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ERROR ANALYSIS FOR THE IN-SITU FABRICATION OF MECHANISMS

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ABSTRACT

Fabrication techniques like Solid Freeform Fabrication (SFF), or Layered Manufacturing, enable the manufacture of completely pre-assembled mechanisms (i.e. those that require no explicit component assembly after fabrication). We refer this manner of building assemblies as *in-situ* fabrication. An interesting issue that arises in this domain is the estimation of errors in the performance of such mechanisms as a consequence of manufacturing variability. Assumptions of parametric independence and stack-up made in conventional error analysis for mechanisms do not hold for this method of fabrication.

In this paper we formulate a general technique for investigating the kinematic performance of mechanisms fabricated *in-situ*. The technique presented admits deterministic and stochastic error estimation of planar and spatial linkages with ideal joints. However, only a planar example is illustrated in this paper. Errors due to joint clearances, or other effects like flexibility and driver-error, are not considered in the analysis - but are part of ongoing research.

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NOMENCLATURE

f	Kinematic function of mechanism
$\mathbf{\Theta}$	Vector of driving inputs
$\mathbf{p}$	Output vector
$\Delta_{\mathbf{p}}$	Variation in the output vector $\mathbf{p}$
$F_P(p)$	CDF of the output function
$\mu_{\mathbf{p}}, \sigma^2_{\mathbf{p}}$	Mean and variance of the output
J_{nom}	Sensitivity Jacobian at nominal config
$\mathbf{l}$	Vector of indep. random parameters l_i
$\Delta_{\mathbf{l}}$	Variation in mechanism parameters
$\mathbf{l}_{nom}$	Nominal values of mechanism parameters
$\phi_{L_i}(l_i)$	PDF of random parameter l_i
$\mu_{l_i}, \sigma^2_{l_i}$	Mean/variance of random parameter l_i
$\phi_{L_1 \ldots L_n}(.)$	Joint PDF of mechanism parameters

INTRODUCTION

Fabrication techniques like Solid Freeform Fabrication (SFF), or Layered Manufacturing, enable the manufacture of completely assembled mechanisms, without the need for explicit assembly. We refer to this manner of manufacture as *in-situ* fabrication. Figure 1 shows some mechanisms intended for robotic applications built using this technique at the Stanford Center for Design Research (Cham *et al*, 1999). Similar mechanisms have also been fabricated at Rutgers University (Alam *et al*, 1999) and the University of Laval (Laliberte *et al*, 1999), and micro-machined *in-situ* mechanisms have been fabricated at Sandia National Labs (Sandia, 2000) and University of California, Berkeley (Burgett *et al*, 1992) - among other places. An interesting issue that arises in this domain is the estimation of errors in the performance of *in-situ* mechanisms, as a consequence of manufacturing variability. Error analysis is particularly relevant to designers of prototype mechanisms, who benefit greatly by early feedback about performance.

Many approaches to error analysis for mechanisms exist - with various simplifying assumptions and different levels of complexity (Hartenberg and Denavit, 1964)(Garrett and Hall, 1969)(Dhande and Chakraborty, 1973)(Tischler and Samuel, 1999). All approaches, however, attempt to solve the same basic problem - *to predict the nature and amount of performance deterioration in mechanisms as a result of non-ideal synthesis, fabrication, materials or componentry.*

Performance of a mechanism is usually taken to mean its accuracy in performing either a positioning, path-generation or function generation task. Clearly, there are many other quantitative and qualitative aspects to mechanism behavior that constitute "ideal" performance. This could include the mechanism's dynamic behavior (including joint friction, vibration characteristics, transmission efficiency etc.), life-cycle issues (e.g. wear and fatigue resistance, serviceability and maintainability) and aesthetics. In this paper, we focus on kinematic performance. In other words, we assume that we are always able to describe the desired task in terms of an output equation of the form:

$$\mathbf{p} = f(\mathbf{l}, \mathbf{\Theta}) \tag{1}$$

where $\mathbf{p}$ denotes an $(m \times 1)$ vector of output end-effector locations, coupler-point positions or crank angles, $\mathbf{\Theta}$ is a $(k \times 1)$ vector of known driving inputs, and $\mathbf{l}$ is a $(n \times 1)$ vector of independent mechanism variables - including deterministic or randomly distributed geometric parameters and/or dimensions. The function $f(\cdot)$ is called the *kinematic function* of the mechanism and is, in general, assumed to be a continuous and differentiable (i.e. smooth) non-linear mapping from the mechanism parameter space to an output space (e.g. a Cartesian workspace). In this treatment, we are only interested in errors that are introduced into the mechanism during fabrication. Subsequently, we assume that the driving inputs are held perfectly to their nominal values, and write a simplified output equation, exclusively in terms of the (variable) mechanism parameters, as follows:

$$\mathbf{p} = f(\mathbf{l}) \tag{2}$$

Conventional error analysis takes component variability (or dimensional/parametric tolerances) as given, and estimates the resulting output deviations using worst-case or stochastic analysis methods. Each assembled component is assumed to vary in an independent and uncorrelated manner. This component-centric approach makes sense for conventional manufacturing, as the process is largely viewed as one involving sequential fabrication and assembly of (slightly faulted) components. However, this approach is unsuitable for *in-situ* fabrication because it is blind to conventional component boundaries. Consequently, the input to the system is not the dimensional variability in links, but the position and orientation variability in fabricated or embedded joints. The mechanism parameters, subsequently, cannot be assumed to vary independently, but do so in a correlated manner.

