that immensely complex spatial topologies may occur in physics has become commonplace. The mathematical investigation of the possible spatial topologies relevant to cosmology has progressed greatly through the work of William Thurston and others.

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Jean-Pierre Luminet is one of those who has been studying the way the different possible spatial topologies may be observationally investigated. One specific proposal he and colleagues have put forward is that the spatial topology may be that of a Poincaré dodecahedral space. This well-written book is a comprehensive introduction to this topic for the lay reader (it contains no equations but has many geometric diagrams illustrating the concepts used). It gives a sound introduction to the relevant cosmological theory and data and discusses in detail the possibilities of complex topologies in a universe where the resulting scale of spatial closure is so small that we have seen right around the universe since the time the universe became transparent, hence seeing many images of the same distant objects in different directions in the night sky. One can in principle study the topology by identifying such multiple images, but this is likely to be very difficult (it is not easy to identify multiple images as being due to the same object, for they will be seen at different distances and at different times in their histories, and effectively from different directions). One can also analyze the statistics of observations of distant sources, but this again may not be conclusive, for example, due to source evolution effects. What is easier in principle is to determine identical circles in the sky in the temperature fluctuations in the cosmic blackbody radiation that comes to us from all directions at an average temperature of 2.75K. This is a difficult task for statistical reasons, but such searches are under way and can in principle eventually be conclusive.

This book introduces all this in a clear way that will appeal to any reader interested in the topic at the *Scientific American* level. It will thereby introduce a very interesting branch of mathematics to nonmathematicians and show how it may relate to the real universe in which we live.

Adventures in Academic Year Undergraduate Research

Kathryn Leonard

This article is the fourth in an occasional series intended for graduate students. The series is coordinated by Associate Editor Lisa Traynor.

When I accepted a position at a university without a Ph.D. program, I knew a first order of business would be to involve undergraduates in my research. For one, original research has become increasingly standard as an expectation for graduate school admittance. For another, I have too many problems to work on and not enough time: I needed to share my wealth. At the time, I had no knowledge of undergraduate research. I was never involved in research as an undergraduate, and, until a few years ago, I was completely unaware that undergraduates performed original research. (I'm not sure what I thought went on in the undergraduate poster session at the Joint Meetings, but certainly not original work.) This article describes my inaugural year of mentoring an undergraduate research group, with pitfalls and advice given their due, and with the hope that the uninitiated and the terrified will be tempted to try it.

Midway through my first year in the new job, a colleague told me of a mini-grant to support an undergraduate research group, funded through the Center for Undergraduate Research in Mathematics (CURM) at Brigham Young University (BYU). A brainchild of CURM director Michael Dorff, the grant provides funding for a course release, a workshop for faculty to build mentoring skills, stipends for student researchers, and travel funds for students and faculty. For me, the faculty workshop was almost more appealing than the course release, as I felt entirely unequipped for mentoring undergraduates. The NSF-supported[1] CURM is designed to replicate BYU's successful undergraduate research program in departments across the country. A few things make the BYU model different: research groups meet during the academic year instead of the more common summer REUs (Research Experiences for Undergraduates), research groups are organized much like lab science groups with more advanced students (including graduate students) mentoring newcomers, students are involved in these groups as early as sophomore year and continue through graduation, and students are paid stipends for their work.

My research group consisted of three juniors and one master's student. We tackled a problem arising from image modeling: modeling deformations of periodic textures. Throughout the year, the group met once per week with me and twice with each other. During the first semester, I also met individually with each student once each week. The group was exhausting but rewarding. As one student put it:

> *The CURM experience for me has been like a rollercoaster. Sometimes I'm frustrated and sometimes I feel like I'm on top of the world. The research is difficult and challenging especially with school going on at the same time. But this challenging experience is addicting! It is great to be given a problem where the answer is not in the back of the book.*

From the professorial point of view, I completely agree. My hope is that, though the answer to being an undergraduate research mentor is not in the back of the book, this article will at least give step-by-step hints for how to begin the problem. Many

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[1] *DMS-0636648.*

suggestions below originated from colleagues at the CURM workshop, whose wisdom has earned my fervent gratitude.

Identify Your Goals

The first step toward building a successful undergraduate research group is to take an honest look at what your goals for the group are. I found out the hard way that goals not explicitly stated will affect your ability to mentor. Mid-year, I became increasingly frustrated that the students were not progressing as quickly as I thought they should. Once I realized the frustration originated in an unstated goal—I wanted the end-of-year product to be publishable—I was able to set the goal aside and focus on helping the students progress at their own speed. I still want a paper to come out of the project, but I've altered my timeline to fit the students' needs and abilities.

Differentiate between goals that benefit the students and goals that benefit you. My personal goals were to involve students in my research in a sustainable way, to raise the on-campus profile of mathematics, and to prioritize research above the noise of the academic year. Of course, some of your goals will be unrealistic. For example, I thought my students would work on small problems while I worked on larger, related problems. It turns out that I could have solved the smaller problems in much less time than it took to mentor the students, and the mentoring took time away from working on the larger problems. But the group did offer a weekly motivation to prevent research from getting lost in the pressures of teaching, and it also spread word of my research to other parts of campus.

My goals for the students were more realistic. I wanted them to experience the beauty of mathematical research, to see that math is alive and relevant, to build confidence in their mathematical abilities, and to learn to communicate effectively. I also wanted to pique their interest enough to apply to graduate school and to consider research mathematics as a possible career.

Find a Problem

I often hear the objection that a colleague's research requires too much machinery to involve undergraduates. Such an objection should not discourage you from supervising undergraduate research. Many participants in the CURM workshop work in esoteric areas, but nonetheless mentor successful undergraduate research projects year after year.

The challenge is to find an interesting yet realistic original research project for the particular students in your group. What is "realistic"? That depends on the students. One CURM participant argued that a good project has enough flexibility to accommodate three levels of research: guided discovery, where the professor expects to take a greater role in offering steps toward a solution; independent investigation, where the student works independently on a simpler problem; and scholarly activity, where the student works independently on a harder problem.

Below are several suggestions from the CURM workshop for where to find good undergraduate projects. I have organized them into broad categories based on relatedness to the professor's research program.

General Resources

- MAA columns with problems for research
- undergraduate journals
- talks in other disciplines
- Pi Mu Epsilon problems
- linear algebra

Resources from Your Research Area

- mistakes in others' proofs
- constructive proofs of others' theorems
- Ph.D. theses
- (re-)interpretation of results for industry applications

Resources from Your Research

- small examples of a larger theorem you're trying to prove
- testing "thresholds" in your own work
- specific examples of a general solution
- spinning research out from a course you teach
- any side calculations

Ideally, the project should have a "hook" problem that the students can understand and start to work on within the first month of the project. If not, the problem probably requires more background than the students can handle. Other pitfalls include choosing a problem you lack the required background in, not sufficiently understanding your students' backgrounds, and not modifying the problem when it proves to be ill-suited to the students' levels of preparation.

The problem I chose was too hard. Toward the end of the fall semester, I performed triage, reformulating the project into something more manageable. We found a partial solution to the new problem this year, and I expect the students will complete the solution shortly after returning in the fall.

Find Students

I found my research students by asking for recommendations from professors who taught sophomore level math courses the year before, then inviting those students to a meeting where we discussed the research project. A few students did not have the time or interest to participate, and a few other students were so flaky that the initial meeting never took place, so the group ended up being self-selecting. Other CURM participants recruit students directly from courses they regularly teach whose content gives relevant background information for the research project. This seems ideal when feasible because then you have direct knowledge of what a student has learned.

My school is still very small, so I did not have the luxury of selecting the best from among a crowd of interested students. If I am ever so lucky, I will try to pick students who are well-matched in personality and ability. In particular, I will look for pairs of students at a similar level but with complementary coursework who seem to have the potential to work well together (though I am unsure how to measure said potential). For me, four students is the ideal number to maintain group momentum without overwhelming my schedule with requests for one-on-one meetings, but the number ranged from two to six among the other CURM participants.

Get Started on Research

At the first group meeting, be prepared to explicitly state what you expect of the students, what the students can expect of you, and what the mechanisms will be for ensuring those expectations are met. Facing the uncertainty inherent in original research will be easier for your students if you clearly state all aspects of the project that can be made concrete. To better prepare my students for the research process, I gave them an article about how solving research problems is different from solving homework. At the very least, it is probably wise to let your students know that feeling stupidly incapable is one of the many delights of research.

I mentioned finding a "hook" problem above, which was arguably the best piece of advice I received during my CURM experience. The hook problem is one that the students can understand almost immediately and that the professor could solve in a lazy afternoon. The point of such a problem is to get the students involved in solving something at the outset, before or at the same time as they learn the necessary background. Ideally, it will be a problem that, once solved, will lead to the meat of the project. Or it might be that the students require the entire year to solve the hook problem. Either way, it gets the students actively working from the start and it helps the professor avoid the problem selection pitfalls mentioned in the previous section.

Another suggestion from the CURM workshop is to start the students on a "fake" problem, a problem possibly unrelated to the project that has a known solution but requires research-like thinking to solve (e.g., using the geometry of rectangles to prove the Pythagorean Theorem). I tried a fake problem with my students but quickly abandoned it because it distracted them from their primary task. Next year, I might try again with a fake problem that relates in some way to the work we are trying to accomplish.

Maintain Momentum

According to CURM director Michael Dorff, the key to a successful research group is maintaining student enthusiasm. Sounds simple, right? Until you remember that students will be experiencing the research cycle for the very first time: the thrill of a new challenge, the satisfaction of growing understanding, the excitement of the first attack, the disappointment when the attack fails, the dullness of days spent staring at equations awaiting new insight, the paralysis of stupidity, the glimmer of hope at a second attack, the disappointment…and so on. Meanwhile, other courses offering the comfort of well-defined tasks and deadlines can lure the students away from the research project. Research will not be a party every day, but here are some ideas for maintaining student focus during the Slough of Despond.

- Intersperse more concrete tasks, such as writing a portion of the final paper or designing a poster, with the open-ended research tasks.
- Schedule a seminar talk for the students to present their research problem so they can see how interesting the project is to other mathematicians.
- Ask students for results from a concrete experiment or computation.
- Require students to maintain records of their work through time sheets or progress logs. Not only will it keep them focused on work, it will allow them to see their progress.
- Do fun things occasionally, like going to lunch or playing a math game or talking about the history of relevant mathematicians.
- End every meeting with a concrete plan for the next few days.
- Rescue students who have struggled in the dark for too long.
- Emphasize that the process of mathematics is the process of making progressively less incorrect mistakes rather than a process of correct answers[2].

Midterms and papers for other courses will still distract some of the students some of the time, but ideally, momentum of the group will carry individual students through.

Manage Group Dynamics

For me, negotiating group dynamics was the most difficult aspect of mentoring. My group of four students consisted of three undergraduates and one master's student, two men (one married), two women, two Hispanic, one Filipina, three who are the first in their family to attend college, one who is the child of academics. These students brought different assumptions, skill levels, and outside responsibilities to the group, as did I, which resulted in some misunderstandings of the behavior

[2] *This suggestion is courtesy of Michael Starbird.*

of other group members. Consequently, the group faced a few difficult moments that could have been avoided if I had anticipated the types of behaviors that would be most damaging.

Need for control. Power struggles will undoubtedly surface in any group, many of which will resolve themselves sensibly. Occasionally, however, a student's unwillingness to cede decision-making to the group will paralyze an otherwise productive group. Not only does the student's controlling behavior disrespect the other students' opinions, it also undermines the independent thought and creativity a research project is meant to foster. The other students will eventually surrender their voices to avoid disdain and argument from the controlling student. Controlling behavior usually indicates deep anxiety or insecurity, so the professor must somehow reduce the student's death grip on control without further damaging his or her self-esteem.

Lack of communication with other group members. Occasionally, a subset of the group may come to an agreement concerning a group decision during a conversation outside regular group meetings. When the subset informs the group of the decision as *fait accompli*, the remainder will likely be upset by their exclusion from the decision making process. Even worse is when the subset acts on the decision without discussion with the other group members, who are perhaps acting on the assumption of a different decision. These miscommunications generate resentment and mistrust among the excluded group members. The research mentor should establish a standard method (email, wiki, blog) for students to maintain open lines of regular communication.

Closure to constructive criticism. Research is challenging and often requires several wrong approaches to find the correct approach. Unfortunately, many students have been conditioned to focus on finding the answer rather than engaging in the *process* of finding the answer. These students will likely struggle with learning from mistakes, particularly when someone else points them out. The transition from homework to research mindset requires the shift of emphasis from answers to understanding. The challenge is to ease the student into viewing mistakes as new information about the problem rather than as an evaluation of student worth.

Sporadic involvement. Some of my students faced substantial responsibilities outside of school, resulting in periods of intense group involvement alternated with periods of absence. Overall, the students worked the same amount, but the students with more consistent involvement lost faith in the others because of their apparent unreliability. I struggled to prevent the absent students from being excluded from important decisions, encouraging the consistent students to empathize with the heavy responsibilities of the sporadic students. At the same time, I urged the sporadic students to hold to a minimum weekly involvement with the group.

I believe the key to addressing these behaviors is to preempt them by making the group process explicit at the outset. Before the group begins the mathematical work for the year, ask students to think about their past group experiences: what characteristics have worked for them? what characteristics have not? how would they like to make group decisions? how would they like to share work? what is the best way for them to hear/give constructive criticism? what if a student misses a meeting? Write a list of these mutually agreed upon norms for group interaction, and return to the list whenever you feel the group dynamic is breaking down. Encourage the students to refer to the list periodically to evaluate whether the group is abiding by its norms. My group could have avoided most of the above scenarios if we had developed methods for addressing them before they arose. Additionally, my interventions would have felt less like scolding if I could have pointed to norms generated by the students themselves.

Enjoy Success

At the end of the year, you may have achieved all—or none—of the goals set out in Step One. Regardless, your students have changed, you have changed, your understanding of the project or the students or both has changed. Change represents success: relish it.

Here at the end of my inaugural year, my students have presented two posters, given five talks, and written a decent draft of a research paper. They can summarize their work efficiently and field questions like pros. All four students plan to apply to Ph.D. programs next year. Already, their understanding of research far outstrips mine upon entering graduate school, so I expect them to excel at the next level. Two of the undergraduates are continuing to work with my group, joining two new students in the fall who will be funded by CURM, and so my dream of a continuous research group has hope of materializing. As a result of the undergraduate grapevine, two students and a colleague in computer science have approached me about my research, resulting in potential research students for the following year and a fledgling faculty research group with the computer scientist and another colleague in physics.

On a larger scale, the dean of faculty at CSUCI has initiated a program to encourage student research groups in all disciplines, thereby institutionalizing undergraduate research on our campus. If our young campus, beset by a state budget crisis, can carve out a small program to support student research, so can yours. Ask your dean for funding, talk to your office of sponsored research, apply for grants, talk to industry partners. Taking the first step may lead you to an elevator.

Things to Try Next Year:

Looking to the coming year of research, I plan to try some assorted techniques that didn't find a home in the above sections:

1. Meet outside my office, preferably in the room where the students meet without me, to free me from distractions and to provide some continuity to the group meetings.
2. Require weekly time cards to be turned in. This year I asked students to keep progress logs but did not collect them, so they dwindled away.
3. Pop in on student meetings from time to time to answer questions and keep the conversation on track.
4. Invite a colleague in the communication department to talk to students about developing productive working groups.
5. Moderate the number of talks and posters so students have longer stretches to focus solely on research.

One final thought: some of the CURM faculty last year were concerned that academic year research programs were not as effective as research programs offered in summer when students and faculty have no other distractions. I believe each is valuable for different reasons. In fact, two of my students will participate this summer in REUs. For me, the pros of academic year research are that it more closely mimics the life of a mathematics researcher, it creates a community of research within the student body, and it allows for the continuity of a research group over the years. In addition, as a pre-tenure faculty member, I want to reserve my summers for my own research. I encourage those who disagree to involve themselves in one of the many wonderful summer REU programs. For now, I am sold on the academic year model.

Other Resources:

- `http://curm.byu.edu/curmresources.htm`
- `http://math.la.asu.edu/~crook/undergrad.html`
- `http://www.maa.org/columns/resources/resources_2_08.html`

Undergraduate Research Journals:

- *Involve*
  `http://pjm.math.berkeley.edu/inv/about/journal/about.html`
- *Rose-Hulman Undergraduate Math Journal*
  `http://www.rose-hulman.edu/mathjournal/who.php`
- *Pi Mu Epsilon*
  `http://www.pme-math.org/journal/overview.html`
- *The Pentagon: The Official Journal of Kappa Mu Epsilon*
  `http://www.kappamuepsilon.org/pages/Pentagon.php`
- *Journal of Undergraduate Sciences*
  `http://www.hcs.harvard.edu/~jus/about.html`
- *Journal of Young Investigators*
  `http://www.jyi.org`
- *Furman University Electronic Journal of Undergraduate Mathematics*
  `http://math.furman.edu/~mwoodard/fuejum/welcome.html`
- *Morehead Electronic Journal of Applicable Mathematics*
  `http://www.morehead-st.edu/mejam/`

About the Cover

Four-color hypermaps

As Georges Gonthier says in his article on the formal proof of the Four-Color Theorem, planar graphs require extra data to encapsulate their planarity in a way that can be used in formal proofs. Hypermaps are what he uses to implement this idea. This as well as other aspects of Gonthier's proof illustrate one of the more remarkable features of formal proofs—that very often the task of making an ordinary proof into one rigorous enough to be checked by computer forces one to make even the original proof much clearer to human beings.

—Bill Casselman, Graphics Editor
(`notices-covers@ams.org`)

Mathematics People

Kitaev Receives MacArthur Fellowship

In September 2008 the MacArthur Foundation named twenty-five new MacArthur Fellows for 2008. Each will receive US$500,000 in "no strings attached" support over the next five years. Among the fellows is Alexei Kitaev of the California Institute of Technology.

Alexei Kitaev is a physicist who explores the mysterious behavior of quantum systems and their implications for developing practical applications, such as quantum computers. Since his days as an undergraduate he has made important theoretical contributions to a wide array of topics within condensed matter physics, including quasicrystals and quantum chaos, among others. More recently Kitaev has devoted considerable attention to the use of quantum physics for performing computation. Upon learning of the first algorithm for factoring numbers (an important aspect of cryptography) with quantum computers, he independently developed an alternative approach using "phase estimation", a solution that generalizes to an even wider range of calculations. At the quantum level, physical interactions often display bizarre, counterintuitive properties that are generally unobservable at the macroscopic level. Kitaev has shown how, under certain circumstances, macroscopic systems can maintain their quantum coherence, even in the presence of external noise. Though his work is focused mainly at the conceptual level, he also participates in efforts to develop working quantum computers. Through his deep insights into the fundamental nature of quantum physics, Kitaev reveals a rich picture of this unfamiliar world, bringing us closer to the realization of the full potential of quantum computing.

Alexei Kitaev received an M.S. (1986) from the Moscow Institute of Physics and Technology and a Ph.D. (1989) from the L. D. Landau Institute of Theoretical Physics in Chernogolovka, Russia. He is a professor of theoretical physics and computer science at the California Institute of Technology. Kitaev served previously as a researcher (1999–2001) at Microsoft Research and as a research associate (1989–1998) at the Landau Institute.

—From MacArthur Foundation announcements

Agrawal Receives Infosys Mathematics Prize

Infosys Technologies Ltd. (Infosys) and the National Institute of Advanced Studies (NIAS) today announced the first-ever winner of the Infosys Mathematics Prize. The winner of this prize for 2008 is Manindra Agrawal, N. Rama Rao Chair Professor in the Department of Computer Science and Engineering at the Indian Institute of Technology, Kanpur. Agrawal will be awarded 1 million rupees (approximately US$22,000) and a medal for his research in complexity theory.

The Infosys Mathematics Prize was jointly instituted by Infosys and NIAS earlier this year to encourage and foster an interest in mathematics. This prize is awarded to a nominated candidate who has made outstanding contributions—fundamental or applied—in any field of mathematics including the areas of pure mathematics, mathematical foundations of computer science and applied mathematics in natural, life, and social sciences.

Manindra Agrawal has been awarded the Infosys Mathematics Prize for his outstanding work in complexity theory, the branch of mathematics concerned with the study of algorithms for solving mathematical and related scientific problems, especially their efficiency and running times. Agrawal is best known for the discovery of a deterministic polynomial time algorithm for primality testing, in his joint paper with his former students. This discovery resolved a long-standing problem of a fast test of primality, which had been the subject of intense study in the field of mathematics and computer science research.

The prize jury consisted of S. R. Srinivasa Varadhan (chair), George C. Papanicolaou, Peter C. Sarnak, Alain Bensoussan, Shigefumi Mori, and M. S. Narasimhan.

—From an Infosys news release

Bartels and Görtz Awarded von Kaven Prize

Arthur Bartels of the University of Münster and Ulrich Görtz of the University of Bonn have been awarded 2008 von Kaven Prizes in Mathematics of the von Kaven Foundation, which is administered by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation). The prizes carry a cash award of €10,000 (approximately US$14,000) each.

Bartels's research is in the field of geometric and algebraic topology and focuses primarily on the so-called Farrell Jones Conjecture and related problems. This conjecture is important to understanding the topology of manifolds. Görtz works in the field of arithmetic algebraic geometry. He is particularly interested in algebraic geometric problems that originate from the Langlands program or the theory of Shimura varieties. This also involves relations to numerous other areas in mathematics—for instance, to algebraic geometry and number theory, and in particular to representation theory.

The von Kaven prize is funded from the proceeds of the von Kaven Foundation, which was established in December 2004 by mathematician Herbert von Kaven.

—From a DFG announcement

Alur and Dill Receive Computer-Aided Verification Award

RAJEEV ALUR of the University of Pennsylvania and DAVID L. DILL of Stanford University have been awarded the 2008 Computer-Aided Verification (CAV) Award for fundamental contributions to the theory of real-time systems verification. This is the first year of this annual award.

The researchers were honored for their joint 1990 article, "Automata for modeling real-time systems". This article laid the theoretical foundation for the computer-aided verification of real-time systems, which are computer systems that are expected to finish their computations by specific deadlines. With the increasing ubiquity of embedded computers, which control everything from aircraft to medical devices, there is an urgent need for a rigorous methodology that can ensure that such systems operate without failures.

During the late 1980s there were several attempts to extend the theory of computer-aided verification to real-time systems. Alur and Dill's work put this research direction on a firm foundational footing. In particular, the formalism of timed automata that they introduced in the prize-winning article has become the standard model for the verification of real-time systems. This article is among the most cited in the field of computer-aided verification.

Rajeev Alur is the Zisman Family Professor in the Department of Computer and Information Science at the University of Pennsylvania. He obtained his bachelor's degree in computer science from the Indian Institute of Technology at Kanpur in 1987 and his Ph.D. in computer science from Stanford University in 1991. Before joining the University of Pennsylvania in 1997, he was with the Computing Science Research Center at Bell Laboratories. Alur's research spans formal modeling and analysis of reactive systems, hybrid systems, model checking, software verification, and design automation for embedded software. His awards include the President of India's Gold Medal for academic excellence, a CAREER award from the National Science Foundation, and an Alfred P. Sloan Faculty Fellowship. He is a fellow of the Association for Computing Machinery (ACM) and of the Institute of Electrical and Electronics Engineers (IEEE) and recently served as the chair of the ACM's Special Interest Group on Embedded Systems (SIGBED).

David L. Dill is a professor of computer science and, by courtesy, of electrical engineering at Stanford University, where he has been on the faculty since 1987. He has an S.B. in electrical engineering and computer science from the Massachusetts Institute of Technology (1979) and a Ph.D. from Carnegie-Mellon University (1987). His research interests cover a variety of areas, including computational systems biology, the theory and application of formal verification techniques to system designs, and voting technology. He has also done research in asynchronous circuit verification and synthesis and in verification methods for hard real-time systems. He was one of the founders and the chief scientist of 0-In Design Automation (later acquired by Mentor Graphics) and the founder of the nonprofit organizations Verified Voting Foundation and VerifiedVoting.org. His awards include the ACM's Distinguished Dissertation award for his Ph.D. thesis, a Presidential Young Investigator Award from the National Science Foundation, a Young Investigator Award from the Office of Naval Research, and the Electronic Frontier Foundation's Pioneer Award (for work in electronic voting). He is a fellow of the IEEE and ACM.

The CAV Award is given annually for a specific fundamental contribution or a series of outstanding contributions to the field of computer-aided verification. CAV is the subdiscipline of computer science that is concerned with ensuring that software and hardware systems operate correctly and reliably. The award was established by the steering committee of the annual CAV conference and carries a cash prize of US$10,000. The first presentation was made at the annual CAV conference on July 10, 2008, in Princeton, New Jersey.

—Aarti Gupta, CAV Committee

MAA Awards

The Mathematical Association of America (MAA) presented several awards for excellence in expository writing and teaching at its Summer Mathfest in Madison, Wisconsin, July 31–August 2, 2008.

The Carl B. Allendoerfer Award is given for articles published in *Mathematics Magazine* and has a cash prize of US$500. The 2008 awardees are EUGENE BOMAN, Pennsylvania State University, RICHARD BRAZIER, Pennsylvania State University, and DEREK SEIPLE, University of Arizona, for their joint article "Mom! There's an asteroid in my closet!", *Mathematics Magazine*, April 2007; and CHRIS CHRISTENSEN, Northern Kentucky University, for the article "Polish mathematicians finding patterns in enigma messages", *Mathematics Magazine*, October 2007.

The Trevor Evans Award is given to authors of expository articles that are accessible to undergraduates and that were published in *Math Horizons*. The prize carries a cash award of US$250. The awardees for 2008 are William Dunham, Muhlenberg College, for his article "Euler's amicable numbers", *Math Horizons*, November 2007; and Robert K. Moniot, Fordham University, for his article "The taxman game", *Math Horizons*, February 2007.

The Lester R. Ford Award is given for articles published in the *American Mathematical Monthly* and has a cash award of US$500. The following authors were honored for 2008: Tom M. Apostol and Mamikon A. Mnatsakanian, both of the California Institute of Technology, for their joint article "Unwrapping curves from cylinders and cones", *American Mathematical Monthly*, May 2007; David Auckly, Kansas State University, for "Solving the quartic with a pencil", *Monthly*, January 2007; Andrew Cohen, University of Massachusetts, and Tanya Leise, Amherst College, for their joint article "Nonlinear oscillators at our fingertips", *Monthly,* January 2007; Thomas C. Hales, University of Pittsburgh, for "The Jordan curve theorem, formally and informally", *Monthly*, December 2007; and Katherine Socha, St. Mary's College of Maryland, for "Circles in circles: Creating a mathematical model of surface water waves", *Monthly*, March 2007.

The George Pólya Award is given for articles published in the *College Mathematics Journal.* It carries a cash award of US$500. The 2008 honorees are Roland Minton, Roanoke College, and Timothy J. Pennings, Hope College, for their joint article "Do dogs know bifurcations?", *College Mathematics Journal*, November 2007; and Andrew J. Simoson, King College, Bristol, Tennessee, for "Pursuit curves for the man in the moone," *College Mathematics Journal*, November 2007.

The Annie and John Selden Prize for Research in Undergraduate Mathematics Education honors a researcher who has established a significant record of published research in undergraduate mathematics education and who has been in the field for at most ten years. The awardee receives a cash prize of US$500. Marilyn Carlson of Arizona State University has been selected to receive the 2008 prize.

The Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member honors beginning college or university faculty members whose teaching has been extraordinarily successful and whose effectiveness in teaching undergraduate mathematics is shown to have had influence beyond their own classrooms. The prize carries a cash award of US$1,000. The prizes for 2008 were awarded to David Brown of Ithaca College; Jacqueline A. Jensen of Sam Houston State University; and Katherine Socha of St. Mary's College of Maryland.

—From an MAA announcement

Petrosyan Awarded Emil Artin Junior Prize

Nansen Petrosyan of the Catholic University of Leuven, Belgium, has been awarded the 2008 Emil Artin Junior Prize in Mathematics. Petrosyan was chosen for his paper "Jumps in cohomology and free group actions", published in the *Journal of Pure and Applied Algebra* **210** (2007), 695–703.

Established in 2001, the Emil Artin Junior Prize in Mathematics carries a cash award of US$1,000 and is presented usually every year to a student or former student of an Armenian university, who is under the age of thirty-five, for outstanding contributions to algebra, geometry, topology, and number theory—the fields in which Emil Artin made major contributions. The prize committee consisted of A. Basmajian, Y. Movsisyan, and V. Pambuccian.

—Artin Prize Committee announcement

Pi Mu Epsilon Student Paper Presentation Awards

Pi Mu Epsilon (PME), the U.S. honorary mathematics society, makes annual awards to recognize the best papers by undergraduate students presented at a PME student paper session. This year PME held a session in conjunction with the Mathematical Association of America MathFest in Madison, Wisconsin, July 30–August 2, 2008. Eight students were designated as 2008 AMS Award Winning Pi Mu Epsilon Student Speakers, each of whom received a check for US$150. Their names, institutions, and paper titles follow.

Samuel Behrend, Ohio Iota Chapter at Denison University, "Determining intrinsic trip linking in straight-edge embeddings of K_9"; Alicia Brinkman, Wisconsin Delta Chapter at Saint Norbert College, "How we roll: The theory and construction of the square wheel bicycle"; Iordan Ganev, Ohio Delta Chapter at Miami University, "Order dimension of subgroups"; Brendan Kelly, New Jersey Chapter at the College of New Jersey, "How to obtain algebraic information from zero-divisor graphs"; Daniel Lithio, Michigan Delta Chapter at Hope College, "Modeling dynamics of a volleyball serve: Choosing the optimal trajectory"; W. Ryan Livingston, Ohio Xi Chapter at Youngstown State University, "Can 2008 be the first digits of 2^n?"; Jared Ruiz, Ohio Xi Chapter at Youngstown State University, "A surprising sum of arctangents"; and Jeremy Thompson, Colorado Gamma Chapter at the United States Air Force Academy, "Numerical semigroups and Wilf's Conjecture".

—From a Pi Mu Epsilon announcement

B. H. Neumann Awards Given

The Board of the Australian Mathematics Trust has named the winners of the B. H. Neumann Awards for 2008. PHILIP SWEDOSH of St. Leonard's College has been a Victorian Certificate of Education (VCE) mathematics examiner and has served on the setting panel for specialist mathematics since 1997. He was also a group leader for the combined Department of Education and Municipal Association of Victoria (MAV) camp, which he directed from 2000 to 2002. He has been a moderator for the Mathematics Challenge for Young Australians since 1993 and Victorian Director of the Australian Mathematics Olympiad Committee since 1998. He was also a member of the organizing committee for the Melbourne Conference of the World Federation of National Mathematics Competitions in 2002.

BEN BURTON has been a tutor at the National Mathematics Summer School and also at the Australian Mathematical Olympiad training camps. He also trained a team of undergraduates at the University of Melbourne for the Association of Computing Machinery (ACM) computer programming competition. He was the inaugural and is still director of training for International Olympiad in Informatics (IOI).

STEVE THORNTON of the University of Canberra has served more than ten years as a member of the Mathematics Challenge for Young Australians Problems Committee. He has served various terms as president and secretary of the Australian Association of Mathematics Teachers and also held various offices with the Mathematics Association of South Australia and the Canberra Mathematics Association. He developed the Australian Mathematics Teacher Enrichment Program, which enabled qualified mathematics teachers to help students who wished to pursue further mathematics study.

The awards, named for Bernhard H. Neumann, are presented each year to mathematicians who have made important contributions over many years to the enrichment of mathematics learning in Australia and its region.

—From a Board of Mathematics Trust announcement

Royal Society of Canada Elections

The following mathematical scientists have been elected to the Royal Society of Canada: IVAR EKELAND, Pacific Institute for the Mathematical Sciences, University of British Columbia; PENGFEI GUAN, McGill University; RAYMOND LAFLAMME, Institute for Quantum Computing, University of Waterloo; and ECKHARD MEINRENKEN, University of Toronto. Chosen as a Specially Elected Fellow was AGNES M. HERZBERG, Queen's University.

—From a Royal Society of Canada announcement

Mathematics Opportunities

NSF Computing Equipment and Instrumentation Programs

The Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) plans a limited number of awards for the support of computing environments for research in the mathematical sciences. SCREMS (Scientific Computing Research Environments for the Mathematical Sciences) supports computing environments dedicated to research in the mathematical sciences. Proposals may request support for the purchase of computing equipment and limited support for professional systems administrators or programmer personnel for research computing needs. These grants are intended to support research projects of high quality that require access to advanced computing resources. Requests for routine upgrades of standard desk-environment workstations or laptop computers are not appropriate for this program. Awards are made to provide support for specific research projects rather than to provide general computing capacity. Proposers are encouraged to include projects involving symbolic and algebraic computations, numerical computations and simulations, and graphical representations (visualization) in aid of the research.

Please see http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5616 for details. The deadline for proposals is **January 22, 2009**.

—From an NSF announcement

Jefferson Science Fellows Program

The Jefferson Science Fellows (JSF) program at the U.S. Department of State is intended to involve the American academic science, technology, and engineering communities in the formulation and implementation of U.S. foreign policy.

Each fellow will spend one year at the U.S. Department of State for an on-site assignment in Washington, D.C., that may also involve extended stays at U.S. foreign embassies and/or missions. Each Fellow will receive a stipend of US$50,000. Following the fellowship year, the Jefferson Science Fellow will return to his or her academic career but will remain available to the U.S. Department of State for short-term projects over the following five years.

The JSF program is administered by the National Academies and supported through a partnership among the MacArthur Foundation; the Carnegie Corporation; the U.S. science, technology, and academic communities; professional scientific societies; and the U.S. Department of State. The deadline for applications is **January 15, 2009**. For further information, email jsf@nas.edu, telephone 202-334-2643, or see the website http://www7.nationalacademies.org/jefferson/.

—From a National Academies announcement

MfA Fellowship Program

The Math for America Foundation (MfA) sponsors the MfA Fellowship Program, which trains mathematically talented individuals to become high school mathematics teachers in New York City, Los Angeles, or San Diego. The fellowship provides an aggregate stipend of up to US$100,000 over five years, a full-tuition scholarship for a master's-level teaching program at one of MfA's partner universities, and ongoing support mechanisms, including mentoring and professional development.

Candidates should hold a bachelor's degree with substantial coursework in mathematics and should be able to demonstrate a strong interest in teaching. Applicants must be willing to commit to a five-year fellowship term in the chosen city. Individuals who are currently teaching or who are certified to teach are not eligible. Candidates must be U.S. citizens or permanent residents of the United States. The deadline for applications for New York City and Los Angeles is **February 13, 2009**; the deadline for applications for San Diego is **June 1, 2009**. For more detailed information, see the website at `http://www.mathforamerica.org/`.

—From an MfA announcement

CMI Liftoff Program for Summer 2009

The Clay Mathematics Institute (CMI) is currently accepting nominations for the 2009 Liftoff program. Through this program, CMI will employ recent Ph.D. recipients as Liftoff Fellows to carry out mathematics research for one month during the summer of 2009. This program provides a transition for young mathematicians from student to faculty member or to a postdoctoral position. Funds for travel to conferences or to visit collaborators are also available to Liftoff Fellows.

Nominations should be made by university mathematics departments; candidates may not apply directly. Criteria for selection are the quality and significance of mathematical research already achieved by the candidate and the potential of the candidate to become a leader in mathematical research.

Nomination packets should include: (1) a cover letter signed by the department chair; (2) two letters of recommendation, including one from the thesis supervisor (existing letters of recommendation already written for job applications can be used); (3) a CV from the nominee, including name, address, telephone, email, date of birth, citizenship, education, thesis title, honors, previous employment, reference to published work or submitted articles, and proposed research; and (4) a one-sentence signed statement from a mathematician agreeing to supervise the nominee on behalf of CMI, with the proposed dates of employment.

Nominations can be sent electronically to the attention of Amanda Battese at `liftoff@claymath.org` or by mail to Clay Mathematics Institute, One Bow Street, Cambridge, MA 02138. The deadline for nominations to be received is **February 15, 2009**. For more information, see the website `http://claymath.org/fas/liftoff`; telephone: 617-995-2600.

—From a CMI announcement

National Academies Christine Mirzayan Graduate Fellowship Program

The Christine Mirzayan Science and Technology Policy Graduate Fellowship Program of the National Academies is designed to engage graduate science, engineering, medical, veterinary, business, and law students in the analysis and creation of science and technology policy and to familiarize them with the interactions of science, technology, and government. As a result, students develop essential skills different from those attained in academia and make the transition from graduate student to professional.

Applications for the fellowships are invited from scholars from graduate through postdoctoral levels in any physical, biological, or social science field or any field of engineering, medicine and health, or veterinary medicine, as well as business, law, education, and other graduate and professional programs. Postdoctoral scholars should have received their Ph.D.'s within the past five years.

The stipend for each 10-week program is US$5,300. The fellowship stipend is intended to cover all living expenses for the period.

Deadlines for receipt of materials for the June program is **March 1, 2009**; for the September program, **June 1, 2009**; and for the January program, **November 1, 2009**. More information and application forms and instructions can be found on the website `http://www7.nationalacademies.org/policyfellows` or by contacting The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: `policyfellows@nas.edu`.

—From a National Academies announcement

Call for Nominations for Waterman Award

Congress established the Alan T. Waterman Award in August 1975 to mark the twenty-fifth anniversary of the National Science Foundation (NSF) and to honor its first director. The annual award recognizes an outstanding young researcher in any field of science or engineering supported by the NSF. In addition to a medal, the awardee receives a grant of US$500,000 over a three-year period for scientific research or advanced study in the mathematical, physical, medical, biological, engineering, social, or other sciences at the institution of the recipient's choice.

Candidates must be U.S. citizens or permanent residents and must be thirty-five years of age or younger or not more than seven years beyond receipt of the Ph.D. degree by December 31 of the year in which they are nominated. Candidates should have demonstrated exceptional individual achievements in scientific or engineering research of sufficient quality to place them at the forefront of their peers. Criteria include originality, innovation, and significant impact on the field.

The deadline for nominations and all supporting material for the award is **December 5, 2008**. For more information, see the website `http://www.nsf.gov/od/waterman/waterman.jsp`.

—From an NSF announcement

Call for Nominations for Ostrowski Prize

The aim of the Ostrowski Foundation is to promote the science of mathematics by periodically awarding an international prize for the best performances in the field of pure mathematics and of the theoretical foundations of numerical mathematics. As a rule, the prize is awarded every two years to the scientist, or group of scientists, who, during the preceding five years, has achieved the highest scientific accomplishments in these fields. It is awarded independently of politics, race, religion, domicile, nationality, or age. The prize in 2007 amounted to 100,000 Swiss francs (approximately US$92,000).

The foundation awards at the same time a scholarship for a talented young mathematician, whose name is to be suggested by the current prizewinners. The scholarship will enable the winner to spend a year of further education (as a postdoctoral fellow) at a university of his or her own choice.

The previous prizewinners are, in chronological order: L. de Branges; J. Bourgain; M. Ratner and M. Laczkovich; A. Wiles; Y. Nesterenko and G. Pisier; A. Beilinson and H. Hofer; H. Iwaniec, P. Sarnak and R. Taylor; P. Seymour; B. Green and T. Tao; and O. Schramm.

The jury invites proposals for candidates for the Ostrowski Prize 2009. The proposals, including a short justification, should be sent to `David.Masser@unibas.ch` before **February 1, 2009**.

—David Masser, University of Basel

News from IPAM

The Institute for Pure and Applied Mathematics (IPAM), located at the University of California, Los Angeles, holds long- and short-term research programs and workshops throughout the academic year for junior and senior mathematicians and scientists who work in academia, the national laboratories, and industry. IPAM also sponsors two summer programs. IPAM's upcoming programs are listed below. Please go to `http://www.ipam.ucla.edu` for detailed information and to find online application and registration forms. IPAM's Science Advisory Board meets in November, when it considers program proposals. Program proposals from the community are encouraged; instructions are available at our website.

IPAM is seeking a new associate director to begin in July 2009. Information about the position and how to apply is available on our website.

Winter 2009 Short Programs. You may apply for support or register for each workshop online.

• *Quantitative and Computational Aspects of Metric Geometry*. January 12–16, 2009.

• *Numerical Approaches to Quantum Many-Body Systems*. January 22–30, 2009.

• *Laplacian Eigenvalues and Eigenfunctions: Theory, Computation, Application*. February 9–13, 2009.

• *Rare Events in High-Dimensional Systems*. February 23–27, 2009.

Quantum and Kinetic Transport Equations: Analysis, Computations, and New Applications, March 9–June 12, 2009. This long program includes the following workshops that are also open for participation. You may apply online for support to be a core participant for the entire program or to attend individual workshops.

• *Tutorials*. March 10–13, 2009.

• *Workshop I: Computational Kinetic Transport and Hybrid Methods*. March 30–April 3, 2009.

• *Workshop II: The Boltzmann Equation: DiPerna-Lions Plus 20 Years*. April 15–17, 2009.

• *Workshop III: Flows and Networks in Complex Media*. April 27–May 1, 2009.

• *Workshop IV: Asymptotic Methods for Dissipative Particle Systems*. May 18–22, 2009.

Research in Industrial Projects for Students (RIPS) 2009. June 21–August 21, 2009. This undergraduate summer research program matches student teams with industrial projects sponsored by industry. Applications are due **February 15, 2009**.

Combinatorics: Methods and Applications in Mathematics and Computer Science. September 8–December 11, 2009. This long program includes the following workshops that are also open for participation. You may apply online for support to be a core participant for the entire program or to attend individual workshops.

• *Tutorials*. September 9–16, 2009.

• *Workshop I: Probabilistic Techniques and Applications*. October 5–9, 2009.

• *Workshop II: Combinatorial Geometry*. October 19–23, 2009.

• *Workshop III: Topics in Graphs and Hypergraphs*. November 2–6, 2009.

• *Workshop IV: Analytical Methods in Combinatorics, Additive Number Theory, and Computer Science*. November 16–20, 2009.

Model and Data Hierarchies for Simulating and Understanding Climate, March 8–June 11, 2010. This long program includes tutorials and four workshops that are also open for participation. You may apply online for support to be a core participant for the entire program or to attend individual workshops.

—From an IPAM announcement

Inside the AMS

Trjitzinsky Memorial Awards Presented

The AMS has made awards to seven undergraduate students through the Waldemar J. Trjitzinsky Memorial Fund. The fund is made possible by a bequest from the estate of Waldemar J., Barbara G., and Juliette Trjitzinsky. The will of Barbara Trjitzinsky stipulates that the income from the bequest should be used to establish a fund in honor of the memory of her husband to assist needy students in mathematics.

For the 2008 awards, the AMS chose seven geographically distributed schools to receive one-time awards of US$3,000 each. The mathematics departments at those schools then chose students to receive the funds to assist them in pursuit of careers in mathematics. The schools are selected in a random drawing from the pool of AMS institutional members.

Waldemar J. Trjitzinsky was born in Russia in 1901 and received his doctorate from the University of California, Berkeley, in 1926. He taught at a number of institutions before taking a position at the University of Illinois, Urbana-Champaign, where he remained for the rest of his professional life. He showed particular concern for students of mathematics and in some cases made personal efforts to ensure that financial considerations would not hinder their studies. Trjitzinsky was the author of about sixty mathematics papers, primarily on quasi-analytic functions and partial differential equations. A member of the AMS for forty-six years, he died in 1973.

Following are the names of the selected schools for 2008, the names of the students receiving Trjitzinsky awards, and brief biographical sketches of the students.

