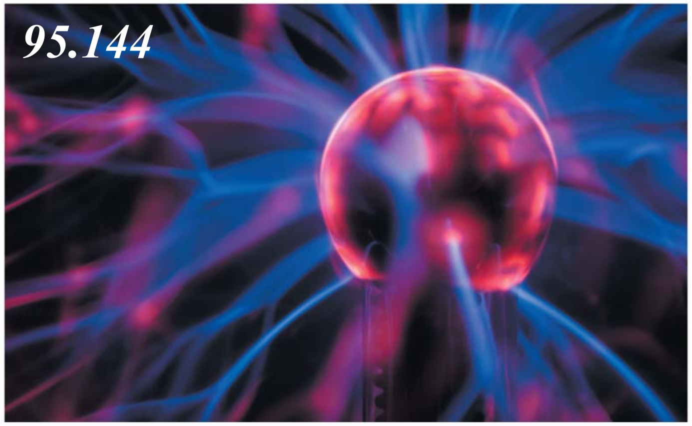
Lecture 2

Physics II

Chapter 26

The Electric Field



Course website:
http://faculty.uml.edu/Andriy_Danylov/Teaching/PhysicsII

Lecture Capture:
http://echo360.uml.edu/danylov201516/physicsII.html

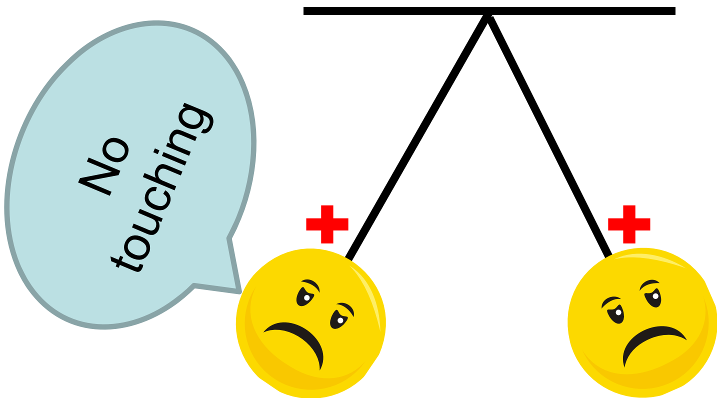
Channel 61 (clicker)

Idea of an Electric Field

Many forces are "contact forces" (tension, friction, normal, …)

But, the gravitational and electrical (magnetic) forces are "long-range forces" (no contact is required).

So, the idea of the field is very useful in these cases.



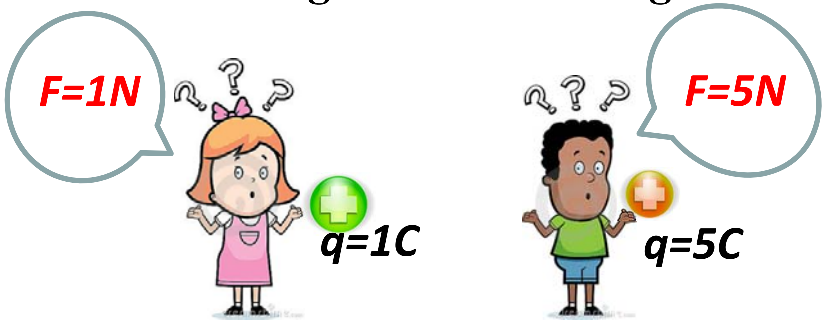
How to define an electric field?

The Electric Field (definition)