This paper reviews conventional error analysis techniques, and builds on the prior work to formulate a general approach for the estimation of output performance errors in the *in-situ* fabrication of mechanisms. Error due to clearances in joints is briefly discussed, but is not considered in the analysis. This work also serves as a first step to the DfM issue of *optimal pose selection* (i.e. finding the best configuration in which to build a mechanism, given certain performance requirements) (Rajagopalan and Cutkosky, 1999). The issues of clearances and optimal pose selection are part of ongoing research at the Center for Design Research at Stanford University.

CONVENTIONAL ERROR ANALYSIS

Conventional error analysis deals with degradation in the performance of a mechanism as a result of parametric or dimensional variations. The parameters considered are typically link lengths for planar linkages, or Denavit-Hartenberg (Denavit and Hartenberg, 1955) type parameters for spatial linkages. The analysis proceeds by taking the Taylor Series expansion of the output function (Equation 1) about the nominal configuration ($\mathbf{l}_{nom}$) of the mechanism. Assuming that the variability in the parameters is much smaller than the actual link dimensions, only the first order terms in the expansion need to be retained:

$$\mathbf{p} \approx f(\mathbf{l}_{nom}) + \frac{\partial f}{\partial \mathbf{l}}]_{nom}(\mathbf{l} - \mathbf{l}_{nom}) \tag{3}$$

or

$$\Delta_{\mathbf{p}} \approx \frac{\partial f}{\partial \mathbf{l}}]_{nom}\Delta_{\mathbf{l}} \tag{4}$$

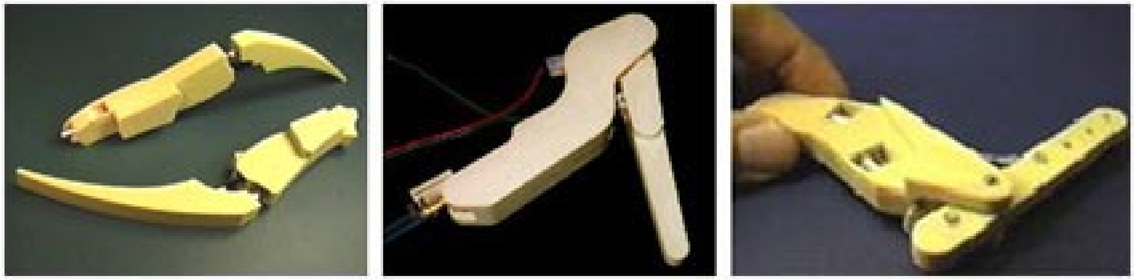

Figure 1. SOME MECHANISMS BUILT IN-SITU, courtesy: Jorge Cham and Mike Binnard, Center for Design Research, Stanford

Letting

$$J_{nom} = \begin{bmatrix} \frac{\partial p_1}{\partial l_1} & \cdots & \frac{\partial p_1}{\partial l_n} \\ \vdots & & \vdots \\ \frac{\partial p_m}{\partial l_1} & \cdots & \frac{\partial p_m}{\partial l_n} \end{bmatrix}_{nom} \quad (5)$$

we get:

$$\Delta_{\mathbf{p}} \approx J_{nom}\Delta_{\mathbf{l}} \quad (6)$$

The quantitiy J_{nom} is known as the *Sensitivity Jacobian* of the mechanism, evaluated at the nominal configuration. This Jacobian relates the first-order component variability ($\Delta_{\mathbf{l}}$) in the mechanism parameter space to the output variation ($\Delta_{\mathbf{p}}$) in Cartesian space. This is classical sensitivity analysis, where all variational effects are bundled into a simple parametric space, and all higher order effects are neglected.

Equation 6 is used as the basis for error analysis and for tolerance allocation. For error analysis, the component variability and sensitivity Jacobian are known for a given mechanism configuration. The output error is then a simple calculation. The variability in the link lengths can be expressed as worst-case deterministic tolerances or as stochastic variation in link parameters (which are assumed to be independent random variables). Both these approaches are discussed further in the following sections.

Worst-Case, Deterministic Error Estimation

In worst-case error estimation, each parameter l_i is assumed to take (exclusively) one of two deterministic values l_i^{min} and l_i^{max}. Furthermore, it is assumed that:

$$l_i^{min} \leq l_i^{nom} \leq l_i^{max}; i = 1, 2, \ldots, n \quad (7)$$

where l_i^{nom} is the nominal value of the i^{th} parameter.

The objective of this kind of error estimation is to determine the worst case envelope of the mechanism performance error. Except for applications where performance within specified limits is absolutely critical, the worst-case analysis results in conservative estimates of error (and thereby, over-design of components). The probability of the worst case combination actually occurring during fabrication is remote. Since the worst performance can occur for any combination of minimum and maximum component parameter values, the technique proceeds by exhaustive calculation of total error for each combination of individual error values. For n parameters, this leads to a search space of 2^n combinations for each mechanism configuration. If the objective is to find the worst-case performance within the entire workspace of the mechanism, then this calculation has to be repeated at each incremental driver position.

An alternative approach is to use dynamic programming techniques (Fenton *et al*, 1989) to estimate the maximum error without computing the total error for every possible combination. While this technique results in significant reduction of the computational burden involved, it is not guaranteed to find the global maximum when the underlying monotonicity assumptions do not hold.