College of Staten Island, City University of New York: Joseph Zancocchio. Zancocchio's mathematical interests include prime numbers, cryptography, and Diophantine equations. He also loves epistemological, logical, and metaphysical philosophy and their relation to underlying mathematical branches. He enjoys studying the history of mathematics. He says "math is the backbone of all science, and for that reason I feel like knowledge of math opens my mind to the knowledge contained in all scientific data."

Georgia Southern University: Phillip D. Lorren. Lorren lives in Snellville, Georgia, and graduated from South Gwinnett High School, where he took numerous honors and advanced placement classes and served as a mathematics tutor to other students. At Georgia Southern he is active in community service, including tutoring in an adult literacy program, and has served as an Honors Ambassador to promote the University Honors Program. He is active in the student chapter of the Mathematical Association of America and participated on the Math Jeopardy Team at the 2008 MAA Southeastern Section meeting. He plans to graduate with a math major in 2010.

Humboldt State University: Hans Parshall. Parshall lives in Eureka, California. He graduated from Arcata High School, Arcata, California, and enrolled in the College of the Redwoods. He took every mathematics course offered in the curriculum and frequently participated in extracurricular problem-solving sessions known as "Pizza & Problems". He entered the Putnam Competition in December 2007 and earned a score of 10, ranking him in the top third of participants. He looks forward to furthering his study of mathematics at Humboldt State University.

Ithaca College: Daksha Shakya. Shakya is a native of Nepal. She is a consistent Dean's List student and has been inducted into two national honorary societies. She enjoys sharing ideas and insights and often helps and works with other students with academic and personal issues. During the summer of 2008 she held a Dana Internship working with a faculty member on the Prisoner Express Program and teaching math to interested members of the Ithaca community.

Luther College. Aaron Peterson. Peterson, a senior, lives in Lindstrom, Minnesota. He began college as a music major but switched to mathematics after taking courses in linear algebra and discrete mathematics. He is currently co-president of the Luther College math club and competes in numerous intercollegiate math competitions. He has held undergraduate research fellowships at the University of Iowa and Texas A&M University and has published a research article. He plans to pursue a Ph.D. degree in mathematics and a career in postsecondary education.

University of Wisconsin, Milwaukee: Amanda J. Mueller. Mueller lives in Waukesha, Wisconsin, and is carrying a double major in mathematics and physics at the university. She graduated from Immanuel Lutheran High School in Eau Claire, Wisconsin, where she was awarded the HEAB Academic Excellence Scholarship. She has worked as a tutor and is currently employed by a developer of computer forensics technology. She enjoys computer programming, singing, theater, and playing piano.

Wright State University: Faith L. Buell. Buell is a first-generation college student working toward a bachelor's degree in mathematics. She was elected to the National Honor Society in high school and is in the honors program at Wright State. She volunteers at food pantries and is involved in many church activities, as well as assisting in tutoring elementary school children in the Dayton area.

—Elaine Kehoe

Epsilon Scholarships Awarded for 2008

The AMS has awarded nine scholarships to students attending programs for mathematically talented high school students held during the summer of 2008. Six students received Ky and Yu-Fen Fan Scholarships, one received a Roderick P. Caldwell Scholarship, and two received Robert H. Oehmke Scholarships.

The Fan Scholarships were awarded to: Zev Rosengarten to attend the Ross Mathematics Program at Ohio State University; Cedric Strickland for the Texas State University Honors Summer Math Camp in San Marcos, Texas; Seth B. Kleinschmidt for the Michigan Math and Science Scholars Summer Program at the University of Michigan, Ann Arbor; Katrina Biele to attend PROMYS (Program in Mathematics for Young Scholars) at Boston University; Tala Huhe for HCSSiM (Hampshire College Summer Studies in Mathematics) in Amherst, Massachusetts; and Kathleen Zhou for MathPath at the University of Vermont, Burlington.

Matej Penciak was awarded the Caldwell Scholarship to attend the Ross Mathematics Program.

The Oehmke Scholarships were awarded to Erin Roach for the Michigan Math and Science Scholars Summer Program and Elizabeth Simon for HCSSiM.

The Epsilon Scholarships are supported by the Ky and Yu-Fen Fan Endowment; by a gift from the Robert H. Oehmke Charitable Fund; and by a gift from Winifred A. Caldwell in memory of her husband, Roderick P. C. Caldwell. For more information on the Epsilon program, see `http://www.ams.org/development/epsilon.html`.

—AMS announcement

From the AMS Public Awareness Office

• **This Mathematical Month—monthly postings of vignettes on people, publications, and mathematics to inform and entertain**. Read about events that occurred in the month of December: Henri Poincaré wrote a letter acknowledging a serious problem in his work on the *n*-body problem, John Nash received the Nobel Prize in Economics Sciences, the first meeting of the Bourbaki group was held, and more, at `http://www.ams.org/ams/thismathmonth-dec.html`.

• **Headlines & Deadlines**. AMS members may sign up to receive twice-monthly email notifications of news and announcements about programs, publications, and events, as well as alerts about deadlines for fellowship and grant applications, calls for proposals, and meeting registrations. The service provides a convenient way to have news that is posted on the AMS page—and some announcements and deadlines before they appear in the *Notices*—delivered directly to you. It's easy to subscribe (and unsubscribe), at `http://www.ams.org/enews`.

• ***Math in the Media***. Explore the current month and archive at `http://www.ams.org/mathmedia/`, including "Tony Phillips' Take on Math in the Media"—in which Phillips (Stony Brook University) offers his perspective on items that have appeared in the media—and "Math Digest", which provides summaries of media coverage of math, edited by Allyn Jackson (deputy editor of the *Notices*). Recent stories in the media were the mountain peak in Colorado named after mathematician Donald Clayton Spencer, the Antikythera Mechanism, girls and math, the "Water Cube"—the National Aquatics Center for the Olympics held in Beijing, to name just a few. *Math in the Media* also includes "Reviews"—links to hundreds of reviews of books, plays, and films. The AMS Public Awareness Office invites members to send links for articles in their local newspapers and on local radio programs related to mathematics to `paoffice@ams.org`.

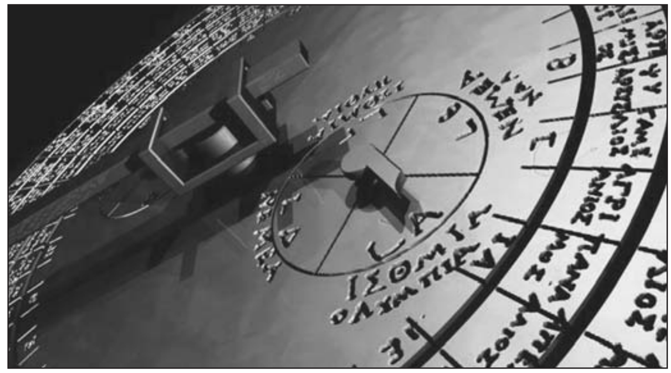

Image ©2008 Tony Freeth and the Antikythera Mechanism Research Project (http://www.antikythera-mechanism.gr/).

—Annette Emerson and Mike Breen, AMS Public Awareness Officers `paoffice@ams.org`

Deaths of AMS Members

Kenneth M. Hoffman, professor emeritus, MIT, died on September 29, 2008. Born on November 30, 1930, he was a member of the Society for 53 years.

Ronald G. Mosier, retired, from Daimler Chrysler, died on September 13, 2008. Born on May 17, 1938, he was a member of the Society for 33 years.

Marcia P. Sward, former executive director of the MAA, died on September 21, 2008. Born on February 1, 1939, she was a member of the Society for 40 years.

William C. Swift, from Crawfordsville, IN, died on September 11, 2008. Born on March 17, 1928, he was a member of the Society for 30 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.ou.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 405-325-7484 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

November 14, 2008: Applications for NRC-Ford Foundation Predoctoral Fellowships. See `http://www7.nationalacademies.org/FORDfellowships/` or contact Fellowships Office, Keck 576, National Research Council, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2872; email: `infofell@nas.edu`.

November 15, 2008: Target date for receipt of applications for NSA Mathematics Sabbatical Program. See `http://www.nsa.gov/msp/index.cfm` or contact the program director, Michelle Wagner (`mdwagn4@nsa.gov`), or the program administrator,

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

AMS Bylaws—*November 2007, p. 1366*

AMS Email Addresses—*February 2008, p. 274*

AMS Ethical Guidelines—*June/July 2006, p. 701*

AMS Officers 2006 and 2007 Updates—*May 2008, p. 629*

AMS Officers and Committee Members—*October 2008, p. 1122*

Conference Board of the Mathematical Sciences—*September 2008, p. 980*

IMU Executive Committee—*December 2008, p. 1441*

Information for *Notices* Authors—*June/July 2008, p. 723*

Mathematics Research Institutes Contact Information—*August 2008, p. 844*

National Science Board—*January 2008, p. 69*

New Journals for 2006, 2007—*June/July 2008, p. 725*

NRC Board on Mathematical Sciences and Their Applications—*March 2008, p. 401*

NRC Mathematical Sciences Education Board—*April 2008, p. 515*

NSF Mathematical and Physical Sciences Advisory Committee—*February 2008, p. 276*

Program Officers for Federal Funding Agencies—*October 2008, p. 1116 (DoD, DoE); December 2007, p. 1359 (NSF); December 2008, p. 1440 (NSF Mathematics Education)*

Program Officers for NSF Division of Mathematical Sciences—*November 2008, p. 1297*

Stipends for Study and Travel—*September 2008, p. 983*

Barbara Johnson (bajohn1@nsa.gov), telephone 301-688-0400.

November 28, 2008: Applications for NRC-Ford Foundation Dissertation and Postdoctoral Fellowships. See http://www7.nationalacademies.org/FORDfellowships/ or contact Fellowships Office, Keck 576, National Research Council, 500 Fifth Street, NW, Washington, DC 20001; telephone: 202-334-2872; email: infofell@nas.edu.

December 1, 2008: Applications for AMS Centennial Fellowships. See http://www.ams.org/employment/centflyer.html; write to the Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; send email to prof-serv@ams.org; or call 401-455-4060.

December 5, 2008: Nominations for the NSF Waterman Award. See "Mathematics Opportunities" in this issue.

December 9, 2008: Applications for East Asia and Pacific Summer Institutes (EAPSI) program. See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5284.

December 15, 2008: Applications for AMS Epsilon Fund grants. See http://www.ams.org/outreach/epsilon.html or contact Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294; telephone: 800-321-4267, ext. 4170; email: prof-serv@ams.org.

January 10, 2009: Applications for AAUW Educational Foundation Fellowships and Grants. See http://www.aauw.org/fga/fellowships_grants/selected.cfm or contact the AAUW Educational Foundation, Selected Professions Fellowships, Dept. 60, 301 ACT Drive, Iowa City, IA 52243-4030; telephone: 319-337-1716, ext. 60; email: aauw@act.org.

January 15, 2009: Applications for Jefferson Science Program at U.S. Department of State. See "Mathematics Opportunities" in this issue.

January 15, 2009: Applications for AMS-AAAS Mass Media Summer Fellowships. See http://www.aaas.org/programs/education/MassMedia/; or contact Stacey Pasco, Manager, Mass Media Program, AAAS Mass Media Science and Engineering Fellows Program, 1200 New York Avenue, NW, Washington, DC 20005; telephone: 202-326-6441; fax: 202-371-9849; email: spasco@aaas.org. Also see http://www.ams.org/government/massmediaann.html or contact the AMS Washington Office, 1527 Eighteenth Street, NW, Washington, DC 20036; telephone: 202-588-1100; fax: 202-588-1853; email: amsdc@ams.org.

January 22, 2009: Proposals for NSF Scientific Computing Research Environments for the Mathematical Sciences (SCREMS). See "Mathematics Opportunities" in this issue.

January 31, 2009: Applications for AMS-AAAS Congressional Fellowship. See http://www.ams.org/government/congressfellowann.html or contact the AMS Washington office at 202-588-1100, email: amsdc@ams.org.

February 1, 2009: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.edu. The postal address is: Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

February 13, 2009: Applications for Math for America Foundation (MfA) Fellowship Program for New York City and Los Angeles. See "Mathematics Opportunities" in this issue.

February 15, 2009: Nominations for Clay Liftoff Program for summer 2009. See "Mathematics Opportunities" in this issue.

February 27, 2009: Submissions for Association for Women in Mathematics (AWM) essay contest. See http://www.awm-math.org/biographies/contest.html.

February 27, 2009: Proposals for DMS New Institute Competition. See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=5302.

March 1, 2009: Applications for the June program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship Program of the National Academies. See "Mathematics Opportunities" in this issue.

March 2, 2009: Applications for EDGE Summer Program. See http://www.edgeforwomen.org/?page_id=5.

April 15, 2009: Applications for fall 2009 semester of Math in Moscow. See http://www.mccme.ru/mathinmoscow or write to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; e-mail: mim@mccme.ru. For information on AMS scholarships see http://www.ams.org/outreach/mimoscow.html or write to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email: student-serv@ams.org.

May 8, 2009: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.edu. The postal address is: Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

June 1, 2009: Applications for the September program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship Program of the National Academies. See "Mathematics Opportunities" in this issue.

June 1, 2009: Applications for the Math for America Foundation (MfA) Fellowship Program in San Diego. See "Mathematics Opportunities" in this issue.

June 2, 2009: Proposals for NSF's Enhancing the Mathematical Sciences Workforce in the Twenty-First Century program. See http://www.nsf.gov/publications/pub_summ.jsp?ods_key=nsf05595.

October 1, 2009: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.edu. The postal address is: Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

November 1, 2009: Applications for the January program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship Program of the National Academies. See "Mathematics Opportunities" in this issue.

NSF Mathematics Education Staff

The Directorate for Education and Human Resources (EHR) of the National Science Foundation (NSF) sponsors a range of programs that support educational projects in mathematics, science, and engineering. Listed below is contact information for those EHR program officers whose fields are in the mathematical sciences or mathematics education. These individuals can provide information about the programs they oversee, as well as information about other EHR programs of interest to mathematicians. The postal address is: Directorate for Education and Human Resources, National Science Foundation, 4201 Wilson Boulevard, Arlington, VA 22230. The EHR web page is `http://www.nsf.gov/dir/index.jsp?org=EHR`.

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Lee Zia
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Office of the Director/Office of Integrative Activities

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`jhlightb@nsf.gov`

IMU Executive Committee

The Executive Committee of the International Mathematical Union (IMU) consists of ten voting members elected for four-year terms: the four officers (president, two vice presidents, and secretary) and six other members. The retiring president is an ex-officio member of the Executive Committee without vote for a period of four years. The current members (terms January 1, 2007, to December 31, 2010) of the IMU Executive Committee are:

President:
László Lovász (Hungary)

Secretary:
Martin Grötschel (Germany)

Vice Presidents:
Zhi-Ming Ma (China),
Claudio Procesi (Italy)

Members at Large:
M. Salah Baouendi (USA),
Manuel de León (Spain),
Ragni Piene (Norway),
Cheryl E. Praeger (Australia),
Victor A. Vassiliev (Russia),
Marcelo Viana (Brazil)

Ex-Officio:
John M. Ball, Past President
(United Kingdom)

Book List

The Book List highlights books that have mathematical themwes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* `notices-booklist@ams.org`.

*Added to "Book List" since the list's last appearance.

An Abundance of Katherines, by John Green. Dutton Juvenile Books, September 2006. ISBN-13:978-0-5254-7688-7. (Reviewed October 2008.)

Amongst Mathematicians: Teaching and Learning Mathematics at University Level, by Elena Nardi. Springer, November 2007. ISBN-13: 978-0-387-37141-2.

The Archimedes Codex, by Reviel Netz and William Noel. Weidenfeld and Nicolson, May 2007. ISBN-13: 978-0-29764-547-4. (Reviewed September 2008.)

The Book of Numbers: The Secret of Numbers and How They Changed the World, by Peter J. Bentley. Firefly Books, February 2008. ISBN-13: 978-15540-736-10.

A Certain Ambiguity: A Mathematical Novel, by Gaurav Suri and Hartosh Singh Bal. Princeton University Press, June 2007. ISBN-13: 978-0-6911-2709-5. (Reviewed February 2008.)

Descartes: A Biography, by Desmond Clarke. Cambridge University Press, March 2006. ISBN 0-521-82301-3. (Reviewed January 2008.)

Digital Dice, by Paul J. Nahin. Princeton University Press, March 2008. ISBN-13: 978-06911-269-82.

Dimensions, by Jos Leys, Etienne Ghys, and Aurélien Alvarez. DVD, 117 minutes. Available at `http://www.dimensions-math.org`.

Discovering Patterns in Mathematics and Poetry, by Marcia Birken and Anne C. Coon. Rodopi, February 2008. ISBN-13: 978-9-0420-2370-3.

Does Measurement Measure Up?: How Numbers Reveal and Conceal the Truth, by John Henshaw. Johns Hopkins University Press, March 2006. ISBN-13: 978-0-8018-8375-0.

Einstein's Mistakes: The Human Failings of Genius, by Hans C. Ohanian. W. W. Norton, September 2008. ISBN-13: 978-0393062939.

Euclidean and Non-Euclidean Geometries: Development and History, fourth revised and expanded edition, by Marvin Jay Greenberg. W. H. Freeman, September 2007. ISBN-13: 978-0-7167-9948-1.

**Fighting Terror Online: the Convergence of Security, Technology and the Law*, by Martin Charles Golumbic. Springer, 2008. ISBN: 978-0-387-73577-1.

Fly Me to the Moon: An Insider's Guide to the New Science of Space Travel, by Edward Belbruno. Princeton University Press, January 2007. ISBN-13: 978-0-6911-2822-1. (Reviewed April 2008.)

Geekspeak: How Life + Mathematics = Happiness, by Graham Tattersall. Collins, September 2008. ISBN-13: 978-00616-292-42.

Geometric Folding Algorithms: Linkages, Origami, Polyhedra, by Erik D. Demaine and Joseph O'Rourke. Cambridge University Press, July 2007. ISBN-13: 978-05218-57574.

The Golden Section: Nature's Greatest Secret (Wooden Books), by Scott Olsen. Walker and Company, October 2006. ISBN-13: 978-08027-153-95.

Group Theory in the Bedroom, and Other Mathematical Diversions, by Brian Hayes. Hill and Wang, April 2008. ISBN-13: 978-0-8090-5219-6.

Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin, by Lawrence Weinstein and John A. Adam. Princeton University Press, April 2008. ISBN-13: 978-0-6911-2949-5.

**Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi: Martin Gardner's First Book of Mathematical Puzzles and Games*, by Martin Gardner. Cambridge University Press, September 2008. ISBN-13: 978-0-521-73525-4.

A History of Abstract Algebra, by Israel Kleiner. Birkhäuser, October 2007. ISBN-13: 978-0-8176-4684-4.

How Round Is Your Circle, by John Bryant and Chris Sangwin. Princeton University Press, January 2008. ISBN-13: 978-0-6911-3118-4.

Impossible?: Surprising Solutions to Counterintuitive Conundrums, by Julian Havil. Princeton University Press, April 2008. ISBN-13: 978-0-6911-3131-3.

The Indian Clerk, by David Leavitt. Bloomsbury USA, September 2007. ISBN-13: 978-15969-1040-9. (Reviewed September 2008.)

Irreligion: A Mathematician Explains Why the Arguments for God Just Don't Add Up, by John Allen Paulos. Hill and Wang, December 2007. ISBN-13: 978-0-8090-591-95. (Reviewed August 2008.)

Kiss My Math: Showing Pre-Algebra Who's Boss, by Danica McKellar. Hudson Street Press, August 2008. ISBN-13: 978-1594630491.

The Last Theorem, by Arthur C. Clarke and Frederik Pohl. Del Rey, August 2008. ISBN-13: 978-0345470218.

The Legacy of Mario Pieri in Geometry and Arithmetic, by Elena Anne Marchisotto and James T. Smith. Birkhäuser, May 2007. ISBN-13: 978-0-8176-3210-6.

Leonhard Euler, A Man to Be Reckoned With, by Andreas K. Heyne and Alice K. Heyne. Birkhäuser, 2007. ISBN-13: 978-3-7643-8332-9. (Reviewed March 2008.)

Logic's Lost Genius: The Life of Gerhard Gentzen, by Eckart Menzler-Trott, Craig Smorynski (translator), Edward R. Griffor (translator). AMS-LMS, November 2007. ISBN-13: 978-0-8218-3550-0.

Making Mathematics Work with Needlework: Ten Papers and Ten Projects, edited by Sarah-Marie Belcastro and Carolyn Yackel. A K Peters, September 2007. ISBN-13: 978-1-5688-1331-8.

Mathematical Mind-Benders, by Peter Winkler. A K Peters, August 2007. ISBN-13: 978-1-5688-1336-3.

Mathematical Omnibus: Thirty Lectures on Classic Mathematics, by Dmitry Fuchs and Serge Tabachnikov. AMS, October 2007. ISBN-13: 978-08218-431-61. (Reviewed in this issue.)

The Mathematician's Brain, by David Ruelle. Princeton University Press, July 2007. ISBN-13 978-0-691-12982-2. (Reviewed November 2008.)

Mathematics and Democracy: Designing Better Voting and Fair-Division Procedures, by Steven J. Brams. Princeton University Press, December 2007. ISBN-13: 978-0691-1332-01.

Mathematics at Berkeley: A History, by Calvin C. Moore. A K Peters, February 2007. ISBN-13: 978-1-5688-1302-8. (Reviewed November 2008.)

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook, by Victor J. Katz et al. Princeton University Press, July 2007. ISBN-13: 978-0-6911-2745-3.

Measuring the World, by Daniel Kehlmann. Pantheon, November 2006. ISBN 0-375-42446-6. (Reviewed June/July 2008.)

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell, 2002. ISBN 0-943396743.

More Sex Is Safer Sex: The Unconventional Wisdom of Economics, by Steven E. Landsburg. Free Press, April 2007. ISBN-13: 978-1-416-53221-7. (Reviewed June/July 2008.)

Number Story: From Counting to Cryptography, by Peter M. Higgins. Springer, February 2008. ISBN-13: 978-1-8480-0000-1

One to Nine: The Inner Life of Numbers, by Andrew Hodges. W. W. Norton, May 2008. ISBN-13: 978-03930-664-18.

**Origami, Eleusis, and the Soma Cube: Martin Gardner's Mathematical Diversions*, by Martin Gardner. Cambridge University Press, September 2008. ISBN-13: 978-0-521-73524-7.

A Passion for Discovery, by Peter Freund. World Scientific, August 2007. ISBN-13: 978-9-8127-7214-5.

Perfect Figures: The Lore of Numbers and How We Learned to Count, by Bunny Crumpacker. Thomas Dunne Books, August 2007. ISBN-13: 978-0-3123-6005-4.

The Poincaré Conjecture: In Search of the Shape of the Universe, by Donal O'Shea. Walker, March 2007. ISBN-13: 978-0-8027-1532-6. (Reviewed January 2008.)

Poincaré's Prize: The Hundred-Year Quest to Solve One of Math's Greatest Puzzles, by George Szpiro. Dutton Adult, June 2007. ISBN-13: 978-0-525-95024-0. (Reviewed January 2008.)

The Presidential Election Game, by Steven J. Brams. A K Peters, December 2007. ISBN-13: 978-1-5688-1348-6.

The Princeton Companion of Mathematics, edited by Timothy Gowers (June Barrow-Green and Imre Leader, associate editors). Princeton University Press, November 2008. ISBN-13: 978-06911-188-02.

The Probability of God: A Simple Calculation That Proves the Ultimate Truth, by Stephen D. Unwin. Three Rivers Press, October 2004. ISBN-13: 978-1-4000-5478-7. (Reviewed February 2008.)

**Professor Stewart's Cabinet of Mathematical Curiosities*, by Ian Stewart. Basic Books, December 2008. ISBN-13: 978-0-465-01302-9.

Pursuit of Genius: Flexner, Einstein, and the Early Faculty at the Institute for Advanced Study, by Steve Batterson. A K Peters, June 2006. ISBN 1-56881-259-0. (Reviewed August 2008.)

Pythagorean Crimes, by Tefcros Michalides. Parmenides Publishing, September 2008. ISBN-13: 978-1930972278.

Random Curves: Journeys of a Mathematician, by Neal Koblitz. Springer, December 2007. ISBN-13: 978-3-5407-4077-3.

**Reminiscences of a Statistician: The Company I Kept*, by Erich Lehmann. Springer, November 2007. ISBN-13: 978-0-387-71596-4.

Roots to Research: A Vertical Development of Mathematical Problems, by Judith D. Sally and Paul J. Sally Jr. AMS, November 2007. ISBN-13: 978-08218-440-38. (Reviewed in this issue.)

Sacred Mathematics: Japanese Temple Geometry, by Fukagawa Hidetoshi and Tony Rothman. Princeton University Press, July 2008. ISBN-13: 978-0-6911-2745-3.

Super Crunchers: Why Thinking-by-Numbers Is the New Way to Be Smart, by Ian Ayres. Bantam, August 2007. ISBN-13: 978-0-5538-0540-6.

Superior Beings: If They Exist, How Would We Know? Game-Theoretic Implications of Omnipotence, Omniscience, Immortality, and Incomprehensibility, by Steven Brams. Springer, second edition, November 2007. ISBN-13: 978-0-387-48065-7. (Reviewed February 2008.)

The Symmetries of Things, by John H. Conway, Heidi Burgiel, and Chaim Goodman-Strauss. A K Peters, May 2008. ISBN-13: 978-1-5688-1220-5.

Symmetry: A Journey into the Patterns of Nature, by Marcus du Sautoy. Harper, March 2008. ISBN-13: 978-0-0607-8940-4.

Symmetry: The Ordering Principle (Wooden Books), by David Wade. Walker and Company, October 2006. ISBN-13: 978-08027-153-88.

Tools of American Math Teaching, 1800–2000, by Peggy Aldrich Kidwell, Amy Ackerberg-Hastings, and David Lindsay Roberts. Johns Hopkins University Press, July 2008. ISBN-13: 978-0801888144.

The Unfinished Game: Pascal, Fermat, and the Seventeenth-Century Letter That Made the World Modern, by Keith Devlin. Basic Books, September 2008. ISBN-13: 978-0-4650-0910-7.

Unknown Quantity: A Real and Imaginary History of Algebra, by John Derbyshire. Joseph Henry Press, May 2006. ISBN 0-309-09657-X. (Reviewed May 2008.)

Useless Arithmetic: Why Environmental Scientists Can't Predict the Future, by Orrin Pilkey and Linda Pilkey-Jarvis. Columbia University Press, February 2007. ISBN 0-231-13212-3. (Reviewed April 2008.)

The Volterra Chronicles: The Life and Times of an Extraordinary Mathematician, by Judith R. Goodstein. AMS, February 2007. ISBN-13: 978-0-8218-3969-0. (Reviewed March 2008.)

The Wraparound Universe, by Jean-Pierre Luminet. A K Peters, March 2008. ISBN-13: 978-1-5688-1309-7. (Reviewed in this issue.)

Zeno's Paradox: Unraveling the Ancient Mystery behind the Science of Space and Time, by Joseph Mazur. Plume, March 2008 (reprint edition). ISBN-13: 978-0-4522-8917-8.

REFERENCE & BOOK LIST

Mathematics Calendar

Please submit conference information for the Mathematics Calendar through the Mathematics Calendar submission form at http://www.ams.org/cgi-bin/mathcal-submit.pl. The most comprehensive and up-to-date Mathematics Calendar information is available on the AMS website at http://www.ams.org/mathcal/.

December 2008

1–4 **SGT-in-Rio: Workshop on Spectral Graph theory with applications on Computer Science, Combinatorial Optimization and Chemistry**, Military Institute of Engineering, Rio de Janeiro, Brazil. (May 2008, p. 635)

1–5 **Nonnegative Matrix Theory: Generalizations and Applications**, American Institute of Mathematics, Palo Alto, California. (Jun./Jul. 2008, p. 740)

5–8 **International Conference on Partial Differential Equations and Applications in honour of Professor Philippe G. Ciarlet's 70th birthday**, City University of Hong Kong, Kowloon, Hong Kong. (Aug. 2008, p. 870)

8–12 **FEMTEC 2008 (Finite Element Methods in Engineering and Science)**, University of Texas at El Paso, Texas. (Jun./Jul. 2008, p. 740)

8–12 **Small Ball Inequalities in Analysis, Probability and Irregularities of Distribution**, American Institute of Mathematics, Palo Alto, California. (May 2008, p. 635)

8–22 **Algebraic Topology, Braids and Mapping Class Groups**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Jun./Jul. 2008, p. 740)

* 10–12 **MACIS 2008 – Third International Conference on Mathematical Aspects of Computer and Information Sciences**, Buenos Aires, Argentina.
Description: Mathematical Aspects of Computer and Information Sciences (MACIS) is a new series of conferences where foundational research on theoretical and practical problems of mathematics for computing and information processing may be presented and discussed. MACIS also addresses experimental and case studies, scientific and engineering computation, design and implementation of algorithms and software systems, and applications of mathematical methods and tools to outstanding and emerging problems in applied computer and information sciences. Each conference focuses on two or three themes. MACIS 2008 themes: (1) Complex Systems and Networks (2) Certified and Reliable Computation (3) Computational Algebraic Geometry.

* 10–12 **Workshop on Triangulated Categories**, Mathematics Department, Swansea University, Swansea, United Kingdom.
Description: This workshop is to bring together mathematicians working in algebraic geometry and topology, mathematical physics, representation theory in order to exchange results, questions and points of view on the role of triangulated categories in their respective fields.
Information: http://www-maths.swan.ac.uk/staff/gg/Workshop/index.html; email: G.Garkusha@swansea.ac.uk.

10–14 **Ninth Pacific Rim Geometry Conference**, National Taiwan University, Taipei, Taiwan. (Jun./Jul. 2008, p. 740)

* 13–14 **Palmetto Number Theory Series VIII**, University of South Carolina, Columbia, South Carolina.
Description: One of three regular number theory meetings per year held in the state of South Carolina.
Information: http://www.math.sc.edu/~boylan/seminars/pants8.html.

13–17 **The 48th American Society for Cell Biology Annual Meeting**, The Moscone Center, San Francisco, California. (May 2008, p. 636)

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

15–19 **4th International Conference on Combinatorial Mathematics and Combinatorial Computing (4ICC)**, University of Auckland, Auckland, New Zealand. (Jun./Jul. 2008, p. 740)

15–19 **The 13th Asian Technology Conference in Mathematics (ATCM 2008)**, Suan Sunandha Rajabhat University, Bangkok, Thailand. (Mar. 2008, p. 413)

17–21 **First Joint International Meeting with the Shanghai Mathematical Society**, Shanghai, China. (Jun./Jul. 2007, p. 784)

18–20 **Pre-ICM International Convention on Mathematical Sciences**, University of Delhi, Delhi, India. (Jun./Jul. 2008, p. 741)

19–21 **Centenary Celebration of Calcutta Mathematical Society: International Symposium on Recent Advances in Mathematics and its Applications: (ISRAMA 2008)**, Calcutta Mathematical Society at AE-374, Sector-1, Salt Lake City Kolkata (Calcutta) 700064, India. (Aug. 2008, p. 870)

22–23 **Mathematical Sciences for Advancement of Science and Technology (MSAST 2008)**, Institute for Mathematics, Bioinformatics, Information Technology and Computer Science (IMBIC), Salt Lake City, Kolkata (Calcutta), India. (Aug. 2008, p. 870)

23–26 **International Conference on Computer Analysis of Science and Technology problems**, Tajik State National University (TSNU), Dushanbe, Tajikistan. (Mar. 2008, p. 413)

* 29–31 **VI International Symposium on Optimization and Statistics (ISOS-2008)**, Aligarh Muslim University, Aligarh, India.
Description: The aim of holding the symposium is to disseminate and highlight the current researches in all the branches of Statistics and Operations Research.
Enquiries: All the queries/correspondence regarding registration, abstract/paper submission, accommodation, travel etc. should be addressed to: Professor A. H. Khan, Director International Symposium 2008, Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh-202 002, India; Phone: +91 571 2701251 (Dept.), +91 571 2720601 (Res.); Cell #: + 919411047042; email: ahamidkhan@rediffmail.com; chairman.stats@gmail.com; or Professor M. J. Ahsan, Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh-202 002, India; Cell #: +919897356448; email: mjahsan2001@yahoo.co.in; http://www.amu.ac.in/shared/sublinkimages/isos2008.pdf.

January 2009

1–March 31 **I-Math Winter School DocCourse Combinatorics and Geometry 2009: Discrete and Computational Geometry**, Centre de Recerca Matematica, Bellaterra, Spain. (Sept. 2008, p. 1028)

1–June 30 **Research Program on Mathematical Biology: Modelling and Differential Equations: An i-MATH activity**, Centre de Recerca Matemática, Bellaterra, Spain. (Nov. 2008, p. 1318)

4–6 **ACM-SIAM Symposium on Discrete Algorithms (SODA09)**, New York Marriott Downtown, New York, New York. (Aug. 2008, p. 870)

5–8 **Joint Mathematics Meetings**, Washington, District of Columbia. (Aug. 2008, p. 870)

4–9 **Workshop on Random Functions and Random Surfaces and Interfaces**, Centre de recherches mathématiques, Université de Montréal, Montréal, Québec, Canada. (Jan. 2008, p. 78)

5–8 **Joint Mathematics Meetings**, Washington, D.C. (May 2008, p. 636)

5–16 **Group Theory, Combinatorics and Computation**, The University of Western Australia, Perth, Australia. (May 2008, p. 636)

* 8–9 **Communicating Complex Statistical Evidence: A conference sponsored by the Cambridge Statistics Initiative and the Winton Programme for the Public Understanding of Risk**, Meeting Room 2, Centre for Mathematical Sciences, Cambridge, United Kingdom.
Theme: Complex statistical models and reasoning can play a major role in informing both policy and individual decisions, but it is not necessarily straightforward to communicate what may be rather subtle statistical issues. This conference will bring together people interested in techniques to maximize the credibility and impact of statistical science in a range of important contexts, including health policy, climate change projections and impact, crime and the law, and epidemic control.
Program: There will be a single strand with no parallel sessions, featuring invited and contributed papers. Contributed papers or posters on a topic relevant to the overall meeting theme are invited.
Conference Chairs: Philip Dawid and David Spiegelhalter. http://ccseconf.org.

8–10 **2009 International Joint Conference**, Singapore (Sept. 2008, p. 1028)

12–16 **Quantitative and Computational Aspects of Metric Geometry**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Aug. 2008, p. 870)

12–May 22 **Algebraic Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (Jun./Jul. 2008, p. 741)

12–June 26 **Algebraic Lie Theory**, Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge CB3 OEH, United Kingdom. (Aug. 2008, p. 870)

14–16 **International Conference on Modeling of Engineering and Technological Problems and 9th National Conference of Indian Society of Industrial & Applied Mathematics**, BMAS Engineering College, Sharda Group, Agra, India. (Aug. 2008, p. 870)

* 17 **The N+4th Southern California Topology Conference**, California Institute of Technology, Pasadena, California.
Description: This regional topology conference has been held on and off for O(N) years. It takes place on a single day, usually in winter, and is typically well-attended by the Southern California topological community. Some travel support for graduate students is anticipated; contact Danny Calegari for details if you are interested.
Information: http://www.its.caltech.edu/~dannyc/sctc/sctc2009.html

19–July 3 **Discrete Integrable Systems**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Aug. 2008, p. 871)

22–24 **Connections for Women: Algebraic Geometry and Related Fields**, Mathematical Sciences Research Institute, Berkeley, California. (Jun./Jul. 2008, p. 741)

22–30 **Numerical Approaches to Quantum Many-Body Systems**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Aug. 2008, p. 871)

26–30 **Classical Algebraic Geometry**, Mathematical Sciences Research Institute, Berkeley, Califonia. (Jun./Jul. 2008, p. 741)

* 30–February 1 **Fifth International Conference on Technology, Knowledge and Society**, Huntsville, Alabama.
Description: This conference will address a range of critically important themes in the various fields that address the relationships between technology, knowledge and society. The conference is cross-disciplinary in scope, a meeting point for technologists with a concern for the social and social scientists with a concern for the technological. The focus is primarily, but not exclusively, on information and communications technologies. As well as impressive line-up of international main speakers, the conference will also include numerous paper, workshop and colloquium presentations by practitioners, teachers and researchers.
Deadline: The deadline for the next round in the call for papers (a title and short abstract) is 9 October 2008. Future deadlines will be announced on the conference website after this date. Proposals are reviewed within two weeks of submission. Full details of the conference, including an online proposal submission form, are to be found at the conference website: http://www.Technology-Conference.com.

February 2009

1–July 15 **Research Programme on Harmonic Analysis, Geometric Measure Theory and Quasiconformal Mappings: An i-MATH activity**, Centre de Recerca Matem†mática, Bellaterra, Spain. (Nov. 2008, p. 1318)

* 2–6 **Advanced Course on Mathematical Biology: Modeling and Differential Equations**, Centre de Recerca Matemàtica, Campus de la Universitat Autònoma, Barcelona, Spain.

Scientific Committee: Rafael Bravo, Àngel Calsina, José A. Carrillo (coordinator), Antoni Guillamon, Benoît Perthame, Angela Stevens.
Information: `http://www.crm.cat/ACMODELING`.

3–7 **30th Linz Seminar on Fuzzy Set Theory**, Bildungszentrum St. Magdalena, Linz, Austria. (Nov. 2008, p. 1319)

* 9–13 **Conference on Mathematical Biology: Modeling and Differential Equations**, Centre de Recerca Matemática, Campus de la Universitat Autònoma Barcelona, Spain.
Scientific Committee: Rafael Bravo, Àngel Calsina (coordinator), José A. Carrillo, Antoni Guillamon, Benoît Perthame, Angela Stevens.
Information: `http://www.crm.cat/CMODELING`.

9–13 **Laplacian Eigenvalues and Eigenfunctions: Theory, Computation, Application**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Aug. 2008, p. 871)

16–20 **Workshop on Complex Geometry**, University of Adelaide, Australia. (Nov. 2008, p. 1319)

16–21 **III School and Workshop on Mathematical Methods in Quantum Mechanics**, Casa della Gioventù, Bressanone/Brixen, Italy. (Oct. 2008, p. 1134)

23–27 **Modern Moduli Theory**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2008, p. 871)

23–27 **Rare Events in High-Dimensional Systems**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Aug. 2008, p. 871)

March 2009

9–June 12 **Quantum and Kinetic Transport: Analysis, Computations, and New Applications**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California. (Apr. 2008, p. 526)

* 10–13 **IPAM/Quantum and Kinetic Transport Equations: Tutorials**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Overview: The tutorials will familiarize participants with issues and techniques involved in quantum and kinetic transport.
Main topics: Mathematical transition from quantum, classical, to hydrodynamics scales; Computational methods for mixed scale problems; Quantum and kinetic models in emerging new applications; Current status of analysis on kinetic equations.
Application/Registration: An application and registration form is available at: `http://www.ipam.ucla.edu/programs/kttut`. Applications received by Jan. 26, 2009, will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. You may also register and attend without IPAM funding.
Organizing Committee: Eric Carlen, Pierre Degond, Irene Gamba, Frank Graziani, Shi Jin, Karl Kempf, Dave Levermore, Peter Markowich, Stanley Osher, Christian Ringhofer, Marshall Slemrod.
Information: `http://www.ipam.ucla.edu/programs/kttut/`; email: `sbeggs@ipam.ucla.edu`.

14 **Statistical Methods for Complex Data: A conference in honor of the 60th birthday of Raymond J. Carroll**, Texas A& M University, Department of Statistics, College Station, Texas. (Sept. 2008, p. 1030)

* 15–17 **Frontier Probability Days 2009**, University of Utah, Salt Lake City, Utah.
Description: "Frontier Probability Days 2009" (FPD'09) is a regional workshop, taking place at the University of Utah, Salt Lake City, UT, on March 15–17, 2009. Its purpose is to bring together mathematicians, both regionally as well as globally, who have an interest in probability and its applications. This will complement other regional conferences that are held annually elsewhere in the U.S.
Information: `http://www.math.utah.edu/~firas/FPD`.

15–20 **ALGORITMY 2009 Conference on Scientific Computing**, Hotel Permon, Podbanske, High Tatra Mountains, Slovak Republic. (Jun./Jul. 2008, p. 741)

18–20 **IAENG International Conference on Scientific Computing ICSC 2009**, Regal Kowloon Hotel, Kowloon, Hong Kong. (Aug. 2008, p. 871)

22–26 **SETIT 2009: The Fifth International Conference Sciences of Electronic, Technologies of Information and Telecommunications**, Hammamet, Tunisia. (Nov. 2008, p. 1319)

23–27 **Combinatorial, Enumerative and Toric Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (Aug. 2008, p. 871)

* 27–28 **35th Annual New York State Regional Graduate Mathematics Conference**, Syracuse University, Syracuse, New York.
Description: This two day conference has been run and organized entirely by the graduate students of the Syracuse University Mathematics Department for the last 35 years. This year our Opening Address will be given by Laszlo Lempert of Purdue University on Friday March 27, 2009, and out Keynote Address will be given by Brian Conrey of the American Institute of Mathematics on Saturday March 28, 2009.
Information: `http://webwork.syr.edu/~mgo/`; email: `tsbleier@syr.edu`.

27–29 **AMS Central Section Meeting**, University of Illinois at Urbana-Champaign, Urbana, Illinois. (Aug. 2008, p. 871)

27–29 **GSCC09: Fifth Annual Graduate Student Combinatorics Conference**, University of Kentucky, Lexington, Kentucky. (Oct. 2008, p. 1134)

* 30–April 3 **Quantum and Kinetic Transport: Computational Kinetic Transport and Hybrid Methods**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Overview: This workshop will focus on computational modeling of kinetic transport models that arise in various kinetic transport problems. Hybridization of computational schemes linking multi-scale and multi-physics will also be addressed. The aim of this workshop is to examine the current states of computational transport, and to foster interdisciplinary interactions among researchers from mathematics, physics, chemistry, engineering, and related disciplines.
Organizing Committee: Pierre Degond, Bjorn Engquist, Frank Graziani, Shi Jin, Caroline Lasser, Anna-Karin Tornberg.
Application/Registration: An application and registration form is available at `http://www.ipam.ucla.edu/programs/ktws1`. Applications received by Feb. 16, 2009, will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. You may also register and attend without IPAM funding.
Information: email: `sbeggs@ipam.ucla.edu`; `http://www.ipam.ucla.edu/programs/ktws1/`.

April 2009

4–5 **AMS Southeastern Section Meeting**, North Carolina State University, Raleigh, North Carolina. (Aug. 2008, p. 871)

6–10 **The 3D Euler and 2D surface quasi-geostrophic equations**, American Institute of Mathematics, Palo Alto, California. (May 2008, p. 636)

* 15–17 **Quantum and Kinetic Transport: The Boltzmann Equation—DiPerna-Lions Plus 20 Years**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Overview: Areas of discussion of this three-day workshop will include the spatially homogeneous and inhomogeneous Boltzmann equation, their hydrodynamic limits, and collisionless Vlasov models in plasma in the classical and relativistic framework with the coupling to Maxwell-Poisson systems.
Organizing Committee: Irene Gamba, Reinhard Illner, Dave Levermore, Laure Saint Raymond, Marshall Slemrod.
Application/Registration: An application and registration form is available at: `http://www.ipam.ucla.edu/programs/ktws2`. Applications received by March 4, 2009, will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. You may also register and attend without IPAM funding.
Information: `http://www.ipam.ucla.edu/programs/ktws2/`; email: `sbeggs@ipam.ucla.edu`.