How to define an electric field of a charge

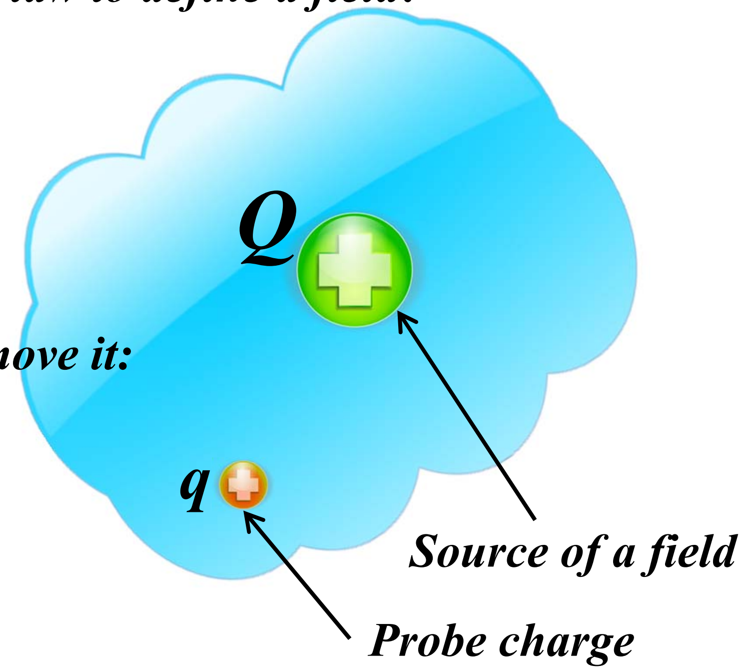
Using Coulomb's law? $F = k\frac{Qq}{r^2}$

What is "not good" about using Coulomb's law to define a field?



The force depends on your probe charge q. Let's remove it:

$$\vec{E} = \frac{\vec{F}_{on\ q}}{q}$$



1) A probe must be positive (convention)

2) A probe must be small so it wouldn't disturb the system created the field

The units of the electric field are N/C.

The magnitude E of the electric field is called the electric field strength.

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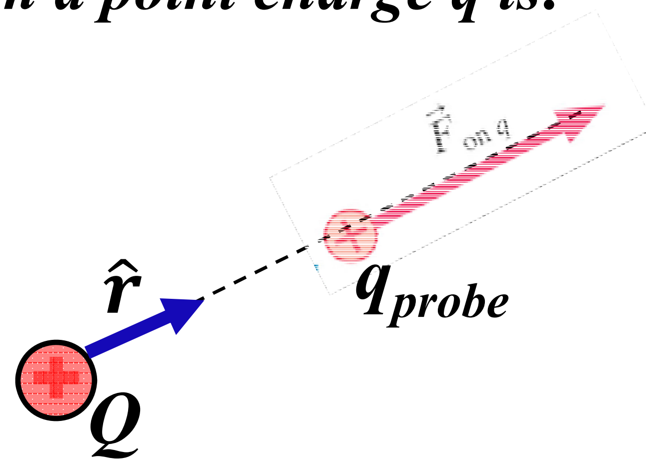
The Electric Field of a Point Charge

Let's find an electric field at a distance r from a point charge q is:

$$\vec{E} = \frac{\vec{F}}{q_{probe}} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{Q q_{probe}}{r^2}\hat{r}}{q_{probe}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

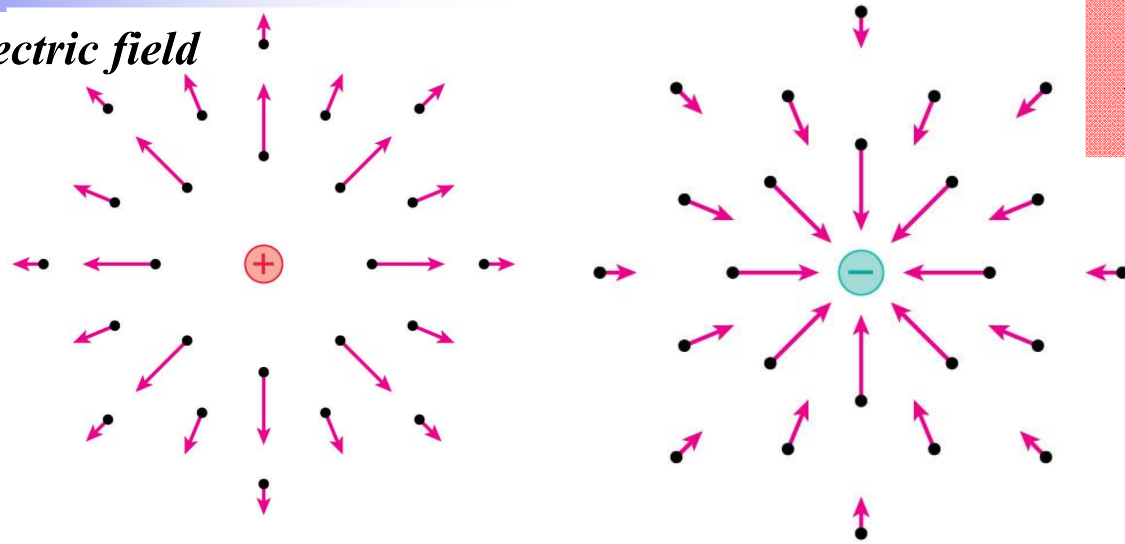


$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

The electric field at a distance r from a point charge Q
($\hat{r}$ - unit vector).