Stochastic Error Estimation

Stochastic error estimation proceeds by assigning a probability distribution function (PDF) to each variable parameter l_i. The component dimension or parameter under consideration is assumed to be a random variable, distributed according to the characteristics of the underlying PDF (denoted as $\phi_{L_i}(l_i)$). The distribution of the output function can then be estimated using standard techniques for stochastic modeling. If certain simplifying assumptions can be made (weakly non-linear function, independent and uncorrelated random variables, identical distributions etc.), an estimate of the distribution and moments of the output function can be made directly from the first and second moments of the random variables. For small variations in the parameters l_i, the output can be approximated as a linear function of the parameters, as follows:

$$\mathbf{p} \approx a + \sum_{i=1}^{n} (\frac{\partial f}{\partial l_i})_\mu (l_i - \mu_{l_i}) \tag{8}$$

where $a \equiv f(\mu_{l_i}; i = 1, 2, \ldots, n)$, and the partials are evaluated at the mean value of the parameters. If the parameters are assumed to vary independently then the output varies approximately Normally with the mean and variance of the distribution given as follows:

$$\begin{aligned} \mu_\mathbf{p} &= a \\ \sigma_\mathbf{p}^2 &= \sum_{i=1}^{n} (\frac{\partial f}{\partial l_i})_\mu^2 \sigma_{l_i}^2 \end{aligned} \tag{9}$$

Here μ_{l_i} and $\sigma_{l_i}^2$ denote the mean and variance, respectively, of the i^{th} random parameter, and $\mu_\mathbf{p}$ and $\sigma_\mathbf{p}^2$ denote the mean and variance of the output function. In the event that the assumption of weak non-linearity of the output function does not hold, then a second order estimate of the mean a variance may yield better results. This is given as (Chakraborty, 1975):

$$\begin{aligned} \mu_\mathbf{p} &= a + \frac{1}{2} \sum_{i=1}^{n} (\frac{\partial^2 f}{\partial l_i^2})_\mu \sigma_{l_i}^2 \\ \sigma_\mathbf{p}^2 &= \sum_{i=1}^{n} (\frac{\partial f}{\partial l_i})_\mu^2 \sigma_{l_i}^2 + \frac{1}{2} \sum_{i=1}^{n} (\frac{\partial^2 f}{\partial l_i^2})_\mu^2 \sigma_{l_i}^2 \\ &\quad + \sum_{i \neq j} (\frac{\partial^2 f}{\partial l_i \partial l_j})_\mu^2 \sigma_{l_i}^2 \sigma_{l_j}^2 \end{aligned} \tag{10}$$

Treatment of Clearances

Consideration of the effects of clearance (or gaps) in the analysis of mechanism error is a natural extension to simple error models which only consider dimensional or parametric variability. The complexity in dealing with clearance comes from the fact that it enables adjacent bodies to break contact with each other - thereby leading to unconstrained relative motion, within some geometric limits (sometimes called *multiple contact* - as opposed to *fixed contact* - kinematics). Since several mechanism analysis models use constraint equations (e.g. loop-closure equations) that are predicated upon persistent point/surface contact and non-interpenetration assumptions, they unravel in the face of joint clearances.

Various techniques have been proposed by researchers to deal with error analysis in the presence of joint clearances. Those proposed in recent years include:

- Equivalent linkage model (Dhande and Chakraborty, 1973)
- Effective length model (Lee and Gilmore, 1991)
- Constrained degree-of-freedom model (Tischler and Samuel, 1999)
- Enumerated contact mode model (Wang and Roth, 1989)
- Configuration space model (Joskowicz *et al*, 1997)

Typically, the simpler clearance models (e.g. the effective length model) are inaccurate, but more generally applicable. The complex models improve prediction accuracy, but may be dependent upon specific joint geometry assumptions which do not hold generically. Clearance models for *in-situ* fabrication differ from conventional methods in a couple of ways. First, better control on actual clearance is possible due to the fact that the clearance geometry is directly manifest in the geometry of the support structure during freeform fabrication (Rajagopalan and Cutkosky, 1998). Second, for embedded joints, clearance and link-length variability cannot be combined into a single factor - as clearance is independent of build/operating configuration, and link-length variability is not. We do not delve further into clearance modeling in this paper, but it remains part of ongoing research.

ERROR ANALYSIS FOR IN-SITU MECHANISMS

The fundamental difference between conventional error analysis, and error analysis for *in-situ* fabrication lies in the form of the inputs into the model. We model *in-situ* fabrication as a process that proceeds by "inserting" joints into a workspace, and forming the links around these embedded joints. Thus, the input to the model is the *variability of the position and/or orientation of the embedded joints, as a function of their location in the build-workspace.* Conventional error-analysis treats parametric variability (i.e. variability in link-lengths etc.) as a given constant input. *In-situ* error analysis estimates parametric variability for each build configuration from the location variability of the joints that make up the linkage. The parametric variability is determined by the sensitivity of each parameter to the joint positions and orientations at a given build pose. The output variability, in turn, is determined by the sensitivity of the output function to the mechanism parameters at each