* 16–18 **New Results on the Discrepancy Function, and Related Results, University of Arkansas Spring Lecture Series**, University of Arkansas, Fayetteville, Arkansas.
Description: New Results on the Discrepancy Function, and Related Results, 34th University of Arkansas Spring Lecture Series, April 16-18 2009. Main Speaker Michael Lacey (Georgia Tech).
Information: `http://math.uark.edu`; email: `lcapogna@uark.edu`.

* 20–22 **Geometry and Physics: Atiyah80**, Edinburgh, Scotland, United Kingdom.
Description: The workshop is planned to coincide with the 80th birthday on April 22, 2009 of Sir Michael Atiyah PRS, PRSE, Fields and Abel Medallist, and undoubtedly the most influential British mathematician of the last 50 years. Perhaps the most exciting interaction between physics and mathematics at the moment is in Quantum Field Theory and String Theory. The proposed workshop is intended to bring the world leaders in this interaction to Edinburgh; to exchange ideas, to give lectures to active practitioners in the fields and to announce their latest results.
Information: `http://www.icms.org.uk/workshops/Atiyah80`; email: `a.ranicki@ed.ac.uk`.

25–26 **AMS Eastern Section Meeting**, Worcester Polytechnic Institute, Worcester, Massachusetts. (Aug. 2008, p. 871)

25–26 **AMS Western Section Meeting**, San Francisco State University, San Francisco, California. (Aug. 2008, p. 872)

27–May 1 **Combinatorial Challenges in Toric Varieties**, American Institute of Mathematics, Palo Alto, California. (Jun./Jul. 2008, p. 741)

* 27–May 1 **Quantum and Kinetic Transport: Flows and Networks in Complex Media**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Organizing Committee: M. C. Carvalho, Karl Kempf, Stephan Mischler, Benedetto Piccoli, Christian Ringhofer.
Overview: This workshop is directed toward particle flows in complex topologies, either in the form of networks and graphs, or in the form of random or quasi-periodic media. The workshop's aim is to bring together an interdisciplinary group of researchers in different areas such as traffic flow simulation, supply chain management and physical flows in random media. These areas share a number of common challenges and require therefore the usage of similar mathematical toolboxes.
Application/Registration: An application and registration form is available at: `http://www.ipam.ucla.edu/programs/ktws3`. Applications received by Mar. 16, 2009, will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. You may also register and attend without IPAM funding.
Information: `http://www.ipam.ucla.edu/programs/ktws3/`; email: `sbeggs@ipam.ucla.edu`.

* 30–May 2 **SIAM International Conference on Data Mining**, John Ascuaga's Nugget, Sparks, Nevada.
Description: SIAM International Conference on Data Mining. Call for Papers Now Available!
Information: The Call for Papers for this conference is now available. Please visit `http://www.siam.org/meetings/sdm09/` for more information.
Deadlines: October 3, 2008: Abstracts Due; October 3, 2008: Proposal for Workshop and Tutorials; October 10, 2008: Manuscripts Due. For additional information, contact SIAM Conference Department at `meetings@siam.org`.

May 2009

* 1–June 20 **INdAM Intensive Period "Geometric Properties of Nonlinear Local and Nonlocal Problems"**, Department of Mathematics "F. Brioschi", Politecnico di Milano, Milan, Italy; Department of Mathematics "F. Casorati", University of Pavia, Pavia, Italy.
Description: It is a Concentration Period sponsored by the Italian INdAM. We plan to have both courses and seminars. The courses will be organized over the whole week, from Monday to Friday: approximately half of them will be held in Milan, the other half in Pavia. In particular, this implies that the partcipants should generally spend at least 4 nights either in Milan or in Pavia. There will be one-hour seminars, naturally linked to the topics covered in the main courses; we plan to have lessons in the morning, possibly followed by questions time, free discussions, and study time, whereas seminars will be generally scheduled in the afternoon. We also aim at shorter contributions from promising young people; therefore we will offer to young PostDocs and Ph.D. students the possibility of giving a brief communication on their current research.
Information: `http://www.imati.cnr.it/gianazza/bimestre`; email: `ugopietro.gianazza@unipv.it`.

4–8 **Stochastic and Deterministic Spatial Modeling in Population Dynamics**, American Institute of Mathematics, Palo Alto, California. (Oct. 2008, p. 1134)

7–9 **8th Mississippi State: UAB Conference on Differential Equations and Computational Simulations**, Mississippi State University, Mississippi State, Mississippi. (Nov. 2008, p. 1319)

10–15 **ICMI Study 19: Proof and Proving in Mathematics Education**, Taipei, Taiwan. (May 2008, p. 636)

* 11–13 **TQC 2009: The 4th Workshop on Theory of Quantum Computation, Communication, and Cryptography**, University of Waterloo, Waterloo, Ontario, Canada.
Description: Quantum computation, quantum communication, and quantum cryptography are subfields of quantum information processing, an interdisciplinary field of information science and quantum mechanics. TQC 2009 focuses on theoretical aspects of these subfields. The objective of the workshop is to bring together researchers so that they can interact with each other and share problems and recent discoveries. It will consist of invited talks, contributed talks, and a poster session.
Information: `http://www.iqc.ca/tqc2009`; email: `simoseve@gmail.com`.

11–15 **Workshop and Advanced Course on Deterministic and Stochastic Modelling in Computational Neuroscience and other Biological Topics**, Barcelona, Spain. (Nov. 2008, p. 1319)

12–14 **7th International Symposium on Hysteresis Modeling and Micromagnetics (HMM-2009)**, Gaithersburg, Maryland. (Nov. 2008, p. 1319)

12–16 (NEW DATE) **First Buea International Conference on the Mathematical Sciences**, University of Buea, Cameroon. (Mar. 2008, p. 408)

17–21 **SIAM Conference on Applications of Dynamical Systems (DS09)**, Snowbird Ski and Summer Resort, Snowbird, Utah. (Oct. 2008, p. 1134)

17–22 **Topology, C*-Algebras, and String Duality - an NSF/CBMS Regional Conference in the Mathematical Sciences**, Texas Christian University, Fort Worth, Texas. (Jun./Jul. 2008, p. 742)

* 18–22 **Quantum and Kinetic Transport: Asymptotic Methods for Dissipative Particle Systems**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Overview: It is the goal of this workshop to bring together researchers from diverse areas, including statistical mechanics, particle systems, probability theory and applications, to discuss developing areas of non-conservative dynamics and the emergence of non-equilibrium statistical states, and to explore potential applications in the natural and social sciences.
Organizing Committee: Irene Gamba, Eric Carlen , Peter Markowich, Anne Nouri, Robert Pego, Mario Pulvirenti, Cederic Villani.
Application/Registration: An application and registration form is available at: `http://www.ipam.ucla.edu/programs/ktws4`. Applications received by April 6, 2009 ,will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. You may also register and attend without IPAM funding.
Information: `http://www.ipam.ucla.edu/programs/ktws4/`; email: `sbeggs@ipam.ucla.edu`.

* 18–22 **SAMPTA'09 (Sampling Theory and Applications)**, Centre International de Rencontres Mathématiques (CIRM), Luminy campus, Marseille, France.

Description: SAMPTA'09 is the 8th international conference on Sampling Theory and Applications. The purpose of SAMPTA's is to bring together mathematicians and engineers interested in sampling theory and its applications to related fields (such as signal and image processing, coding theory, control theory, complex analysis, harmonic analysis, differential equations) to exchange recent advances and to discuss open problems. SAMPTA09 will be organized around plenary lectures, general sessions on sampling and applications, and special sessions on selected topics. Ample time will be left for discussions. Please notice that a number of scholarchips will be will be available for young researchers (Ph.D. students and PostDocs), to cover their stay at CIRM.
Information: `http://www.latp.univ-mrs.fr/SAMPTA09`.

18–23 **Workshop on Interacting Stochastic Particle Systems**, Centre de recherches mathématiques, Université de Montréal, Montréal, Québec, Canada. (Jan. 2008, p. 78)

25–29 **6th European Conference on Elliptic and Parabolic Problems**, Hotel Serapo, Gaeta, Italy. (Oct. 2008, p. 1134)

25–29 **14th International Conference on Gambling & Risk Taking**, Harrah's Lake Tahoe, Stateline, Nevada. (Oct. 2008, p. 1134)

27–June 1 **The International Conference "Infinite Dimensional Analysis and Topology"**, Yaremche, Ivano-Frankivsk, Ukraine. (May 2008, p. 636)

June 2009

* 1–3 **Global Conference on Power Control and Optimization (PCO'2009)**, Bali, Indonesia.
Information: `http://www.engedu2.net`; email: `vasantglobal@gmail.com`.

* 1–5 **2nd Chaotic Modeling and Simulation International Conference (CHAOS2009)**, MAICh Conference Center, Chania, Crete, Greece.
Description: The general topics and the special sessions proposed for the Conference (Chaos2009) include but are not limited to: Chaos and Nonlinear Dynamics, Stochastic Chaos, Chemical Chaos, Data Analysis and Chaos, Hydrodynamics, Turbulence and Plasmas, Optics and Chaos, Chaotic Oscillations and Circuits, Chaos in Climate Dynamics, Geophysical Flows, Biology and Chaos, Neurophysiology and Chaos, Hamiltonian Systems, Chaos in Astronomy and Astrophysics, Chaos and Solitons, Micro- and Nano- Electro-Mechanical Systems, Neural Networks and Chaos, Ecology and Economy.
Information: For more information and submission details please visit the conference website `http://www.chaos2009.net`.

3–5 **Conference on Character Theory of Finite Groups in honor of Martin Isaacs**, Universitat de Valencia, Spain. (Sept. 2008, p. 1031)

3–15 **Interactions Between Hyperbolic Geometry, Quantum Topology and Number Theory Workshop**, Columbia University, New York, New York. (Oct. 2008, p. 1134)

* 8–11 **MAMERN09: 3rd International Conference on Approximation Methods and Numerical Modeling in Environment and Natural Resources**, University of Pau, Pau, France.
Description: The International Conference MAMERN, organized by the University of Pau & CNRS (France), in collaboration with the universities of Granada (Spain) and Mohammed I University (Morocco) is intended to take place every two years. The Third conference will be held in Pau, France, and focus on the following topics: Approximation and modeling applied to environment sciences and natural resources; New applications and developments in approximation methods; Mathematics and computation in geosciences; Modeling of ecosystems; Oceanographic and coastal engineering; Numerical modeling of flow and transport in porous media; Mathematical analysis of models in porous media; Multi-scale modeling of flow and transport in porous media.
Contact: `mamern@univ-pau.fr`.

8–12 **Computational Methods and Function Theory 2009**, Bilkent University, Ankara, Turkey. (Jun./Jul. 2008, p. 742)

8–13 **Workshop on Disordered Systems: Spin Glasses**, Centre de recherches mathématiques, Université de Montréal, Montréal, Québec, Canada. (Jan. 2008, p. 78)

* 8–19 **Geometry and Arithmetic around Galois Theory**, Galatasaray University, Istanbul, Turkey.
Description: This is a research level school on algebraic covers and their moduli spaces (Hurwitz spaces) in relation with inverse Galois theory, descent theory, constructions of covers, deformation methods and geometric Galois theory. There will be lectures on construction of Hurwitz spaces, counting points of a Hurwitz space, diophantine aspects of modular towers, modular towers and torsion on abelian varieties, fundamental groups and Galois action.
Information: `http://math.gsu.edu.tr/GAGT/index.html`; email: `celalcem@gmail.com`.

* 11–13 **Representation Theory**, Institut de Recherche Mathématique Avancée, Université de Strasbourg, 7 rue René Descacrtes, Strasbourg, France.
Description: The meeting is Number 83 in the series "Encounter Between Mathematicians and Theoretical Physicists". The focus is on representation theory. There will be survey lectures and specialized talks.
Invited speakers: C. Bachas (ENS Paris), V. Fock (Strasbourg and Aarhus), K. Gawedzki (ENS Lyon), T. Kobayashi (Tokyo), B. Kroetz, (MPI, Bonn), J.-P. Labesse (Aix-Marseille II), E. Opdam (Amsterdam), V. Ovsienko (Lyon), V. Schomerus (Hamburg), J. Teschner (Hamburg), V. Turaev (Bloomington).
Organization and information: A. Papadopoulos and S. Souaifi (Strasbourg); email: `papadop@math.u-strasbg.fr`; `souaifi@math.u-strasbg.fr`; `http://www-irma.u-strasbg.fr/article717.html`;

* 14–20 **47th International Symposium on Functional Equations**, Gargnano, Italy.
Topics: Functional equations and inequalities, mean values, functional equations on algebraic structures, Hyers-Ulam stability, regularity properties of solutions, conditional functional equations, functional-differential equations, iteration theory; applications of the above, in particular to the natural, social, and behavioral sciences.
Local Organizer: Gian Luigi Forti, Dipt. di Matematica, Via C. Saldini 50, Univ. d. Studi I-20133 Milano; email: `GianLuigi.Forti@mat.unimi.it`.
Scientific Committee: J. Aczél (Honorary Chair; Waterloo, ON, Canada), W. Benz (Honorary Member, Hamburg, Germany), Z. Daroczy (Honorary Member, Debrecen, Hungary), R. Ger (Chair; Katowice, Poland), Zs. Pales (Debrecen, Hungary), J. Rätz (Bern, Switzerland), L. Reich (Graz, Austria), and A. Sklar (Chicago, IL, USA).
Information: Participation at these annual meetings is by invitation only. Those wishing to be invited to this or one of the following meetings should send details of their interest and, preferably, publications (paper copies) and/or manuscripts with their postal and email address to: R. Ger, Institute of Mathematics, Silesian University, Bankowa 14, PL-40-007 Katowice, Poland; `romanger@us.edu.pl`, before December 15, 2008.

14–27 **ESI workshop on large cardinals and descriptive set theory**, Esi, Vienna, Austria. (Oct. 2008, p. 1134)

15–18 **The 5th International Conference "Dynamical Systems and Applications"**, "Ovidius" University of Constantza, Constantza, Romania. (Nov. 2008, p. 1319)

* 15–18 **SIAM Conference on Mathematical & Computational Issues in the Geosciences**, Leipziger Kubus Conference Center, Helmholtz - Centre for Environmental Research - UFZ, Leipzig, Germany.
Invited Speakers: Martin Blunt, Imperial College London; Chris Farmer, Schlumberger and University of Oxford; Rupert Klein, Potsdam Institute for Climate Impact Research and Free University of Berlin; Rosemary Knight, Stanford University, CA; Peter Lemke, University of Bremen, Germany; Barbara Romanowicz, University of California, Berkeley; Joannes J. Westerink, University of Notre Dame, IN.
Information and Deadlines: The Call for Papers for this conference is now available. Please visit `http://www.siam.org/meetings/gs09/index.php.ion`. November 14, 2008: Minisymposium proposals. December 15, 2008: Abstracts for contributed and minisymposium speakers. For additional information, contact SIAM Conference Department at `meetings@siam.org`.

15–19 **Conference on Harmonic Analysis, Geometric Measure Theory and Quasiconformal Mappings**, Barcelona, Spain. (Nov. 2008, p. 1319)

15–19 **Waves 2009: The 9th International Conference on Mathematical and Numerical Aspects of Waves Propagation**, Pau, France. (Jun./Jul. 2008, p. 742)

18–19 **2nd IMA International Conference on Mathematics in Sport**, University of Groningen, The Netherlands. (Oct. 2008, p. 1134)

21–27 **Eighth International Conference Symmetry in Nonlinear Mathematical Physics**, Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv (Kiev), Ukraine. (Nov. 2008, p. 1319)

22–26 (NEW DATE) **5th Asian Mathematical Conference (AMC 2009)**, Penang /Kulalumpur, Malaysia. (Jun./Jul. 2008, p. 742)

22–26 **The 10th European Congress of Stereology and Image Analysis**, University of Milan, 20133 Milan, Italy. (Oct. 2008, p. 1135)

28–July 18 **IAS/Park City Mathematics Institute (PCMI) 2009 Summer Session: Arithmetic of L-functions**, Park City, Utah. (Sept. 2008, p. 1032)

* 29–July 1 **1st Rapid Modelling Conference**, Neuchâtel, Switzerland.
Description: The objective of this conference is to provide an international, multidisciplinary platform for researchers and practitioners to create and exchange knowledge on increasing competitiveness through speed. Lead time reduction (through techniques ranging from quick response manufacturing to lean production) is achieved through a mutually reinforcing combination of changed mindset and analytical tools.
Information: `http://www.unine.ch/rmc09`.

July 2009

1–3 **International Conference of Applied and Engineering Mathematics 2009**, Imperial College London, London, United Kingdom. (Oct. 2008, p. 1135)

* 5–10 **22nd British Combinatorial Conference**, University of St. Andrews, Fife, Scotland.
Invited Speakers: Arrigo Bonisoli (Universita di Modena e Reggio Emilia); Peter J. Cameron (Queen Mary, University of London); Willem H. Haemers (Tilburg University); Gholamreza B. Khosrovshahi (IPM); Alexandr V. Kostochka (University of Illinois at Urbana-Champaign); Daniela Kühn (University of Birmingham); Marc Noy (Universitat Politècnica de Catalunya); Oliver Riordan (University of Oxford); Gordon Royle (University of Western Australia).
Programme: The speakers above will each give a one-hour talk. These talks are intended to be accessible to postgraduate students, postdoctoral fellows, and researchers in all areas of combinatorics. In addition participants are invited to give a talk of twenty minutes on any combinatorial topic. A problem session will be held on the last day.
Information: `http://bcc2009.mcs.st-and.ac.uk`; email: `colva@mcs.st-and.ac.uk`.

* 6–8 **SIAM Conference on Control and Its Applications**, Sheraton Denver Hotel, Denver, Colorado.
Description: This conference is a continuation of a series of meetings started in 1989 in San Francisco. The field of control theory is central to a wide range of aeronautic, aerospace, industrial, automotive and advanced technological systems and increasingly recognized as fundamental for emerging fields ranging from nanotechnology to cell regulation. In addition to its ubiquity for process regulation in the physical sciences, control concepts now pervade the biological, computer, and social sciences.
Topics: This conference will showcase a wide range of topics in control and systems theory. The topics and applications include real-time optimization and data assimilation, cellular and biological regulation, control techniques for financial mathematics, cooperative control for unmanned autonomous vehicles, biomedical control, risk sensitive control and filtering, control of smart systems, flow control and quantum control.
Information: `http://www.siam.org/meetings/ct09/`; email: `wilden@siam.org`.

6–10 **26th Journées Arithmétiques**, Université de Saint-Etienne, Saint-Etienne, France. (Jun./Jul. 2008, p. 742)

6–10 **First PRIMA Pacific Rim Congress of Mathematicians**, University of New South Wales, Sydney, Australia. (Jun./Jul. 2008, p. 742)

6–10 **Journées de Géométrie Arithmétique de Rennes**, Institut de Recherche Mathématique de Rennes, Université de Rennes 1, Rennes, France. (Sept. 2008, p. 1032)

* 6–11 **International Conference on Semigroups and Related Topics**, Faculty of Sciences of the University of Porto, Porto, Portugal. (Nov. 2008, p. 1319)
Description: This conference aims to present to the participants the state of the art in algebraic theory of semigroups and to approach related fields with fruitful connections.
Information: `http://www.fc.up.pt/ICSrt2009/`; email: `mdelgado@fc.up.pt`.

13–17 **9th International Conference on Finite Fields and Applications**, University College Dublin, Dublin, Ireland. (Jun./Jul. 2008, p. 742)

14–24 **The 19th International Conference on Banach algebras**, Bedlewo, Poland. (Oct. 2008, p. 1135)

14–January 15 **Call for Papers: 14th International Conference on Gambling and Risk Taking**, University of Nevada, Reno, at Harrah's Lake Tahoe, Nevada. (Oct. 2008, p. 1135)

* 20–24 **21st International Conference on Formal Power Series & Algebraic Combinatorics**, Research Institute for Symbolic Computation, Hagenberg, Austria.
Description: Topics include all aspects of combinatorics and their relations with other parts of mathematics, physics, computer science, and biology.
Conference Contents: Invited lectures, contributed presentations, poster session, and software demonstrations. As usual, there will be no parallel sessions. The official languages of the conference are English and French.

20–24 **AIP (Applied Inverse Problems)**, Vienna, Austria. (Nov. 2008, p. 1319)

20–24 **Equadiff 12**, Brno, Czech Republic. (Aug. 2008, p. 872)

20–December 18 **Non-Abelian Fundamental Groups in Arithmetic Geometry**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Aug. 2008, p. 872)

27–30 **The Society for Mathematical Biology Annual Meeting**, University of British Columbia, Vancouver, Canada. (Nov. 2008, p. 1319)

27–31 **33rd Conference on Stochastic Processes and their Applications**, Berlin, Germany. (May 2008, p. 636)

29–July 24 **The Cardiac Physiome Project**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Aug. 2008, p. 872)

August 2009

* 10–12 **Continuing Statistics Education Workshop**, Statistics Online Computational Resource (SOCR), University of California, Los Angeles, California.
Description: The 2009 SOCR aims to demonstrate the functionality, utilization and assessment of the current SOCR resources in probability and statistics curricula at different levels. The SOCR Motto "It's Online, Therefore it Exists!" clearly indicates that all of these tools, activities and materials are openly and anonymously available over the Internet to the entire community. This workshop will be of most value to AP teachers and college instructors of probability and statistics classes who have interests in exploring novel technology-enhanced approaches for improving statistics education. The workshop will provide an interactive forum for exchange of ideas and recommendations for strategies to integrate computers, modern pedagogical approaches, the Internet and new student assessment techniques.
Registration: Registration and participation is free and all participants will be provided with free room and board for the duration of the workshop. There is a limit of the participants we can admit, but the workshop will be web-streamed live.
Information: `http://www.SOCR.ucla.edu`.

* 10–12 **Workshop on Technology-Enhanced Probability and Statistics Education Using SOCR Resources**, Statistics Online Computational Resource (SOCR), University of California, Los Angeles, California.
Summary: The 2009 SOCR aims to demonstrate the functionality, utilization and assessment of the current SOCR resources in probability and statistics curricula at different levels. The SOCR Motto "It's Online, Therefore it Exists!" clearly indicates that all of these tools, activities and materials are openly and anonymously available over the Internet to the entire community. This workshop will be of most value to AP teachers and college instructors of probability and statistics classes who have interests in exploring novel technology-enhanced approaches for improving statistics education. The workshop will provide an interactive forum for exchange of ideas and recommendations for strategies to integrate computers, modern pedagogical approaches, the Internet and new student assessment techniques.
Registration: Registration and participation is free and all participants will be provided with free room and board for the duration of the workshop. There is a limit of the participants we can admit, but the workshop will be web-streamed live.
Information: `http://wiki.stat.ucla.edu/socr/index.php/SOCR_Events_Aug2009`; `http://www.SOCR.ucla.edu`; email: `dinov@stat.ucla.edu`.

12–December 18 **Dynamics of Discs and Planets**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Aug. 2008, p. 872)

17 **Symplectic and Contact Geometry and Topology**, Mathematical Sciences Research Institute, Berkeley, California. (Sept. 2008, p. 1033)

17–21 **Modular forms on noncongruence groups**, American Institute of Mathematics, Palo Alto, California. (Aug. 2008, p. 872)

17–December 18 **Tropical Geometry**, Mathematical Sciences Research Institute, Berkeley, California. (Sept. 2008, p. 1033)

24–28 **Relative trace formula and periods of automorphic forms**, American Institute of Mathematics, Palo Alto, California. (Sept. 2008, p. 1033)

September 2009

11–17 (NEW DATE) **Models in Developing Mathematics Education**, Dresden University of Applied Sciences, Dresden, Germany. (Apr. 2007, p. 498)

15–18 **Bogolyubov Kyiv Conference: "Modern Problems of Theoretical and Mathematical Physics"**, Bogolyubov Institute for Theoretical Physics, Kyiv, Ukraine. (Nov. 2008, p. 1319)

October 2009

* 5–8 **2009 SIAM/ACM Joint Conference on Geometric Design and Solid & Physical Modeling**, Hilton San Francisco Financial District, San Francisco, California.
Description: The 2009 Joint Conference on Geometric and Solid & Physical Modeling seeks high quality, original research contributions that strive to advance all aspects of geometric and physical modeling, and their application in design, analysis and manufacturing, as well as in biomedical, geophysical, digital entertainment, and other areas. A shared objective of both the SIAM GD and ACM SPM communities is a desire to highlight work of the highest quality on the problems of greatest relevance to industry and science.
Information: `http://www.siam.org/meetings/gdspm09/`.

* 5–9 **Combinatorics: Probabilistic Techniques and Applications**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Description: The probabilistic approach has been successful in combinatorics, graph theory, combinatorial number theory, optimization and theoretical computer science. This workshop will focus on several main research directions of probabilistic combinatorics, including the application of probability to solve combinatorial problems, the study of random combinatorial objects and the investigation of randomized algorithms.
Organizing Committee: Alan Frieze, Nathan (Nati) Linial, Angelika Steger, Benjamin Sudakov, Prasad Tetali.
Application/Registration: An application and registration form is available at: `http://www.ipam.ucla.edu/programs/cmaws1`. Applications received by August 24, 2009, will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. You may also register and attend without IPAM funding.
Information: `http://www.ipam.ucla.edu/programs/cmaws1/`; email: `sbeggs@ipam.ucla.edu`.

* 12–16 **Algebra, Geometry, and Mathematical Physics**, The Bedlewo Mathematical Research and Conference Center, Bedlewo, Poland.
Description: Contemporary hot trends in algebra, geometry, and mathematical physics.
Organizing Committee: V. Abramov, J. Grabowski, E. Paal (Vice-Chair), A. Stolin, A. Tralle (Chair), P. Urbanski.
Information: `http://www.agmf.astralgo.eu/bdl09/`.

* 14–16 **The 9th Conference Shell Structures Theory and Applications**, Neptun Hotel, Hel Peninsula, Baltic Sea, Jurata, Poland.
Description: The aim of the SSTA 2009 Conference is to bring together scientists, designers, engineers and other specialists of shell structures in order to discuss important results and new ideas in this broad field of activity. The previous one - 8th SSTA 2005 - was attended by 109 participants from 16 countries.
Conference Topics: The theory and analysis of shells, numerical analysis of shell structures and elements, design and maintenance of shell structures, special surface-related mechanical problems. The conference program will include general lectures and contributed oral presentations. The main language of the conference will be English.
Publications and Deadline: All accepted papers (full-length article in English) will appear in the hard-cover volume of Proceedings published by CRC Press/Balkema, Taylor & Francis Group. Deadline for submission of the full paper is February 28, 2009.

16–18 **AMS Central Section Meeting**, Baylor University, Waco, Texas. (Aug. 2008, p. 872)

* 19–23 **Combinatorics: Combinatorial Geometry**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Organizing Committee: Alexander Barvinok, Gil Kalai, Janos Pach, Jozsef Solymosi, Emo Welzl.
Overview: Combinatorial geometry deals with the structure and complexity of discrete geometric objects and is closely related to computational geometry, which deals with the design of efficient computer algorithms for manipulation of these objects. The focus of this workshop will be on the study of discrete geometric objects, their combinatorial structure, stressing the connections between discrete geometry and combinatorics, number theory, analysis and computer science.
Application/Registration: An application and registration form is available at: `http://www.ipam.ucla.edu/programs/cmaws2`. Applications received by Sept. 7, 2009, will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. You may also register and attend without IPAM funding.
Information: `http://www.ipam.ucla.edu/programs/cmaws2`; email: `sbeggs@ipam.ucla.edu`.

* 19–23 **Higher Reidemeister Torsion**, American Institute of Mathematics, Palo Alto, California.
Description: This workshop, sponsored by AIM and the NSF, will focus on connections between different constructions for invariants of fiber bundles.
Information: `http://aimath.org/ARCC/workshops/reidemeister.html`; email: `ebasor@aimath.org`.

24–25 **AMS Eastern Section Meeting**, Pennsylvania State University, University Park, Pennsylvania. (Aug. 2008, p. 872)

30–November 1 **AMS Southeastern Section Meeting**, Florida Atlantic University, Boca Raton, Florida. (Aug. 2008, p. 872)

November 2009

7–8 **AMS Western Section Meeting**, University of California, Riverside, California. (Aug. 2008, p. 872)

* 16–20 **Combinatorics: Analytical Methods in Combinatorics, Additive Number Theory and Computer Science**, Institute for Pure and Applied Mathematics (IPAM), UCLA, Los Angeles, California.
Overview: This workshop will focus on the interplay between combinatorics, discrete probability, additive number theory and computer science with emphasis on a wide spectrum of analytical tools that are used there. One of the workshop's aims is to foster interaction between researchers in these areas, discuss recent progress and communicate new results and ideas. We would also like to utilize this forum to make the state-of-the-art analytical techniques accessible to a broader audience.
Organizing Committee: Irit Dinur, Ben Green, Gil Kalai, Alex Samorodnitsky, Terence Tao, Van Vu.
Application/Registration: An application and registration form is available at: http://www.ipam.ucla.edu/programs/cmaws4. Applications received by Oct. 5, 2009, will receive fullest consideration. Encouraging the careers of women and minority mathematicians and scientists is an important component of IPAM's mission and we welcome their applications. You may also register and attend without IPAM funding.
Information: http://www.ipam.ucla.edu/programs/cmaws4/; email: sbeggs@ipam.ucla.edu.

December 2009

* 16–18 **The 4th Indian International Conference on Artificial Intelligence: (IICAI-09)**, Tumkur (near Bangalore), India.
Description: The conference consists of paper presentations, special workshops, sessions, invited talks and local tours, etc. and it is one of the biggest AI events in the world. We invite draft paper submissions.
Information: For details visit: http://www.iiconference.org.

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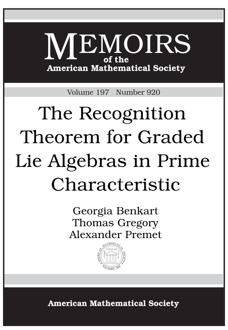
Algebra and Algebraic Geometry



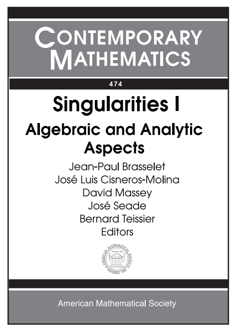
The Recognition Theorem for Graded Lie Algebras in Prime Characteristic

Georgia Benkart, *University of Wisconsin-Madison, WI*, **Thomas Gregory**, *Ohio State University, Mansfield, OH*, and **Alexander Premet**, *University of Manchester, England*

Contents: Graded Lie algebras; Simple Lie algebras and algebraic groups; The contragredient case; The noncontragredient case; Bibliography.

Memoirs of the American Mathematical Society, Volume 197, Number 920

January 2009, 145 pages, Softcover, ISBN: 978-0-8218-4226-3, LC 2008039455, 2000 *Mathematics Subject Classification:* 17B50, 17B70, 17B20; 17B05, **Individual member US$43**, List US$72, Institutional member US$58, Order code MEMO/197/920



Singularities I

Algebraic and Analytic Aspects

Jean-Paul Brasselet, *Institut de Mathématiques de Luminy-CNRS, Marseille, France*, **José Luis Cisneros-Molina**, *Universidad Nacional Autónoma de México, Cuernavaca, Mexico*, **David Massey**, *Northeastern University, Boston, MA*, **José Seade**, *Universidad Nacional Autónoma de México, Cuernavaca, Mexico*, and **Bernard Teissier**, *Institut de Mathématiques de Jussieu-CNRS, Paris, France*, Editors

This is the first part of the proceedings of the "School and Workshop on the Geometry and Topology of Singularities", held in Cuernavaca, Mexico, from January 8 to 26, 2007, in celebration of the 60th birthday of Lê Dũng Tráng.

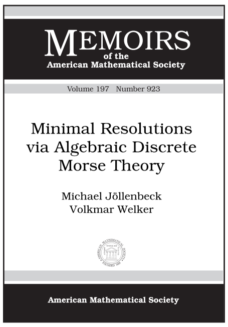
This volume contains fourteen cutting-edge research articles on algebraic and analytic aspects of singularities of spaces and maps. By reading this volume, and the accompanying volume on geometric and topological aspects of singularities, the reader should gain an appreciation for the depth, breadth, and beauty of the subject and also find a rich source of questions and problems for future study.

This item will also be of interest to those working in analysis.

Contents: **E. A. Bartolo**, **P. Cassou-Noguès**, **I. Luengo**, and **A. Melle-Hernández**, On the log-canonical threshold for germs of plane curves; **S. E. Cappell**, **L. Maxim**, and **J. L. Shaneson**, Intersection cohomology invariants of complex algebraic varieties; **F. El Zein**, Topology of algebraic morphisms; **T. Gaffney**, Non-isolated complete intersection singularities and the A_f condition; **H. H. Khoai**, Unique range sets and decomposition of meromorphic functions; **D. B. Massey**, Enriched relative polar curves and discriminants; **L. Maxim** and **J. Schürmann**, Hodge-theoretic Atiyah-Meyer formulae and the stratified multiplicative property; **L. J. McEwan**, Vertical monodromy and spectrum of a Yomdin series; **Z. Mebkhout**, Structures de Frobenius et €xposants de la Monodromie p-adique des Équations Différentielles; **L.N. Macarro**, Linearity conditions on the Jacobian ideal and logarithmic-meromorphic comparison for free divisors; **A. Némethi**, Poincaré series associated with surface singularities; **G. Rond**, Approximation de Artin cylindrique et morphismes d'algèbres analytiques; **C. Sabbah**, An explicit stationary phase formula for the local formal Fourier-Laplace transform; **M. J. Saia** and **C. H. Soares, Jr.**, On modified C^{ℓ}-trivialization of $C^{\ell+1}$-real germs of functions.

Contemporary Mathematics, Volume 474

December 2008, 349 pages, Softcover, ISBN: 978-0-8218-4458-8, LC 2008028179, 2000 *Mathematics Subject Classification:* 14B05, 14E15, 14J17, 32Sxx, 34M35, 35A20, **AMS members US$79**, List US$99, Order code CONM/474



Minimal Resolutions via Algebraic Discrete Morse Theory

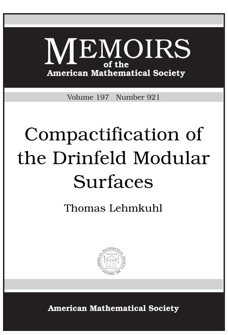
Michael Jöllenbeck, *Phillips-Universität Marburg, Germany*, and **Volkmar Welker**, *Philipps-Universität Marburg, Germany*

This item will also be of interest to those working in discrete mathematics and combinatorics.

Contents: Introduction; Algebraic discrete Morse theory; Resolution of the residue field in the commutative case; Resolution of the residue field in the non-commutative case; Application to acyclic Hochschild complex; Minimal (cellular) resolutions for (p-) Borel fixed ideals; Appendix A. The bar and the Hochschild complex; Appendix B. Proofs for algebraic discrete Morse theory; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 197, Number 923

January 2009, 74 pages, Softcover, ISBN: 978-0-8218-4257-7, LC 2008039850, 2000 *Mathematics Subject Classification:* 13D02, 05E99, **Individual member US$37**, List US$62, Institutional member US$50, Order code MEMO/197/923



Compactification of the Drinfeld Modular Surfaces

Thomas Lehmkuhl

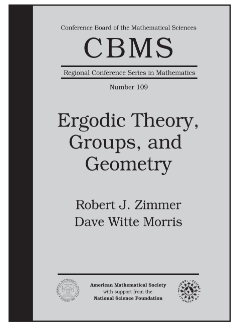
This item will also be of interest to those working in number theory.

Contents: Line bundles; Drinfeld modules; Deformation theory; Tate uniformization; Compactification of the modular surfaces; Appendix; Bibliography; Glossary of notations; Index.

Memoirs of the American Mathematical Society, Volume 197, Number 921

January 2009, 94 pages, Softcover, ISBN: 978-0-8218-4244-7, LC 2008039489, 2000 *Mathematics Subject Classification:* 11G09, 13D10, 14B20, **Individual member US$39**, List US$65, Institutional member US$52, Order code MEMO/197/921

Analysis



Ergodic Theory, Groups, and Geometry

Robert J. Zimmer, *University of Chicago, IL*, and **Dave Witte Morris**, *University of Lethbridge, AB, Canada*

The study of group actions on manifolds is the meeting ground of a variety of mathematical areas. In particular, interesting geometric insights can be obtained by applying measure-theoretic techniques. This book provides an introduction to some of the important methods, major developments, and open problems on the subject. It is slightly expanded from lectures given by Zimmer at the CBMS conference at the University of Minnesota. The main text presents a perspective on the field as it was at that time. Comments at the end of each chapter provide selected suggestions for further reading, including references to recent developments.

A co-publication of the AMS and CBMS.

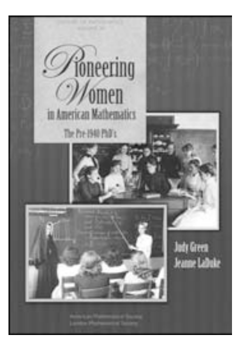
This item will also be of interest to those working in algebra and algebraic geometry and geometry and topology.

Contents: Introduction; Actions in dimension 1 or 2; Geometric structures; Fundamental groups I; Gromov representation; Superrigidity and first applications; Fundamental groups II (Arithmetic theory); Locally homogeneous spaces; Stationary measures and projective quotients; Orbit equivalence; Background material; Name index; Index.

CBMS Regional Conference Series in Mathematics, Number 109

December 2008, 87 pages, Softcover, ISBN: 978-0-8218-0980-8, LC 2008037157, 2000 *Mathematics Subject Classification:* 22F10, 37A15, 53C10, 57S20, **All Individuals US$23**, List US$29, Order code CBMS/109

General and Interdisciplinary



Pioneering Women in American Mathematics

The Pre-1940 PhD's

Judy Green, *Marymount University, Arlington, VA*, and **Jeanne LaDuke**, *DePaul University, Chicago, IL*

More than 14 percent of the PhD's awarded in the United States during the first four decades of the twentieth century went to women, a proportion not achieved again until the 1980s. This book is the result of a study in which the authors identified all of the American women who earned PhD's in mathematics before 1940, and collected extensive biographical and bibliographical information about each of them. By reconstructing as complete a picture as possible of this group of women, Green and LaDuke reveal insights into the larger scientific and cultural communities in which they lived and worked.

The book contains an extended introductory essay, as well as biographical entries for each of the 228 women in the study. The authors examine family backgrounds, education, careers, and other professional activities. They show that there were many more women earning PhD's in mathematics before 1940 than is commonly thought. Extended biographies and bibliographical information are available from the companion website for the book: **www.ams.org/bookpages/hmath-34**.

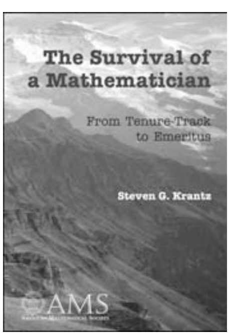
The material will be of interest to researchers, teachers, and students in mathematics, history of mathematics, history of science, women's studies, and sociology. The data presented about each of the 228 individual members of the group will support additional study and analysis by scholars in a large number of disciplines.

Co-published with the London Mathematical Society beginning with Volume 4. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

Contents: Introduction; Family background and precollege education; Undergraduate education; Graduate education; Employment issues; Career patterns; Scholarly and professional contributions; Epilogue; Biographical entries; Abbreviations; Archives and manuscript collections; Selected bibliography; Index to the essay.

History of Mathematics, Volume 34

January 2009, 345 pages, Hardcover, ISBN: 978-0-8218-4376-5, LC 2008035318, 2000 *Mathematics Subject Classification:* 01A70, 01A60, 01A80, 01A73, 01A55, 01A05, 01A99, **AMS members US$63**, List US$79, Order code HMATH/34



The Survival of a Mathematician

From Tenure-Track to Emeritus

Steven G. Krantz, *Washington University in St. Louis, MO*

A successful mathematical career involves doing good mathematics, to be sure, but also requires a wide range of skills that are not normally taught in graduate school. The purpose of this book is to provide guidance to the professional mathematician in how to develop and survive in the profession. There is information on how to begin a research program, how to apply for a grant, how to get tenure, how to teach, and how to get along with one's colleagues. After tenure, there is information on how to direct a Ph.D. student, how to serve on committees, and how to serve in various posts in the math department. There is extensive information on how to serve as Chairman. There is also material on trouble areas: sexual harassment, legal matters, disputes with colleagues, dealing with the dean, and so forth.

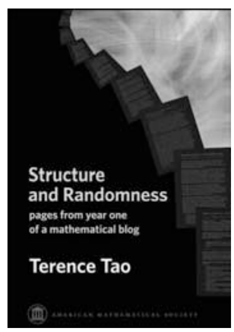
One of the themes of the book is how to have a fulfilling professional life. In order to achieve this goal, Krantz discusses keeping a vigorous scholarly program going and finding new challenges, as well as dealing with the everyday tasks of research, teaching, and administration.

In short, this is a survival manual for the professional mathematician—both in academics and in industry and government agencies. It is a sequel to the author's *A Mathematician's Survival Guide*.

Steven G. Krantz is an accomplished mathematician and an award-winning author. He has published more than 150 research articles and over 50 books. He has worked as an editor of several book series, research journals, and for the *Notices of the AMS*.

Contents: *Simple steps for little feet:* The meaning of life; Your duties; Sticky wickets; *Living the life:* Research; Beyond research; Being department chair; *Looking ahead:* Living your life; Glossary; Bibliography; Index.

January 2009, approximately 301 pages, Softcover, ISBN: 978-0-8218-4629-2, LC 2008036232, 2000 *Mathematics Subject Classification:* 00-01; 00A99, 00A30, 00A05, 00A06, **AMS members US$31**, List US$39, Order code MBK/60



Structure and Randomness

pages from year one of a mathematical blog

Terence Tao, *University of California, Los Angeles, CA*

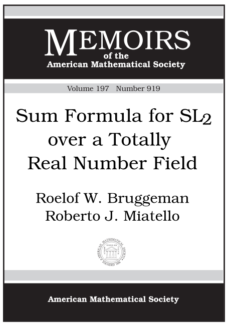
There are many bits and pieces of folklore in mathematics that are passed down from advisor to student, or from collaborator to collaborator, but which are too fuzzy and non-rigorous to be discussed in the formal literature. Traditionally, it was a matter of luck and location as to who learned such folklore mathematics. But today, such bits and pieces can be communicated effectively and efficiently via the semiformal medium of research blogging. This book grew from such a blog.

In 2007, Terry Tao began a mathematical blog, as an outgrowth of his own website at UCLA. This book is based on a selection of articles from the first year of that blog. These articles discuss a wide range of mathematics and its applications, ranging from expository articles on quantum mechanics, Einstein's equation $E = mc^2$, or compressed sensing, to open problems in analysis, combinatorics, geometry, number theory, and algebra, to lecture series on random matrices, Fourier analysis, or the dichotomy between structure and randomness that is present in many subfields of mathematics, to more philosophical discussions on such topics as the interplay between finitary and infinitary in analysis. Some selected commentary from readers of the blog has also been included at the end of each article. While the articles vary widely in subject matter and level, they should be broadly accessible to readers with a general graduate mathematics background; the focus in many articles is on the "big picture" and on informal discussion, with technical details largely being left to the referenced literature.

Contents: Expository articles; Lectures; Open problems; Bibliography.

January 2009, approximately 310 pages, Softcover, ISBN: 978-0-8218-4695-7, 2000 *Mathematics Subject Classification:* 00-02, **AMS members US$28**, List US$35, Order code MBK/59

Number Theory



Sum Formula for SL_2 over a Totally Real Number Field

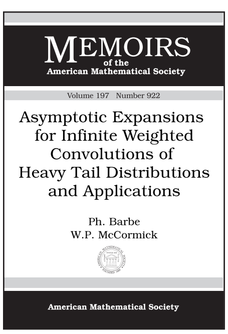
Roelof W. Bruggeman, *Universiteit Utrecht, The Netherlands*, and **Roberto J. Miatello**, *Universidad Nacional de Córdoba, Argentina*

Contents: Introduction; Spectral sum formula; Kloosterman sum formula; Appendix A. Sum formula for the congruence subgroup $\Gamma_1(I)$; Appendix B. Comparisions; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 197, Number 919

January 2009, 81 pages, Softcover, ISBN: 978-0-8218-4202-7, LC 2008039456, 2000 *Mathematics Subject Classification:* 11F72, 11F30, 11F41, 11L05, 22E30, **Individual member US$37**, List US$62, Institutional member US$50, Order code MEMO/197/919

Probability



Asymptotic Expansions for Infinite Weighted Convolutions of Heavy Tail Distributions and Applications

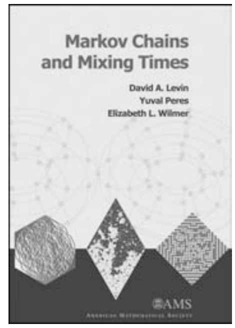
Ph. Barbe, *CNRS, Paris, France*, and **W. P. McCormick**, *University of Georgia, Athens*

Contents: Introduction; Main result; Implementing the expansion; Applications; Preparing the proof; Proof in the positive case; Removing the sign restriction on the random variables; Removing the sign restriction on the constants; Removing the smoothness restriction; Appendix. **Maple** code; Bibliography.

Memoirs of the American Mathematical Society, Volume 197, Number 922

January 2009, 117 pages, Softcover, ISBN: 978-0-8218-4259-1, LC 2008039491, 2000 *Mathematics Subject Classification:* 41A60, 60F99; 41A80, 44A35, 60E07, 60G50, 60K05, 60K25, 62E17, 62G32, **Individual member US$40**, List US$67, Institutional member US$54, Order code MEMO/197/922

COURSE ADOPTION



Markov Chains and Mixing Times

David A. Levin, *University of Oregon, Eugene, OR*, **Yuval Peres**, *Microsoft Research, Redmond, WA, and University of California, Berkeley, CA*, and **Elizabeth L. Wilmer**, *Oberlin College, OH*

This book is an introduction to the modern approach to the theory of Markov chains. The main goal of this approach is to determine the rate of convergence of a Markov chain to the stationary distribution as a function of the size and geometry of the state space. The authors develop the key tools for estimating convergence times, including coupling, strong stationary times, and spectral methods. Whenever possible, probabilistic methods are emphasized. The book includes many examples and provides brief introductions to some central models of statistical mechanics. Also provided are accounts of random walks on networks, including hitting and cover times, and analyses of several methods of shuffling cards. As a prerequisite, the authors assume a modest understanding of probability theory and linear algebra at an undergraduate level. *Markov Chains and Mixing Times* is meant to bring the excitement of this active area of research to a wide audience.

Contents: *Part I:* Introduction to finite Markov chains; Classical (and useful) Markov chains; Markov chain Monte Carlo: Metropolis and Glauber chains; Introduction to Markov chain mixing; Coupling; Strong stationary times; Lower bounds on mixing times; The symmetric group and shuffling cards; Random walks on networks; Hitting times; Cover times; Eigenvalues; *Part II:* Eigenfunctions and comparison of chains; The transportation metric and path coupling; The Ising model; From shuffling cards to shuffling genes; Martingales and evolving sets; The cut-off phenomenon; Lamplighter walks; Continuous-time chains; Countable state-space chains; Coupling from the past; Open problems; Appendix A: Notes on notation; Appendix B: Background material; Appendix C: Introduction to simulation; Appendix D: Solutions to selected exercises; Bibliography; Index.

December 2008, 364 pages, Hardcover, ISBN: 978-0-8218-4739-8, LC 2008031811, 2000 *Mathematics Subject Classification:* 60J10, 60J27, 60B15, 60C05, 65C05, 60K35, 68W20, 68U20, 82C22, **AMS members US$52**, List US$65, Order code MBK/58

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Abelian Varieties

David Mumford, *Brown University, Providence, RI*

This is a reprinting of the revised second edition (1974) of David Mumford's classic 1970 book. It gives a systematic account of the basic results about abelian varieties. It includes expositions of analytic methods applicable over the ground field of complex numbers, as well as of scheme-theoretic methods used to deal with inseparable isogenies when the ground field has positive characteristic. A self-contained proof of the existence of the dual abelian variety is given. The structure of the ring of endomorphisms of an abelian variety is discussed. These are appendices on Tate's theorem on endomorphisms of abelian varieties over finite fields (by C. P. Ramanujam) and on the Mordell–Weil theorem (by Yuri Manin).

David Mumford was awarded the 2007 AMS Steele Prize for Mathematical Exposition.

According to the citation: "*Abelian Varieties* ... remains the definitive account of the subject ... the classical theory is beautifully intertwined with the modern theory, in a way which sharply illuminates both ... [It] will remain for the foreseeable future a classic to which the reader returns over and over."

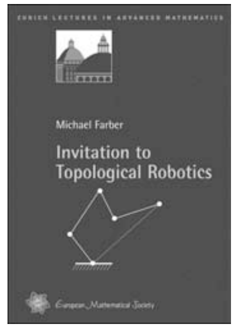
A publication of the Tata Institute of Fundamental Research. Distributed worldwide except in India, Bangladesh, Bhutan, Maldavis, Nepal, Pakistan, and Sri Lanka.

Contents: Analytic theory; Alegebraic theory via varieties; Algebraic theory via schemes; Hom (X, X) and l-adic representation; Appendix I: The theorem of Tate by C.P. Ramanujam; Appendix II: Mordell–Weil theorem by Yuri Manin; Bibliography; Index.

Tata Institute of Fundamental Research

August 2008, 263 pages, Hardcover, ISBN: 978-81-85931-86-9, 2000 *Mathematics Subject Classification:* 14K20, 14K05, **AMS members US$24**, List US$30, Order code TIFR/13

Analysis



Invitation to Topological Robotics

Michael Farber, *University of Durham, England*

This book discusses several selected topics of a new emerging area of research on the interface between topology and engineering. The first main topic is topology of configuration spaces of mechanical linkages. These manifolds arise in various fields of mathematics and in other sciences, e.g., engineering, statistics, molecular biology. To compute Betti numbers of these configuration spaces the author applies a new technique of Morse theory in the presence of an involution. A significant result of topology of linkages presented in this book is a solution of a conjecture of Kevin Walker which states that the relative sizes of bars of a linkage are determined, up to certain equivalence, by the cohomology algebra of the linkage configuration space.

This book also describes a new probabilistic approach to topology of linkages which treats the bar lengths as random variables and studies mathematical expectations of Betti numbers. The second main topic is topology of configuration spaces associated to polyhedra. The author gives an account of a beautiful work of S. R. Gal, suggesting an explicit formula for the generating function encoding Euler characteristics of these spaces. Next the author studies the knot theory of a robot arm, focusing on a recent important result of R. Connelly, E. Demain, and G. Rote. Finally, he investigates topological problems arising in the theory of robot motion planning algorithms and studies the homotopy invariant **TC**(X) measuring navigational complexity of configuration spaces.

This book is intended as an appetizer and will introduce the reader to many fascinating topological problems motivated by engineering.

This item will also be of interest to those working in geometry and topology.

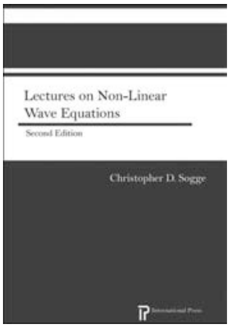
A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Linkages and polygon spaces; Euler characteristics of configuration spaces; Knot theory of the robot arm; Navigational complexity of configuration spaces; Recommendations for further reading; Bibliography; Index.

Zurich Lectures in Advanced Mathematics, Volume 8

August 2008, 144 pages, Softcover, ISBN: 978-3-03719-054-8, 2000 *Mathematics Subject Classification:* 58E05, 57R70, 57R30, **AMS members US$31**, List US$39, Order code EMSZLEC/8

Differential Equations



Lectures on Non-Linear Wave Equations

Second Edition

Christopher D. Sogge, *Johns Hopkins University, Baltimore, MD*

This much-anticipated revised second edition of Christopher Sogge's 1995 work provides a self-contained account of the basic facts concerning the linear wave equation and the methods from harmonic analysis that are necessary when studying nonlinear hyperbolic differential equations. Sogge examines quasilinear equations with small data where the Klainerman–Sobolev inequalities and weighted space-time estimates are introduced to prove global existence results. New simplified arguments are given in the current edition that allow one to handle quasilinear systems with multiple wave speeds.

The next topic concerns semilinear equations with small initial data. John's existence theorem for $\mathbf{R}^{1+3}$ is discussed with blow-up problems and some results for the spherically symmetric case. After this, general Strichartz estimates are treated. Proofs of the endpoint Strichartz estimates of Keel and Tao and the Christ–Kiselev lemma are given, the material being new in this edition. Using the Strichartz estimates, the critical wave equation in $\mathbf{R}^{1+3}$ is studied.

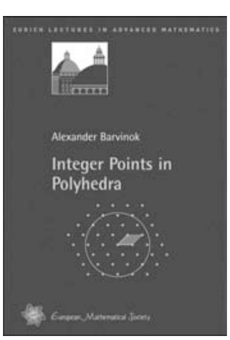
A publication of International Press. Distributed worldwide by the American Mathematical Society.

Contents: Background and groundwork; Quasilinear equations with small data; Semilinear equations with small data; General Strichartz estimates; Global existence for semilinear equations with large data; Appendix: Some tools from classical analysis.

International Press

July 2008, 203 pages, Hardcover, ISBN: 978-1-57146-173-5, 2000 *Mathematics Subject Classification:* 35L70; 42B25, **AMS members US$55**, List US$69, Order code INPR/72

Geometry and Topology



Integer Points in Polyhedra

Alexander Barvinok, *University of Michigan, Ann Arbor, MI*

This is a self-contained exposition of several core aspects of the theory of rational polyhedra with a view towards algorithmic applications to efficient counting of integer points, a problem arising in many areas of pure and applied mathematics. The approach is based on the consistent development and application of the apparatus of generating functions and the algebra of polyhedra. Topics range from classical, such as the Euler characteristic, continued fractions, Ehrhart polynomial, Minkowski Convex Body Theorem, and the Lenstra–Lenstra–Lovász lattice reduction algorithm, to recent advances such as the Berline–Vergne local formula.

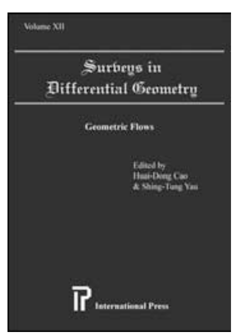
The text is intended for graduate students and researchers. Prerequisites are a modest background in linear algebra and analysis as well as some general mathematical maturity. Numerous figures, exercises of varying degree of difficulty as well as references to the literature and publicly available software make the text suitable for a graduate course.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction; The algebra of polyhedra; Linear transformations and polyhedra; The structure of polyhedra; Polarity; Tangent cones. Decompositions modulo polyhedra with lines; Open polyhedra; The exponential valuation; Computing volumes; Lattices, bases, and parallelepipeds; The Minkowski Convex Body Theorem; Reduced basis; Exponential sums and generating functions; Totally unimodular polytopes; Decomposing a 2-dimensional cone into unimodular cones via continued fractions; Decomposing a rational cone of an arbitrary dimension into unimodular cones; Efficient counting of integer points in rational polytopes; The polynomial behavior of the number of integer points in polytopes; A valuation on rational cones; A "local" formula for the number of integer points in a polytope; Bibliography; Index.

Zurich Lectures in Advanced Mathematics, Volume 9

August 2008, 200 pages, Softcover, ISBN: 978-3-03719-052-4, 2000 *Mathematics Subject Classification:* 52C07, 52B20, 05A15, 52B45, 52B55, 52C45, 11H06, **AMS members US$35**, List US$44, Order code EMSZLEC/9



Surveys in Differential Geometry, Volume XII

Geometric Flows

Huai-Dong Cao, *Lehigh University, Bethlehem, PA*, and **Shing-Tung Yau**, *Harvard University, Cambridge, MA*, Editors

Geometric flows are non-linear parabolic differential equations which describe the evolution of geometric structures. Inspired by Hamilton's Ricci flow, the field of geometric flows has seen tremendous progress in the past 25 years and yields important applications to geometry, topology, physics, nonlinear analysis, etc. Of course, the most spectacular development is Hamilton's theory of Ricci flow and its application to three-manifold topology, including the Hamilton-Perelman proof of the Poincaré conjecture.

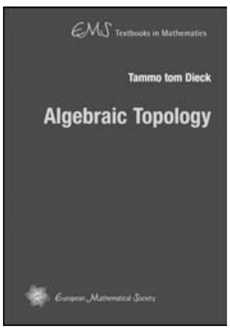
This twelfth volume of the annual *Surveys in Differential Geometry* examines recent developments on a number of geometric flows and related subjects, such as Hamilton's Ricci flow, formation of singularities in the mean curvature flow, the Kähler-Ricci flow, and Yau's uniformization conjecture.

A publication of International Press. Distributed worldwide by the American Mathematical Society.

Contents: S. Brendle, On the conformal scalar curvature equation and related problems; **A. Chau** and **L.-F. Tam**, A survey of the Kähler–Ricci flow and Yau's uniformization conjecture; **H.-D. Cao**, **B.-L. Chen**, and **X.-P. Zhu**, Recent developments on the Hamilton's Ricci flow; **C. Gerhardt**, Curvature flows in semi-Riemannian manifolds; **J. Krieger**, Global regularity and singularity development for wave maps; **V. Moncrief**, Relativistic Teichmüller theory: A Hamilton–Jacobi approach to 2 + 1-dimensional Einstein gravity; **L. Ni**, Monotonicity and Li–Yau–Hamilton inequalities; **C. Sinestrari**, Singularities of mean curvature flow and flow with surgeries; **M.-T. Wang**, Some recent developments in Lagrangian mean curvature flows.

International Press

July 2008, 356 pages, Hardcover, ISBN: 978-1-57146-118-6, 2000 *Mathematics Subject Classification:* 53C44, **AMS members US$68**, List US$85, Order code INPR/71



Algebraic Topology

Tammo tom Dieck, *University of Göttingen, Germany*

This book is written as a textbook on algebraic topology. The first part covers the material for two introductory courses about homotopy and homology. The second part presents more advanced applications and concepts (duality, characteristic classes, homotopy groups of spheres, bordism). The author recommends starting an introductory course with homotopy theory. For this purpose, classical results are presented with new elementary proofs. Alternatively, one could start more traditionally with singular and axiomatic homology. Additional chapters are devoted to the geometry of manifolds, cell complexes and fibre bundles. A special feature is the rich supply of nearly 500 exercises and problems. Several sections include topics which have not appeared before in textbooks as well as simplified proofs for some important results.

Prerequisites are standard point set topology (as recalled in the first chapter), elementary algebraic notions (modules, tensor product), and some terminology from category theory. The aim of the book is to introduce advanced undergraduate and graduate (master's) students to basic tools, concepts and results of algebraic topology. Sufficient background material from geometry and algebra is included.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Topological spaces; The fundamental group; Covering spaces; Elementary homotopy theory; Cofibrations and fibrations; Homotopy groups; Stable homotopy. Duality; Cell complexes; Singular homology; Homology; Homological algebra; Cellular homology; Partitions of unity in homotopy theory; Bundles; Manifolds; Homology of manifolds; Cohomology; Duality; Characteristic classes; Homology and homotopy; Bordism; Bibliography; Symbols; Index.

EMS Textbooks in Mathematics, Volume 8

September 2008, 578 pages, Hardcover, ISBN: 978-3-03719-048-7, 2000 *Mathematics Subject Classification:* 55-01, 57-01, **AMS members US$62**, List US$78, Order code EMSTEXT/8

Fukaya Categories and Picard–Lefschetz Theory

Paul Seidel, *Massachusetts Institute of Technology, Cambridge, MA*

The central objects in the book are Lagrangian submanifolds and their invariants, such as Floer homology and its multiplicative structures, which together constitute the Fukaya category. The relevant aspects of pseudo-holomorphic curve theory are covered in some detail, and there is also a self-contained account of the necessary homological algebra.

Generally, the emphasis is on simplicity rather than generality. The last part discusses applications to Lefschetz fibrations and contains many previously unpublished results. The book will be of interest to graduate students and researchers in symplectic geometry and mirror symmetry.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: A_∞-categories; Fukaya categories; Picard–Lefschetz theory; Bibliography; Symbols; Index.

Zurich Lectures in Advanced Mathematics, Volume 10

June 2008, 334 pages, Softcover, ISBN: 978-3-03719-063-0, 2000 *Mathematics Subject Classification:* 53D40; 32Q65, 53D12, 16E45, **AMS members US$46**, List US$58, Order code EMSZLEC/10

Number Theory

Lectures on Algebraic Independence

Yu. V. Nesterenko, *Moscow State University, Russia*

This book is an expanded version of the notes of a course of lectures given by at the Tata Institute of Fundamental Research in 1998. It deals with several important results and methods in transcendental number theory.

First, the classical result of Lindemann–Weierstrass and its applications are dealt with. Subsequently, Siegel's theory of *E*-functions is developed systematically, culminating in Shidlovskii's theorem on the algebraic independence of the values of the *E*-functions satisfying a system of differential equations at certain algebraic values. Proof of the Gelfond–Schneider Theorem is given based on the method of interpolation determinants introduced in 1992 by M. Laurent.

The author's famous result in 1996 on the algebraic independence of the values of the Ramanujan functions is the main theme of the reminder of the book. After deriving several beautiful consequences of his result, the author develops the algebraic material necessary for the proof. The two important technical tools in the proof are Philippon's criterion for algebraic independence and zero bound for Ramanujan functions. The proofs of these are covered in detail.

The author also presents a direct method, without using any criterion for algebraic independence as that of Philippon, by which one can obtain lower bounds for transcendence degree of finitely generated field $\mathbb{Q}(\omega_1, \ldots, \omega_m)$. This is a contribution towards Schanuel's conjecture.

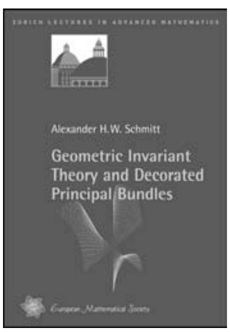
The book is self-contained and the proofs are clear and lucid. A brief history of the topics is also given. Some sections intersect with Chapters 3 and 10 of *Introduction to Algebraic Independence Theory*, Lecture Notes in Mathematics, Springer, 1752, edited by Yu. V. Nesterenko and P. Philippon.

Narosa Publishing House for the Tata Institute of Fundamental Research. Distributed worldwide except in India, Bangladesh, Bhutan, Maldavis, Nepal, Pakistan, and Sri Lanka.

Contents: Lindemann–Weierstrass theorem; E-functions and Shidlovskii's theorem; Small transcendence degree (exponential function); Small transcendence degree (modular functions); Algebraic fundamentals; Philippon's criterion of algebraic independence; Fields of large transcendence degree; Multiplicity estimates.

Tata Institute of Fundamental Research

November 2008, 157 pages, Softcover, ISBN: 978-81-7319-984-4, 2000 *Mathematics Subject Classification:* 11J81; 11J85, **AMS members US$32**, List US$40, Order code TIFR/14



Geometric Invariant Theory and Decorated Principal Bundles

Alexander H. W. Schmitt, *Freie Universität, Berlin, Germany*

The book starts with an introduction to Geometric Invariant Theory (GIT). The fundamental results of Hilbert and Mumford are exposed as well as more recent topics such as the instability flag, the finiteness of the number of quotients, and the variation of quotients.

In the second part, GIT is applied to solve the classification problem of decorated principal bundles on a compact Riemann surface. The solution is a quasi-projective moduli scheme which parameterizes those objects that satisfy a semistability condition originating from gauge theory. The moduli space is equipped with a generalized Hitchin map.

Via the universal Kobayashi–Hitchin correspondence, these moduli spaces are related to moduli spaces of solutions of certain vortex type equations. Potential applications include the study of representation spaces of the fundamental group of compact Riemann surfaces.

The book concludes with a brief discussion of generalizations of these findings to higher dimensional base varieties, positive characteristic, and parabolic bundles.

The text is fairly self-contained (e.g., the necessary background from the theory of principal bundles is included) and features numerous examples and exercises. It addresses students and researchers with a working knowledge of elementary algebraic geometry.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

Contents: Introduction; Geometric invariant theory; Decorated principal bundles; Bibliography; Index.

Zurich Lectures in Advanced Mathematics, Volume 11

July 2008, 397 pages, Softcover, ISBN: 978-3-03719-065-4, 2000 *Mathematics Subject Classification:* 14L24, 14H60; 13A50, 14D20, 14F05, 14L30, **AMS members US$50**, List US$62, Order code EMSZLEC/11

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For more information about the department and the university, visit our website: `http://www.math.ua.edu/`.

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000107

CALIFORNIA INSTITUTE OF TECHNOLOGY
Mathematics Department
Scott Russell Johnson
Senior Postdoctoral Scholar in Mathematics

Description: There are three terms in the Caltech academic year. The fellow is expected to teach one course in two terms each year, and is expected to be in residence even during terms when not teaching. The initial appointment is for three years with an additional three-year terminal extension expected.

Eligibility: Offered to a candidate within six years of having received the Ph.D. who shows strong research promise in one of the areas in which Caltech's mathematics faculty is currently active.

Deadline: January 1, 2009.

Application information: Please apply online at: `http://mathjobs.org`. To avoid duplication of paperwork, your application will also be considered for an Olga Taussky and John Todd Instructorship and a Harry Bateman Research Instructorship.

Caltech is an Affirmative Action/Equal Opportunity Employer. Women, minorities, veterans, and disabled persons are encouraged to apply.

000108

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2008 rate is $110 per inch or fraction thereof on a single column (one-inch minimum), calculated from top of headline. Any fractional text of 1/2 inch or more will be charged at the next inch rate. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: January 2009 issue–October 28, 2008; February 2009 issue–November 26, 2008; March 2009 issue–December 29, 2008; April 2009 issue–January 29, 2009; May 2009–February 27, 2009; June/July 2009 issue–April 28. 2009.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 20904. Advertisers will be billed upon publication.

CALIFORNIA INSTITUTE OF TECHNOLOGY
Mathematics Department
Harry Bateman Research Instructorships in Mathematics

Description: Appointments are for two years. The academic year runs from approximately October 1 to June 1. Instructors are expected to teach one course per quarter for the full academic year and to devote the rest of their time to research. During the summer months there are no duties except research.

Eligibility: Open to persons who have recently received their doctorates in mathematics.

Deadline: January 1, 2009.

Application information: Please apply online at: `http://mathjobs.org`. To avoid duplication of paperwork, your application may also be considered for an Olga Taussky and John Todd Instructorship.

Caltech is an Affirmative Action/Equal Opportunity Employer. Women, minorities, veterans, and disabled persons are encouraged to apply.

000109

UNIVERSITY OF CALIFORNIA LOS ANGELES
Institute for Pure and Applied Mathematics

The Institute for Pure and Applied Mathematics (IPAM) at UCLA is seeking an Associate Director (AD), to begin a two-year appointment on July 1, 2009. The AD is expected to be an active and established research mathematician or scientist in a related field, with experience in conference organization. The primary responsibility of the AD will be running individual programs in coordination with the organizing committees. More information on IPAM's programs can be found at `http://www.ipam.ucla.edu`. The selected candidate will be encouraged to continue his or her personal research program within the context of the responsibilities to the institute. For a detailed job description and application instructions, go to `http://www.ipam.ucla.edu/jobopenings/assocdirector.aspx`. Applications will receive fullest consideration if received by February 1, 2009, but we will accept applications as long as the position remains open. UCLA is an Equal Opportunity/Affirmative Action Employer.

000145

UNIVERSITY OF SOUTHERN CALIFORNIA
Department of Mathematics

The University of Southern California Department of Mathematics seeks to fill the following three positions. The start date for all three positions is August 2009.

Tenure-Track Assistant Professorship. Subject area: open. Candidates should have demonstrated excellence in research and a strong commitment to graduate and undergraduate education.

Busemann Assistant Professorship. Subject area: geometry and/or topology. Candidates should demonstrate great promise in research in geometry/topology and evidence of strong teaching. This is a three-year non-tenure-track appointment with a three-course-per-year teaching load.

Assistant Professor Non-Tenure-Track. Subject area: any field of mathematics of interest to senior members of the department. Candidates should demonstrate great promise in research and evidence of strong teaching. This is a three-year non-tenure-track appointment with a four-course-per-year teaching load.

To apply, please submit the following materials: letter of application and curriculum vitae, including your email address, telephone and fax numbers, preferably with the standardized AMS Cover Sheet. Candidates should also arrange for at least three letters of recommendation to be sent, one of which addresses teaching skills. Applications through MathJobs at `http://www.mathjobs.org` are preferred. Otherwise, all materials should be mailed to:

Search Committee
Department of Mathematics
College of Letters Arts and Sciences
University of Southern California
3620 Vermont Avenue, KAP 108
Los Angeles, CA 90089-2532.

Review of applications will begin November 15, 2008, and will continue until the positions are filled. Additional information about the USC College Department of Mathematics can be found at our website `http://www.usc.edu/schools/college/mathematics/`. USC strongly values diversity and is committed to equal opportunity in employment. Women and men and members of all racial and ethnic groups are encouraged to apply.

000147

CONNECTICUT

FAIRFIELD UNIVERSITY
Department of Mathematics and Computer Science

The Department of Mathematics and Computer Science at Fairfield University invites applications for one tenure-track position in mathematics, at the rank of assistant professor, to begin in September 2009. We seek a highly qualified candidate with a commitment to and demonstrated excellence in teaching, and strong evidence of research potential. A doctorate in mathematics is required. The teaching load is 3 courses/9 credit hours per semester and consists primarily of courses at the undergraduate level. The successful candidate will be expected to teach a wide variety of courses from elementary calculus and statistics to graduate level courses; in particular, Fairfield University's core curriculum includes two semesters of mathematics for all undergraduates.

Fairfield University, the Jesuit University of Southern New England, is a comprehensive university with about 3,200 undergraduates and a strong emphasis on liberal arts education. The department has an active faculty of 14 full-time tenured or tenure-track members. We offer a BS and an MS in mathematics, as well as a BS in computer science. The MS program is an evening program and attracts students from various walks of life—secondary school teachers, eventual Ph.D. candidates, and people working in industry, among others.

Fairfield offers competitive salaries and compensation benefits. The picturesque campus is located on Long Island Sound in southwestern Connecticut, about 50 miles from New York City. Fairfield is an Affirmative Action/Equal Opportunity Employer. For more information see the department webpage at: `http://www.fairfield.edu/macs_index.html`. Applicants should send a letter of application, a curriculum vitae, teaching and research statements, and three letters of recommendation commenting on the applicant's experience and promise as a teacher and scholar, to Matt Coleman, Chair of the Department of Mathematics and Computer Science, Fairfield University, 1073 N. Benson Rd., Fairfield CT 06824-5195. Full consideration will be given to complete applications received by December 12, 2008. We will be interviewing at the Joint Mathematics Meetings in Washington DC, January 5-8, 2009. Please let us know if you will be attending.

000102

DISTRICT OF COLUMBIA

AMERICAN UNIVERSITY
Department of Mathematics

Two tenure-track positions in the Mathematics/Statistics Department at American University at the rank of assistant professor or associate professor, beginning fall 2009. Qualified candidates will have a strong background in mathematics or statistics with a Ph.D., teaching experience is required. American University is an EEO/AA employer. Minority and women candidates are encouraged to apply. See `http://math.american.edu/positions`, or contact the Department of Mathematics and Statistics at (202) 885-3120 for details.

000151

GEORGIA

GEORGIA INSTITUTE OF TECHNOLOGY
School of Mathematics

The School of Mathematics at Georgia Tech is continuing an ambitious faculty recruitment program begun five years ago. During the past five years, fifteen appointments were made, including four tenured appointments, two at the full professor level and two at the associate professor level. Building on past successes, this recruiting effort is intended to make rapid advances in the scope and quality of our research and graduate education programs. Candidates will be considered at all ranks, with priority given to those candidates who (1) bring exceptional quality research credentials to Georgia Tech; (2) complement existing strengths in the School of Mathematics; (3) reinforce bridges to programs in engineering and the physical, computing, and life sciences; (4) have strong potential for external funding; and (5) have a demonstrated commitment to high quality teaching at both the undergraduate and graduate levels. Consistent with these priorities, candidates will be considered in all areas of pure and applied mathematics and statistics. Applications should consist of a curriculum vitae, including a list of publications; summary of future research plans; and at least three letters of reference. Applications should also include evidence of teaching skills. Applications should preferably be electronically submitted directly to: `http://www.mathjobs.org`. If a candidate cannot submit an application electronically, then it may be sent to the Hiring Committee, School of Mathematics, Georgia Institute of Technology, Atlanta, GA, 30332-0160. Candidates for associate and full professor positions should also submit a statement outlining their vision for service as a senior faculty member at Georgia Tech. Review of applications will begin in October 2008, and the roster of candidates being considered will be updated on a continual basis. Georgia Tech, an institution of the University System of Georgia, is an Equal Opportunity/Affirmative Action Employer.

000096

ILLINOIS

UNIVERSITY OF ILLINOIS AT CHICAGO
Department of Mathematics, Statistics, and Computer Science

The department has active research programs in a broad spectrum of centrally important areas of pure mathematics, computational and applied mathematics, combinatorics, mathematical computer science and scientific computing, probability and statistics, and mathematics education. See `http://www.math.uic.edu` for more information.

Applications are invited for the following position, effective August 16, 2009, subject to budgetary approval.

Research Assistant Professorship. This is a non-tenure-track position, normally renewable annually to a maximum of three years. This position carries a teaching responsibility of one course per semester, and the expectation that the incumbent play a significant role in the research life of the department. The salary for AY 2008–2009 for this position is \$54,500, the salary for AY 2009–2010 may be higher. Applicants must have a Ph.D. or equivalent degree in mathematics, computer science, statistics, mathematics education or related field, and evidence of outstanding research potential. Preference will be given to candidates in areas related to number theory or dynamical systems.

Send vita and at least three (3) letters of recommendation, clearly indicating the position being applied for, to: Appointments Committee, Dept. of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, 851 S. Morgan (m/c 249), Box R, Chicago, IL 60607. Applications through `http://mathjobs.org` are encouraged. No email applications will be accepted. To ensure full consideration, materials must be received by December 31, 2008. However, we will continue considering candidates until all positions have been filled. Minorities, persons with disabilities, and women are particularly encouraged to apply. UIC is an AA/EOE.

000137

UNIVERSITY OF ILLINOIS AT CHICAGO
Department of Mathematics, Statistics, and Computer Science

The department has active research programs in a broad spectrum of centrally important areas of pure mathematics, computational and applied mathematics, combinatorics, mathematical computer science and scientific computing, probability and statistics, and mathematics education. See `http://www.math.uic.edu` for more information.

Applications are invited for the following positions, effective August 16, 2009, subject to budgetary approval.

Tenure-track positions. Candidates in all areas of interest to the department will be considered. The position is at the assistant professor level.

Tenured position. Candidates in mathematical logic, with a preference for model theory or descriptive set theory, will be considered. The position is at the Associate Professor or Professor level.

Applicants must have a Ph.D. or equivalent degree in mathematics, computer science, statistics, mathematics education or related field, an outstanding research record, and evidence of strong teaching ability. The salary is negotiable.

Send vita and at least three (3) letters of recommendation, clearly indicating the position being applied for, to: Appointments Committee, Dept. of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, 851 S. Morgan (m/c 249), Box T, Chicago, IL 60607. Applications through `http://mathjobs.org` are encouraged. No email applications will be accepted. To ensure full consideration, materials must be received by November 11, 2008. However, we will continue considering candidates until all positions have been filled. Minorities, persons with disabilities, and women are particularly encouraged to apply. UIC is an AA/EOE.

000138

INDIANA

INDIANA UNIVERSITY SOUTH BEND
Department of Mathematical Sciences

The Department of Mathematical Sciences invites applications for a tenure-track, assistant professor position in mathematics. See the advertisement at `http://www.mathjobs.org/jobs/IUSB/1392`.

000142

UNIVERSITY OF NOTRE DAME
Department of Mathematics
Robert Lumpkins Instructorship in Mathematics and Notre Dame Instructorship in Mathematics

The Department of Mathematics of the University of Notre Dame invites applications from recent doctorates for the positions of 1) Robert Lumpkins Instructor in Mathematics, 2) Notre Dame Instructor in Mathematics. Candidates in any specialty compatible with the research interests of the department will be considered. The teaching load and salary will be competitive with those of distinguished instructorships at other AMS Group I universities. This position is for a term of three years beginning August 22, 2009; it is not renewable and is not tenure-track. Applications, including a curriculum vitae and a completed AMS standard cover sheet, should be filed through MathJobs (`http://www.MathJobs.org`). Applicants should also arrange for at least three letters of recommendation to be submitted through the MathJobs system. These letters should address the applicant's research accomplishments and supply evidence that the applicant has the ability to communicate articulately and teach effectively. Notre Dame is an Equal Opportunity Employer, and we particularly welcome applications from women and minority candidates. The evaluation of candidates will begin December 1, 2008. Information about the department is available at http://math.nd.edu.

AND

The Department of Mathematics of the University of Notre Dame invites applications from recent doctorates in mathematical logic for a postdoctoral position. Candidates in any area of mathematical logic compatible with the research interests of the logicians in the department will be considered with preference given to candidates in model theory. The position is contingent upon the availability of funding and, if funded, will extend for a term of three years beginning August 22, 2009. It is not renewable and is not tenure-track; the teaching load is one course per semester. The salary will be competitive with those of distinguished instructorships at other AMS Group I universities, and the position includes summer research support for each of the first two summers and some discretionary funding each year. Applications, including a curriculum vitae and a completed AMS standard cover sheet, should be filed through MathJobs (http://www.MathJobs.org). Applicants should also arrange for at least three letters of recommendation to be submitted through the MathJobs system. These letters should address the applicant's research accomplishments and supply evidence that the applicant has the ability to communicate articulately and teach effectively. Notre Dame is an Equal Opportunity Employer, and we particularly welcome applications from women and minority candidates. The evaluation of candidates will begin December 1, 2008. Information about the department is available at http://math.nd.edu.

000095

KANSAS

UNIVERSITY OF KANSAS
Department of Mathematics

Applications are invited for a tenure-track assistant professor position in numerical analysis expected to begin as early as August 18, 2009. Ph.D. or terminal degree in math or a related field is expected by the starting date of the appointment. For complete position announcement go to http://www.math.ku.edu/jobs or contact kumath@math.ku.edu. Letter of application, detailed vita, research description, teaching statement, completed AMS application form, and at least three recommendation letters (teaching ability must be addressed in at least one letter) should be mailed to Jack Porter, Chair, Department of Mathematics, 1460 Jayhawk Boulevard, University of Kansas, Lawrence, KS 66045-7523 (or fax to 785-864-5255).

Deadline: Review of applications will begin on November 15, 2008, and will continue until the position is filled. EO/AA Employer.

000134

MARYLAND

JOHNS HOPKINS UNIVERSITY
Department of Mathematics
Full Professor

The Department of Mathematics invites applications for one or more positions at the associate professor or full professor level in general areas of analysis, algebra, topology, number theory, and mathematical physics beginning fall 2009 or later.

To submit your applications go to: http://www.mathjobs.org/jobs/jhu. Applicants are strongly advised to submit their other materials electronically at this site.

If you do not have computer access, you may mail your application directly to: Appointments Committee, Department of Mathematics, Johns Hopkins University, 404 Krieger Hall, Baltimore, MD 21218. Application should include a vita, at least four letters of recommendation of which one specifically comments on teaching, and a description of current and planned research. Write to cpoole@jhu.edu for questions concerning these positions. Applications received by November 17, 2008, will be given priority. The Johns Hopkins University is an Affirmative Action/Equal Opportunity Employer. Minorities and women candidates are encouraged to apply.

000045

JOHNS HOPKINS UNIVERSITY
Department of Mathematics
Non-Tenure-Track J. J. Sylvester Assistant Professor

Subject to availability of resources and administrative approval, the Department of Mathematics solicits applications for non-tenure-track assistant professor positions beginning fall 2009.

The J. J. Sylvester Assistant Professorship is a three-year position offered to recent Ph.D.'s with outstanding research potential. Candidates in all areas of pure mathematics, including analysis, mathematical physics, geometric analysis, complex and algebraic geometry, number theory, and topology are encouraged to apply. The teaching load is three courses per academic year.

To submit your applications go to: http://www.mathjobs.org/jobs/jhu. Applicants are strongly advised to submit their other materials electronically at this site.

If you do not have computer access, you may mail your application directly to: Appointments Committee, Department of Mathematics, Johns Hopkins University, 404 Krieger Hall, Baltimore, MD 21218. Application should include a vita, at least four letters of recommendation of which one specifically comments on teaching, and a description of current and planned research. Write to cpoole@jhu.edu for questions concerning these positions. Applications received by November 17, 2008, will be given priority. The Johns Hopkins University is an Affirmative Action/Equal Opportunity Employer. Minorities and women candidates are encouraged to apply.

000046

MASSACHUSETTS

BOSTON COLLEGE
Department of Mathematics
Tenure-Track Positions in Number Theory and in Geometry/Topology

The Department of Mathematics at Boston College invites applications for two tenure-track positions at the level of assistant professor beginning in September 2009, one in number theory and the second in geometry/topology. In exceptional cases, a higher level appointment may be considered. The teaching load for each position is three semester courses per year.

Requirements include a Ph.D. or equivalent in mathematics awarded in 2007 or earlier, a record of strong research combined with outstanding research potential, and demonstrated excellence in teaching mathematics.

A completed application should contain a cover letter, a description of research plans, a statement of teaching philosophy, curriculum vitae, and at least four letters of recommendation. One or more of the letters of recommendation should directly comment on the candidate's teaching credentials.

Applications completed no later than December 1, 2008, will be assured our fullest consideration. Please submit all application materials through http://mathjobs.org. If necessary, printed materials may otherwise be sent to:

Chair, Search Committee in Number Theory
(resp. in Geometry/Topology)
Department of Mathematics
Boston College
Chestnut Hill, MA 02467-3806

Applicants may learn more about the department, its faculty and its programs at http://www.bc.edu/math. Electronic inquiries concerning these positions may be directed to math-search-nt@bc.edu or math-search-gt@bc.edu. Boston College is an Affirmative Action/Equal Opportunity Employer. Applications from women, minorities, and individuals with disabilities are encouraged.

000060

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mathematics

The Mathematics Department at MIT is seeking to fill positions at the level of assistant professor or higher for September 2009. Appointments are based on exceptional research contributions in pure mathematics. Appointees will be expected to fulfill teaching duties and pursue their own research program. Ph.D. required by employment start date. We request that applications and other materials, including (a) curriculum vitae, (b) research description, and (c) three letters of recommendation be submitted online at `http://www.mathjobs.org`. Applications should be complete by **December 1, 2008**, to receive full consideration. We request that your reference letters be submitted by reviewers online via `http://mathjobs`. We will also accept recommendations sent as PDF attachments to `pure@math.mit.edu`, or in hardcopy mailed to: Pure Mathematics Committee, Room 2-345, Department of Mathematics, MIT, 77 Massachusetts Ave., Cambridge, MA 02139-4307. Please do not mail or email duplicates of items already submitted via `mathjobs`.

MIT is an Equal Opportunity, Affirmative Action Employer.

000073

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mathematics
C.L.E. Moore Instructorships in Mathematics

These positions for September 2009 are open to mathematicians who show definite promise in research. Applicants with Ph.D.'s after June 2008 are strongly preferred. Appointees will be expected to fulfill teaching duties and pursue their own research program. We request that applications and other materials, including (a) curriculum vitae, (b) research description, and (c) three letters of recommendation, be submitted online at `http://www.mathjobs.org`. Applications should be complete by **December 1, 2008**, to receive full consideration. We request that your letters of reference be submitted by the reviewers online via `http://mathjobs`. We will also accept recommendations either as PDF attachments sent to: `pure@math.mit.edu`, or as paper copies mailed to: Pure Mathematics Committee, Room 2-345, Department of Mathematics, MIT, 77 Massachusetts Ave., Cambridge, MA 02139-4307. Please do not mail or email duplicates of items already submitted via `mathjobs`.

MIT is an Equal Opportunity, Affirmative Action Employer.

000074

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mathematics
Applied Mathematics

The applied mathematics group at MIT is seeking to fill combined teaching and research positions at the level of instructor, assistant professor or higher, beginning September 2009. Ph.D. required by employment start date. Appointments are mainly based on exceptional research qualifications. Candidates in all areas of applied mathematics, including physical applied mathematics, computational molecular biology, numerical analysis, scientific computation, and theoretical computer science will be considered. Current activities of the group include: combinatorics, operations research, theory of algorithms, numerical analysis, astrophysics, condensed matter physics, computational physics, fluid dynamics, geophysics, nonlinear waves, theoretical and computational molecular biology, material sciences, quantum computing and quantum field theory, but new hiring may involve other areas as well. We request that applications and other materials, including (a) curriculum vitae, (b) research description, and (c) three letters of recommendation be submitted online at: `http://www.mathjobs.org`, preferably well in advance of our deadline of **January 1, 2009**, since we will begin our deliberations in December. We request that your reference letters be submitted by reviewers online via `http://mathjobs`. We will also accept recommendations sent as PDF attachments to: `applied@math.mit.edu`, or in hardcopy mailed to: Applied Mathematics Committee, Room 2-345, Department of Mathematics, MIT, 77 Massachusetts Ave., Cambridge, MA 02139-4307. Please do not mail or email duplicates of items already submitted via `mathjobs`.

MIT is an Equal Opportunity, Affirmative Action Employer.

000075

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mathematics
Statistics

The Department of Mathematics at MIT is seeking to fill combined teaching and research positions at the level of instructor, assistant professor, or higher in STATISTICS or APPLIED PROBABILITY, beginning September 2009. Appointments are mainly based on exceptional research qualifications. Ph.D. required by employment start date. We request that applications and other materials, including (a) curriculum vitae, (b) research description, and (c) three letters of recommendation be submitted online at: `http://www.mathjobs.org`. Applications should be complete by **January 1, 2009**, to receive full consideration. We request that your reference letters be submitted by reviewers online via `http://mathjobs`. We will also accept recommendations sent as PDF attachments to: `statistics@math.mit.edu`, or in hardcopy mailed to: Committee on Statistics, Room 2-345, Department of Mathematics, MIT, 77 Massachusetts Ave., Cambridge, MA 02139-4307. Please do not mail or email duplicates of items already submitted via `mathjobs`.

MIT is an Equal Opportunity, Affirmative Action Employer.

000076

WILLIAMS COLLEGE
Mathematics & Statistics

The Williams College Department of Mathematics and Statistics invites applications for one tenure-track position in mathematics, beginning fall 2009, at the rank of assistant professor (in an exceptional case, a more advanced appointment may be considered). We are seeking a highly qualified candidate who has demonstrated excellence in teaching and research, and who will have a Ph.D. by the time of appointment.