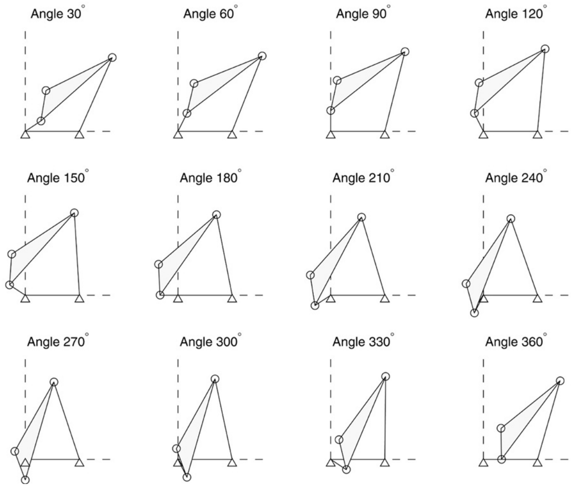

Figure 3. MULTIPLE POSITIONS OF THE EXAMPLE 4-BAR, 30 deg increments of θ

operating configuration.

Error analysis involves estimating the variability in the link parameters l_i, and then applying sensitivity analysis techniques to determine the error in the output function (at various operating configurations) for a mechanism that is fabricated *in-situ*. In the following sections, this process is illustrated for the specific planar 4-bar mechanism shown in Figure 2. To aid with discussion of the results, the mechanism is also shown in various configurations (i.e. specific values of the driving angle θ) in Figure 3.

Worst-Case, Deterministic Error Estimation

Given the mechanism nominal build configuration (or *pose*), the extremal values of the position of each joint are known. From the worst-case joint positions, it is possible to compute all the parameters l_i of the mechanism (where the joints are assumed to lie exclusively at one of their extremal positions). For each combination of worst-case parameters, the error in the output function ($\mathbf{p}$) is evaluated. This calculation is repeated for all operating angles, for each candidate build pose.

Figure 4 illustrates the results of the worst-case error estimation for the example 4-bar mechanism for a two candidate build poses ($\theta = 90$ deg and $\theta = 270$ deg). The coupler-point location is shown as a cloud of points in the vicinity of the nominal coupler-point. There are 1024 points in each cloud, with each point corresponding to one combination of worst-case joint locations. Figure 5 plots the worst-case variability (i.e one-half the perimeter of the bounding-box for each cloud in Figure 4) of the coupler-point location as a function of the build configuration.

Stochastic Error Estimation

We assume that the joint coordinates (positions and orientations) are independent random variables with known distributions. Mechanism parameters (like link-lengths, joint angles, joint offsets and skew angles) are functions of the random joint coordinates.

The stochastic properties of a function of random variables can, in principle, be derived exactly from the given distributions of the random variables. However, in practice, the exact derivation is intractable in the absence of certain simplifying assumptions. For a weakly non-linear function of independent and uncorrelated random variables, the mean and variance of the function can be approximated directly from the mean and variance of the underlying random variables, as illustrated in Equations 9 and 10. When the simplifying assumptions do not hold, the function properties needs to be determined analytically (see Equation 13), by modifying the first-order approximation equation to account for correlation effects (see Equation 14), by experimentation, or by using Monte Carlo simulation techniques (Hammersley and Handscomb, 1964). In general, the analytical technique is not tractable for all but the simplest of cases. In this paper, we develop a modified first-order error estimation technique, and compare it with the results obtained by Monte Carlo simulation. Monte Carlo simulation is the customary validation technique in this type of stochastic analysis of error (Dhande and Chakraborty, 1973) (Liang and Xian, 1989) (Lee and Gilmore, 1991) (Choi *et al*, 1998), since experimental validation requires the reproduction of several thousand identical prototypes under controlled conditions to obtain statistically significant results.

Equation 9 can be applied directly to the mechanism parameters (l_i), given the stochastic properties (i.e. mean and variance) of the joint variables (x_k). The parameters are simple functions (i.e. sums, products and differences) of the joint variables, which are assumed independent and uncorrelated. Moreover, the variance in any joint variable can be assumed to be much smaller than its mean (for macro-scale devices), since the precision of fabrication equipment is typically several orders-of-magnitude smaller than the part dimensions. This implies that the variability in the mechanism parameters can be approximated as a linear function (weighted by the sensitivity coefficients) of the variability

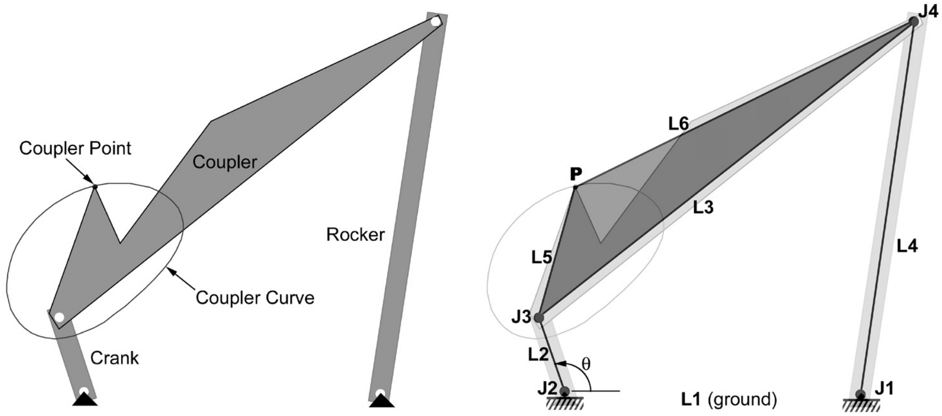

Figure 2. ACTUAL AND SCHEMATIC DIAGRAMS OF THE PLANAR 4-BAR CRANK-ROCKER MECHANISM USED AS AN EXAMPLE IN THIS PAPER ($L_1 = 15cm, L_2 = 5cm, L_3 = 25cm, L_4 = 20cm, L_5 = 7.5cm, L_6 = 20cm$)

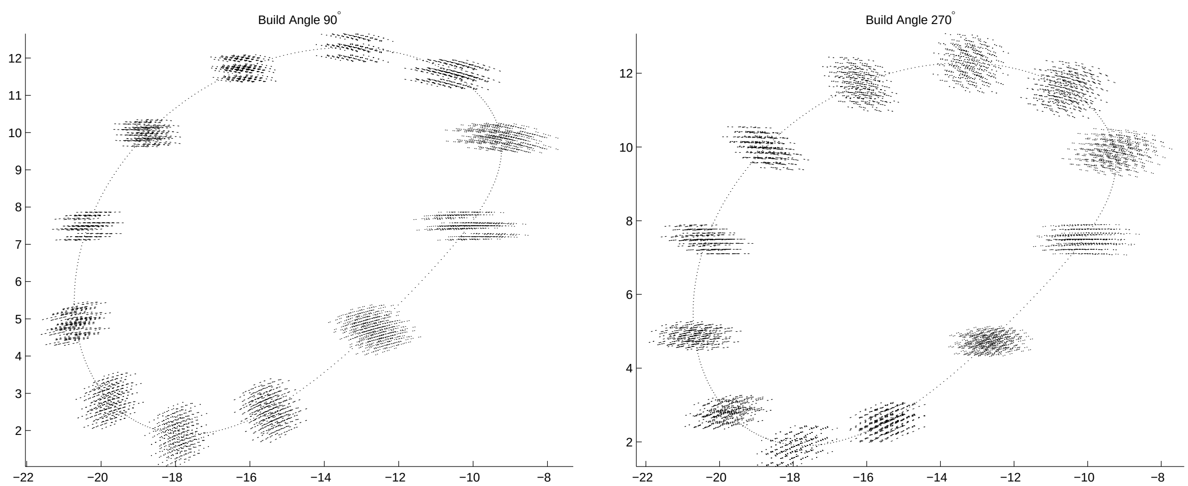

Figure 4. WORST CASE COUPLER-POINT POSITIONAL ERROR ON THE COUPLER PATH

in the input, as follows:

$$\sigma_{l_i}^2 \approx \sum_k (\frac{\partial l_i}{\partial x_k})_\mu^2 \sigma_{x_k}^2; \;\; i = 1, 2, \ldots n \tag{11}$$

where $\sigma_{l_i}^2$ is the variance of the i^{th} mechanism parameter, and x_k represents the k^{th} joint variable, and $\sigma_{x_k}^2$ represents the variance of the k^{th} joint variable. If the joint variables follow Normal distributions (typical for most physical random processes involving many noise factors), then the parameters too will follow a Normal distribution.

The parameters l_i, however, are correlated random variables. The correlation coefficients (ρ_{ij}) of each parameter pair (l_i, l_j) can be approximated using the sensitivity coefficients as follows:

$$\rho_{ij} \approx \frac{\sum_k (\frac{\partial l_i}{\partial x_k})_\mu (\frac{\partial l_j}{\partial x_k})_\mu \sigma_{x_k}^2}{\sigma_{l_i} \sigma_{l_j}} \tag{12}$$

Figure 6 compares the first order estimate of link-length variability against that obtained by Monte Carlo simulation, for four of the links in the example 4-bar in Figure 2, with an equal joint-position variance of 0.01 cm in the X and Y directions. Figure 7 compares the pairwise correlation coefficients obtained for the approximation in Equation 12 against that obtained by Monte Carlo simulation, for the same four links of the example 4-bar. In both cases, the approximation yields results that are very close to the simulation - illustrating the validity of the assumption of independence.