Williams College is a private, coeducational, residential, highly selective liberal arts college with an undergraduate enrollment of approximately 2,000 students. The teaching load is two courses per 12-week semester and a winter term course every other January. In addition to excellence in teaching, an active and successful research program is expected.

Applicants are asked to supply a vita and have three letters of recommendation on teaching and research sent. Teaching and research statements are also welcome. Applications may be made online (`http://www.mathjobs.org/jobs`). Alternately, application materials and letters of recommendation may be sent to Olga R. Beaver, Chair of the Hiring Committee, Department of Mathematics and Statistics, Williams College, Williamstown, MA 01267. Evaluation of applications will begin on or after November 15 and will continue until the position is filled. For more information on the Department of Mathematics and Statistics, please visit `http://www.williams.edu/Mathematics`.

Williams College is committed to building and supporting a diverse population of faculty, staff, and students; to fostering a varied and inclusive curriculum; and to providing a welcoming intellectual environment for all. As an EEO/AA Employer, Williams encourages applications from all backgrounds. To learn more about Williams College, please visit `http://www.williams.edu`.

000048

WORCESTER POLYTECHNIC INSTITUTE
Department of Mathematical Sciences
Applied Mathematics Position

The Mathematical Sciences Department of Worcester Polytechnic Institute (WPI) invites applications for one tenure-track assistant professor position, to begin in the fall of 2009. Exceptionally well-qualified candidates may be considered for appointment at a higher rank. Preferred research interests are in the areas of: partial differential equations with applications to fluid and solid mechanics, mathematical biology, numerical analysis, optimization, and stochastic PDEs.

An earned Ph.D. or equivalent degree is required. Successful candidates must demonstrate strong research potential and evidence of quality teaching, and will be expected to contribute to the department's research activities and to its innovative, project-based educational programs.

WPI is a private and highly selective technological university with an enrollment of over 3,000 undergraduates and 1,100 full- and part-time graduate students. The Mathematical Sciences Department has 23 tenured/tenure-track faculty and supports BS, MS, and Ph.D. programs in applied, financial and industrial mathematics and applied statistics (see `http://www.wpi.edu/+math`).

Qualified applicants should send a detailed curriculum vitae, a brief statement of specific teaching and research objectives, and three letters of recommendation at least one of which addresses teaching potential, to: Math Search Committee, Mathematical Sciences Department, WPI, 100 Institute Road, Worcester, MA 01609-2280, USA.

Applicants will be considered on a continuing basis until the position is filled. Review of applications will start December 1, 2008.

To enrich education through diversity, WPI is an Affirmative Action, Equal Opportunity Employer.

000155

WORCESTER POLYTECHNIC INSTITUTE
Department of Mathematical Sciences
Applied/Industrial Mathematics Position

The Mathematical Sciences Department of Worcester Polytechnic Institute (WPI) invites applications for one tenure-track associate professor position, to begin in the fall of 2009. Exceptionally well-qualified candidates may be considered for appointment at a higher rank.

Successful candidates must have a strong research program in applied mathematics and experience in industrial mathematics projects with business and industry. They are expected to contribute to the department's research activities and its innovative project-based program, within the Center for Industrial Mathematics and Statistics.

WPI is a private and highly selective technological university with an enrollment of over 3,000 undergraduates and 1,100 full- and part-time graduate students. The Mathematical Sciences Department has 23 tenured/tenure-track faculty and supports BS, MS, and Ph.D. programs in applied, financial and industrial mathematics and applied statistics (see `http://www.wpi.edu/+math`).

Qualified applicants should send a detailed curriculum vitae, a brief statement of specific teaching and research objectives, and three letters of recommendation at least one of which addresses teaching potential, to: Math Search Committee, Mathematical Sciences Department, WPI, 100 Institute Road, Worcester, MA 01609-2280, USA.

Applicants will be considered on a continuing basis until the position is filled. Review of applications will start December 1, 2008.

To enrich education through diversity, WPI is an Affirmative Action, Equal Opportunity Employer.

000156

MICHIGAN

WAYNE STATE UNIVERSITY
Department of Mathematics

Pending authorization, the Department of Mathematics invites applications for a possible tenure-track position commencing in fall 2009. Applications should include a signed, detailed vita; description of current research interests; and four letters of recommendation, one of which should address teaching. Solid evidence of teaching at the undergraduate level is preferred to a statement of teaching philosophy. There is also a possibility of a visiting position for the 2009-2010 academic year. A Ph.D. in mathematics and a strong interest in research and teaching are required for all positions. Applications received by December 1, 2008, will be given priority. Wayne State University—People working together to provide quality service. Applicants must apply online at: `http://jobs.wayne.edu`. For further information, please consult the department's website: `http://www.math.wayne.edu`.

Wayne State University is an Equal Opportunity/Affirmative Action Employer. Women and members of underrepresented minority groups are especially encouraged to apply.

000099

MISSISSIPPI

MISSISSIPPI STATE UNIVERSITY
Mathematics and Statistics
Faculty Positions in Mathematics

Applications are invited for two or more tenure-track positions in pure and applied mathematics at the rank of assistant, associate, or full professor starting August, 2009. The department offers the Ph.D. degree in mathematical sciences and is highly research oriented. Requirements include a doctoral degree in an area of mathematical sciences, demonstrated success or a strong potential for research and a commitment to effective undergraduate and graduate teaching. Preference will be given to candidates with interests that closely fit well with current strengths in the department. Mississippi State University is the largest university in Mississippi and a land-grant Carnegie Doctoral/Research-extensive institution. Further information about the department may be found at `http://www.msstate.edu/dept/math`. Salary is competitive and commensurate with experience and qualifications.

For full consideration, all supporting material should be submitted electronically through `http://www.mathjobs.org/jobs`, by January 5, 2009. Supporting materials should include a detailed résumé with detailed research accomplishments, summaries of research plans and teaching philosophy, a completed AMS Standard Cover Sheet (`http://www.ams.org/employment`), and three letters of recommendation. One reference letter should address the applicant's teaching. In addition, a cover letter should be submitted through `http://mathjobs.org` addressed to: Chair, Mathematics Search Committee, Department of Mathematics and Statistics, P. O. Drawer MA, Mississippi State, MS 39762. All applicants must also complete the online Personal Information Data Form located at `http://www.jobs.msstate.edu`. Select "Create Application" and choose "Personal Information Data Form". (Apply to Req. #4224). Screening will begin January 6, 2009, and will continue until the positions are filled. Mississippi State University is an AA/EOE.

000132

NEBRASKA

UNIVERSITY OF NEBRASKA-LINCOLN
Department of Mathematics

Applications are invited for two tenure-track positions and one postdoctoral position in mathematics, starting in August 2009, as follows:

1. Modern Analysis. (Requisition #080764) One tenure-track assistant professor position in modern analysis.

2. Mathematical Biology. (Requisition #080765) One tenure-track assistant professor position in mathematical biology.

3. Research Assistant Professor. (Requisition #080763) One three-year (non-tenure-track) position in mathematics.

For all positions, use of the AMS application cover sheet is encouraged. First review of applications will begin on December 5, 2008. Successful candidates for each position should have a Ph.D. in mathematics and outstanding potential for research and teaching in mathematics. For the mathematical biology position applicants with a Ph.D. in a field related to mathematical biology will also be considered. Applicants should submit a letter of application, a CV, statements addressing their research and teaching, and at least three letters of reference, at least one of which should address teaching, to: Search Committee Chair (position title), Department of Mathematics, University of Nebraska-Lincoln, Lincoln, NE 68588-0130. To be considered for the position, applicants must complete the Faculty/Administrative Information Form at `http://employment.unl.edu`, (appropriate requisition #). For more information see the department's website: `http://www.math.unl.edu`. The University of Nebraska is committed to a pluralistic campus community through affirmative action, equal opportunity, work-life balance, and dual careers. Contact Marilyn Johnson at (402) 472-8822 for assistance.

000152

NEW HAMPSHIRE

DARTMOUTH COLLEGE
John Wesley Young Research Instructorship

The John Wesley Young Instructorship is a postdoctoral, two- to three-year appointment intended for promising Ph.D. graduates with strong interests in both research and teaching and whose research interests overlap a department member's. Current research areas include applied mathematics, combinatorics, geometry, logic, non-commutative geometry, number theory, operator algebras, probability, set theory, and topology. Instructors teach four ten-week courses distributed over three terms, though one of these terms in residence may be free of teaching. The assignments normally include introductory, advanced undergraduate, and graduate courses. Instructors usually teach at least one course in their own specialty. This appointment is for 26 months with a monthly salary of US$4,833 and a possible 12 month renewal. Salary includes two-month research stipend for instructors in residence during two of the three summer months. To be eligible for a 2009-2011 instructorship, candidate must be able to complete all requirements for the Ph.D. degree before September 2009. Applications may be obtained at `http://www.math.dartmouth.edu/recruiting/` or `http://www.mathjobs.org`. Position ID: 237-JWY. General inquiries can be directed to Annette Luce, Department of Mathematics, Dartmouth College, 6188 Kemeny Hall, Hanover, New Hampshire 03755-3551. At least one referee should comment on applicant's teaching ability; at least two referees should write about applicant's research ability. Applications received by January 5, 2009, receive first consideration; applications will be accepted until position is filled. Dartmouth College is committed to diversity and strongly encourages applications from women and minorities.

000078

NEW JERSEY

RAMAPO COLLEGE OF NEW JERSEY
Assistant Professor of Mathematics

Tenure track position for fall 2009.

JOB DESCRIPTION: Responsibilities encompass a wide range of undergraduate mathematics courses, and the teaching and development of General Education mathematics courses. REQUIREMENTS: Ph.D. in Mathematics by September 1, 2009, is required. College teaching experience preferred.

Faculty members are expected to maintain active participation in research, scholarship, college governance, service, academic advisement and professional development activities.

All applications must be completed online at: `http://www.ramapo.edu/hrjobs`. Attach resume, cover letter, statement of teaching philosophy, research interests and a list of three references to your completed application. Since its beginning, Ramapo College has had an intercultural/international mission. Please tell us how your background, interest and experience can contribute to this mission, as well as to the specific position for which you are applying.

Review of applications will begin immediately and continue until the position is filled. Position offers excellent state benefits. To request accommodations, call 201-684-7734. Additional supportive materials in non-electronic format may be sent to Dr. Lawrence D'Antonio, Search Committee Chair.

RAMAPO COLLEGE OF NEW JERSEY
505 Ramapo Valley Road
Mahwah, NJ 07430

New Jersey's Public Liberal Arts College

Ramapo College is a member of the Council of Public Liberal Arts Colleges (COPLAC), a national alliance of leading liberal arts colleges in the public sector.

Ramapo College of New Jersey is located in the beautiful foothills of the Ramapo Valley Mountains approximately 25 miles northwest of New York City. Ramapo College is a comprehensive institution of higher education dedicated to the promotion of teaching and learning within a strong liberal arts based curriculum, thus earning the designation "New Jersey's Public Liberal Arts College". Its curricular emphasis includes the liberal arts and sciences, social sciences, fine and performing arts, and the professional programs within a residential and sustainable living and learning environment. Organized into thematic learning communities, Ramapo College provides academic excellence through its interdisciplinary curriculum, international education, intercultural understanding, and experiential learning opportunities. EEO/AFFIRMATIVE ACTION.

000130

RUTGERS UNIVERSITY, CAMDEN
Department of Mathematical Sciences
Joseph and Loretta Lopez Endowed Chair in Mathematics

Applications and nominations are invited for the Joseph and Loretta Lopez Chair in Mathematics. The department seeks a distinguished scholar in mathematics with international reputation, well-established research and teaching record, and demonstrated ability to generate external funding. This endowed chair is the first at the Camden Campus of Rutgers University. It is a tenured faculty position and the chair is for a 5-year renewable term. The holder of this chair will be a senior faculty member and a vigorous participant in the research, instruction, and service work of the Department of Mathematical Sciences. The holder will also be expected to play a vital role in the recently established Center for Computational and Integrative Biology. Applicants must demonstrate evidence of interest in the areas of mathematical and/or computational biology.

The appointment will commence on September 1, 2009. The salary and startup funds are highly competitive and negotiable. The department will begin reviewing applications on December 17 and continue its review until the position is filled. Applications should be sent to:

Professor Gabor Toth, Chair, Search Committee, Department of Mathematical Sciences, Rutgers University, Camden, Camden, New Jersey, 08102.

Applicants should also arrange for at least four letters of recommendation to be sent. Rutgers University, Camden, is an Affirmative Action/Equal Opportunity Employer and encourages applications from women and minority group members.

000077

RUTGERS UNIVERSITY-NEW BRUNSWICK
Mathematics Department

The Mathematics Department of Rutgers University-New Brunswick invites applications for the following positions which may be available September 2009.

TENURED POSITION: Subject to availability of funding, the department expects one or more openings at the level of associate professor or professor. Candidates must have the Ph.D. and show a sustained record of outstanding research accomplishments in pure or applied mathematics, and concern for teaching. Outstanding candidates in any field of pure or applied mathematics will be considered. In addition, candidates should show outstanding leadership in research. The normal annual teaching load for research-active faculty is 2-1, that is, two courses for one semester, plus one course for the other semester. Review of applications begins immediately.

TENURE-TRACK ASSISTANT PROFESSORSHIP: Subject to availability of funding, the department may have one or more openings at the level of tenure-track assistant professor. Candidates must have the Ph.D. and show a sustained record of outstanding research accomplishments in pure or applied mathematics, and concern for teaching. Outstanding candidates in any field of pure or applied mathematics will be considered. The normal annual teaching load for research-active faculty is 2-1, that is, two courses for one semester, plus one course for the other semester. Review of applications begins November 1.

HILL ASSISTANT PROFESSORSHIP and NON-TENURE-TRACK ASSISTANT PROFESSORSHIP: These are both three-year nonrenewable positions. Subject to availability of funding, the department may have one or more open positions of these types. The Hill Assistant Professorship carries a reduced teaching load of 2-1 for research; candidates for it should have received the Ph.D., show outstanding promise of research ability in pure or applied mathematics, and have concern for teaching. The non-tenure-track assistant professorship carries a teaching load of 2-2; candidates for it should show evidence of superior teaching accomplishments and promise of research ability. Review of applications begins December 1, 2008.

Applicants for the above position(s) should submit a curriculum vitae (including a publication list) and arrange for four letters of reference to be submitted, one of which evaluates teaching.

Applicants should first go to the website `https://www.mathjobs.org/jobs` and fill out the AMS Cover Sheet electronically. It is essential to fill out the cover sheet completely, including naming the positions being applied for (TP, TTAP, HILL, NTTAP, respectively) giving the AMS Subject Classification number(s) of area(s) of specialization, and answering the question about how materials are being submitted. The strongly preferred way to submit the CV, references, and any other application materials is online at `https://www.mathjobs.org/jobs`. If necessary, however, application materials may instead be mailed to: Search Committee, Dept. of Math-Hill Center, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854-8019.

Review of applications will begin on the dates indicated above, and will continue until openings are filled. Updates on these positions will appear on the Rutgers Mathematics Department webpage at `http://www.math.rutgers.edu`.

Rutgers is an Affirmative Action/Equal Opportunity Employer and encourages applications from women and minority-group members.

000144

NEW YORK

CLARKSON UNIVERSITY
Division of Mathematics and Computer Science

The Division of Mathematics and Computer Science (`http://www.clarkson.edu/mcs`) invites applications for a tenure-track position in applied mathematics starting in August 2009 at the associate or full professor level.

We are especially interested in candidates with expertise in computational areas of applied mathematics, including statistics, or dynamical systems, but all areas of applied mathematics will be considered. Responsibilities will include teaching undergraduate and graduate level mathematics courses and directing graduate students. For this position, demonstrated excellence in both research, including a record of funding, and teaching are required. In addition, the candidate should be able to interact with other faculty in the department and the university.

Applications including vita and three reference letters should be submitted to Prof. P. A. Turner, Department of Mathematics and Computer Science, Clarkson University, Potsdam, NY 13699-5815. Completed applications will be reviewed starting immediately. Women and minorities are urged to apply. Clarkson University is an AA/EOE Employer. (Pos. # 41-08)

000133

NORTH CAROLINA

NORTH CAROLINA STATE UNIVERSITY
Department of Mathematics
Faculty Position in Mathematics with a Focus on Education

The Mathematics Department at North Carolina State University invites applications for a tenure-track, rank-open position beginning fall 2009. We seek an individual to provide leadership in developing and implementing innovative educational activities in the Mathematics Department. The new faculty member will be expected to have a record of research in mathematics or mathematics education and a commitment to scholarship in the teaching and learning of college-level mathematics. Candidates must have a Ph.D. in the mathematical sciences, a record of successful postdoctoral experience, and a commitment to effective teaching at the undergraduate and graduate levels. Senior-level applicants must demonstrate a strong record of grant support and advising of research students. More information about this position and the department can be found at `http://www.math.ncsu.edu`.

To submit your application materials, go to `http://www.mathjobs.org/jobs/ncsu`. Include a vita, at least three letters of recommendation, and a description of current and planned research. You will then be given instructions to go to `http://jobs.ncsu.edu/applicants/Central?quickFind=81837` and complete a Faculty Profile for the position. Write to `math-jobs@math.ncsu.edu` for questions concerning this position.

NC State University is an Equal Opportunity and Affirmative Action Employer. In addition, NC State welcomes all persons without regard to sexual orientation. The College of Physical and Mathematical Sciences welcomes the opportunity to work with candidates to identify suitable employment opportunities for spouses or partners. Review of applications will begin on January 1, 2009.

000135

WAKE FOREST UNIVERSITY
Department of Mathematics

Applications are invited for two tenure-track positions in mathematics at the assistant professor level beginning August 2009. We seek highly qualified candidates who have a commitment to excellence in both teaching and research. A Ph.D. in mathematics or a related area is required. Candidates with research interests in number theory, combinatorics, or algebra will receive first consideration. The department has 20 members and offers both a B.A. and a B.S. in mathematics, with an optional concentration in statistics, and a B.S. in each of mathematical business and mathematical economics. The department has a graduate program offering an M.A. in mathematics. A complete application will include a letter of application, curriculum vitae, teaching statement, research statement, graduate transcripts, and three letters of recommendation. Applicants are encouraged to post materials electronically at: `http://www.mathjobs.org`. Hard copy can be sent to Stephen Robinson, Wake Forest University, Department of Mathematics, P.O. Box 7388, Winston-Salem, NC 27109. (sbr@wfu.edu, http://www.math.wfu.edu). AA/EO Employer.

000093

THE UNIVERSITY OF NORTH CAROLINA AT CHAPEL HILL
Department of Mathematics

The Department of Mathematics at the University of North Carolina at Chapel Hill seeks to fill a distinguished chaired position, the Linker Professorship. An exceptional research record is the fundamental criterion for this position. Applications are invited from mathematicians working in the areas of analysis and geometry. The search will continue in earnest until the position is filled. We expect interviews to commence after January 1, 2009. Applicants must apply online at http://hr.unc.edu/jobseekers/ and search for recruitment ID# 1001086. For further information on the department, please visit our website at http://www.math.unc.edu or contact Professor Patrick Eberlein at pbe@email.unc.edu. UNC-CH is an Equal Opportunity Employer.

000148

OHIO

THE OHIO STATE UNIVERSITY
Assistant Professor in Mathematical Biology

The Department of Mathematics invites applications for a tenure-track position in mathematical biology in the Colleges of Biological and Mathematical and Physical Sciences. The position will be 80% in the Department of Mathematics and 20% in an appropriate biological sciences department, which will be determined based on the successful candidate's area of specialization. The Department of Mathematics will serve as the tenure initiating unit. Preference will be given to mathematicians with interest in areas such as ecology, population genetics, and evolutionary biology. The appointee will be part of a growing faculty in the area of mathematical biology at The Ohio State University, with opportunities to participate in the activities of the Mathematical Biosciences Institute, a National Science Foundation-funded national institute located at The Ohio State University. Through research and teaching, the appointee will contribute to bridging biology and mathematics at The Ohio State University. Flexible work options available.

Applicants should submit their curriculum vitae, statement of research and teaching interests, and contact information for three references online to: http://www.mathjobs.org. If you cannot apply online, please contact facultysearch@math.ohio-state.edu.

Review of applications begins November 17, 2008, and will continue until a suitable candidate is hired.

To build a diverse workforce Ohio State encourages applications from minorities, veterans, women, and individuals with disabilities. EEO/AA employer.

000141

THE OHIO STATE UNIVERSITY
College of Mathematical and Physical Sciences
Department of Mathematics

The Department of Mathematics in the College of Mathematical and Physical Sciences at The Ohio State University expects to have openings at both the junior and senior level in the area of mathematical and computational biology.

Applicants should have a Ph.D. in mathematics or a related area, such as mathematical sciences, biomathematics, biology, chemistry, computer science, physics, and engineering and should show outstanding promise and/or accomplishments in both research and teaching. The successful candidate will be expected to teach courses in the Mathematics Department and actively participate in the Mathematical Biosciences Institute. Further information on the department and the MBI can be found at http://www.math.ohio-state.edu and http://mbi.osu.edu. Flexible work options available.

Applications should be submitted online at http://www.mathjobs.org. Applications are considered on a continuing basis but the annual review process begins November 17, 2008. If you cannot apply online, please contact facultysearch@math.ohio-state.edu.

Senior candidates should arrange for at least five letters of recommendation and junior candidates should arrange for at least three letters of recommendation.

To build a diverse workforce, Ohio State encourages applications from minorities, veterans, women, and individuals with disabilities. EEO/AA Employer.

000131

UNIVERSITY OF DAYTON
Department of Mathematics

Applications are invited for a tenure-track position in the Department of Mathematics at the assistant professor level starting in August 2009. Candidates must have a Ph.D. in mathematics. Applicants must have a strong commitment to research, and the potential to become an effective teacher. Responsibilities include developing and maintaining a research agenda, teaching a broad range of mathematics courses, advising, and curriculum development. The applicant will also be expected to participate in directing undergraduate students working on research projects and to work with students from diverse backgrounds. We are open to all areas of research, but fundamental areas such as algebra, number theory, topology, and set theory will receive special attention.

The selection process begins December 9, 2008. To receive full consideration, all materials must be received by January 12, 2009. A complete application consists of a resume, three letters of recommendation, a statement of research agenda, a statement of teaching philosophy, and an unofficial graduate school transcript. Both teaching abilities and research abilities should be addressed in the letters of recommendation. Please include an email address in your correspondence.

We strongly encourage applicants to submit materials through http://mathjobs.org. Applications can also be sent directly to: Dr. Robert Gorton, Chair of the Mathematics Search Committee, Department of Mathematics, University of Dayton, Dayton, OH 45469-2316. Contact the search committee at: Robert.Gorton@notes.udayton.edu.

Obtain further information at: http://campus.udayton.edu/~mathdept.

The University of Dayton, a comprehensive Catholic university founded by the Society of Mary in 1850, is an Equal Opportunity/Affirmative Action Employer. Women, minorities, individuals with disabilities, and veterans are strongly encouraged to apply. The University of Dayton is firmly committed to the principle of diversity.

000101

PENNSYLVANIA

BRYN MAWR COLLEGE
Department of Mathematics

The Department of Mathematics invites applications for a continuing non-tenure-track position of Math Program Coordinator to begin July 1, 2009. The position is a three-year appointment. It can be renewed for multiple terms. An MA/MS in mathematics is required, though a Ph.D. in mathematics is preferred. We are seeking an enthusiastic individual with excellent teaching, communication, and administrative skills. The successful candidate will work with our department to provide our students with a comfortable transition from high school to college, will encourage them to pursue mathematics beyond the elementary level, and will coordinate departmental organizations and activities. Applicants must share our dedication to opening young women's minds to mathematics while enhancing their abilities to think both deeply and broadly. See http://www.brynmawr.edu/math for a more detailed description of the department and the position.

Applicants with excellent teaching and administrative skills should arrange to have a cover letter, a curriculum vita, a statement of teaching interests and philosophy, a list of mathematics courses taken and taught, official undergraduate and graduate transcripts, and at least three reference letters sent to: Search Committee, Department of Mathematics, Bryn Mawr College, 101 N. Merion Avenue, Bryn Mawr, PA 19010-2899.

Applications may also be submitted online through http://mathjobs.org. Applications should be complete by December 15, 2008.

Located in suburban Philadelphia, Bryn Mawr College is a highly selective liberal arts college for women who share an intense intellectual commitment, a self-directed and purposeful vision of their lives, and a desire to make meaningful contributions to the world. Bryn Mawr comprises an undergraduate college with 1,200 students, as well as coeducational graduate schools in some humanities, sciences, and social work. The College participates in a consortium together with Haverford and Swarthmore Colleges and the University of Pennsylvania. Bryn Mawr College is an Equal Opportunity, Affirmative Action Employer. Minority candidates and women are especially encouraged to apply.

000111

MILLERSVILLE UNIVERSITY
Department of Mathematics
Geometry/Topology

Full-time, tenure-track assistant professor position beginning August 2009. Duties include an annual 24-hour teaching load, scholarly activity, student advisement, curriculum development and committee work. Required: Ph.D. (or completion by date of appointment) in mathematics with expertise in geometry or topology. Must exhibit evidence of strong commitment to excellence in teaching and continued scholarly activity; must be prepared to teach a broad spectrum of undergraduate mathematics courses and to teach undergraduate geometry as it relates to the preparation of teachers. Must complete successful interview and teaching demonstration. Preferred: Evidence of commitment to working in a diverse environment. Full consideration given to applications received by January 21, 2009. Email applications will not be accepted. Send application letter that addresses the position requirements, vita, copies of undergraduate and graduate transcripts and three letters of reference (at least two of which attest to recent teaching effectiveness) to: Dr. Noel Heitmann, Search Committee, Department of Mathematics/AMS1208, Millersville University, P.O. Box 1002, Millersville, PA 17551-0302. An EO/AA Institution, http://www.millersville.edu.

000150

UNIVERSITY OF PITTSBURGH
Department of Mathematics

The Department of Mathematics at the University of Pittsburgh invites applications for a postdoctoral appointment starting the fall term 2009. The appointment is renewable annually to a maximum of three years. The position is funded jointly by the University of Pittsburgh and a new NSF Research Training Group (RTG) grant on complex biological systems across multiple space and time scales, see http://www.math.pitt.edu/~cbsg/.

The research areas covered by the RTG include (i) the development and analysis of mathematical models and computational algorithms for solving spatio-temporal problems arising in biology and (ii) the applications of these and other methods to problems arising in inflammation and neuroscience.

To be successful, a candidate must demonstrate excellence in research, and must also have strong commitment to excellence in teaching at both the undergraduate and graduate levels. Candidates should be willing to work closely with experimentalists and clinicians.

All applications must include the following: (1) a curriculum vita, (2) a personal statement addressing their research agenda, (3) a statement of teaching philosophy, (4) a completed AMS Standard Cover Sheet form and (5) at least three letters of recommendation. Applications should be submitted electronically through http://www.mathjobs.org. If the candidate is unable to submit electronically, materials may be sent to: Postdoctoral Search Committee in Complex Biological Systems, Department of Mathematics, University of Pittsburgh, Pittsburgh, PA 15260. Review of completed files will begin on January 10, 2009, and continue until the position is filled.

The University of Pittsburgh is an Affirmative Action, Equal Opportunity Employer. Women and members of minority groups underrepresented in academia are especially encouraged to apply. NSF restrictions require that eligible candidates must be U.S. citizens or permanent residents.

000136

TEXAS

SOUTHERN METHODIST UNIVERSITY
Clements Chair of Mathematics

Applications are invited for the Clements Chair of Mathematics to begin in the fall semester of 2009. Preference will be given to senior scholars with outstanding records of research who also have a strong commitment to teaching including an established history of advising Ph.D. theses. Applicants in all areas of applied and computational mathematics are encouraged. The Department of Mathematics, which offers an active doctoral program in applied and computational mathematics, is in an exciting period of transition having hired four faculty last year. Visit http://www.smu.edu/math for more information about the department.

To apply, send a letter of application with a curriculum vitae, a list of publications, research and teaching statements, and the names of three references (references will not be contacted before receiving approval of the candidate) to: The Faculty Search Committee, Department of Mathematics, Southern Methodist University, P.O. Box 750156, Dallas, Texas, 75275-0156. The Search Committee can be contacted by sending email to math-search@mail.smu.edu. (Tel: 214-768-2452; Fax: 214-768-2355).

To ensure full consideration for the position, the application must be received by January 9, 2009, but the committee will continue to accept applications until the position is filled. The committee will notify applicants of its employment decision after the position is filled.

SMU, a private university with an engineering school, is situated in a quiet residential section of Dallas. Dallas is home to the University of Texas Southwestern Medical Center and its new Systems Biology Center.

SMU will not discriminate on the basis of race, color, religion, national origin, sex, age, disability or veteran status. SMU is also committed to nondiscrimination on the basis of sexual orientation.

000140

THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Mathematics
Austin, Texas 78712

Expected openings for fall include: (**a**) Instructorships, some that have R.H. Bing Faculty Fellowships attached to them, and (**b**) possibly two or more positions at the tenure-track/tenure level.

(**a**) Instructorships at The University of Texas at Austin are postdoctoral appointments, renewable for two additional years. It is assumed that applicants for instructorships will have completed all Ph.D. requirements by August 17, 2009. Other factors being equal, preference will be given to those whose doctorates were conferred in 2008 or 2009. Candidates should show superior research ability and have a strong commitment to teaching. Consideration will be given only to persons whose research interests have some overlap with those of the permanent faculty. Duties consist of teaching undergraduate or graduate courses and conducting independent research. The projected salary is $44,000 for the nine-month academic year.

Each **R.H. Bing Fellow** holds an instructorship in the Mathematics Department, with a teaching load of two courses in one semester and one course in the other.

The combined Instructorship-Fellowship stipend for nine months is US$52,000, which is supplemented by a travel allowance of US$1,000. Pending satisfactory performance of teaching duties, the Fellowship can be renewed for two additional years. Applicants must show outstanding promise in research. Bing Fellowship applicants will automatically be considered for other departmental openings at the postdoctoral level, so a separate application for such a position is unnecessary.

Those wishing to apply for instructor positions are asked to send a vita and a brief research summary to the above address c/o Instructor Committee. Transmission of the preceding items via the Internet (URL: `http://www.ma.utexas.edu/jobs/application`) is encouraged.

(**b**) An applicant for a **tenure-track** or **tenured** position must present a record of exceptional achievement in her or his research area and must demonstrate a proficiency at teaching. In addition to the duties indicated above for instructors, such an appointment will typically entail the supervision of Ph.D. students. The salary will be commensurate with the level at which the position is filled and the qualifications of the person who fills it.

Those wishing to apply for **tenure-track**/**tenured** positions are asked to send a vita and a brief research summary to the above address, c/o Recruiting Committee. Transmission of the preceding items via the Internet (URL: `http://www.ma.utexas.edu/jobs/application/TenureTrack`) is encouraged.

All applications should be supported by four or more letters of recommendation, at least one of which speaks to the applicant's teaching credentials. The screening of applications will begin on December 1, 2008.

Background check will be conducted on the applicant selected. The University of Texas at Austin is an Affirmative Action/Equal Opportunity Employer.

000146

VIRGINIA

VIRGINIA TECH
Department of Mathematics
Faculty Position

The Department of Mathematics at Virginia Tech invites applications for one tenure-track position in algebraic combinatorics and related areas to start in the fall of 2009, subject to budgetary approval. The position is designated at the assistant professor level, but outstanding candidates at more senior levels are encouraged to apply. The search is part of the College of Science's Computational Science Cluster initiative.

Applications must be submitted online at `http://jobs.vt.edu/` (posting #081045). Instructions for submitting the application appear at `http://www.math.vt.edu/people/phaskell/search.htm`, which also provides information about the Computational Science Cluster. In addition to submitting the online application, applicants should arrange to have four letters of support sent to: Professor Mark Shimozono, Search Chair, Department of Mathematics, Virginia Tech, Blacksburg, VA 24061-0123.

Applications received by December 1, 2008, will receive full consideration. Virginia Tech is an AA/EEO employer; applications from members of underrepresented groups are especially encouraged. Individuals with disabilities desiring accommodations in the application process should notify Ms. Sandy Blevins (`Blevins\_AT\_math.vt.edu`), (540) 231-8267) or call TTY 1-800-828-1120.

000153

WYOMING

UNIVERSITY OF WYOMING
Department of Mathematics
Position in Algebra, Combinatorics or Number Theory

The University of Wyoming Mathematics Department is seeking outstanding candidates for a tenure-track position in algebra, combinatorics or number theory to start August 2009. UW has a strong research group in algebra, combinatorics & number theory that includes faculty in mathematics and computer science, and benefits from collaborative ties with faculty in the Rocky Mountain Region. For more information, see `http://math.uwyo.edu`.

Preference will be given to candidates with expertise and experience in research topics that overlap, while providing new expertise and directions to the research interests of our current faculty. Applicants must possess a Ph.D. in mathematics or a related field, and should have outstanding accomplishments in both research and teaching.

Candidates must be strongly committed to shaping and developing the department's research, curricular, and service roles, and supervising graduate students.

Salary will be competitive and commensurate with qualifications. Interested applicants should submit their application, including curriculum vitae, a statement of research and teaching interests, and electronic copies of up to three publications that represent their best work, through the website `http://math.uwyo.edu/acnt-position.asp`. They should arrange for three letters of recommendation to also be submitted through this site, including at least one letter that addresses their teaching abilities and experience. Review of applications will begin January 5, 2009. Send inquiries to `algcombn@uwyo.edu`. The University of Wyoming is a Carnegie Foundation Research/Doctoral Extensive Institution, is an AA/EEO and encourages women and underrepresented minorities to apply.

000149

INDIA

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH MOHALI, INDIA

Indian Institute of Science Education and Research (IISER) Mohali is in the process of building a School of Mathematics and Computer Science. The institute has started an integrated five-year MS program in basic sciences as well as a Ph.D. program from August 2007. The focus of the institute is to combine cutting edge research with pedagogy of science. Outstanding candidates having a flair for teaching and a strong research background are encouraged to apply for faculty positions. For details see: `http://www.iisermohali.ac.in/faculty_openings.htm`. IISER Mohali is funded by the Ministry of Human Resource Development, Government of India, and is a degree-granting institution. Mohali is adjacent to Chandigarh near the foothills of the Himalayas.

000139

FOR SALE

Intoduction to Integral Equations with Applications—revised second edition.

The most applicable intoductory book on the subject, *Intro. to Integral Equations with Applications*, by A. Jerri, comes in a revised second edition in 452 p. with *Student's Solution Manual* and excellent reviews (Math. Rev. and Zentrablatt Math.). Both for US$84.95 plus $7 S&H. Paypal payment (to Sampling Publishing account) is preferred or email `jerria12@yahoo.com`.

000143

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Shanghai, People's Republic of China

Fudan University

December 17–21, 2008

Wednesday – Sunday

Meeting #1045

First Joint International Meeting between the AMS and the Shanghai Mathematical Society

Associate secretary: Susan J. Friedlander

Announcement issue of *Notices*: June 2008

Program first available on AMS website: Not applicable

Program issue of electronic *Notices*: Not applicable

Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions: Not applicable

For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/internmtgs.html`.

Invited Addresses

Robert J. Bryant, University of California Berkeley, *Title to be announced.*

L. Craig Evans, University of California Berkeley, *Title to be announced.*

Zhi-Ming Ma, Chinese Academy of Sciences, *Title to be announced.*

Richard Schoen, Stanford University, *Title to be announced.*

Xiaoping Yuan, Fudan University, *Title to be announced.*

Weiping Zhang, Chern Institute, *Title to be announced.*

Special Sessions

Biomathematics: Newly Developed Applied Mathematics and New Mathematics Arising from Biosciences, **Banghe Li**, Chinese Academy of Sciences, **Reinhard C. Laubenbacher**, Virginia Bioinformatics Institute, and **Jianjun Paul Tian**, College of William and Mary.

Combinatorics and Discrete Dynamical Systems, **Reinhard C. Laubenbacher**, Virginia Bioinformatics Institute, **Klaus Sutner**, Carnegie Mellon University, and **Yaokun Wu**, Shanghai Jiao Tong University.

Differential Geometry and Its Applications, **Jianguo Cao**, University of Notre Dame, and **Yu Xin Dong**, Fudan University.

Dynamical Systems Arising in Ecology and Biology, **Qishao Lu**, Beijing University of Aeronautics & Astronautics, and **Zhaosheng Feng**, University of Texas-Pan American.

Elliptic and Parabolic Nonlinear Partial Differential Equations, **Changfeng Gui**, University of Connecticut, and **Feng Zhou**, East China Normal University.

Harmonic Analysis and Partial Differential Equations with Applications, **Yong Ding**, Beijing Normal University, **Guo-Zhen Lu**, Wayne State University, and **Shanzhen Lu**, Beijing Normal University.

Integrable System and Its Applications, **En-Gui Fan**, Fudan University, **Sen-Yue Lou**, Shanghai Jiao Tong

University and Ningbo University, and **Zhi-Jun Qiao**, University of Texas-Pan American.

Integral and Convex Geometric Analysis, **Deane Yang**, Polytechnic University, and **Jiazu Zhou**, Southwest University.

Lie Algebras, Vertex Operator Algebras and Related Topics, **Hu Nai Hong**, East China Normal University, and **Yi-Zhi Huang**, Rutgers University.

Nonlinear Systems of Conservation Laws and Related Topics, **Gui-Qiang Chen**, Northwestern University, and **Shuxing Chen** and **Yi Zhou**, Fudan University.

Optimization and Its Application, **Shu-Cherng Fang**, North Carolina State University, and **Xuexiang Huang**, Fudan University.

Quantum Algebras and Related Topics, **Naihuan N. Jing**, North Carolina State University, **Quanshui Wu**, Fudan University, and **James J. Zhang**, University of Washington.

Recent Developments in Nonlinear Dispersive Wave Theory, **Jerry Bona**, University of Illinois at Chicago, **Bo Ling Guo**, Institute of Applied Physics and Computational Mathematics, **Shu Ming Sun**, Virginia Polytech Institute and State University, and **Bingyu Zhang**, University of Cincinnati.

Representation of Algebras and Groups, **Birge K. Huisgen-Zimmermann**, University of California Santa Barbara, **Jie Xiao**, Tsinghua University, **Jiping Zhang**, Beijing University, and **Pu Zhang**, Shanghai Jiao Tong University.

Several Complex Variables and Applications, **Siqi Fu**, Rutgers University, **Min Ru**, University of Houston, and **Zhihua Chen**, Tongji University.

Several Topics in Banach Space Theory, **Gerard J. Buskes** and **Qingying Bu**, University of Mississippi, and **Lixin Cheng**, Xiamen University.

Stochastic Analysis and Its Application, **Jiangang Ying**, Fudan University, and **Zhenqing Chen**, University of Washington.

Topics in Partial Differential Equations and Mathematical Control Theory, **Xiaojun Huang**, Rutgers University, **Gengsheng Wang**, Wuhan University of China, and **Stephen S.-T. Yau**, University of Illinois at Chicago.

Washington, District of Columbia

Marriott Wardman Park Hotel and Omni Shoreham Hotel

January 5–8, 2009

Monday – Thursday

Meeting #1046

Joint Mathematics Meetings, including the 115th Annual Meeting of the AMS, 92nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Bernard Russo
Announcement issue of *Notices*: October 2008
Program first available on AMS website: November 1, 2008
Program issue of electronic *Notices*: January 2009
Issue of *Abstracts*: Volume 30, Issue 1

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/national.html`.

AMS-MAA Invited Addresses

The Joint AMS Committee on Science Policy-MAA Science Policy Committee Government Speaker on Wednesday afternoon has been cancelled.

AMS Sessions

The title of the Committee on the Profession Panel Discussion on Tuesday at 2:30 p.m. is **What I Wish I Had Known or Studied Before Going to Graduate School** and will be moderated by **Craig L. Huneke**, University of Kansas.

The title of the presentation sponsored by the Committee on Science Policy on Wednesday at 2:30 p.m. is **Future Federal Science and Technology Budgets**. **Kei Koizumi**, American Association for the Advancement of Science, will outline the latest news on this topic as seen from his position as director of the R&D Budget and Policy Program for the AAAS.

MAA Sessions

Career Fair, Monday, 9:00 a.m.—11:00 a.m., organized by **Robert W. Vallin**, MAA. This event will help answer the eternal questions, “Who is hiring people with math degrees?” and “How can I get in contact with them?” All students, whether they are earning Bachelor’s, Master’s, or Ph.D. degrees, are invited to participate. Representatives from companies in government and industry will take part in the event. Participants will have the opportunity to make contacts, hand out resumes or curricula vita, and explore the many kinds of careers they may pursue in the future. A human resources professional will also be on hand to critique resumes. Exhibitors for this event may participate for a registration fee of US$100; JMM exhibitors may participate for US$50. Please contact Stephen DeSanto at `sdesanto@maa.org` to register, and Robert Vallin at `RVallin@maa.org` for any other questions.

Panelists for the **Actuarial Education** session at 5:00 p.m. on Wednesday are **James W. Daniel**, University of Texas at Austin; **Ken Guthrie**, Society of Actuaries; **Bryan Hearsey**, Lebanon Valley College; **Emily Kessler**, Society of Actuaries; and **Hwa Chi Liang**, Washburn University. **Kevin E. Charlwood** will serve as moderator.

Panelists for the **Mathematicians and Public Policy** discussion at 2:30 p.m. on Thursday include **Vernon J. Ehlers** U.S. Congressman, Michigan, and House Subcommittee on Research and Science Education; **Jerry McNerney**, U.S. Congressman, California; **Douglas N. Arnold**, University of Minnesota and president-elect, Society for Industrial and Applied Mathematics; **Daniel H. Ullman**, George Washington University and former AMS/AAAS Congressional Fellow. The moderator is **Philippe Tondeur**, former director of the Division of Mathematical Sciences, NSF.

Special Interest Groups of the MAA

SIGMAA on Circles (SIGMAA MCST) Business Meeting, Wednesday, 6:00 p.m.–7:00 p.m. See information on a reception in the "Social Events" section.

Other Organizations

Mathematical Art Exhibit Prize, to be awarded on Tuesday in the exhibit area at a time to be announced, was established in 2008 through an endowment provided to the AMS by an anonymous donor, who wishes to acknowledge those whose works demonstrate the beauty and elegance of mathematics expressed in a visual art form. First, second and third prizes will be awarded by a panel of judges chosen by the AMS and MAA for aesthetically pleasing works that combine mathematics and visual art exhibited in the 2009 Joint Mathematics Meetings Mathematical Art Exhibit. See your program for times the exhibit area is open.

Social Events

Brigham Young University Mathematics Department Reception, Monday, 6:00 p.m.–8:00 p.m. Alumni and friends of BYU Mathematics and its program (e.g., CURM, REU) are invited to attend a reception with light refreshments. Please contact Michael Dorff for more information at `mdorff@math.byu.edu`.

Claremont Colleges Alumni Reception, Tuesday, 6:00 p.m.–8:00 p.m. Please join your fellow Claremont College math faculty, alumni, students, and special guests. Hors d'oeurves and drinks will be provided. Please send your RSVP to `alumni@home.edu`.

Lehigh University Reception, Tuesday, 5:45 p.m.–7:00 p.m. All friends and graduates are invited.

MAA Open House, Wednesday, 3:00 p.m.–6:00 p.m. Come see the MAA Dolciani Mathematical Center, the Halmos Carriage House, and the River of Bricks. Refreshments will be served. Special bus service has been arranged for every half hour to and from the MAA beginning, at 3:00 p.m. The last bus back is at 6:00 p.m. Pick up/drop off is at the 24th Street entrance to the Marriott right next to Harry's Pub on the lobby level.

National Association of Math Circles (NAMC), and NAMC and SIGMAA on Circles Joint Reception, Wednesday, 7:00 p.m.–9:00 p.m.

University of Michigan Alumni and Friends Reception, Wednesday, 5:00 p.m.–6:30 pm.

NSA Women in Mathematics Society Networking Session, Tuesday, 6:00 p.m.–8:00 p.m. All participants are welcome to this annual event. Please stop by the NSA booth in the exhibit hall for the exact location.

North Carolina State University Reception, Tuesday, 6:00 p.m.–8:00 p.m. All alumni, family, spouses, and friends of the Department of Mathematics are invited to come and meet old friends and to hear of recent events in the department.

University of Wisconsin-Madison Department of Mathematics Reception, Tuesday, 6:00 p.m.–8:00 p.m. All alumni and friends are invited to visit with new and old friends.

MAA Ancillary Workshop

Teaching Introductory Data Analysis through Modeling, presented by **Daniel Kaplan**, Macalester College, Sunday, 8:30 a.m.–5:00 p.m. This hands-on workshop intended for teachers of introductory statistics in colleges and universities will present a new way of teaching introductory data analysis that gives a central role to modeling techniques. Modeling provides a strong unifying framework for statistics and at the same time ties statistics closely to the scientific method and the demands of realistic multivariable data. The workshop will introduce the ways in which models can be used for description, the interpretation of models in terms of association, change, and partial change (that is, change in one variable while holding others constant). In place of the usual matrix-based theory of linear models, the workshop will present a geometrical approach to theory that is accessible to introductory students and fully illuminates important ideas in data analysis: fitting, confounding and Simpson's paradox, correlation and collinearity. Inference is introduced using resampling and simulation, from which it is straightforward to transition to a general framework for inference, analysis of covariance. Computation (using the free package R) will feature prominently in hands-on activities; participants should bring laptop computers if possible. Participants do **not** need to have previous experience with R or with statistical modeling. Our students can learn it and so can you!

There is no registration fee for this workshop. Note that it will be **held the day before** the Joint Mathematics Meetings actually begin in the Marriott Wardman Park Hotel. Workshop materials and lunch during the workshop will be provided. Workshop participants are encouraged to bring their own laptops. Workshop participants are responsible for their own transportation and lodging. Enrollment is limited to 40. For registration and additional details go to `http://www.causeweb.org/workshop/modeling_jmm09/`.

Urbana, Illinois

University of Illinois at Urbana-Champaign

March 27–29, 2009

Friday – Sunday

Meeting #1047

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: January
Program first available on AMS website: February 12, 2009
Program issue of electronic *Notices*: March
Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: December 9, 2008
For abstracts: February 3, 2009

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Jeffrey C. Lagarias, University of Michigan, *Title to be announced* (Erdős Memorial Lecture).

Jacob Lurie, Massachusetts Institute of Technology, *Title to be announced*.

Gilles Pisier, Texas A&M University, *Title to be announced*.

Akshay Venkatesh, New York University-Courant Institute, *Title to be announced*.

Special Sessions

Algebra, Geometry and Combinatorics (Code: SS 10A), **Rinat Kedem**, University of Illinois at Urbana-Champaign, and **Alexander T. Yong**, University of Minnesota.

Algebraic Methods in Statistics and Probability (Code: SS 3A), **Marlos A. G. Viana**, University of Illinois at Chicago.

Complex Dynamics and Value Distribution (Code: SS 11A), **Aimo Hinkkanen** and **Joseph B. Miles**, University of Illinois at Urbana-Champaign.

Concrete Aspects of Real Positive Polynomials (Code: SS 20A), **Victoria Powers**, Emory University, and **Bruce Reznick**, University of Illinois at Urbana-Champaign.

Differential Geometry and Its Applications (Code: SS 16A), **Stephanie B. Alexander**, University of Illinois at Urbana-Champaign, and **Jianguo Cao**, University of Notre Dame.

Geometric Function Theory and Analysis on Metric Spaces (Code: SS 6A), **Sergiy Merenkov**, **Jeremy Taylor Tyson**, and **Jang-Mei Wu**, University of Illinois at Urbana-Champaign.

Geometric Group Theory (Code: SS 2A), **Sergei V. Ivanov**, **Ilya Kapovich**, **Igor Mineyev**, and **Paul E. Schupp**, University of Illinois at Urbana-Champaign.

Graph Theory (Code: SS 4A), **Alexander V. Kostochka** and **Douglas B. West**, University of Illinois at Urbana-Champaign.

Holomorphic and CR Mappings (Code: SS 9A), **John P. D'Angelo**, **Jiri Lebl**, and **Alex Tumanov**, University of Illinois at Urbana-Champaign.

Hyperbolic Geometry and Teichmuller Theory (Code: SS 18A), **Jason Deblois**, University of Illinois at Chicago, **Richard P. Kent IV**, Brown University, and **Christopher J. Leininger**, University of Illinois at Urbana-Champaign.

Local and Homological Methods in Commutative Algebra (Code: SS 13A), **Florian Enescu**, Georgia State University, and **Sandra Spiroff**, University of Mississippi.

Mathematical Visualization (Code: SS 7A), **George K. Francis**, University of Illinois at Urbana-Champaign, **Louis H. Kauffman**, University of Illinois at Chicago, **Dennis Martin Roseman**, University of Iowa, and **Andrew J. Hanson**, Indiana University.

Nonlinear Partial Differential Equations and Applications (Code: SS 21A), **Igor Kukavica**, University of Southern California, and **Anna L. Mazzucato**, Pennsylvania State University.

Number Theory in the Spirit of Erdős (Code: SS 14A), **Kevin Ford** and **A. J. Hildebrand**, University of Illinois at Urbana-Champaign.

Operator Algebras and Operator Spaces (Code: SS 8A), **Zhong-Jin Ruan**, **Florin P. Boca**, and **Marius Junge**, University of Illinois at Urbana-Champaign.

Probabilistic and Extremal Combinatorics (Code: SS 5A), **Jozsef Balogh** and **Zoltan Furedi**, University of Illinois at Urbana-Champaign.

The Interface Between Number Theory and Dynamical Systems (Code: SS 17A), **Florin Boca**, University of Illinois at Urbana-Champaign, **Jeffrey Lagarias**, University of Michigan, and **Kenneth Stolarsky**, University of Illinois at Urbana-Champaign.

The Logic and Combinatorics of Algebraic Structures (Code: SS 22A), **John Snow**, Concordia University, and **Jeremy Alm**, Illinois College.

Time, Scale and Frequency Methods in Harmonic Analysis (Code: SS 15A), **Richard S. Laugesen**, University of Illinois at Urbana-Champaign, and **Darrin M. Speegle**, St. Louis University.

Topological Dynamics and Ergodic Theory (Code: SS 19A), **Alica Miller**, University of Louisville, and **Joseph Rosenblatt**, University of Illinois at Urbana-Champaign.

Topological Field Theories, Representation Theory, and Algebraic Geometry (Code: SS 12A), **Thomas Nevins**, University of Illinois at Urbana-Champaign, and **David Ben-Zvi**, University of Texas at Austin.

q-Series and Partitions (Code: SS 1A), **Bruce Berndt**, University of Illinois at Urbana-Champaign, and **Ae Ja Yee**, Pennsylvania State University.

Raleigh, North Carolina

North Carolina State University

April 4–5, 2009
Saturday – Sunday

Meeting #1048

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: January 2009
Program first available on AMS website: February 19, 2009
Program issue of electronic *Notices*: April 2009
Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: December 16, 2008
For abstracts: February 10, 2009

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Nathan Dunfield, University of Illinois at Urbana-Champaign, *Surfaces in finite covers of 3-manifolds: The Virtual Haken Conjecture*.

Reinhard C. Laubenbacher, Virginia Bioinformatics Institute, *Algebraic models in systems biology*.

Jonathan C. Mattingly, Duke University, *Stochastically forced fluid equations: Transfer between scales and ergodicity*.

Raman Parimala, Emory University, *Title to be announced*.

Special Sessions

Advancements in Turbulent Flow Modeling and Computation (Code: SS 8A), **Leo G. Rebholz**, Clemson University, and **Traian Iliescu**, Virginia Polytechnic Institute and State University.

Algebraic Groups and Symmetric Spaces (Code: SS 19A), **Stacy Beun**, Cabrini College, and **Aloysius Helminck**, North Carolina State University.

Applications of Algebraic and Geometric Combinatorics (Code: SS 2A), **Seth M. Sullivant**, Harvard University, and **Carla D. Savage**, North Carolina State University.

Applications of Dynamical Systems to Problems in Biology (Code: SS 16A), **John E. Franke** and **James F. Selgrade**, North Carolina State University.

Brauer Groups, Quadratic Forms, Algebraic Groups, and Lie Algebras (Code: SS 12A), **Eric S. Brussel** and **Skip Garibaldi**, Emory University.

Commutative Rings and Monoids (Code: SS 17A), **Scott T. Chapman**, Sam Houston State University, and **James B. Coykendall**, North Dakota State University.

Computational Methods in Lie Theory (Code: SS 10A), **Eric Sommers**, University of Massachusetts, Amherst, and **Molly Fenn**, North Carolina State University.

Deferred Correction Methods and Their Applications (Code: SS 20A), **Elizabeth L. Bouzarth** and **Anita T. Layton**, Duke University.

Enumerative Geometry and Related Topics (Code: SS 7A), **Richard L. Rimanyi**, University of North Carolina, Chapel Hill, and **Leonardo C. Mihalcea**, Duke University.

Galois Module Theory and Hopf Algebras (Code: SS 13A), **Robert G. Underwood**, Auburn University Montgomery, and **James E. Carter**, College of Charleston.

Geometry of Differential Equations (Code: SS 9A), **Thomas A. Ivey**, College of Charleston, and **Irina A. Kogan**, North Carolina State University.

Homotopical Algebra with Applications to Mathematical Physics (Code: SS 3A), **Thomas J. Lada**, North Carolina State University, and **Jim Stasheff**, University of North Carolina, Chapel Hill.

Kac-Moody Algebras, Vertex Algebras, Quantum Groups, and Applications (Code: SS 1A), **Bojko N. Bakalov**, **Kailash C. Misra**, and **Naihuan N. Jing**, North Carolina State University.

Low-Dimensional Topology and Geometry (Code: SS 4A), **Nathan M. Dunfield**, University of Illinois at Urbana-Champaign, **John B. Etnyre**, Georgia Institute of Technology, and **Lenhard Ng**, Duke University.

Mathematical Progress and Challenges for Biological Materials (Code: SS 18A), **Mansoor A. Haider**, North Carolina State University, and **Gregory Forest**, University of North Carolina, Chapel Hill.

Mathematics of Immunology and Infectious Diseases (Code: SS 14A), **Stanca M. Ciupe**, Duke University.

Nonlinear Dynamics and Control (Code: SS 11A), **Anthony M. Bloch**, University of Michigan, Ann Arbor, and **Dmitry Zenkov**, North Carolina State University.

Numerical Solution of Partial Differential Equations and Applications (Code: SS 15A), **Alina Chertock** and **Zhilin Li**, North Carolina State University.

Recent Advances in Symbolic Algebra and Analysis (Code: SS 5A), **Michael F. Singer** and **Agnes Szanto**, North Carolina State University.

Rings, Algebras, and Varieties in Combinatorics (Code: SS 6A), **Patricia Hersh**, North Carolina State University, **Christian Lenart**, SUNY Albany, and **Nathan Reading**, North Carolina State University.

Worcester, Massachusetts

Worcester Polytechnic Institute

April 25–26, 2009

Saturday – Sunday

Meeting #1050

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: February 2009
Program first available on AMS website: March 12, 2009
Program issue of electronic *Notices*: April 2009
Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: January 6, 2009
For abstracts: March 3, 2009

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Octav Cornea, Université de Montréal, *Title to be announced*.

Fengbo Hang, Courant Institute of New York University, *Title to be announced*.

Umberto Mosco, Worcester Polytechnic Institute, *Title to be announced*.

Kevin Whyte, University of Illinois at Chicago, *Title to be announced*.

Special Sessions

Algebraic Graph Theory, Association Schemes, and Related Topics (Code: SS 8A), **William J. Martin**, Worcester Polytechnic Institute, and **Sylvia A. Hobart**, University of Wyoming.

Discrete Geometry and Combinatorics (Code: SS 5A), **Egon Schulte**, Northeastern University, and **Brigitte Servatius**, Worcester Polytechnic Institute.

Effective Dynamics and Interactions of Localized Structures in Schrodinger Type Equations (Code: SS 10A), **Fridolin Ting**, Lakehead University.

Number Theory (Code: SS 4A), **John T. Cullinan**, Bard College, and **Siman Wong**, University of Massachusetts, Amherst.

Quasi-Static and Dynamic Evolution in Fracture Mechanics (Code: SS 6A), **Christopher J. Larsen**, Worcester Polytechnic Institute.

Real and Complex Dynamics of Rational Difference Equations with Applications (Code: SS 9A), **M. R. S. Kulenovic** and **Orlando Merino**, University of Rhode Island.

Scaling, Irregularities, and Partial Differential Equations (Code: SS 7A), **Umberto Mosco** and **Bogdan M. Vernescu**, Worcester Polytechnic Institute.

Symplectic and Contact Topology (Code: SS 1A), **Peter Albers**, Purdue University/ETH Zurich, and **Basak Gurel**, Vanderbilt University.

The Mathematics of Climate Change (Code: SS 3A), **Catherine A. Roberts** and **Gareth E. Roberts**, College of the Holy Cross, **Mary Lou Zeeman**, Bowdoin College.

Topological Robotics (Code: SS 2A), **Li Han** and **Lee N. Rudolph**, Clark University.

San Francisco, California

San Francisco State University

April 25–26, 2009

Saturday – Sunday

Meeting #1049

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2009
Program first available on AMS website: March 12, 2009
Program issue of electronic *Notices*: April 2009
Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: January 6, 2009
For abstracts: March 3, 2009

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Yehuda Shalom, University of California Los Angeles, *Title to be announced*.

Roman Vershynin, University of California Davis, *Title to be announced*.

Karen Vogtmann, Cornell University, *Title to be announced*.

Efim Zelmanov, University of California Los Angeles, *Title to be announced*.

Special Sessions

Advances in the Theory of Integer Linear Optimization and its Extensions (Code: SS 7A), **Matthias Koeppe** and **Peter Malkin**, University of California Davis.

Algebra and Number Theory with Polyhedra (Code: SS 11A), **Matthias Beck**, San Francisco State University, and **Christian Haase**, Freie Universität Berlin.

Applications of Knot Theory to the Entanglement of Biopolymers (Code: SS 10A), **Javier Arsuaga**, San Francisco

State University, **Kenneth Millett**, University of California Santa Barbara, and **Mariel Vazquez**, San Francisco State University.

Aspects of Differential Geometry (Code: SS 9A), **David Bao**, San Francisco State University, and **Lei Ni**, University of California San Diego.

Banach Algebras, Topological Algebras and Abstract Harmonic Analysis (Code: SS 1A), **Thomas V. Tonev**, University of Montana-Missoula, and **Fereidoun Ghahramani**, University of Manitoba.

Concentration Inequalities (Code: SS 3A), **Sourav Chatterjee**, University of California Berkeley, and **Roman Vershynin**, University of California Davis.

Geometry and Topology of Orbifolds (Code: SS 6A), **Elizabeth Stanhope**, Lewis & Clark University, and **Joseph E. Borzellino**, California State University San Luis Obispo.

Lie group actions, Teichmuller Flows and Number Theory (Code: SS 12A), **Jayadev Athreya**, Yale University, **Yitwah Cheung**, San Francisco State University, and **Anton Zorich**, Rennes University.

Matroids in Algebra and Geometry (Code: SS 8A), **Federico Ardila**, San Francisco State University, and **Lauren Williams**, Harvard University.

Nonlinear Dispersive Equations (Code: SS 4A), **Sebastian Herr**, University of California Berkeley, and **Jeremy L. Marzuola**, Columbia University.

Recent Progress in Geometric Group Theory (Code: SS 2A), **Seonhee Lim** and **Anne Thomas**, Cornell University.

Waco, Texas

Baylor University

October 16–18, 2009

Friday – Sunday

Meeting #1051

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2009
Program first available on AMS website: September 3, 2009
Program issue of electronic *Notices*: October 2009
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 17, 2009
For consideration of contributed papers in Special Sessions: June 30, 2009
For abstracts: August 25, 2009

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

David Ben-Zvi, University of Texas at Austin, *Title to be announced.*

Alexander A. Kiselev, University of Wisconsin, *Title to be announced.*

Michael C. Reed, Duke University, *Title to be announced.*

Igor Rodnianski, Princeton University, *Title to be announced.*

Special Sessions

Commutative Algebra: Module and Ideal Theory (Code: SS 4A), **Lars W. Christensen**, Texas Tech University, **Louiza Fouli**, University of Texas at Austin, and **David Jorgensen**, University of Texas at Arlington.

Dynamic Equations on Time Scales: Analysis and Applications (Code: SS 1A), **John M. Davis**, **Ian A. Gravagne**, and **Robert J. Marks**, Baylor University.

Mathematical Models of Neuronal and Metabolic Mechanisms (Code: SS 3A), **Janet Best**, Ohio State University, and **Michael Reed**, Duke University.

Numerical Solutions of Singular or Perturbed Partial Differential Equation Problems with Applications (Code: SS 2A), **Peter Moore**, Southern Methodist University, and **Qin Sheng**, Baylor University.

Topological Methods for Boundary Value Problems for Ordinary Differential Equations (Code: SS 5A), **Richard Avery**, Dakota State University, **Paul W. Eloe**, University of Dayton, and **Johnny Henderson**, Baylor University.

University Park, Pennsylvania

Pennsylvania State University

October 24–25, 2009

Saturday – Sunday

Meeting #1052

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: August 2009
Program first available on AMS website: September 10, 2009
Program issue of electronic *Notices*: October 2009
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 24, 2009
For consideration of contributed papers in Special Sessions: July 7, 2009
For abstracts: September 1, 2009

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael K. H. Kiessling, Rutgers University, *Title to be announced.*

Kevin R. Payne, Universita degli di Milano, *Title to be announced*.

Laurent Saloff-Coste, Cornell University, *Title to be announced*.

Robert C. Vaughan, Penn State University, *Title to be announced*.

Boca Raton, Florida

Florida Atlantic University

October 30 – November 1, 2009

Friday – Sunday

Meeting #1053

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: August 2009
Program first available on AMS website: September 17, 2009
Program issue of electronic *Notices*: October 2009
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 30, 2009
For consideration of contributed papers in Special Sessions: July 14, 2009
For abstracts: September 8, 2009

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Spyros Alexakis, Princeton University, *Title to be announced*.

Kai-Uwe Bux, University of Virginia, *Title to be announced*.

Dino J. Lorenzini, University of Georgia, *Title to be announced*.

Eduardo D. Sontag, Rutgers University, *Title to be announced*.

Special Sessions

Commutative Ring Theory (Code: SS 3A), **Alan Loper**, Ohio State University, and **Lee C. Klingler**, Florida Atlantic University.

Concentration, Functional Inequalities, and Isoperimetry (Code: SS 2A), **Mario Milman**, Florida Atlantic University, **Christian Houdre**, Georgia Institute of Technology, and **Emanuel Milman**, Institute for Advanced Study.

Constructive Mathematics (Code: SS 1A), **Robert Lubarsky**, **Fred Richman**, and **Martin Solomon**, Florida Atlantic University.

Riverside, California

University of California

November 7–8, 2009

Saturday – Sunday

Meeting #1054

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: September 2009
Program first available on AMS website: September 24, 2009
Program issue of electronic *Notices*: November 2009
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 6, 2009
For consideration of contributed papers in Special Sessions: July 21, 2009
For abstracts: September 15, 2009

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Christopher Hacon, University of Utah, *Title to be announced*.

Birge Huisgen-Zimmerman, University of California Santa Barbara, *Title to be announced*.

Jun Li, Stanford University, *Title to be announced*.

Joseph Teran, University of California Los Angeles, *Title to be announced*.

Special Sessions

Algebraic Geometry (Code: SS 1A), **Christopher Hacon**, University of Utah, and **Ziv Ran**, University of California Riverside.

Fluid Mechanics (Code: SS 5A), **James Kelliher** and **Qi Zhang**, University of California Riverside.

History and Philosophy of Mathematics (Code: SS 4A), **Shawnee L. McMurran**, California State University San Bernardino, and **James J. Tattersall**, Providence College.

Noncommutative Geometry (Code: SS 2A), **Vasiliy Dolgushev** and **Wee Liang Gan**, University of California Riverside.

Representation Theory (Code: SS 3A), **Vyjayanthi Chari**, **Wee Liang Gan**, and **Jacob Greenstein**, University of California Riverside.

San Francisco, California

Moscone Center West and the San Francisco Marriott

January 13–16, 2010
Wednesday – Saturday
Joint Mathematics Meetings, including the 116th Annual Meeting of the AMS, 93rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society of Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2009
Program first available on AMS website: November 1, 2009
Program issue of electronic *Notices*: January 2010
Issue of *Abstracts*: Volume 31, Issue 1

Deadlines
For organizers: April 1, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lexington, Kentucky

University of Kentucky

March 27–28, 2010
Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: August 28, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

St. Paul, Minnesota

Macalester College

April 10–11, 2010
Saturday – Sunday
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 10, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Albuquerque, New Mexico

University of New Mexico

April 17–18, 2010
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines
For organizers: September 17, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Berkeley, California

University of California Berkeley

June 2–5, 2010
Wednesday – Saturday
Eighth Joint International Meeting of the AMS and the Sociedad Matemática Mexicana.
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: February 2010
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Notre Dame, Indiana

Notre Dame University

September 18–19, 2010

Saturday – Sunday
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: February 19, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Los Angeles, California

University of California Los Angeles

October 9–10, 2010

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 10, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 5–8, 2011

Wednesday – Saturday
Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2010
Program first available on AMS website: November 1, 2010
Program issue of electronic *Notices*: January 2011
Issue of *Abstracts*: Volume 32, Issue 1

Deadlines

For organizers: April 1, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Statesboro, Georgia

Georgia Southern University

March 12–13, 2011

Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 12, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012, *Wednesday – Saturday*

Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: April 1, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013
Wednesday – Saturday

Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center

January 15–18, 2014
Wednesday – Saturday

Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015
Saturday – Tuesday

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The 2008 *Notices* Index

January issue: 1–200
February issue: 201–336
March issue: 337–440
April issue: 441–552
May issue: 553–656
June/July issue: 657–768
August issue: 769–912
September issue: 913–1064
October issue: 1065–1216
November issue: 1217–1360
December issue: 1361–1504

Index Section	Page Number
The AMS	1484
Announcements	1485
Articles	1485
Authors	1486
Deaths of Members of the Society	1487
Grants, Fellowships, Opportunities	1489
Letters to the Editor	1490
Meetings Information	1491
New Publications Offered by the AMS	1491
Obituaries	1491
Officers and Committee Members	1491
Opinion	1491
Prizes and Awards	1492
Prizewinners	1494
Reference and Book List	1501
Reviews	1501
Surveys	1501
Tables of Contents	1501

About the Cover

41, 225, 380, 549, 595, 710, 813, 959, 1160, 1300, 1427

The AMS

2007 Annual Survey of the Mathematical Sciences (First Report), 253
2007 Annual Survey of the Mathematical Sciences (First Report, Part II), 387
2007 Annual Survey of the Mathematical Sciences in the United States (Second Report), 814
2007 Annual Survey of the Mathematical Sciences in the United States (Third Report), 1271
2007 Election Results, 301
2008 American Mathematical Society Election (Special Section), 996
2008 AMS Election—Nominations by Petition, 302
2008 Award for Distinguished Public Service, 508
2008 Award for an Exemplary Program or Achievement in a Mathematics Department, 598
2008 Bôcher Prize, 499
2008 Cole Prize in Number Theory, 497
2008 Conant Prize, 491
2008 Doob Prize, 503
2008 Eisenbud Prize, 505
2008 Morgan Prize, 494
2008 Steele Prizes, 486
2008–2009 AMS Centennial Fellowship Awarded, 715
American Mathematical Society Centennial Fellowships, 973, 1288
American Mathematical Society—Contributions, 623
AMS-AAAS Mass Media Summer Fellowships, 62, 1110
AMS Announces Congressional Fellow, 721
AMS Congressional Fellowship, 1289
AMS Department Chairs Workshop, 1112
AMS Email Support for Frequently Asked Questions, 274
AMS Epsilon Fund, 1110
AMS Holds Workshop for Department Chairs, 721
AMS Honors Mathematics Teachers, 841
AMS Scholarships for "Math in Moscow", 973
AMS Short Course in Washington, DC, 1161
AMS Sponsors Exhibition on Mathematics and Cardiology, 1112
AMS Sponsors NExT Fellows, 1294
Applications and Nominations Invited for Position of AMS Associate Secretary, 866
Biographies of Candidates, 2008, 1002
Call for Applications and Nominations for Editor of the *Notices*, 665, 797
Call for Nominations for the Albert Leon Whiteman Memorial Prize, 521
Call for Nominations for AMS Award for Mathematics Programs That Make a Difference, 300
Call for Nominations for the AMS-MAA-SIAM Frank and Brennie Morgan Prize, 520
Call for Nominations for the Frank Nelson Cole Prizes, 519

Call for Nominations for the George David Birkhoff Prize, 519
Call for Nominations for Leroy P. Steele Prizes, 404
Call for Nominations for the Levi L. Conant Prize, 519
Call for Nominations for the Ruth Lyttle Satter Prize, 521
Call for Suggestions for 2009 AMS Elections, 1014
Current Events Session at Joint Meetings, 66
Epsilon Awards for 2008, 617
Epsilon Scholarships Awarded for 2008, 1438
Erdős Memorial Lecture, 66
Fan China Exchange Program Names Awardees, 2008, 721
From the AMS Public Awareness Office, 273, 399, 512, 617, 722, 841, 978, 1113, 1294, 1438
General Information Regarding Meetings & Conferences of the AMS, 95
Math in Moscow Scholarships Awarded, 273, 841
Mathematical Sciences Employment Center in Washington, DC, 1046, 1157
MathSciNet Links to Mathematics Genealogy Project, 618
New *Mathematical Moments* Available, 66
Nominations by Petition for 2009 AMS Election, 1015
Officers and Committee Members, 1122
Officers of the Society 2007 and 2008 Updates, 629
Reciprocity Agreements, 1306
Report of the Executive Director, State of the AMS, 2008, 850
Report of the Treasurer (2007), 858
Search for an Executive Director for the American Mathematical Society, 1075, 1222
Statistics on Women Mathematicians Compiled by the AMS, 1132
Trjitzinsky Memorial Awards Presented, 1437
Voting Information for 2008 AMS Election, 865

Announcements

DoE Report on Applied Mathematics, 976
Everett Pitcher Lectures, 398
McCarthy Appointed AWM Executive Director, 976
News from PIMS, 270
Noticed: Kevin Short, 618
Noticed: Manil Suri, 618
Program for ICM2010, Hyderabad, 271
Santosa Appointed IMA Director, 1292
STIX Fonts Project Completes Design Phase, 270

Articles

— Features

(A) Centennial Celebration of Two Great Scholars: Heiberg's Translation of the Lost Palimpsest of Archimedes—1907; Heath's Publication on Euclid's Elements—1908 (Shirley B. Gray), 776
Computable Fields and Galois Theory (Russell Miller), 798
Cross-Cultural Analysis of Students with Exceptional Talent in Mathematical Problem Solving (Titu Andreescu, Joseph A. Gallian, Jonathan M. Kane, and Janet E. Mertz), 1248
Crystals That Nature Might Miss Creating (Toshikazu Sunada), 208
(An) Evaluation of Mathematics Competitions Using Item Response Theory (Jim Gleason), 8
(The) Father of the Father of American Mathematics (Steve Batterson), 352
Formal Proof (Thomas C. Hales), 1370
Formal Proof—The Four-Color Theorem (Georges Gonthier), 1382
Formal Proof—Getting Started (Freek Wiedijk), 1408
Formal Proof—Theory and Practice (John Harrison), 1395
From the Fundamental Theorem of Algebra to Astrophysics: A "Harmonious" Path (Dmitry Khavinson and Genevra Neumann), 666
Henri's Crystal Ball (Philip J. Davis and David Mumford), 458
Interview with Martin Davis (Allyn Jackson), 560
Irving Kaplansky's Role in Mid-Twentieth Century Functional Analysis (Richard V. Kadison), 216
Jobs in Mathematics Education in Institutions of Higher Education in the United States (Robert E. Reys), 676
Mathematics and Voting (Donald G. Saari), 448
Möbius Transformations Revealed (Douglas N. Arnold and Jonathan Rogness), 1226
Old and New on the Exceptional Group G_2 (Ilka Agricola), 922
(A) Remarkable Collection of Babylonian Mathematical Texts (Jöran Friberg), 1076
Uncovering a New L-function (Andrew R. Booker), 1088
Undecidability in Number Theory (Bjorn Poonen), 344
Victor L. Klee (1925–2007) (Peter Gritzmann and Bernd Sturmfels), 467
Who Is Alexander Grothendieck? (Winfried Scharlau), 930
Your Hit Parade: The Top Ten Most Fascinating Formulas in Ramanujan's Lost Notebook (George E. Andrews and Bruce C. Berndt), 18

— Featured Communications

Ask Professor Nescio, 590
Evansville Honors the First Black Ph.D. in Mathematics and His Family (Talitha M. Washington), 588
Visualizing the Sieve of Eratosthenes (David Cox), 579

— Communications

2007 Annual Survey of the Mathematical Sciences (First Report), (Polly Phipps, James W. Maxwell, and Colleen Rose), 253
2007 Annual Survey of the Mathematical Sciences (First Report, Part II), (Polly Phipps, James W. Maxwell, and Colleen Rose), 387
2007 Annual Survey of the Mathematical Sciences (Third Report, Part III), (Polly Phipps, James W. Maxwell, and Colleen A. Rose), 1271
2008 Award for Distinguished Public Service, 508
2008 Award for an Exemplary Program or Achievement in a Mathematics Department, 598
2008 Bôcher Prize, 499
2008 Cole Prize in Number Theory, 497

2008 Conant Prize, 491
2008 Doob Prize, 503
2008 Eisenbud Prize, 505
2008 JPBM Communications Award, 605
2008 Morgan Prize, 494
2008 Steele Prizes, 486
Adventures in Academic Year Undergraduate Research (Kathryn Leonard), 1422
Arnold and Faddeev Receive 2008 Shaw Prize, 966
Ask Professor Nescio, 1263
AWM Awards Given in San Diego, 609
Building a Research Career: Mathematics Research Communities (Allyn Jackson), 247
Citation Statistics: An IMU Report, 968
Clarke, Emerson, and Sifakis Receive 2007 ACM Turing Award, 709
Climate Change: A Research Opportunity for Mathematics? (Patricia Clark Kenschaft), 695
Deligne, Griffiths, and Mumford Receive 2008 Wolf Prize (Allyn Jackson), 594
Differential Geometry, Strasbourg, 1953 (Michèle Audin), 366
Donaldson Receives Nemmers Prize, 808
Fiscal Year 2009 Budget Request for the National Science Foundation (Allyn Jackson), 711
Grothendieck at 80, IHES at 50 (Allyn Jackson), 962
Hardy's "Small" Discovery Remembered (Indika Rajapakse, Lindsey Muir, and Paul Martin), 384
Interview with George Csicsery (Bill Casselman), 576
Interview with Srinivasa Varadhan (Martin Raussen and Christian Skau), 238
James Eells (1926–2007) (Domingo Toledo), 704
Karp Receives 2008 Kyoto Prize, 1102
Kontsevich and Witten Receive 2008 Crafoord Prize in Mathematics, 593
(The) Last Poem of James Clerk Maxwell (Daniel S. Silver), 1266
MAA Prizes Presented in San Diego, 606
Major Gift Launches New Geometry and Physics Center (Allyn Jackson), 685
Mathematical Sciences in the FY 2009 Budget (Samuel M. Rankin III), 809
Mathematics Programs That Make a Difference, 603
Memories of Shourik (Valentin Poénaru), 964
Michel Kervaire, 1927–2007 (Shalom Eliahou, Pierre de la Harpe, Jean-Claude Hausmann, Claude Weber), 960
My Summer at the Voice of America (Adriana Salerno), 249
My Trip to the Hill (Sastry G. Pantula), 54
(A) Note on the *Geometry* and Descartes's Mathematical Work (Michel Serfati), 50
(A) Photographic Look at the Joint Meetings, San Diego, 2008, 476
Taubes Receives NAS Award in Mathematics (Allyn Jackson), 596
Thompson and Tits Receive 2008 Abel Prize, 707
University of Iowa Wins Exemplary Program Award (Allyn Jackson), 599
(A) Valuable Diversion (John Haws), 251
(A) View on the Transition from Academia to Finance (Catherine O'Neil), 700
WHAT IS…a Cross Ratio? (François Labourie), 1234
WHAT IS…Forcing? (Thomas Jech), 692
WHAT IS…an ∞-Category? (Jacob Lurie), 949
WHAT IS…JPEG? (David Austin), 226
WHAT IS…Outer Space? (Karen Vogtmann), 784
WHAT IS…a Period Domain? (James Carlson and Phillip Griffiths), 1418
WHAT IS…Property A? (Piotr Nowak and Guoliang Yu), 474
WHAT IS…Quantum Chaos? (Ze'ev Rudnick), 32
WHAT IS…Stable Commutator Length? (Danny Calegari), 1100
WHAT IS…a Systole? (Marcel Berger), 374
WHAT IS…a Toric Variety? (Ezra Miller), 586
Where Are Journals Headed? Why We Should Worry About Author-Pay (John Ewing), 381
William J. LeVeque (1923–2007) (James W. Maxwell), 1261

Authors

Agricola, Ilka, 922
Andreescu, Titu, 1248
Andrews, George E., 18
Arnold, Douglas N., 1069, 1226
Audin, Michèle, 366
Austin, David, 226
Batterson, Steve, 352
Berger, Marcel, 374
Berggren, J. L., 943
Berndt, Bruce C., 18
Booker, Andrew R., 1088
Bourguignon, Jean-Pierre, 1221
Calegari, Danny, 235, 1100
Carlson, James, 1418
Casselman, Bill, 576
Corfield, David, 1243
Cox, David N., 579
Davis, Philip J., 458
de la Harpe, Pierre, 960
Ekeland, Ivar, 371
Eliahou, Shalom, 960
Ellis, George F. R., 1421
Ewing, John, 381
Friberg, Jöran, 1076
Gallian, Joseph A., 1248
Gleason, Jim, 8
Gonthier, Georges, 1384
Gray, Shirley B., 776
Griffiths, Phillip, 1418
Gritzmann, Peter, 467
Häggström, Olle, 789
Halberstam, Heini, 952
Hales, Thomas C., 1370
Hall, Peter, 920
Harrison, John, 1395
Hausmann, Jean-Claude, 960
Haws, John, 251

Jackson, Allyn, 247, 445, 560, 594, 596, 599, 685, 711, 962
Jech, Thomas, 692
Johnson, Jesse, 557
Jones, Christopher K. R. T., 481
Kadison, Richard V., 216
Kalai, Gil, 687
Kane, Jonathan M., 1248
Kenschaft, Patricia Clark, 695
Khavinson, Dmitry, 666
Kirby, Rob, 1237
Labourie, François, 1234
Leonard, Kathryn, 1422
Lickorish, W. B. Raymond, 37
Lurie, Jacob, 949
Magid, Andy, 5, 661, 1367
Martin, Paul, 384
Maxwell, James W., 253, 387, 814, 1261, 1271
Mehta, Hemant, 231
Mertz, Janet E., 1248
Miller, Ezra, 586
Miller, Russell, 798
Muir, Lindsey, 384
Mumford, David, 458, 919
Nathanson, Melvyn B., 773
Neumann, Genevra, 666
Nir, Oaz, 1096
Nowak, Piotr, 474
O'Neil, Catherine, 700
Oort, Frans, 681
Pantula, Sastry G., 54
Phipps, Polly, 253, 387, 814, 1271
Poénaru, Valentin, 964
Pollatsek, Harriet, 1415
Poonen, Bjorn, 344
Pyenson, Lewis, 793
Rajapakse, Indika, 384
Rankin, Samuel M., III, 809
Raussen, Martin, 238
Reys, Robert E., 676
Rogness, Jonathan, 1226
Rose, Colleen A., 253, 387, 814, 1271
Ross, Shane, 478
Rudnick, Ze'ev, 32
Saari, Donald G., 448
Salerno, Adriana, 249
Sandberg, Irwin W., 377
Scharlau, Winfried, 930
Segal, Sanford L., 583
Serfati, Michel, 44, 50
Silver, Daniel S., 1266
Skau, Christian, 238
Sturmfels, Bernd, 467
Sunada, Toshikazu, 208
Toledo, Domingo, 704
Ullman, Daniel, 205
Vogtmann, Karen, 784
Washington, Talitha M., 588
Weber, Claude, 960
Wiedijk, Freek, 1408
Wood, Carol, 573
Yu, Guoliang, 474
Ziegler, Günter M., 341

Backlog of Mathematics Research Journals, 1301

Book Reviews — *See* **Reviews**

Classified Advertisements

86, 317, 423, 537, 644, 754, 886, 1042, 1145, 1330, 1462

Corrections

Correction to "2008 AWM Shafer Prize" (May 2008), 837
Correction to "Doctoral Degrees Conferred" (February 2008), 510
Correction to "Old and New on the Exceptional Group G_2" article (September 2008), 1293
Correction to Short Course, Washington, DC, information, 1356
Correction to "U.S. High School Girls Compete at China Girls' Math Olympiad" (November 2007), 267

Deaths of AMS Members

Amir-Moez, Ali R., 67
Anderson, Richard D., 618
Arenberg, C. Arne, 1113
Ax, James, 67
Azbelev, Nikolay V., 618
Ballou, Donald H., 1294
Barr, William J., 842
Bear, Herbert S., 842
Beck, William A., 1295
Berger, Melvyn S., 842
Blitch, Patricia M., 1113
Boland, James W., 67
Brobeck, Barbara Caldwell, 842
Bromberg, Eleazer, 512
Buianouckas, Francis R., 842
Butnariu, Dan, 1113
Byers, Ralph, 512
Cartan, Henri, 1295
Chen, Y. W., 842
Clark, Charles L., 618
Cook, E. Allen, 1295
Cooke, Kenneth L., 67
Cullen, Helen F., 618
D'Attorre, Leonardo, 1113
DeMarr, Ralph E., 512
Devinatz, Allen, 842
Diener, Karl-Heinz, 512
Dowson, Henry R., 512
Druet, Robert L., 67
Dubisch, Roy, 512
Duren, William L., Jr., 722
D'yakonov, Eugene Georg, 512
Eigel, Edwin G., 1113
Etter, Daniel O., 67
Evans, W. Buell, 1295
Ewing, Richard E., 512
Farkas, Miklos, 67

Farley, Belmont G., 618
Faulkner, Frank D., 67
Faust, Sr. Claude Marie, 513
Feibleman, James K., 1113
Forbes, Donald, 842
Forman, William, 1113
Frankild, Anders, 67
Free, Norman S., 842
Freedman, Marvin, 722
Gabriel, Richard F., 513
Gale, David, 618
Garcia, Mariano, 842
Geiger, David S., 67
Goldengershel, Esfir Jos-Gersh, 722
Goldie, Alfred W., 618
Golomb, Michael, 722
Gromoll, Detlef, 1113
Gucker, Frank F., 842
Gupta, Narain D., 1113
Gurney, Margaret, 67
Gustafson, William H., 67
Hailpern, Raoul, 618
Hall, Robert A., 1295
Hamilton, John R., 1295
Harris, Theodore E., 67
Havens, Jim, 1295
Hayes, Charles A., 67
Heinonen, Juha, 67
Heller, Alex, 842
Hirsch, Warren M., 67
Hoffman, Ken, 1438
Horn, Alfred, 1295
Householder, James E., 618
Howard, Bernard E., 67
Hunt, Gilbert, 842
Hunter, Louise S., 67
Isaacson, Eugene, 842
Johnson, Robert S., 513
Kaplan, Wilfred, 1295
Karlin, Sam, 513
Keller, Charles Lewis, 842
Keller, Herbert B., 513
Kizuka, Takashi, 722
Klee, Victor, 67
Kluk, Dennis S., 722
Koch, Charles F., 842
Komkov, Vadim, 1113
Krall, Allan M., 1295
Kramer, Raymond F., Jr., 618
Kyburg, Henry E., Jr., 513
Lebel, Jean E., 513
Leroux, Pierre, 1295
Levene, Howard, 842
LeVeque, William J., 513
Levine, Eugene, 842
Levine, Jack, 67
Light, F. W., Jr., 67
Lloyd, Stuart P., 513
MacCarthy, Justin G., 67
Machover, Maurice, 722
Madrecki, Andrzej, 1295
Manis, Merle E., 722
Mansfield, Ralph, 513
Marcou, R. J., 67
Mark, J. C., 1113
Mason, Robert M., 722
Mather, Bertha W., 67
McAuley, Van A., 842
McDermot, Robert G., 67
McMillan, Audrey W., 722
Mendenhall, Robert V., 67
Mielke, Paul T., 1295
Mosier, Ronald G., 1438
Moyls, Benjamin N., 618
Noble, Andrewa R., 67
Odom, William E., 1287
Pilling, Donald L., 1113
Posey, Eldon E., 1113
Potts, Donald H., 67
Protter, Murray H., 722
Reisel, Robert B., 513
Richardson, Stanley, 722
Rider, Daniel, 1113
Roberts, Richard C., 1295
Rogulski, Jan S., 722
Rose, Gene F., 722
Rosenberg, Alex, 513
Rubin, Robert J., 513
Sacksteder, Richard C., 513
Sadowski, Eugene, 67
Samuels, Beth, 267
Schlesinger, Ernest C., 842
Schlosser, Jon A., 1295
Schnitzer, Franz J., 67
Schramm, Oded, 1295
Sholander, Marlow C., 842
Sinclair, Annette, 1295
Smith, William K., 513
Solitar, Donald M., 842
Sousa Ramos, Jose, 722
Steinberger, Meir, 722
Stokes, H. Christine B., 842
Stong, Robert E., 722
Sunouchi, Gen-ichiro, 618
Sunseri, Mary V., 67
Supnick, Fred, 842
Sward, Marcia P., 1438
Swift, William C., 1438
Szüsz, Peter, 618
Takeuchi, Masaru, 513
Treybig, L. Bruce, 1295
Tropp, Henry S., 513
Tsutsumi, Akira, 722
Wallace, Andrew H., 513
Wayne, Alan, 67
Winter, Eva A., 1113
Wirszup, Izaak, 513
Wojciechowski, Krzysztof P., 1113

Doctoral Degrees Conferred 2006–2007, 280

Supplementary List, 826

Election Information (AMS)

2007 Election Results, 301
2008 American Mathematical Society Election (Special Section), 996
2008 AMS Election Nominations by Petition, 302
Biographies of Candidates, 2008, 1002
Call for Suggestions for 2009 AMS Elections, 1014
Nominations by Petition for 2009 AMS Election, 1015
Voting Information for 2008 AMS Election, 865