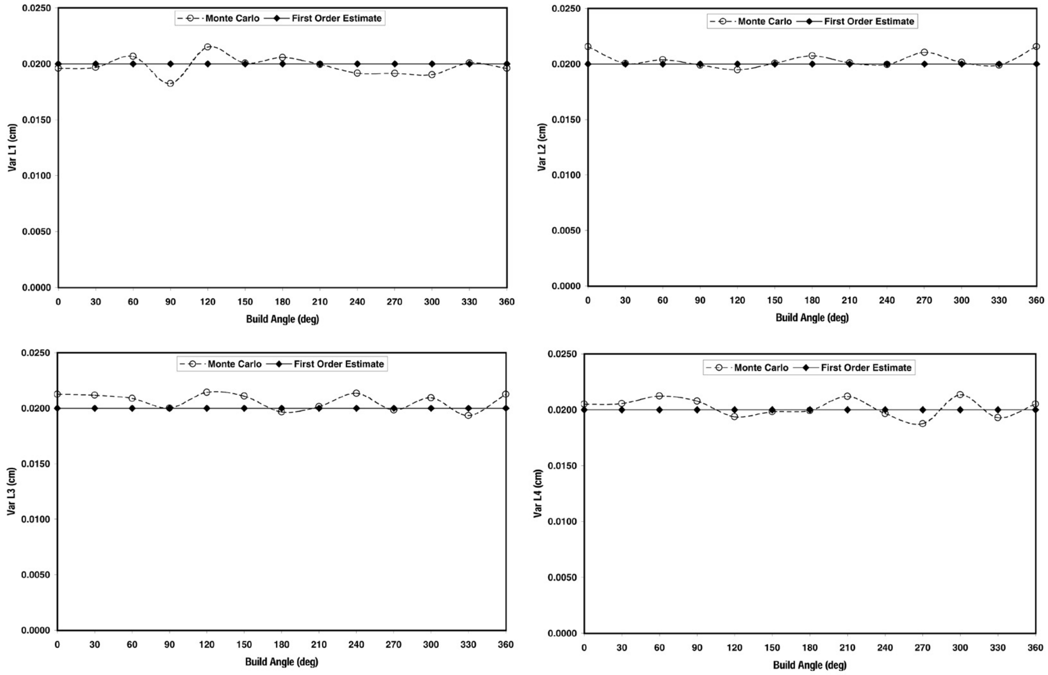

Figure 6. FIRST ORDER ESTIMATES OF THE LINK LENGTH VARIANCE COMPARED TO THE RESULTS OF A MONTE CARLO SIMULATION

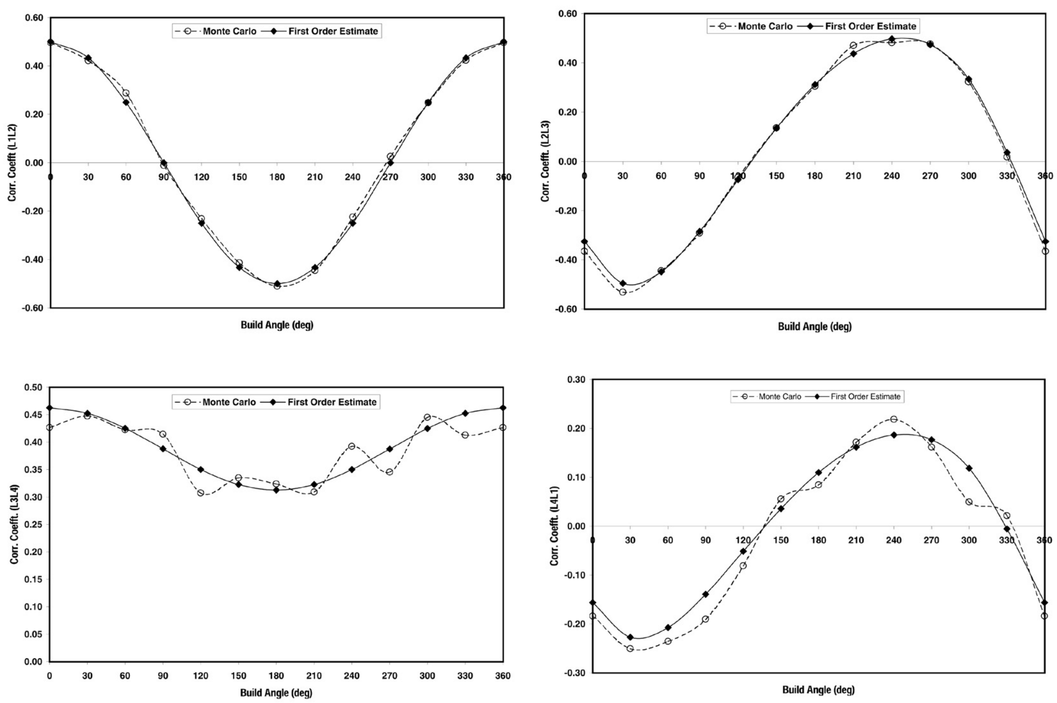

Figure 7. PAIRWISE CORRELATION COEFFICIENTS OF THE LINK LENGTHS - FIRST-ORDER RESULTS COMPARED TO THE MONTE CARLO SIMULATION

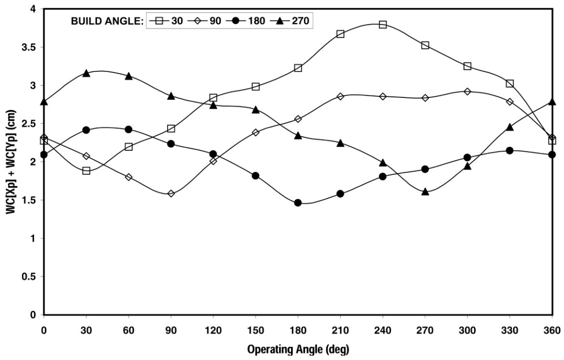

Figure 5. WORST CASE COUPLER-POINT POSITIONAL ERROR AGAINST OPERATING ANGLE

Our real interest in this treatment, however, is in the behavior of the output function ($\mathbf{p}$) during operation. As indicated earlier, the output is a function of the mechanism parameters which, being dependent functions of the given independent random variables (i.e. the joint variables), are themselves correlated random variables. Thus, the simplifying assumptions which could be made for the estimation of parametric variability are not applicable for the estimation of output variability. No simple analytical technique exists for the determination of the distribution of a general function of correlated random variables. In theory, the cumulative distribution function of the output can be evaluated as follows:

$$F_P(p) = \int_{-\infty}^{l_1} \cdots \int_{-\infty}^{l_n} f(l_1, \ldots l_n).\phi_{L_1 \ldots L_n}(l_1, \ldots l_n)\, dl_1 \ldots dl_n \tag{13}$$

However, the joint distribution function $\phi_{L_1 \ldots L_n}(l_1, \ldots l_n)$ is not easy to determine when the random variables l_i are correlated. Furthermore, the upper limits of the multiple integral need to be expressed in terms of the output variables, which is not analytically feasible except for the simplest of cases.