Employment Center

Mathematical Sciences Employment Center in Washington, DC, 1046, 1157

Grants, Fellowships, Opportunities

AAUW Educational Foundation Fellowships and Grants, 1290
American Mathematical Society Centennial Fellowships, 973, 1288
AMS-AAAS Mass Media Summer Fellowships, 62, 1110
AMS Congressional Fellowship, 1289
AMS Epsilon Fund, 1110
AMS Scholarships for “Math in Moscow”, 973
AP Calculus Readers Sought, 511
AWM Essay Contest, 1111
AWM Travel Grants for Women, 974
Call for Applications and Nominations for Editor of the *Notices*, 665
Call for Nominations for 2008 IBC Prize, 268
Call for Nominations for the Albert Leon Whiteman Memorial Prize, 521
Call for Nominations for AMS Award for Mathematics Programs That Make a Difference, 300
Call for Nominations for the AMS-MAA-SIAM Frank and Brennie Morgan Prize, 520
Call for Nominations for Clay Research Fellows, 975
Call for Nominations for Computer-Aided Verification Award, 61
Call for Nominations for the Frank Nelson Cole Prizes, 519
Call for Nominations for the George David Birkhoff Prize, 519
Call for Nominations for Heineman Prize, 720
Call for Nominations for ICTP Ramanujan Prize, 720
Call for Nominations for Information-Based Complexity Young Researcher Award, 839
Call for Nominations for Leroy P. Steele Prizes, 404
Call for Nominations for the Levi L. Conant Prize, 519
Call for Nominations for Ostrowski Prize, 1435
Call for Nominations for the Ruth Lyttle Satter Prize, 521
Call for Nominations for SASTRA Ramanujan Prize, 720
Call for Nominations for Sloan Fellowships, 720
Call for Nominations for TWAS Prizes, 397
Call for Nominations for Waterman Award, 1434
Call for Proposals for 2009 NSF-CBMS Regional Conferences, 615
Clay Institute 2008 Summer School, 64
CMI Liftoff Program for Summer 2009, 1434
Departments Coordinate Job Offer Deadlines, 63
DMS Opens New Institute Competition, 838, 1290
DMS Workforce Program in the Mathematical Sciences, 511
EDGE Summer Program, 1289
Enhancing the Mathematical Sciences Workforce in the Twenty-First Century, 1110
Graduate Student Travel Grants to 2009 JMM, 1288
IMU Seeks Permanent Location, 64
International Mathematics Competition for University Students, 615
Invitation for Bids for ICM 2014, 720
Jefferson Science Fellows Program, 1433
MfA Fellowship Program, 1433
National Academies Christine Mirzayan Graduate Fellowship Program, 1434
National Academies Research Associateship Programs, 62, 268
News from AIM, 1111
News from the CIRM Trento, 840
News from the Clay Institute, 268
News from the CRM Montreal, 62
News from the Fields Institute, 63, 975
News from the IMA, 615
News from the Institut Mittag-Leffler, 1291
News from IPAM, 616, 1435
NRC-Ford Foundation Diversity Fellowships, 1290
NSA Mathematical Sciences Grants and Sabbaticals Program, 839
NSF CAREER Program Guidelines Available, 839
NSF-CBMS Regional Conferences, 2008, 614
NSF Computing Equipment and Instrumentation Programs, 1433
NSF Focused Research Groups, 838
NSF Graduate Research Fellowships, 1289
NSF International Research Fellow Awards, 974
NSF Mathematical Sciences Postdoctoral Research Fellowships, 839
NSF Postdoctoral Research Fellowships, 614, 1110
NSF Support for Undergraduate Training in Biological and Mathematical Sciences, 397
Pi Mu Epsilon Student Paper Presentation Awards, 2008, 1430
PIMS Postdoctoral Fellows, 1111
Program Director Positions at NSF, 398
Project NExT: New Experiences in Teaching, 511
Proposal Due Dates at the DMS, 61
Research Experiences for Undergraduates, 974
Research Opportunities for U.S. Graduate Students in Asia and Australia, 1289
Stipends for Study and Travel, 983
Summer Program for Women Undergraduates, 268

Invited Speakers

Alexakis, Spyros (Boca Raton, FL), 764
Arnold, Douglas N. (Washington, DC), 762
Behrens, Mark (Huntsville, AL), 105
Bennett, Michael A. (Claremont, CA), 103

Ben-Zvi, David (Waco, TX), 907
Bloch, Anthony Michael (Huntsville, AL), 105
Bryant, Robert J. (Shanghai, People's Republic of China), 1053
Bux, Kai-Uwe (Boca Raton, FL), 764
Caffarelli, Luis A. (Washington, DC), 762
Calderer, M. Carme (Kalamazoo, MI), 105
Camassa, Roberto (Huntsville, AL), 105
Chudnovsky, Maria (Baton Rouge, LA), 101; (Washington, DC), 1172
Conder, Marston (Wellington, New Zealand), 97
Cornea, Octav (Worcester, MA), 906
Deift, Percy (Washington, DC), 1348
Downey, Rodney G. (Wellington, New Zealand), 97
Dunfield, Nathan (Raleigh, NC), 763
Dyson, Freeman (Vancouver, Canada), 104
E, Weinan (New York, NY), 98
Evans, L. Craig (Shanghai, People's Republic of China), 105
Exel, Ruy (Rio de Janeiro, Brazil), 103
Freedman, Michael H. (Wellington, New Zealand), 97
Galatius, Soren (Baton Rouge, LA), 101
Gowers, William Timothy (New York, NY), 98
Hacon, Christopher (Riverside, CA), 1480
Hang, Fengbo (Worcester, MA), 906
Huisgen-Zimmerman, Birge (Riverside, CA), 1480
Ionescu, Alexandru (Kalamazoo, MI), 105
Jin, Shi (Bloomington, IN), 102
Jurdjevic, Velimir (Rio de Janeiro, Brazil), 103
Kapovich, Ilya (New York, NY), 98
Kenyon, Richard (Vancouver, Canada), 104
Khare, Chandrashekhar (Claremont, CA), 103
Khovanov, Mikhail (Washington, DC), 762
Kiessling, Michael K. H. (University Park, PA), 1479
Kiselev, Alexander A. (Waco, TX), 907
Kleiner, Bruce J. (Wellington, New Zealand), 97
Kleshchev, Alexander S. (Vancouver, Canada), 104
Koskela, Pekka (Middletown, CT), 432
Lagarias, Jeffrey C. (Urbana, IL), 1194
Larsen, Michael J. (Bloomington, IN), 102
Laubenbacher, Reinhard C. (Raleigh, NC), 763
Lewis, Mark (Vancouver, Canada), 104
Li, Jun (Riverside, CA), 1480
Lin, Huaxin (Claremont, CA), 103
Lorenzini, Dino J. (Boca Raton, FL), 764
Ludwig, Monika (Middletown, CT), 432
Lurie, Jacob (Urbana, IL), 905
Ma, Zhi-Ming (Shanghai, People's Republic of China), 105
Margulis, Grigorii A. (Washington, DC), 762
Martin, Gaven J. (Wellington, New Zealand), 97
Mattingly, Jonathan C. (Raleigh, NC), 763
Mirzakhani, Maryam (Washington, DC), 1054
Mordukhovich, Boris S. (Kalamazoo, MI), 105
Mosco, Umberto (Worcester, MA), 906
Mustata, Mircea (Bloomington, IN), 102
Nachbin, Andre (Rio de Janeiro, Brazil), 103
Nadler, David (Kalamazoo, MI), 105
Naor, Assaf (Wellington, New Zealand), 97
Ono, Ken (Washington, DC), 762
Papadimitriou, Christor (Washington, DC), 898
Parimala, Raman (Raleigh, NC), 763
Payne, Kevin R. (University Park, PA), 1480
Peterson, Ivars (Washington, DC), 1172
Phong, Duong Hong (Middletown, CT), 432
Pisier, Gilles (Urbana, IL), 905
Reed, Michael C. (Waco, TX), 907
Rockmore, Daniel C. (Washington, DC), 1172
Rodnianski, Igor (Waco, TX), 907
Saloff-Coste, Laurent (University Park, PA), 1480
Sapir, Mark V. (Huntsville, AL), 105
Sarnak, Peter (Washington, DC), 1172
Savin, Ovidiu (New York, NY), 98
Scanlon, Thomas W. (Middletown, CT), 432
Schilling, Anne (Claremont, CA), 103
Schoen, Richard M. (Rio de Janeiro, Brazil), 103; (Shanghai, People's Republic of China), 105
Schramm, Oded (Washington, DC), 650
Sethian, James A. (Washington, DC), 762
Shalom, Yehuda (San Francisco,CA), 763
Shen, Zhongwei (Baton Rouge, LA), 101
Shestakov, Ivan P. (Rio de Janeiro, Brazil), 103
Shimozono, Mark (Baton Rouge, LA), 101
Slaman, Theodore A. (Wellington, New Zealand), 97
Sontag, Eduardo D. (Boca Raton, FL), 764
Strogatz, Steven H. (Washington, DC), 1169
Taylor, Richard (Shanghai, People's Republic of China), 105
Teran, Joseph (Riverside, CA), 1480
Terras, Audrey A. (Vancouver, Canada), 104
Vakil, Ravi (New York, NY), 98
Vaughan, Robert C. (University Park, PA), 1480
Venkatesh, Akshay (Urbana, IL), 905
Vershynin, Roman (San Francisco, CA), 763
Visser, Matthew J. (Wellington, New Zealand), 97
Vogtmann, Karen (San Francisco, CA), 763
Whyte, Kevin (Worcester, MA), 906
Wilkinson, Amie (Rio de Janeiro, Brazil), 103
Winkler, Peter M. (Washington, DC), 1172
Wright, Margaret H. (Bloomington, IN), 102
Yuan, Xiaoping (Shanghai, People's Republic of China), 105
Zelmanov, Efim (San Francisco, CA), 763
Zhang, Weiping (Shanghai, People's Republic of China), 105

Letters to the Editor

Aczel, Janos (Paying for Paid Referees), 558
Ahmed, Elsayed (Abusing Mathematical Models), 664
Ayoub, Raymond (Memories of Elbert Cox and Removing Barriers), 774
Barzilai, Jonathan (Value of a Game), 446
Bharali, Gautam (The Mathematical Research Communities), 558
Boas, Harold P. (Remembering Alex Rosenberg), 774
Boyadzhiev, Khristo (Non-English Names of Prominent Mathematicians), 446
Chaffee, Jim (Varieties of Mathematical Truth), 1368
Chakerian, Don (The Pythagoras Game), 206
Chow, Timothy (Resurrecting Out-of-Print Books), 1223

Cox, David (More on "Visualizing the Sieve of Eratosthenes"), 774
Deift, Percy (On the Mumford Donation), 1368
Griffiths, Peter L. (What Bombelli Knew), 774
Haber, Seymour (Access to Electronic Resources), 921
Hearsey, Bryan V. (Mathematical Attitude), 343
Herzberg, Agnes M. (Which Way Is Gauss Really Facing?), 1223
Izmirlian, Grant (Open Source Software), 342
Juricevic, Robert (Visualizing Eratosthenes in 1949), 921
Kang, Ming-chang (Kaplansky's Lecture Notes), 446
Khavinson, Dmitry (Correction to "From the Fundamental Theorem of Algebra to Astrophysics…"), 1072
Khovanova, Tanya (Senderov's "Killer" Problems), 343
Kleiman, Steven L. (Heironymus Georg Zeuthen), 1223
Koblitz, Neal (Reply to Wigderson), 7
Lipschutz, Seymour (Other Affordable Textbooks), 6
Lubinsky, Doron (On the Mumford Donation), 1368
Magid, Andy (Editor's Note: More on Cake Cutting), 1072
Mumford, David (Response to Deift, Lubinsky, and Nevai), 1369
Neumann, Genevra (Correction to "From the Fundamental Theorem of Algebra to Astrophysics…"), 1072
Nevai, Paul (On the Mumford Donation), 1368
Pambuccian, Victor (F. Schur, not I. Schur), 206
Read, Allan E. (Which Way Is Gauss Really Facing?), 1223
Reid, Constance (Julia and Raphael Robinson), 1369
Robert, Alain M. (The Pythagoras Game), 206
Sabbagh, Gabriel (Some Statements in the Klein Protocols), 342
Schoen, Alan H. (On the Graph (10-3)-a), 663
Serre, Jean-Pierre (Octonion Algebras and Cohomology Classes), 446
Small, Lance (Government Funding Facts), 664
Sormani, Christina (The Pythagoras Game), 206
Sunada, Toshikazu (Correction: "Crystals That Nature Might Miss Creating"), 343
Szyld, Daniel B. (Referendum Results Remain Relevant), 664
Tamburini, Matteo (My Teach for America Experience), 558
Tschinkel, Yuri (Reply to Sabbagh), 342
Vogan, David (History of the Kazhdan-Lusztig Conjectures), 342
Waterhouse, William C. (Hardy and Biology), 663
Wigderson, Avi (Brief History of the Foundations of Cryptography), 6

Mathematics Calendar

74, 304, 406, 522, 630, 729, 867, 1017, 1133, 1318, 1445

Meeting Announcements, Presenters of Papers, and Programs

Baton Rouge, LA, March 2008 (announcement), 320
Bloomington, IN, April 2008 (announcement), 322
Claremont, CA, May 2008 (announcement), 428
Huntsville, AL, October 2008 (announcement), 895
Kalamazoo, MI, October 2008 (announcement), 892
Middletown, CT, October 2008 (announcement), 890
New York, NY, March 2008 (announcement), 98
Rio de Janeiro, Brazil, June 2008 (announcement), 326
San Diego, CA, January 2008 (presenters and program), 109
Shanghai, China, December 2008 (announcement), 759
Vancouver, Canada, October 2008 (announcement), 887
Washington, DC, January 2009 (program), 1169; (timetable), 1201

Meetings and Conferences*

97, 319, 425, 539, 645, 756, 887, 1050, 1165, 1346, 1473

(*Page numbers given are to the main meeting announcements. See `http://www.ams.org/meetings` for full details and updates.)

AAAS Meeting, Features Interdisciplinary Mathematics Program, February 14–18, 2008, Boston, MA, 94
General Information Regarding Meetings & Conferences of the AMS, 95
Tenth Conference on p-adic and Non-Archimedean Analysis, June 30–July 3, 2008, Michigan State University, 335
Meetings and Conferences Table of Contents, 199, 335, 439, 551, 655, 767, 911, 1063, 1215, 1359, 1486

New Publications Offered by the AMS

79, 309, 415, 527, 637, 743, 873, 1034, 1136, 1320, 1453

Noticed

Kevin Short Receives 2008 Grammy Award, 618
Manil Suri Publishes Second Novel, 618
Science Magazine Publishes Article Highlighting *Notices* Author Robert Reys' Conclusions About Mathematics Education Doctorates, 1369

Obituaries

Alex Rosenberg (1926–2007), 613
Beth Samuels (1975–2007), 267
James Eells (1926–2007), 704
Juha Heinonen (1960–2007), 836
Krzysztof P. Wojciechowski (1953–2008), 1109
Michel Kervaire (1927–2007), 960
Murray H. Protter (1918–2008), 836
Victor L. Klee (1925–2007), 467
William J. LeVeque (1923–2007), 1261
William E. Odom (1932–2008), 1287

Officers of the Society

AMS Officers and Committee Members, 1122
Officers of the Society 2007 and 2008 Updates, 629

Opinion

Challenges for Math in Australia (Peter Hall), 920
DARPA and Hilbert (Allyn Jackson), 445
Desperately Seeking Mathematical Truth (Melvyn B. Nathanson), 773
Infrastructures and Policies for Mathematicians (Jean-Pierre Bourguignon), 1221
Is the Public Hungry for Math? (Douglas N. Arnold), 1069
Job Talk (Andy Magid), 1367

Math Year in Germany (Günter M. Ziegler), 341
Mathematics Doctoral Programs, Then and Now (Andy Magid), 5
Reflections on Meetings (Andy Magid), 661
Triumphs and Struggles in a Mathematics Classroom (Jesse Johnson), 557
What Does an AMS Congressional Fellow Do? (Daniel Ullman), 205
(The) Wolf Prize and Supporting Palestinian Education (David Mumford), 919

Prizes and Awards

AAAS Fellows (Carlos Castillo-Chavez, Tony F. Chan, Bruno Nachtergaele, Lawrence Sirovich, Robert J. Zimmer), 267
AAAS Mentor Award, 2007 (Carlos Castillo Chavez), 611
Abel Prize, 2008 (John Griggs Thompson, Jacques Tits), 707
ACM A. M. Turing Award, 2007 (Edmund M. Clarke, E. Allen Emerson, Joseph Sifakis), 709
ACM Gödel Prize (Daniel A. Spielman, Shang-Hua Teng), 970
ACM Paris Kanellakis Theory and Practice Award (Bruno Buchberger), 834
ACM-W Athena Award, 2008–2009 (Shafi Goldwasser), 833
Adams Prize, 2008 (Tom Bridgeland, David Tong), 833
AIM Five-Year Fellowship, 2008 (Travis Schedler), 611
American Academy of Arts and Sciences Elections, 2008 (Ruzena Bajcsy, Emily A. Carter, Sun-Yung Alice Chang, Tobias Colding, Vladimir Drinfeld, David Gottlieb, John Guckenheimer, Larry V. Hedges, Richard H. Herman, David Kazhdan, James H. Simons, Charles Simonyi), 836
AMS Centennial Fellowship, 2008–2009 (Christopher Hoffman), 715
AMS Karl Menger Awards (Alexander Churchill, Shravani Mikkilineni, David Rosengarten, Eric Larson, Alex Chen, Paul Kominers, Matthew Wage, Swara Kopparty, Sana Raoof, Nurlan Taiganov, Artem Timoshenko, Sarah Sellers), 971
André-Aisenstadt Mathematics Prize, 2008 (Jozsef Solymosi, Jonathan Taylor), 266
André Lichnerowicz Prize, 2008 (Henrique Bursztyn, Marius Crainic), 1285
ASL Sacks Prizes, 2007 (Adrien Deloro, Wojciech Moczydlowski), 509
Award for an Exemplary Program or Achievement in a Mathematics Department, 2008 (University of Iowa), 598
AWM Awards (2008 Louise Hay Award—Harriet S. Pollatsek; Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman—Galyna Dobrovolska, Alison Miller; Honorable Mention—Naomi Brownstein, Reagin Taylor McNeill, Mary Wootters), 609
AWM Essay Contest Winners, 2007 (Leena Shah, Sarah Budrus, Elizabeth Faiella, Haley Kossek, Helen A. Rawlins), 510
B. H. Neumann Awards, 2008, (Philip Swedosh, Ben Burton, Steve Thornton), 1431
Blackwell-Tapia Prize, 2008 (Juan C. Meza), 1107
Bôcher Prize, 2008 (Alberto Bressan, Charles Fefferman, Carlos Kenig), 499
Bolyai Prize (László Lovász), 264
Brin Prize in Dynamical Systems (Giovanni Forni), 715
Carl Sagan Prize (Keith Devlin), 394
Clay Research Awards, 2008 (Clifford Taubes, Claire Voisin), 612
Clay Research Fellowships, 2008 (Spyros Alexakis, Adrian Ioana, Xinyi Yuan), 612
CMS Prizes (2009 Krieger-Nelson Prize—Yael Karshon; 2009 Jeffery-Williams Prize—Stephen Kudla; 2008 Coxeter-James Prize—Ravi Vakil), 717
Cole Prize in Number Theory, 2008 (Manjul Bhargava), 497
Communicator Award, 2008 (Günter M. Ziegler), 834
Computer-Aided Verification Award, 2008 (Rajeev Alur, David L. Dill), 1429
Conant Prize, 2008 (J. Brian Conrey, Shlomo Hoory, Nathan Linial, Avi Wigerson), 491
Crafoord Prize in Mathematics, 2008 (Maxim Kontsevich, Edward Witten), 593
CRM-Fields-PIMS Prize, 2008 (Allan Borodin), 394
Dannie Heineman Prize in Mathematical Physics, 2008 (Mitchell J. Feigenbaum), 611
Dirac Medal, 2008 (Juan Martín Maldacena, Joseph Polchinski, Cumrun Vafa), 1284
Distinguished Public Service Award, 2008 (Herbert Clemens), 508
Doob Prize, 2008 (Enrico Bombieri, Walter Gubler), 503
Eisenbud Prize, 2008 (Hirosi Ooguri, Andrew Strominger, Cumrun Vafa), 505
Emanuel and Carol Parzen Prize, 2008 (Nancy Reid, Marvin Zelen), 717
Emil Artin Junior Prize, 2008 (Nansen Petrosyan), 1430
European Mathematical Society Prizes (Artur Avila, Alexei Borodin, Ben Green, Olga Holtz, Boáz Klartag, Alexander Kuznetsov, Assaf Naor, Laure Saint-Raymond, Agata Smoktunowicz, Cédric Villani, Josselin Garnier), 1106
Fermat Prize, 2007 (Chandrashekhar Khare), 265
Ferran Sunyer i Balaguer Prize, 2008 (Luis Barreira), 715
Ford Foundation Diversity Fellowships, 2007 (Manuel L. Reyes), 972
Fulbright Foreign Scholarships, 2007–2008 (Gregory R. Conner, Michael T. Lacey, Regina Y. Liu, Michael R. Penkava, Philip E. Protter), 718
Guggenheim Fellows, 2008 (Douglas N. Arnold, Ovidiu Costin, Chandrashekhar B. Khare, Paolo Mancosu, Peter Ozsváth, Marc A. Suchard), 718
Hans Schneider Prize in Linear Algebra, 2008 (Beresford Parlett, Cleve Moler), 971
ICA Euler Medal, 2007 (Stephen Milne), 717
ICCM Awards, 2007 (Chern Prize in Mathematics—Shiu-Yuen Cheng, Mu-Tao Wang; International Cooperation Award—Stanley Osher; Morningside Gold Medal of Mathematics—Jianqing Fan, Xujia Wang; Morningside Silver Medal of Mathematics—Chiu-Chu Liu, Lizhen Ji, Shi Jin, Chiun-Chuan Chen, Ye Tian), 509
ICMI Medals, 2007 (Felix Klein Medal—Jeremy Kilpatrick; Hans Freudenthal Medal—Anna Sfard), 716

ICRA Award (Osamu Iyama), 612
ICTP/IMU Srinivasa Ramanujan Prize, 2007 (Jorge Lauret), 265
Information-Based Complexity Prize, 2008 (Anargyros Papageorgiou), 833
Information-Based Complexity Young Researcher Award, 2007 (Andreas Neuenkirch), 266
Infosys Mathematics Prize, 2008 (Manindra Agrawal), 1428
Intel Science Talent Search Scholarships, 2008 (Katherine Banks, Philip Mocz), 719
International Mathematical Olympiad, 2008 (Alex Zhai, Colin Sandon, Krishanu Roy Sankar, Shaunak Kishore, Evan O'Dorney, Paul Christiano), 1286
IUPAP Boltzmann Medal, 2007 (Kurt Binder, Giovanni Gallavotti), 718
Jerome Sacks Award, 2008 (John Rice), 1284
John von Neumann Theory Prize, 2007 (Arthur F. Veinott Jr.), 716
JPBM Communications Award, 2008 (Carl Bialik), 605
Kyoto Prize, 2008 (Richard M. Karp), 1102
L. E. J. Brouwer Prize, 2008 (Phillip A. Griffiths), 833
London Mathematical Society Prizes, 2008 (Pólya Prize—David Preiss; Fröhlich Prize—Nicholas Higham; Senior Berwick Prize—Kevin Buzzard; Whitehead Prizes—Timothy Browning, Tamás Hausel, Martin Hairer, Nina Snaith), 1107
MAA Awards, 2008 (Carl B. Allendoerfer Award—Eugene Boman, Richard Brazier, Derek Seiple, Chris Christensen; Trevor Evans Award—William Dunham, Robert K. Moniot; Lester R. Ford Award—Tom M. Apostol, Mamikon A. Mnatsakanian, David Auckly, Andrew Cohen, Tanya Leise, Thomas C. Hales, Katherine Socha; George Pólya Award—Roland Minton, Timothy J. Pennings, Andrew J. Simoson; Annie and John Selden Prize for Research in Undergraduate Mathematics Education—Marilyn Carlson; Henry Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member—David Brown, Jacqueline A. Jensen, Katherine Socha), 1429
MAA Awards for Mathematical Modeling, 2008 (Jason Chen, Joonhahn Cho, Brian Choi, Martin Hunt, Christopher Pong, George Tucker), 1286
MAA Prizes Presented in San Diego, 2008 (Gung and Hu Award for Distinguished Service—Lida K. Barrett; Haimo Awards for Teaching—Annalisa Crannell, Kenneth I. Gross, James Morrow; Chauvenet Prize—Andrew Granville; Beckenbach Book Prize—William Dunham; Euler Book Prize—Benjamin H. Yandell; David P. Robbins Prize—Neil J. A. Sloane; Certificates of Meritorious Service—Herbert Kasube, Donald E. Bennett, Victor Gummersheimer, Leonard F. Klosinski, H. Joseph Straight, Andrew Matchett), 608
MacArthur Fellowship, 2008 (Alexei Kitaev), 1428
Mathematics Education Trust Lifetime Achievement Awards, 2008 (Frank K. Lester Jr., Robert E. Reys), 834
Moody Foundation Mega Math Challenge, 2008 (Summa Cum Laude—Thomas Jackson, Kelly Roache, Afanasiy Yermakov, Jason Zukus; Magna Cum Laude—Michael Bacsik, Jephthah Liddie, Joshua Newman, Thomas Sozzi, Kevin Tien; Cum Laude—Anand Desai, Ruby Lee, Shengzhi Li, Anirvan Mukherjee, Lingke Wang; Meritorious—Eric Chung, Alaap Parikh, Ashutosh Singhal; Exemplary—Brandon Comella, Gawain Lau, Kelvin Mei, Nevin Raj, Yiwen Zhan; First Honorable Mention—Brett Musco, Cameron Musco, Christopher Musco, Christopher Shaw, Karan Takhar), 835
Morgan Prize, 2008 (Nathan Kaplan), 494
NAS Award for Initiatives in Research, 2008 (Anna C. Gilbert), 611
NAS Award in Mathematics, 2008 (Clifford H. Taubes), 596
National Academy of Engineering Elections (Jon M. Kleinberg, Prabhakar Raghavan, Vladimir Rokhlin, James A. Sethian, Arthur John Robin Gorell Milner, Ekkehard Ramm), 612
National Academy of Sciences Elections (Emily A. Carter, Helmut Hofer, Peter W. Jones, Frank T. Leighton, Thomas M. Liggett, Nathan Seiberg, Elizabeth A. Thompson, H. Keith Moffatt, Terence C. Tao), 836
National Award in Statistics for Senior Statisticians, 2007–2008 (B. L. S. Prakasa Rao), 1108
Nemmers Prize in Mathematics, 2008 (Simon Donaldson), 808
NSF Career Awards, 2007 (Hao-Min Zhou, Huibin Zhou, Hao Zhang, Dongbin Xiu, Zhiqiang Tan, Scott R. Sheffield, Brian C. Rider, Peter J. Mucha, Jiashun Jin, Anil N. Hirani, Serkan Gugercin, Carlos Garcia-Cervera, Alexander Gamburd, David M. Fisher, Patrick Cheridito, Indira L. Chatterji), 267
NSF Graduate Research Fellowships, 2008 (June Andrews, Michael J. Barany, Adam L. Boocher, Naomi Brownstein, Rex T. Cheung, Dustin T. Clausen, Kelly B. Funk, Ilya Grigoriev, Ian R. Haken, Joseph Hirsh, Kenji Y. Kozai, Aaron D. Kleinman, Anand P. Kulkarni, Brandon W. Levin, Alison B. Miller, Anand U. Oza, Aaron Silberstein, Pablo R. Solis, Claire M. Tomesch, Kevin H. Wilson, Jesse Wolfson, Elena Yudovina, Inna I. Zakharevich), 719
NSF Postdoctoral Fellowships, 2007 (Sami H. Assaf, Dmitriy S. Boyarchenko, Patrick C. Clarke, Calder Daenzer, Jason DeBlois, Amanda L. Folsom, Jayce R. Getz, Thomas C. Hangelbroek, Benjamin J. Howard, Rizwanur R. Khan, Kay L. Kirkpatrick, Troy J. Lee, Yi-Kai Liu, Jason D. Lotay, Larson E. Louder, Cheng Ly, Danielle J. Lyles, Jeremy L. Marzuola, David B. McReynolds, Jeffrey A. Mermin, Gregg J. Musiker, Alvaro Pelayo, Kathleen A. Ponto, Karl E. Schwede, Jake P. Solomon, Jeffrey D. Streets, Vincent R. Vatter, Benjamin T. Webster, Paul A. Wright), 59
NSF Postdoctoral Research Fellowships, 2008 (Jarod D. Alper, John A. Baldwin, Nawaf Bou-Rabee, Jeremy S. Brandman, Steven K. Butler, Matthew B. Day, Inessa Epstein, Joel W. Fish, David S. Freeman, William D. Gillam, Mark Hoefer, William P. Hooper, Angela B. Hugeback, Justin C. Kao, Sara C. Koch, Alex Kontorovich, Karen M. Lange, Lionel Levine, Joel C. Miller, Erin C. Munro, Scott A. Norris, Katharine A. Ott, Manish M. Patnaik, Jonathon R. Peterson, Paul P. Pollack, Brendon P. Rhoades, Matthew D. Rogers, Yanir A. Rubinstein, Susan J. Sierra, Katherine E. Stange, Samuel N. Stechmann, Brian T. Street, John R. Taylor, Frank H. Thorne, Ian I. Tice,

Robert E. Waelder, Jared Weinstein, Jonathan Wise, Tatiana Yarmola, Josephine T. Yu, Jessica V. Zuniga), 1286
Oberwolfach Prize, 2007 (Ngô Bao Châu), 266
ONR Young Investigators Award, 2008 (Joel A. Tropp, Justin Romberg, Carlos A. Guestrin, David Kempe, Adrian Lew, Leigh McCue), 1285
Packard Fellowship, 2007 (Akshay Venkatesh), 395
Paul Erdős Awards, 2008 (Hans-Dietrich (Dieter) Gronau, Bruce Henry, Leou Shian), 1108
PECASE Award, 2006 (Kiran Kedlaya), 265
Pi Mu Epsilon Student Paper Presentation Awards, 2008 (Samuel Behrend, Alicia Brinkman, Iordan Ganev, Brendan Kelly, Daniel Lithio, W. Ryan Livingston, Jared Ruiz, Jeremy Thompson), 1430
Prizes of the Canadian Mathematical Society (2008 Adrien Pouliot Award—Harley Weston; 2008 Excellence in Teaching Award—Edward Bierstone; 2007 G. de B. Robinson Award—Ronald van Luijk), 1284
Prizes of the Mathematical Society of Japan (Autumn Prize—Tadahisa Funaki; Geometry Prizes—Shigeyuki Morita, Kenichi Yoshikawa; Analysis Prizes—Shigeki Aida, Toshiaki Hishida, Takeshi Hirai), 395
Prizes of the Mathematical Society of Japan, 2008 (Spring Prize—Hideo Takaoka; Publication Prize—Susumu Otake, Takeshi Kitano, Takahiko Yamaguchi, Mitsuo Sugiura; Algebra Prize—Osamu Iyama, Yoshinori Namikawa, Toshiyuki Tanisaki), 972
Professor of the Year Awards (Outstanding Community Colleges Professor for 2007—Rosemary M. Karr; Outstanding Master's Universities and Colleges Professor for 2007—Carlos G. Spaht; State Professor of the Year— Frank Jones), 395
Putnam Prizes (Jason C. Bland, Brian R. Lawrence, Aaron C. Pixton, Qingchun Ren, Xuancheng Shao, Arnav Tripathy, Alison B. Miller), 719
Rhodes Scholarships (Adam M. Levine, Shayak Sarkar), 395
Ribenboim Prize in Number Theory, 2008 (Adrian Iovita), 1108
Rolf Schock Prize (Endre Szemerédi), 1284
Rollo Davidson Prize, 2008 (Brian Rider, Bálint Virág), 716
Royal Society of Canada Elections (Ivar Ekeland, Pengfei Guan, Raymond Laflamme, Eckhard Meinrenken, Agnes Herzberg), 1431
Ruth I. Michler Memorial Prize (Irina Mitrea), 832
Salem Prize, 2007 (Akshay Venkatesh), 716
SASTRA Ramanujan Prize, 2007 (Ben Green), 58
Science and Engineering Visualization Challenge, 2007 (Douglas Arnold, Jonathan Rogness), 58
Shaw Prize, 2008 (Vladimir Arnold, Ludwig Faddeev), 966
SIAM Prizes, 2008 (George Pólya Prize—Van H. Vu; W. T. and Idalia Reid Prize in Mathematics—Max Gunzburger; Distinguished Service Prize—Philippe Tondeur; John von Neumann Lecturer—David I. Gottlieb; DiPrima Prize—Daan Huybrechs; Outstanding Paper Prize—Vicent Caselles, Antonin Chambolle, Matteo Novaga, Subhash Khot, Todd Kapitula, P. G. Kevrekidis, Zhigang Chen; Mathematical Contest in Modeling—Amy M. Evans, Tracy L. Stepien, Christopher Chang, Zhou Fan, Yi Sun; Student Paper Prizes—Jeremy Brandman, Roland Griesmaier, David Ketcheson), 1105
Siemens Competition in Math, Science, and Technology (Jacob Steinhardt, Ayon Sen, Alexander C. Huang), 396
Sigma Xi Young Investigator Award (Mason Porter), 716
Sloan Research Fellowships, 2008 (Dmytro Arinkin, Wolfgang Bangerth, Valentin Blomer, Paolo Cascini, Timothy P. Chartier, Kevin J. Costello, Laura DeMarco, Kirsten Eisentrager, Noureddine El Karoui, Inwon C. Kim, Marcus A. Khuri, Joachim Krieger, Jean-François Lafont, Fengyan Li, Mauro Maggioni, Laura F. Matusevich, Toufic Mubadda Suidan, Natasa Pavlovic, Ben Weinkove, Andrej Zlatos), 718
Société Mathématique de France Prizes, 2008 (d'Alembert Prize—Marie-José Pestel; Anatole Decerf Prize—Robert Ferréol), 1107
Steele Prizes, 2008 (Neil Trudinger, Endre Szemerédi, George Lusztig), 486
Templeton Award, 2008 (Michael Heller), 833
Timoshenko Medal (Thomas J. R. Hughes), 718
TWAS Prize in Mathematics, 2007 (Shrikrishna Dani), 265
USA Mathematical Olympiad, 2008 (David Benjamin, TaoRan Chen, Paul Christiano, Samuel Elder, Shaunak Kishore, Delong Meng, Evan O'Dorney, Qinxuan Pan, David Rolnick, Colin Sandon, Krishanu Sankar, Alex Zhai), 835
von Kaven Prize, 2007 (Gitta Kutyniok), 265
von Kaven Prize, 2008 (Arthur Bartels, Ulrich Görtz), 1428
Waterman Award, 2008 (Terence Tao), 832
Wolf Prize, 2008 (Pierre R. Deligne, Phillip A. Griffiths, David B. Mumford), 594
YouTube Video Contest (Mace Mateo), 1107

Prizewinners

Agrawal, Manindra (2008 Infosys Mathematics Prize), 1428
Aida, Shigeki (Analysis Prize of the MSJ), 395
Alexakis, Spyros (2008 Clay Research Fellowship), 612
Alper, Jarod D. (2008 NSF Postdoctoral Research Fellowship), 1286
Alur, Rajeev (2008 Computer-Aided Verification Award), 1429
Andrews, June (2008 NSF Graduate Research Fellowship), 719
Apostol, Tom M. (2008 MAA Lester R. Ford Award), 1430
Arinkin, Dmytro (2008 Sloan Research Fellowship), 718
Arnold, Douglas N. (2007 Science and Engineering Visualization Challenge), 58; (2008 Guggenheim Fellow), 718
Arnold, Vladimir (2008 Shaw Prize), 966
Assaf, Sami H. (2007 NSF Postdoctoral Fellowship), 59
Auckly, David (2008 MAA Lester R. Ford Award), 1430
Avila, Artur (European Mathematical Society Prize), 1106
Bacsik, Michael (2008 Moody Foundation Mega Math Challenge Magna Cum Laude), 835

Bajcsy, Ruzena (2008 American Academy of Arts and Sciences Elections), 836
Baldwin, John A. (2008 NSF Postdoctoral Research Fellowship), 1286
Bangerth, Wolfgang (2008 Sloan Research Fellowship), 718
Barany, Michael J. (2008 NSF Graduate Research Fellowship), 719
Barreira, Luis (2008 Ferran Sunyer i Balaguer Prize), 715
Barrett, Lida K. (2008 MAA Gung and Hu Award for Distinguished Service), 606
Bartels, Arthur (2008 von Kaven Prize), 1428
Behrend, Samuel (2008 Pi Mu Epsilon Student Paper Presentation Award), 1430
Benjamin, David (2008 USA Mathematical Olympiad), 835
Bennett, Donald E. (2008 MAA Certificate of Meritorious Service), 608
Bhargava, Manjul (2008 Cole Prize), 497
Bialik, Carl (2008 JPBM Communications Award), 605
Bierstone, Edward (CMS 2008 Excellence in Teaching Award), 1285
Binder, Kurt (2007 IUPAP Boltzmann Medal), 718
Bland, Jason C. (Putnam Prize), 719
Blomer, Valentin (2008 Sloan Research Fellowship), 718
Boman, Eugene (2008 MAA Carl B. Allendoerfer Award), 1429
Bombieri, Enrico (2008 Doob Prize), 503
Boocher, Adam L. (2008 NSF Graduate Research Fellowship), 719
Borodin, Alexei (European Mathematical Society Prize), 1106
Borodin, Allan (2008 CRM-Fields-PIMS Prize), 394
Bou-Rabee, Nawaf (2008 NSF Postdoctoral Research Fellowship), 1286
Boyarchenko, Dmitriy S. (2007 NSF Postdoctoral Fellowship), 59
Brandman, Jeremy S. (2008 SIAM Student Paper Prize), 1105; (2008 NSF Postdoctoral Research Fellowship), 1286
Brazier, Richard (2008 MAA Carl B. Allendoerfer Award), 1429
Bressan, Alberto (2008 Bôcher Prize), 499
Bridgeland, Tom (2008 Adams Prize), 833
Brinkman, Alicia (2008 Pi Mu Epsilon Student Paper Presentation Award), 1430
Brown, David (2008 MAA Henry Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member), 1430
Browning, Timothy (2008 LMS Whitehead Prize), 1107
Brownstein, Naomi (2008 AWM Schafer Prize Honorable Mention), 609; (2008 NSF Graduate Research Fellowship), 719
Buchberger, Bruno (ACM Paris Kanellakis Theory and Practice Award), 834
Budrus, Sarah (2007 AWM Essay Contest Winner), 510
Bursztyn, Henrique (2008 André Lichnerowicz Prize), 1285
Burton, Ben (2008 B. H. Neumann Award), 1431
Butler, Steven K. (2008 NSF Postdoctoral Research Fellowship), 1286
Buzzard, Kevin (2008 LMS Senior Berwick Prize), 1107
Carlson, Marilyn (2008 MAA Annie and John Selden Prize for Research in Undergraduate Mathematics Education), 1430
Carter, Emily A. (2008 American Academy of Arts and Sciences Elections; National Academy of Sciences Elections), 836
Cascini, Paolo (2008 Sloan Research Fellowship), 718
Caselles, Vicent (SIAM Outstanding Paper Prize), 1105
Castillo-Chavez, Carlos (AAAS Fellow), 267; (2007 AAAS Mentor Award), 611
Chambolle, Antonin (SIAM Outstanding Paper Prize), 1105
Chan, Tony F. (AAAS Fellow), 267
Chang, Sun-Yung Alice (2008 American Academy of Arts and Sciences Elections), 836
Chartier, Timothy P. (2008 Sloan Research Fellowship), 718
Chatterji, Indira L. (2007 NSF CAREER Award), 267
Châu, Ngô Bao (2007 Oberwolfach Prize), 266
Chen, Alex (AMS Karl Menger Award), 971
Chen, Chiun-Chuan (2007 ICCM Morningside Silver Medal of Mathematics), 509
Chen, Jason (2008 MAA Award for Mathematical Modeling), 1286
Chen, TaoRan (2008 USA Mathematical Olympiad), 835
Chen, Zhigang (SIAM Outstanding Paper Prize), 1105
Cheng, Shiu-Yuen (2007 ICCM Chern Prize in Mathematics), 509
Cheridito, Patrick (2007 NSF CAREER Award), 267
Cheung, Rex T. (2008 NSF Graduate Research Fellowship), 719
Cho, Joonhahn (2008 MAA Award for Mathematical Modeling), 1286
Choi, Brian (2008 MAA Award for Mathematical Modeling), 1286
Christensen, Chris (2008 MAA Carl B. Allendoerfer Award), 1429
Christiano, Paul (2008 USA Mathematical Olympiad), 835; (2008 International Mathematical Olympiad), 1286
Chung, Eric (2008 Moody Foundation Mega Math Challenge Meritorious), 835
Churchill, Alexander (AMS Karl Menger Award), 971
Clarke, Edmund M. (2007 ACM Turing Award), 709
Clarke, Patrick C. (2007 NSF Postdoctoral Fellowship), 59
Clausen, Dustin T. (2008 NSF Graduate Research Fellowship), 719
Clemens, Herbert (2008 Award for Distinguished Public Service), 508
Cohen, Andrew (2008 MAA Lester R. Ford Award), 1430
Colding, Tobias (2008 American Academy of Arts and Sciences Elections), 836
Comella, Brandon (2008 Moody Foundation Mega Math Challenge Exemplary), 835
Conner, Gregory R. (2007–2008 Fulbright Foreign Scholarship), 719
Conrey, J. Brian (2008 Conant Prize), 491
Costello, Kevin J. (2008 Sloan Research Fellowship), 718

Costin, Ovidiu (2008 Guggenheim Fellow), 718
Crainic, Marius (2008 André Lichnerowicz Prize), 1285
Crannell, Annalisa (2008 MAA Haimo Award for Teaching), 606
Daenzer, Calder (2007 NSF Postdoctoral Fellowship), 59
Dani, Shrikrishna (2007 TWAS Prize in Mathematics), 265
Day, Matthew B. (2008 NSF Postdoctoral Research Fellowship), 1286
DeBlois, Jason (2007 NSF Postdoctoral Fellowship), 59
Deligne, Pierre R. (2008 Wolf Prize), 594
Deloro, Adrien (2007 ASL Sacks Prizes), 509
DeMarco, Laura (2008 Sloan Research Fellowship), 718
Desai, Anand (2008 Moody Foundation Mega Math Challenge Cum Laude), 835
Devlin, Keith (Carl Sagan Prize), 394
Dill, David L. (2008 Computer-Aided Verification Award), 1429
Dobrovolska, Galyna (2008 AWM Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman), 609
Donaldson, Simon (2008 Nemmers Prize in Mathematics), 808
Drinfeld, Vladimir (2008 American Academy of Arts and Sciences Elections), 836
Dunham, William (2008 MAA Beckenbach Book Prize), 607; (2008 MAA Trevor Evans Award), 1430
Eisentrager, Kirsten (2008 Sloan Research Fellowship), 718
Ekeland, Ivar (Royal Society of Canada), 1431
Elder, Samuel (2008 USA Mathematical Olympiad), 835
El Karoui, Noureddine (2008 Sloan Research Fellowship), 718
Emerson, E. Allen (2007 ACM Turing Award), 709
Epstein, Inessa (2008 NSF Postdoctoral Research Fellowship), 1286
Faddeev, Ludwig (2008 Shaw Prize), 966
Faiella, Elizabeth (2007 AWM Essay Contest Winner), 510
Fan, Jianqing (2007 ICCM Morningside Gold Medal of Mathematics), 509
Fefferman, Charles (2008 Bôcher Prize), 500
Feigenbaum, Mitchell J. (2008 Dannie Heineman Prize in Mathematical Physics), 611
Ferréol, Robert (2008 SMF Anatole Decerf Prize), 1107
Fish, Joel W. (2008 NSF Postdoctoral Research Fellowship), 1286
Fisher, David M. (2007 NSF CAREER Award), 267
Folsom, Amanda L. (2007 NSF Postdoctoral Fellowship), 59
Forni, Giovanni (Brin Prize in Dynamical Systems), 715
Freeman, David S. (2008 NSF Postdoctoral Research Fellowship), 1286
Funaki, Tadahisa (Autumn Prize of the MSJ), 395
Funk, Kelly B. (2008 NSF Graduate Research Fellowship), 719
Gallavotti, Giovanni (2007 IUPAP Boltzmann Medal), 718
Gamburd, Alexander (2007 NSF CAREER Award), 267
Ganev, Iordan (2008 Pi Mu Epsilon Student Paper Presentation Award), 1430
Garcia-Cervera, Carlos (2007 NSF CAREER Award), 267
Garnier, Josselin (European Mathematical Society Felix Klein Prize), 1107
Getz, Jayce R. (2007 NSF Postdoctoral Fellowship), 59
Gilbert, Anna C. (2008 NAS Award for Initiatives in Research), 611
Gillam, William D. (2008 NSF Postdoctoral Research Fellowship), 1286
Goldwasser, Shafi (2008–2009 ACM-W Athena Award), 833
Görtz, Ulrich (2008 von Kaven Prize), 1428
Gottlieb, David I. (2008 American Academy of Arts and Sciences Elections), 836; (SIAM John von Neumann Lecturer), 1105
Granville, Andrew (2008 MAA Chauvenet Prize), 607
Green, Ben (2007 SASTRA Ramanujan Prize), 58; (European Mathematical Society Prize), 1106
Griesmaier, Roland (2008 SIAM Student Paper Prize), 1105
Griffiths, Phillip A. (2008 L. E. J. Brouwer Prize), 833; (2008 Wolf Prize), 594
Grigoriev, Ilya (2008 NSF Graduate Research Fellowship), 719
Gronau, Hans-Dietrich (Dieter) (2008 Paul Erdős Award), 1108
Gross, Kenneth I. (2008 MAA Haimo Award for Teaching), 606
Guan, Pengfei (Royal Society of Canada), 1431
Gubler, Walter (2008 Doob Prize), 504
Guckenheimer, John (2008 American Academy of Arts and Sciences Elections), 836
Guestrin, Carlos A. (2008 ONR Young Investigators Award), 1285
Gugercin, Serkan (2007 NSF CAREER Award), 267
Gummersheimer, Victor (2008 MAA Certificate of Meritorious Service), 608
Gunzburger, Max (SIAM W. T. and Idalia Reid Prize in Mathematics), 1105
Hairer, Martin (2008 LMS Whitehead Prize), 1107
Haken, Ian R. (2008 NSF Graduate Research Fellowship), 719
Hales, Thomas C. (2008 MAA Lester R. Ford Award), 1430
Hangelbroek, Thomas C. (2007 NSF Postdoctoral Fellowship), 59
Hausel, Tamás (2008 LMS Whitehead Prize), 1107
Hedges, Larry V. (2008 American Academy of Arts and Sciences Elections), 836
Heller, Michael (2008 Templeton Award), 833
Henry, Bruce (2008 Paul Erdős Award), 1108
Herman, Richard H. (2008 American Academy of Arts and Sciences Elections), 836
Herzberg, Agnes M. (Royal Society of Canada), 1431
Higham, Nicholas (2008 LMS Fröhlich Prize), 1107
Hirai, Takeshi (Analysis Prize of the MSJ), 395
Hirani, Anil N. (2007 NSF CAREER Award), 267
Hirsh, Joseph (2008 NSF Graduate Research Fellowship), 719
Hishida, Toshiaki (Analysis Prize of the MSJ), 395
Hoefer, Mark (2008 NSF Postdoctoral Research Fellowship), 1286

Hofer, Helmut (National Academy of Sciences Elections), 836
Hoffman, Christopher (2008–2009 AMS Centennial Fellowship), 715
Holtz, Olga (European Mathematical Society Prize), 1106
Hooper, William P. (2008 NSF Postdoctoral Research Fellowship), 1286
Hoory, Shlomo (2008 Conant Prize), 491
Howard, Benjamin J. (2007 NSF Postdoctoral Fellowship), 59
Huang, Alexander C. (Siemens Competition in Math, Science, and Technology), 396
Hugeback, Angela B. (2008 NSF Postdoctoral Research Fellowship), 1286
Hughes, Thomas J. R. (Timoshenko Medal), 718
Hunt, Martin (2008 MAA Award for Mathematical Modeling), 1286
Huybrechs, Daan (SIAM DiPrima Prize), 1105
Ioana, Adrian (2008 Clay Research Fellowship), 612
Iovita, Adrian (2008 Ribenboim Prize in Number Theory), 1108
Iyama, Osamu (ICRA Award), 612; (2008 Algebra Prize of the MSJ), 972
Jackson, Thomas (2008 Moody Foundation Mega Math Challenge Summa Cum Laude), 835
Jensen, Jacqueline A. (2008 MAA Henry Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member), 1430
Ji, Lizhen (2007 ICCM Morningside Silver Medal of Mathematics), 509
Jin, Jiashun (2007 NSF CAREER Award), 267
Jin, Shi (2007 ICCM Morningside Silver Medal of Mathematics), 509
Jones, Frank (State Professor of the Year), 395
Jones, Peter W. (National Academy of Sciences Elections), 836
Kao, Justin C. (2008 NSF Postdoctoral Research Fellowship), 1286
Kapitula, Todd (SIAM Outstanding Paper Prize), 1105
Kaplan, Nathan (2008 Morgan Prize), 494
Karp, Richard M. (2008 Kyoto Prize), 1102
Karr, Rosemary M. (Outstanding Community Colleges Professor for 2007), 395
Karshon, Yael (2009 CMS Krieger-Nelson Prize), 717
Kasube, Herbert (2008 MAA Certificate of Meritorious Service), 608
Kazhdan, David (2008 American Academy of Arts and Sciences Elections), 836
Kedlaya, Kiran (2006 PECASE Award), 265
Kelly, Brendan (2008 Pi Mu Epsilon Student Paper Presentation Award), 1430
Kempe, David (2008 ONR Young Investigators Award), 1285
Kenig, Carlos (2008 Bôcher Prize), 501
Ketcheson, David (2008 SIAM Student Paper Prize), 1105
Kevrekidis, P. G. (SIAM Outstanding Paper Prize), 1105
Khan, Rizwanur R. (2007 NSF Postdoctoral Fellowship), 59
Khare, Chandrashekhar B. (2007 Fermat Prize), 265; (2008 Guggenheim Fellow), 718
Khot, Subhash (SIAM Outstanding Paper Prize), 1105
Khuri, Marcus A. (2008 Sloan Research Fellowship), 718
Kilpatrick, Jeremy (2007 ICMI Felix Klein Medal), 716
Kim, Inwon C. (2008 Sloan Research Fellowship), 718
Kirkpatrick, Kay L. (2007 NSF Postdoctoral Fellowship), 59
Kishore, Shaunak (2008 USA Mathematical Olympiad), 835; (2008 International Mathematical Olympiad), 1286
Kitaev, Alexei (2008 MacArthur Fellowship), 1428
Kitano, Takeshi (2008 Publication Prize of the MSJ), 972
Klartag, Boáz (European Mathematical Society Prize), 1106
Kleinberg, Jon M. (National Academy of Engineering Elections), 612
Kleinman, Aaron D. (2008 NSF Graduate Research Fellowship), 719
Klosinski, Leonard F. (2008 MAA Certificate of Meritorious Service), 608
Koch, Sara C. (2008 NSF Postdoctoral Research Fellowship), 1286
Kominers, Paul (AMS Karl Menger Award), 971
Kontorovich, Alex (2008 NSF Postdoctoral Research Fellowship), 1286
Kontsevich, Maxim (2008 Crafoord Prize in Mathematics), 593
Kopparty, Swara (AMS Karl Menger Award), 971
Kossek, Haley (2007 AWM Essay Contest Winner), 510
Kozai, Kenji Y. (2008 NSF Graduate Research Fellowship), 719
Krieger, Joachim (2008 Sloan Research Fellowship), 718
Kudla, Stephen (2009 CMS Jeffery-Williams Prize), 717
Kulkarni, Anand P. (2008 NSF Graduate Research Fellowship), 719
Kutyniok, Gitta (2007 von Kaven Prize), 265
Kuznetsov, Alexander (European Mathematical Society Prize), 1106
Lacey, Michael T. (2007–2008 Fulbright Foreign Scholarship), 719
Laflamme, Raymond (Royal Society of Canada), 1431
Lafont, Jean-François (2008 Sloan Research Fellowship), 718
Lange, Karen M. (2008 NSF Postdoctoral Research Fellowship), 1286
Larson, Eric (AMS Karl Menger Award), 971
Lau, Gawain (2008 Moody Foundation Mega Math Challenge Exemplary), 835
Lauret, Jorge (2007 ICTP/IMU Ramanujan Prize), 265
Lawrence, Brian R. (Putnam Prize), 719
Lee, Ruby (2008 Moody Foundation Mega Math Challenge Cum Laude), 835
Lee, Troy J. (2007 NSF Postdoctoral Fellowship), 59
Leighton, Frank T. (National Academy of Sciences Elections), 836
Leise, Tanya (2008 MAA Lester R. Ford Award), 1430
Lester, Frank K., Jr. (2008 Mathematics Education Trust Lifetime Achievement Award), 834
Levin, Brandon W. (2008 NSF Graduate Research Fellowship), 719
Levine, Adam M. (Rhodes Scholarship), 395

Levine, Lionel (2008 NSF Postdoctoral Research Fellowship), 1286
Lew, Adrian (2008 ONR Young Investigators Award), 1285
Li, Fengyan (2008 Sloan Research Fellowship), 718
Li, Shengzhi (2008 Moody Foundation Mega Math Challenge Cum Laude), 835
Liddie, Jephthah (2008 Moody Foundation Mega Math Challenge Magna Cum Laude), 835
Liggett, Thomas M. (National Academy of Sciences Elections), 836
Linial, Nathan (2008 Conant Prize), 491
Lithio, Daniel (2008 Pi Mu Epsilon Student Paper Presentation Award), 1430
Liu, Chiu-Chu (2007 ICCM Morningside Silver Medal of Mathematics), 509
Liu, Regina Y. (2007–2008 Fulbright Foreign Scholarship), 719
Liu, Yi-Kai (2007 NSF Postdoctoral Fellowship), 59
Livingston, W. Ryan (2008 Pi Mu Epsilon Student Paper Presentation Award), 1430
Lotay, Jason D. (2007 NSF Postdoctoral Fellowship), 59
Louder, Larson E. (2007 NSF Postdoctoral Fellowship), 59
Lovász, László (Bolyai Prize), 264
Lusztig, George (2008 Steele Prize), 486
Ly, Cheng (2007 NSF Postdoctoral Fellowship), 59
Lyles, Danielle J. (2007 NSF Postdoctoral Fellowship), 59
Maggioni, Mauro (2008 Sloan Research Fellowship), 718
Maldacena, Juan Martín (2008 Dirac Medal), 1284
Mancosu, Paolo (2008 Guggenheim Fellow), 718
Marzuola, Jeremy L. (2007 NSF Postdoctoral Fellowship), 59
Matchett, Andrew (2008 MAA Certificate of Meritorious Service), 608
Mateo, Mace (YouTube Video Contest), 1107
Matusevich, Laura F. (2008 Sloan Research Fellowship), 718
McCue, Leigh (2008 ONR Young Investigators Award), 1285
McNeill, Reagin Taylor (2008 AWM Schafer Prize Honorable Mention), 609
McReynolds, David B. (2007 NSF Postdoctoral Fellowship), 59
Mei, Kelvin (2008 Moody Foundation Mega Math Challenge Exemplary), 835
Meinrenken, Eckhard (Royal Society of Canada), 1431
Meng, Delong (2008 USA Mathematical Olympiad), 835
Mermin, Jeffrey A. (2007 NSF Postdoctoral Fellowship), 59
Meza, Juan C. (2008 Blackwell-Tapia Prize), 1107
Mikkilineni, Shravani (AMS Karl Menger Award), 971
Miller, Alison B. (2008 AWM Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman), 609; (2008 NSF Graduate Research Fellowship; Putnam Prize), 719
Miller, Joel C. (2008 NSF Postdoctoral Research Fellowship), 1286
Milne, Stephen (2007 ICA Euler Medal), 717
Milner, Arthur John Robin Gorell (National Academy of Engineering Elections), 612
Minton, Roland (2008 MAA George Pólya Award), 1430
Mitrea, Irina (Ruth I. Michler Memorial Prize), 832
Mnatsakanian, Mamikon A. (2008 MAA Lester R. Ford Award), 1430
Moczydlowski, Wojciech (2007 ASL Sacks Prizes), 509
Moffatt, H. Keith (National Academy of Sciences Elections), 836
Moler, Cleve (2008 Hans Schneider Prize), 971
Moniot, Robert K. (2008 MAA Trevor Evans Award), 1430
Morita, Shigeyuki (Geometry Prize of the MSJ), 395
Morrow, James (2008 MAA Haimo Award for Teaching), 606
Mucha, Peter J. (2007 NSF CAREER Award), 267
Mukherjee, Anirvan (2008 Moody Foundation Mega Math Challenge Cum Laude), 835
Mumford, David B. (2008 Wolf Prize), 594
Munro, Erin C. (2008 NSF Postdoctoral Research Fellowship), 1286
Musco, Brett (2008 Moody Foundation Mega Math Challenge First Honorable Mention), 835
Musco, Cameron (2008 Moody Foundation Mega Math Challenge First Honorable Mention), 835
Musco, Christopher (2008 Moody Foundation Mega Math Challenge First Honorable Mention), 835
Musiker, Gregg J. (2007 NSF Postdoctoral Fellowship), 59
Nachtergaele, Bruno (AAAS Fellow), 267
Namikawa, Yoshinori (2008 Algebra Prize of the MSJ), 972
Naor, Assaf (European Mathematical Society Prize), 1106
Neuenkirch, Andreas (2007 Information-Based Complexity Young Researcher Award), 266
Newman, Joshua (2008 Moody Foundation Mega Math Challenge Magna Cum Laude), 835
Norris, Scott A. (2008 NSF Postdoctoral Research Fellowship), 1286
Novaga, Matteo (SIAM Outstanding Paper Prize), 1105
O'Dorney, Evan (2008 USA Mathematical Olympiad), 835; (2008 International Mathematical Olympiad), 1286
Ooguri, Hirosi (2008 Eisenbud Prize), 505
Osher, Stanley (2007 ICCM International Cooperation Award), 509
Otake, Susumu (2008 Publication Prize of the MSJ), 972
Ott, Katharine A. (2008 NSF Postdoctoral Research Fellowship), 1286
Oza, Anand U. (2008 NSF Graduate Research Fellowship), 719
Ozsváth, Peter (2008 Guggenheim Fellow), 718
Pan, Qinxuan (2008 USA Mathematical Olympiad), 835
Papageorgiou, Anargyros (2008 Information-Based Complexity Prize), 833
Parikh, Alaap (2008 Moody Foundation Mega Math Challenge Meritorious), 835
Parlett, Beresford (2008 Hans Schneider Prize), 971
Patnaik, Manish M. (2008 NSF Postdoctoral Research Fellowship), 1286
Pavlovic, Natasa (2008 Sloan Research Fellowship), 718
Pelayo, Alvaro (2007 NSF Postdoctoral Fellowship), 59
Penkava, Michael R. (2007–2008 Fulbright Foreign Scholarship), 719

Pennings, Timothy J. (2008 MAA George Pólya Award), 1430
Pestel, Marie-José (2008 SMF d'Alembert Prize), 1107
Peterson, Jonathon R. (2008 NSF Postdoctoral Research Fellowship), 1286
Petrosyan, Nansen (2008 Emil Artin Junior Prize), 1430
Pixton, Aaron C. (Putnam Prize), 719
Polchinski, Joseph (2008 Dirac Medal, 2008), 1284
Pollack, Paul P. (2008 NSF Postdoctoral Research Fellowship), 1286
Pollatsek, Harriet S. (2008 AWM Louise Hay Award), 609
Pong, Christopher (2008 MAA Award for Mathematical Modeling), 1286
Ponto, Kathleen A. (2007 NSF Postdoctoral Fellowship), 59
Porter, Mason (Sigma Xi Young Investigator Award), 716
Preiss, David (2008 LMS Pólya Prize), 1107
Protter, Philip E. (2007–2008 Fulbright Foreign Scholarship), 719
Raghavan, Prabhakar (National Academy of Engineering Elections), 612
Raj, Nevin (2008 Moody Foundation Mega Math Challenge Exemplary), 835
Ramm, Ekkehard (National Academy of Engineering Elections), 612
Rao, B. L. S. Prakasa (2007–2008 National Award in Statistics for Senior Statisticians), 1108
Raoof, Sana (AMS Karl Menger Award), 971
Rawlins, Helen A. (2007 AWM Essay Contest Winner), 510
Reid, Nancy (2008 Parzen Prize), 717
Ren, Qingchun (Putnam Prize), 719
Reyes, Manuel L. (Ford Foundation Diversity Fellowship), 972
Reys, Robert E. (2008 Mathematics Education Trust Lifetime Achievement Award), 834
Rhoades, Brendon P. (2008 NSF Postdoctoral Research Fellowship), 1286
Rice, John (2008 Jerome Sacks Award), 1284
Rider, Brian C. (2007 NSF CAREER Award), 267; (2008 Rollo Davidson Prize), 716
Roache, Kelly (2008 Moody Foundation Mega Math Challenge Summa Cum Laude), 835
Rogers, Matthew D. (2008 NSF Postdoctoral Research Fellowship), 1286
Rogness, Jonathan (2007 Science and Engineering Challenge), 58
Rokhlin, Vladimir (National Academy of Engineering Elections), 612
Rolnick, David (2008 USA Mathematical Olympiad), 835
Romberg, Justin (2008 ONR Young Investigators Award), 1285
Rosengarten, David (AMS Karl Menger Award), 971
Rubinstein, Yanir A. (2008 NSF Postdoctoral Research Fellowship), 1286
Ruiz, Jared (2008 Pi Mu Epsilon Student Paper Presentation Award), 1430
Saint-Raymond, Laure (European Mathematical Society Prize), 1106
Sandon, Colin (2008 USA Mathematical Olympiad), 835; (2008 International Mathematical Olympiad), 1286
Sankar, Krishanu Roy (2008 USA Mathematical Olympiad), 835; (2008 International Mathematical Olympiad), 1286
Sarkar, Shayak (Rhodes Scholarship), 395
Schedler, Travis (2008 AIM Five-Year Fellowship), 611
Schwede, Karl E. (2007 NSF Postdoctoral Fellowship), 59
Seiberg, Nathan (National Academy of Sciences Elections), 836
Seiple, Derek (2008 MAA Carl B. Allendoerfer Award), 1429
Sellers, Sarah (AMS Karl Menger Award), 971
Sen, Ayon (Siemens Competition in Math, Science, and Technology), 396
Sethian, James A. (National Academy of Engineering Elections), 612
Sfard, Anna (2007 ICMI Hans Freudenthal Medal), 717
Shah, Leena (2007 AWM Essay Contest Winner), 510
Shao, Xuancheng (Putnam Prize), 719
Shaw, Christopher (2008 Moody Foundation Mega Math Challenge First Honorable Mention), 835
Sheffield, Scott R. (2007 NSF CAREER Award), 266
Shian, Leou (2008 Paul Erdős Award), 1108
Sierra, Susan J. (2008 NSF Postdoctoral Research Fellowship), 1286
Sifakis, Joseph (2007 ACM Turing Award), 709
Silberstein, Aaron (2008 NSF Graduate Research Fellowship), 719
Simons, James H. (2008 American Academy of Arts and Sciences Elections), 836
Simonyi, Charles (2008 American Academy of Arts and Sciences Elections), 836
Simoson, Andrew J. (2008 MAA George Pólya Award), 1430
Singhal, Ashutosh (2008 Moody Foundation Mega Math Challenge Meritorious), 835
Sirovich, Lawrence (AAAS Fellow), 267
Sloane, Neil J. A. (2008 MAA David P. Robbins Prize), 608
Smoktunowicz, Agata (European Mathematical Society Prize), 1106
Snaith, Nina (2008 LMS Whitehead Prize), 1107
Socha, Katherine (2008 MAA Henry Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member), 1430; (2008 MAA Lester R. Ford Award), 1430
Solis, Pablo R. (2008 NSF Graduate Research Fellowship), 719
Solomon, Jake P. (2007 NSF Postdoctoral Fellowship), 59
Solymosi, Jozsef (2008 André-Aisenstadt Mathematics Prize), 266
Sozzi, Thomas (2008 Moody Foundation Mega Math Challenge Magna Cum Laude), 835
Spaht, Carlos G. (2007 Outstanding Master's Universities and Colleges Professor), 395
Spielman, Daniel A. (ACM Gödel Prize), 970
Stange, Katherine E. (2008 NSF Postdoctoral Research Fellowship), 1286
Stechmann, Samuel N. (2008 NSF Postdoctoral Research Fellowship), 1286

Steinhardt, Jacob (Siemens Competition in Math, Science, and Technology), 396
Straight, H. Joseph (2008 MAA Certificate of Meritorious Service), 608
Street, Brian T. (2008 NSF Postdoctoral Research Fellowship), 1286
Streets, Jeffrey D. (2007 NSF Postdoctoral Fellowship), 59
Strominger, Andrew (2008 Eisenbud Prize), 507
Suchard, Marc A. (2008 Guggenheim Fellow), 718
Sugiura, Mitsuo (2008 Publication Prize of the MSJ), 972
Suidan, Toufic Mubadda (2008 Sloan Research Fellowship), 718
Swedosh, Philip (2008 B. H. Neumann Award), 1431
Szemerédi, Endre (2008 Steele Prize), 486; (Rolf Schock Prize), 1284
Taiganov, Nurlan (AMS Karl Menger Award), 971
Takaoka, Hideo (2008 Spring Prize of the MSJ), 972
Takhar, Karan (2008 Moody Foundation Mega Math Challenge First Honorable Mention), 835
Tan, Zhiqiang (2007 NSF CAREER Award), 266
Tanisaki, Toshiyuki (2008 Algebra Prize of the MSJ), 972
Tao, Terence (2008 Waterman Award), 832; (National Academy of Sciences Elections), 836
Taubes, Clifford (2008 NAS Award in Mathematics), 596; (2008 Clay Research Awards), 612
Taylor, John R. (2008 NSF Postdoctoral Research Fellowship), 1286
Taylor, Jonathan (2008 André-Aisenstadt Mathematics Prize), 266
Teng, Shang-Hua (ACM Gödel Prize), 970
Thompson, Elizabeth A. (National Academy of Sciences Elections), 836
Thompson, Jeremy (2008 Pi Mu Epsilon Student Paper Presentation Award), 1430
Thompson, John Griggs (2008 Abel Prize), 707
Thorne, Frank H. (2008 NSF Postdoctoral Research Fellowship), 1286
Thornton, Steve (2008 B. H. Neumann Award), 1431
Tian, Ye (2007 ICCM Morningside Silver Medal of Mathematics), 509
Tice, Ian I. (2008 NSF Postdoctoral Research Fellowship), 1286
Tien, Kevin (2008 Moody Foundation Mega Math Challenge Magna Cum Laude), 835
Timoshenko, Artem (AMS Karl Menger Award), 971
Tits, Jacques (2008 Abel Prize), 707
Tomesch, Claire M. (2008 NSF Graduate Research Fellowship), 719
Tondeur, Philippe (SIAM Prize for Distinguished Service to the Profession), 1105
Tong, David (2008 Adams Prize), 833
Tripathy, Arnav (Putnam Prize), 719
Tropp, Joel A. (2008 ONR Young Investigators Award), 1285
Trudinger, Neil (2008 Steele Prize), 486
Tucker, George (2008 MAA Award for Mathematical Modeling), 1286
Vafa, Cumrun (2008 Eisenbud Prize), 507; (2008 Dirac Medal), 1284
Vakil, Ravi (2008 CMS Coxeter-James Prize), 717
van Luijk, Ronald (CMS 2007 G. de B. Robinson Award), 1285
Vatter, Vincent R. (2007 NSF Postdoctoral Fellowship), 59
Veinott, Arthur F., Jr. (2007 John von Neumann Theory Prize), 716
Venkatesh, Akshay (2007 Packard Fellowship), 395; (2007 Salem Prize), 716
Villani, Cédric (European Mathematical Society Prize), 1107
Virág, Bálint (2008 Rollo Davidson Prize), 716
Voisin, Claire (2008 Clay Research Award), 612
Vu, Van H. (SIAM George Pólya Prize), 1105
Waelder, Robert E. (2008 NSF Postdoctoral Research Fellowship), 1286
Wage, Matthew (AMS Karl Menger Award), 971
Wang, Lingke (2008 Moody Foundation Mega Math Challenge Cum Laude), 835
Wang, Mu-Tao (2007 ICCM Chern Prize in Mathematics), 509
Wang, Xujia (2007 ICCM Morningside Gold Medal of Mathematics), 509
Webster, Benjamin T. (2007 NSF Postdoctoral Fellowship), 59
Weinkove, Ben (2008 Sloan Research Fellowship), 718
Weinstein, Jared (2008 NSF Postdoctoral Research Fellowship), 1286
Weston, Harley (CMS 2008 Adrien Pouliot Award), 1284
Wigderson, Avi (2008 Conant Prize), 491
Wilson, Kevin H. (2008 NSF Graduate Research Fellowship), 719
Wise, Jonathan (2008 NSF Postdoctoral Research Fellowship), 1286
Witten, Edward (2008 Crafoord Prize in Mathematics), 593
Wolfson, Jesse (2008 NSF Graduate Research Fellowship), 719
Wootters, Mary (2008 AWM Schafer Prize Honorable Mention), 609
Wright, Paul A. (2007 NSF Postdoctoral Fellowship), 59
Xiu, Dongbin (2007 NSF CAREER Award), 266
Yamaguchi, Takahiko (2008 Publication Prize of the MSJ), 972
Yandell, Benjamin H. (2008 MAA Euler Book Prize), 607
Yarmola, Tatiana (2008 NSF Postdoctoral Research Fellowship), 1286
Yermakov, Afanasiy (2008 Moody Foundation Mega Math Challenge Summa Cum Laude), 835
Yoshikawa, Kenichi (Geometry Prize of the MSJ), 395
Yu, Josephine T. (2008 NSF Postdoctoral Research Fellowship), 1286
Yuan, Xinyi (2008 Clay Research Fellowship), 612
Yudovina, Elena (2008 NSF Graduate Research Fellowship), 719
Zakharevich, Inna I. (2008 NSF Graduate Research Fellowship), 719
Zelen, Marvin (2008 Parzen Prize), 717
Zhai, Alex (2008 USA Mathematical Olympiad), 835; (2008 International Mathematical Olympiad), 1286

Zhan, Yiwen (2008 Moody Foundation Mega Math Challenge Exemplary), 835
Zhang, Hao (2007 NSF CAREER Award), 266
Zhou, Hao-Min (2007 NSF CAREER Award), 266
Zhou, Huibin (2007 NSF CAREER Award), 266
Ziegler, Günter M. (2008 Communicator Award), 834
Zimmer, Robert J. (AAAS Fellow), 267
Zlatos, Andrej (2008 Sloan Research Fellowship), 718
Zukus, Jason (2008 Moody Foundation Mega Math Challenge Summa Cum Laude), 835
Zuniga, Jessica V. (2008 NSF Postdoctoral Research Fellowship), 1286

Reciprocity Agreements, 1306

Reference and Book List

68, 275, 400, 514, 619, 723, 843, 979, 1115, 1296, 1439

Reviews

(An) Abundance of Katherines (Reviewed by Oaz Nir), 1096
(The) Archimedes Codex (Reviewed by J. L. Berggren), 943
(A) Certain Ambiguity (Reviewed by Danny Calegari), 235
Descartes, the Pioneer of the Scientific Revolution (Reviewed by Michel Serfati), 44
Economics and Common Sense (Reviewed by Gil Kalai), 687
Fly Me to the Moon (Reviewed by Shane Ross), 478
In Puris Naturalibus (Reviewed by Lewis Pyenson), 793
(The) Indian Clerk (Reviewed by Heini Halberstam), 952
Irreligion (Reviewed by Olle Häggström), 789
Is Mathematics Misapplied to the Environment? (Reviewed by Christopher K. R. T. Jones), 481
Julia Robinson and Hilbert's Tenth Problem (Reviewed by Carol Wood), 573
Leonhard Euler, A Man to be Reckoned With (Reviewed by Ivar Ekeland), 371
Mathematical Omnibus: Thirty Lectures on Classic Mathematics (Reviewed by Harriet Pollatsek), 1415
(The) Mathematician's Brain (Reviewed by David Corfield), 1243
Mathematics at Berkeley: A History (Reviewed by Rob Kirby), 1237
Measuring the World (Reviewed by Frans Oort), 681
(The) Poincaré Conjecture and *Poincaré's Prize* (Reviewed by W. B. Raymond Lickorish), 37
(The) Probability of God and *Superior Beings* (Reviewed by Hemant Mehta), 231
Roots to Research: A Vertical Development of Mathematical Problems (Reviewed by Harriet Pollatsek), 1417
Unknown Quantity (Reviewed by Sanford L. Segal), 583
(The) Volterra Chronicles (Reviewed by Irwin W. Sandberg), 377
(The) Wraparound Universe (Reviewed by George F. R. Ellis), 1421

Stipends for Study and Travel, 983

Surveys

2007 Annual Survey of the Mathematical Sciences (First Report), 253
2007 Annual Survey of the Mathematical Sciences (First Report, Part II), 387
2007 Annual Survey of the Mathematical Sciences in the United States (Second Report), 814
2007 Annual Survey of the Mathematical Sciences in the United States (Third Report), 1271
Doctoral Degrees Conferred 2006–2007, 280
Doctoral Degrees Conferred 2006–2007, Supplementary List, 826
Statistics on Women Mathematicians Compiled by the AMS, 1132

Tables of Contents

3, 203, 339, 443, 555, 659, 771, 915, 1067, 1219, 1363

WHAT IS…

WHAT IS…a Cross Ratio? (François Labourie), 1234
WHAT IS…Forcing? (Thomas Jech), 692
WHAT IS…an ∞-Category? (Jacob Lurie), 949
WHAT IS…JPEG? (David Austin), 226
WHAT IS…Outer Space? (Karen Vogtmann), 784
WHAT IS…a Period Domain? (James Carlson and Phillip Griffiths), 1418
WHAT IS…Property A? (Piotr Nowak and Guoliang Yu), 474
WHAT IS…Quantum Chaos? (Ze'ev Rudnick), 32
WHAT IS…Stable Commutator Length? (Danny Calegari), 1100
WHAT IS…a Systole? (Marcel Berger), 374
WHAT IS…a Toric Variety? (Ezra Miller), 586

Meetings and Conferences of the AMS

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The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at www.ams.org/meetings/.**

Meetings:

2008		
December 17–21	Shanghai, People's Republic of China	p. 1473
2009		
January 5–8	Washington, DC Annual Meeting	p. 1474
March 27–29	Urbana, Illinois	p. 1476
April 4–5	Raleigh, North Carolina	p. 1477
April 25–26	Worcester, Massachusetts	p. 1478
April 25–26	San Francisco, California	p. 1478
Oct. 16–18	Waco, Texas	p. 1479
Oct. 24–25	University Park, Pennsylvania	p. 1479
Oct. 30–Nov. 1	Boca Raton, Florida	p. 1480
Nov. 7–8	Riverside, California	p. 1480
2010		
January 13–16	San Francisco, California Annual Meeting	p. 1481
March 27–28	Lexington, Kentucky	p. 1481
April 10–11	St. Paul, Minnesota	p. 1481
April 17–18	Albuquerque, New Mexico	p. 1481
June 2–5	Berkeley, California	p. 1481
September 18–19	Notre Dame, Indiana	p. 1482
October 9–10	Los Angeles, California	p. 1482
2011		
January 5–8	New Orleans, Louisiana Annual Meeting	p. 1482
March 12–13	Statesboro, Georgia	p. 1482
2012		
January 4–7	Boston, Massachusetts Annual Meeting	p. 1483
2013		
January 9–12	San Diego, California Annual Meeting	p. 1483
2014		
January 15–18	Baltimore, Maryland Annual Meeting	p. 1483
2015		
January 10–13	San Antonio, Texas Annual Meeting	p. 1483

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 95 in the January 2008 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

2009 Joint Meetings Advance Registration/Housing Form

Joint Mathematics Meetings
Washington, DC
January 5–8, 2009

Name ____________________
(please write name as you would like it to appear on your badge)

Mailing Address ____________________

____

Telephone ____________________ Fax: ____________________

In case of emergency (for you) at the meeting, call: Day # ____________ Evening #: ____________

Email Address ____________________
(Acknowledgment of this registration will be sent to the email address given here, unless you check this box: *Send by U.S. Mail* ❑)

Affiliation for badge ____________________

Nonmathematician guest badge name ____________________
(please note charge below)

Membership
✓ all that apply. First column is eligible for member registration fee

❑ AMS	❑ ASA
❑ MAA	❑ AWM
❑ ASL	❑ NAM
❑ CMS	❑ YMN
❑ SIAM	

❑ **I DO NOT want my program and badge to be mailed to me on 12/12/08. (Materials will be mailed unless you check this box.)**

Registration Fees

Joint Meetings	**by Dec 15**	**at mtg**	**Subtotal**
❑ Member AMS, ASL, CMS, MAA, SIAM	US $216	US $282	
❑ Nonmember	US $335	US $435	
❑ Graduate Student	US $ 44	US $ 54	
❑ Undergraduate Student	US $ 30	US $ 40	
❑ High School Student	US $ 5	US $ 10	
❑ Unemployed	US $ 43	US $ 53	
❑ Temporarily Employed	US $174	US $202	
❑ Developing Countries Special Rate	US $ 43	US $ 53	
❑ Emeritus Member of AMS or MAA	US $ 43	US $ 53	
❑ High School Teacher	US $ 43	US $ 53	
❑ Librarian	US $ 43	US $ 53	
❑ Nonmathematician Guest	US $ 15	US $ 15	
		$	________

AMS Short Course: *Quantum Computation and Quantum Information (1/3–1/4)*

❑ Member of AMS or MAA	US $ 96	US $130	
❑ Nonmember	US $130	US $160	
❑ Student, Unemployed, Emeritus	US $ 44	US $ 65	
		$	________

MAA Short Course: *Data Mining & New Trends in Teaching Statistics. (1/3–1/4)*

❑ Member of MAA or AMS	US $125	US $140	
❑ Nonmember	US $175	US $190	
❑ Student, Unemployed, Emeritus	US $ 50	US $ 60	
		$	________

MAA Minicourses (see listing in text)

I would like to attend: ❑ One Minicourse ❑ Two Minicourses
Please enroll me in MAA Minicourse(s) #______ and/or #______
In order of preference, my alternatives are: #______ and/or #______
Price: US $60 for each minicourse.
(For more than 2 minicourses call or email the MMSB.) $ ________

Employment Center

Applicant résumé forms and employer job listing forms can be found at www.ams.org/emp-reg/.

❑ Employer—Quiet Area Table (2 interviewers)	US $250	US $330	
❑ Employer— Additional Quiet Area Table	US $100	N/A	
❑ Employer— Committee Table (3-6 interviewers)	US $350	US $425	
❑ Employer— Curtained Booth (1-3 interviewers)	US $425	N/A	
❑ Applicant	US $ 25	US $40	
		$	________

Graduate School Fair

❑ Graduate School Fair Table	US $ 50	N/A	
		$	________

Events with Tickets

MER Banquet (1/6) US $53.00 #____Regular #____Veg #____Kosher
NAM Banquet (1/7) US $52.00 #____Regular #____Veg #____Kosher
AMS Banquet (1/8) US $52.50 #____Regular #____Veg #____Kosher
Luncheon for Jim Tattersall (1/8) US $36.00 #____Reg #____Veg #____Kosher
$ ________

Other Events

❑ Graduate Student/First Time Attendee Reception (1/5) (no charge)

Total for Registrations and Events $ ________

Registration for the Joint Meetings is not required for the Short Courses, but it is required for the Minicourses and the Employment Center.

Payment

Registration & Event Total (total from column on left) $ ________
Hotel Deposit (only if paying by check) $ ________

Total Amount To Be Paid **$** ________
(Note: A US $5 processing fee will be charged for each returned check or invalid credit card. Debit cards are not accepted.)

Method of Payment

❑ Check. Make checks payable to the AMS. Checks drawn on foreign banks must be in equivalent foreign currency at current exchange rates.
❑ Credit Card. VISA, MasterCard, AMEX, Discover (no others accepted)

Card number: ____________________

Exp. date: ________ Zipcode of credit card billing address: ____________

Signature: ____________________

Name on card: ____________________

❑ Purchase order # ____________________ (please enclose copy)

Other Information

Mathematical Reviews field of interest # ____________

How did you hear about this meeting? Check one: ❑ Colleague(s) ❑ *Notices* ❑ *Focus* ❑ Internet

❑ This is my first Joint Mathematics Meetings.
❑ I am a mathematics department chair.
❑ For planning purposes for the MAA Two-year College Reception, please check if you are a faculty member at a two-year college.
❑ I would like to receive promotions for future JMM meetings.
❑ **Please do not include my name on any promotional mailing list.**
❑ Please ✓ this box if you have a disability requiring special services.

Mail to:

Mathematics Meetings Service Bureau (MMSB)
P. O. Box 6887
Providence, RI 02940-6887 Fax: 401-455-4004
Questions/changes call: 401-455-4143 or 1-800-321-4267 x4143; mmsb@ams.org

Deadlines *Please register by the following dates for:*

Résumés/job descriptions printed in the *Winter Lists*	**Oct. 22, 2008**
To be eligible for the complimentary room drawing:	**Oct. 31, 2008**
For housing reservations, badges/programs mailed:	**Nov. 14, 2008**
For housing changes/cancellations through MMSB:	**Dec. 5, 2008**
For advance registration for the Joint Meetings, Employment Center, Short Courses, MAA Minicourses, & Tickets:	**Dec. 15, 2008**
For 50% refund on banquets, cancel by:	**Dec. 22, 2008***
For 50% refund on advance registration, Minicourses & Short Courses, cancel by:	**Dec. 30, 2008***

***no refunds after this date**

Washington DC Joint Mathematics Meetings Hotel Reservations

To ensure accurate assignments, please rank hotels in order of preference by writing 1, 2, 3, etc., in the column on the left and by circling the requested room type and rate. If the rate or the hotel requested is no longer available, you will be assigned a room at a ranked or unranked hotel at a comparable rate. Please call the MMSB for details on suite configurations, sizes, availability, etc. Suite reservations can only be made through the MMSB to receive the convention rate. Reservations at the following hotels must be made through the MMSB to receive the convention rates listed. Reservations made directly with the hotels at the JMM rate will be changed to a higher rate. All rates are subject to a 14.5% sales tax.. **Guarantee requirements: First night deposit by check (add to payment on reverse of form) or a credit card guarantee. The Hilton will charge credit cards for the first night deposit immediately upon receipt of reservations.**

❏ Deposit enclosed (see front of form) ❏ Hold with my credit card Card Number ____________ Exp. Date ________ Signature ____________

Date and Time of Arrival ____________ **Date and Time of Departure** ____________

Name of Other Room Occupant ____________ **Arrival Date** ________ **Departure Date** ________ **Child (give age(s)** ________

Name of Other Room Occupant ____________ **Arrival Date** ________ **Departure Date** ________ **Child (give age(s)** ________

Order of choice	Hotel	Single	Double 1 bed	Double 2 beds	Triple 2 beds	Triple 2 beds w/cot	Triple - king or queen w/cot	Quad 2 beds	Quad 2 beds w/cot	Suites Starting rates
	Marriott Wardman Park Hotel									
	Marriott Regular Rate	US $150	US $150	US $150	US $165	US $165	US $165	US $180	US $180	US $450
	Student Rate	US $120	US $120	US $120	US $128	US $128	US $128	US $136	US $136	N/A
	Omni Shoreham Hotel									
	Omni Regular Rate	US $150	US $150	US $150	US $170	US $195	US $195	US $170	US $195	US $325
	Student Rate	US $120	US $120	US $120	US $136	US $161	US $161	US $136	US $161	N/A
	Hilton Washington									
	Hilton Rate 1*	US $109	US $109	US $109	US $109	N/A	US $134	US $109	N/A	US $359
	Hilton Rate 2*	US $119	US $119	US $119	US $119	N/A	US $144	US $119	N/A	US $359

* Please note: Hilton 1 and Hilton 2 rooms are identical.

Special Housing Requests:.

❏ I have disabilities as defined by the ADA that require a sleeping room that is accessible to the physically challenged. My needs are: ____________

❏ Other requests: ____________

❏ I am a member of a hotel frequent-travel club and would like to receive appropriate credit.

The hotel chain and card number are: ____________

Email confirmations (no paper) will be sent by the Marriott and the Hilton. (The Omni will not send confirmations.)

Please provide your email address for Marriott & Hilton confirmations:

____

If you are not making a reservation, please check off one of the following:

❏ I plan to make a reservation at a later date.

❏ I will be making my own reservations at a hotel not listed. Name of hotel: ____________

❏ I live in the area or will be staying privately with family or friends.

❏ I plan to share a room with ____________, who is making the reservations.

IMA INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS

Membership opportunities

in connection with the 2009-2010 thematic program on

COMPLEX FLUIDS AND COMPLEX FLOWS

IMA GENERAL MEMBERSHIPS provide an opportunity for mathematicians and scientists employed elsewhere to spend a period of one month to one year in residence at the IMA, and to participate in the 2009-2010 thematic program. The residency should fall in the period September 2009 through June 2010 (in special cases extending into the summer months). Logistic support such as office space, computer facilities, and secretarial support will be provided, and local expenses may be provided.

IMA POSTDOCTORAL FELLOWSHIPS provide an excellent opportunity for mathematical scientists near the beginning of their career who have a background in and/or an interest in learning about applied and computational aspects of Complex Fluids and Complex Flows. IMA postdoctoral fellowships run one to two years, at the option of the holder, starting September 1, 2009. Deadline January 4, 2009.

IMA INDUSTRIAL POSTDOCTORAL FELLOWSHIPS are designed to prepare mathematicians for research careers in industry or involving industrial interaction. IMA industrial postdoctoral fellowships run two years starting September 1, 2009. They are funded jointly by the IMA and an industrial sponsor, and holders devote 50% effort working with industrial scientists and 50% effort on a combination of their own research and the IMA activities. Deadline January 4, 2009.

IMA NEW DIRECTIONS RESEARCH PROFESSORSHIPS provide an extraordinary opportunity for established mathematicians–typically mid-career faculty at US universities–to branch into new directions and increase the impact of their research by spending the 2009-2010 academic year immersed in the thematic program at the IMA. Research Professors will enjoy an excellent research environment and stimulating scientific program connecting Complex Fluids and Complex Flows and related areas of mathematics with a broad range of fields of application. New Directions Research Professors are expected to be in resident and active participants in the program, but are not assigned formal duties. Deadline January 16, 2009.

For more information and application materials see
www.ima.umn.edu/docs/ or phone 612-624-6066.

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