The assumption that makes this problem tractable, once again, is that of weak-nonlinearity in the output function. In other words, if we can assume that the second and higher-order terms in the Taylor Series expansion of the output function can be discarded, then it is possible to derive an expression that directly produces an approximate estimate for the output variance, given the variance ($\sigma_{l_i}^2$) and correlation coefficients (ρ_{ij}) of the mechanism parameters. Furthermore, if the total number of parameters are large (i.e. $n > 5$), then, according to the Central Limit Theorem, the output function will follow an approximately Normal distribution, regardless of the individual parameter distributions (Feller, 1957). Thus, by making the linear approximation, we completely side-step the evaluation of the extremely problematic multiple integral in Equation 13. The output error estimate in the presence of correlated inputs is given as:

$$\sigma_{\mathbf{p}}^2 \approx \sum_{i=1}^{n} (\frac{\partial f}{\partial l_i})_{\mu}^2 \sigma_{l_i}^2 + 2 \sum_i \sum_j \frac{\partial f}{\partial l_i}]_{\mu} \frac{\partial f}{\partial l_j}]_{\mu} \rho_{ij} \sigma_{l_i} \sigma_{l_j} \tag{14}$$

where $i = 1, 2, \ldots n$ and $j \neq i$. In the special case where only *adjacent* parameters share a joint variable, $\rho_{ij} = 0$ for non-adjacent parameters, and the above equation needs to be evaluated only for the cases where $j = i - 1$. Note that all the sensitivity coefficients in the above equation are evaluated at the nominal operating configuration (μ) of the mechanism. Comparison of Equation 14 and Equation 9 reveals that they differ only in the second term on the RHS. This term, then, is the adjustment term that accounts for the correlation effect that results from the co-dependence of the mechanism parameters on the same joint coordinates. Reduction of manufacturing errors can be achieved by either reducing the joint-location variance $\sigma_{x_k}^2$ (and subsequently, parametric variances $\sigma_{l_i}^2$), or by choosing a build configuration such that the subtractive effect of the second RHS term in Equation 14 is maximized. We refer to this problem as that of "optimal pose selection," and the solution is part of ongoing work (Rajagopalan, 2000).

Summarizing, the first order approximations are the only tractable, general purpose estimates of the output function variability. Equation 14 indicates that the output error depends upon the output function sensitivity coefficients (evaluated at the nominal operating configuration), the parametric variances, and the pairwise correlation coefficients of the parameters. The parametric variances and the correlation coefficients are functions of the mechanism build pose, during *in-situ* fabrication.

Figure 8 compares the first order estimated coupler-point error for the example 4-bar fabricated *in-situ* against the Monte Carlo simulations of the same quantity. Also included are the estimates using the conventional approach, which does not include the consideration of correlation effects. Comparisons can also be made between these results, and those of the worst case error estimate presented earlier (see Figure 5). The worst-case and stochastic estimates for a specific build angle are compared in Figure 9. It is clear from the comparison that the worst-case method is significantly more conservative in its estimation of output error.

Figure 10 plots the simulated coupler-point variance against the number of random trials. This helps with the

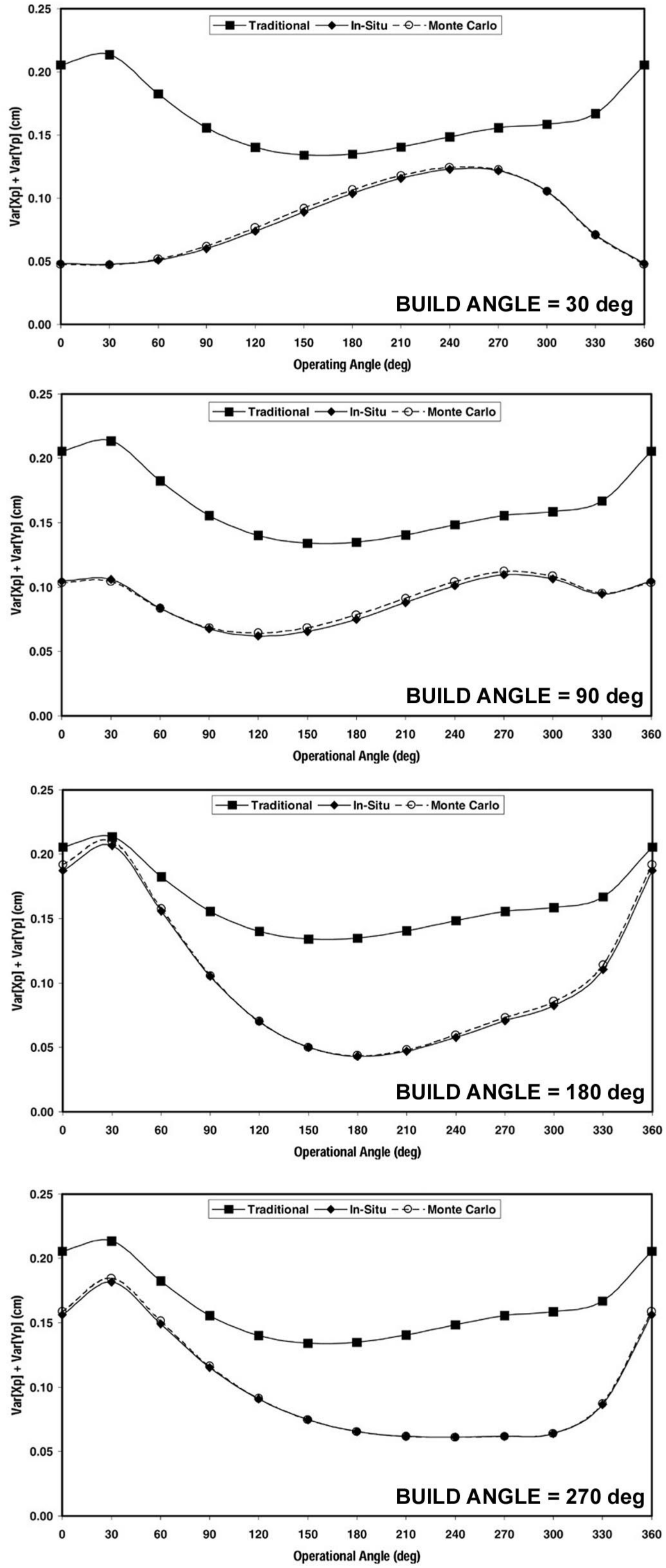

Figure 8. FIRST-ORDER ESTIMATES OF COUPLER-POINT VARIANCE, CONVENTIONAL AND *IN-SITU*, COMPARED TO SIMULATED RESULTS

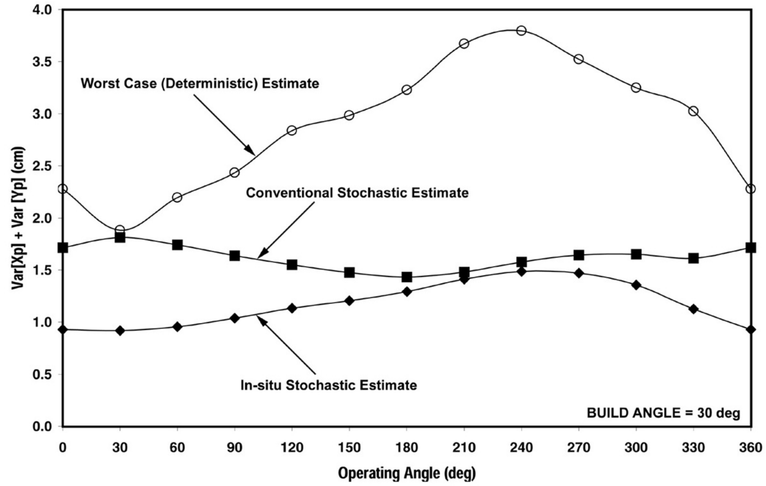

Figure 9. COMPARISON OF 3σ STOCHASTIC AND WORST-CASE ERROR ESTIMATES

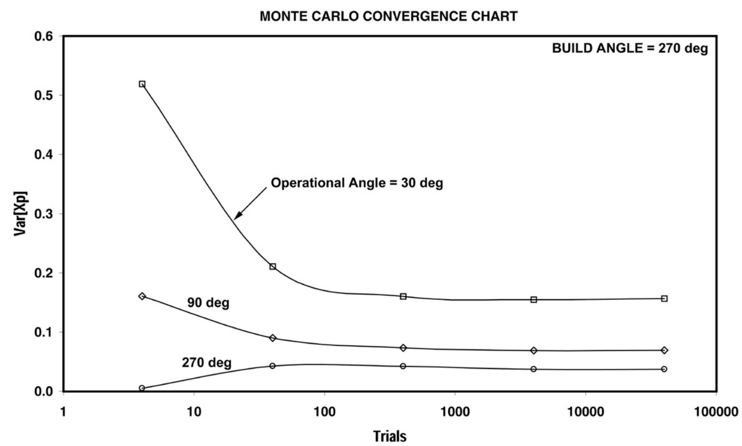

Figure 10. CONVERGENCE RATE FOR THE MONTE-CARLO SIMULATION

estimation of the minimum number of trials needed in order for the random estimates to converge to a steady value (between 4000 and 10,000 in this case).

CONCLUSIONS AND FUTURE WORK

In this paper, we have demonstrated a general technique for the estimation of output performance error in mechanisms that are fabricated *in-situ*. Unlike conventional fabrication and error estimation techniques, the configuration (or pose) in which a mechanism is built has a large impact on the accuracy of the mechanism. Furthermore, due to the correlations between the random mechanism parameters for *in-situ* mechanisms, conventional first-order stochastic error estimation techniques do not hold. A modified estimation equation is presented, and the results compared to Monte Carlo simulations - with good agreement.

Ongoing and future work in this area includes illustrating this approach for general spatial mechanisms, analysis of clearance effects, and using this error estimation technique as a basis for the optimal pose selection problem. Several of these issues have been dealt with as of the submission of this paper, and interested readers are referred to (Rajagopalan, 2000).

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