Electric field lines (one charge)

Mapping of an electric field



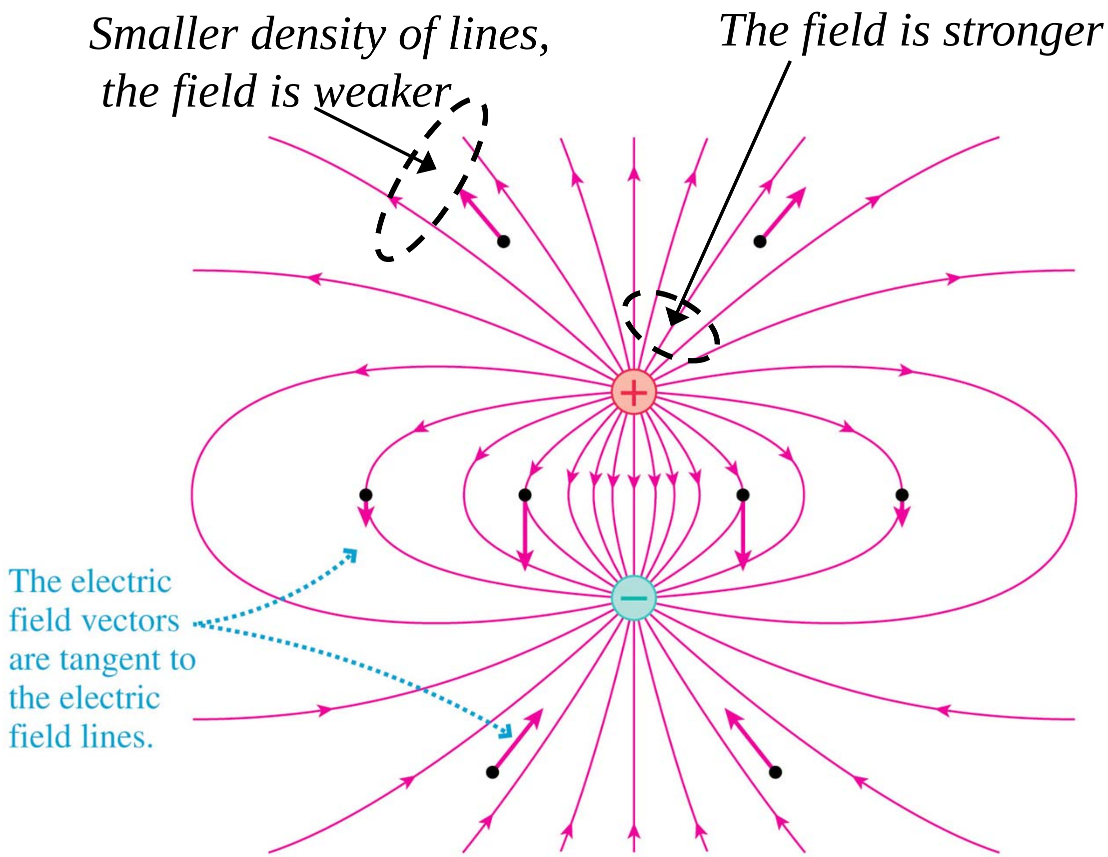
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

It is very useful to picture an electric field using electric field lines.
This concept was introduced by M. Faraday as an aid in visualizing electric fields.

The lines emanate from a positive charge and terminate on a negative one.

Electric field lines (two charges/dipole)

Field lines of two equal but opposite charges



1) ***The electric field vector is tangent to the field line at that point***
2) ***The lines emanate from a positive charge and terminate on a negative one.***

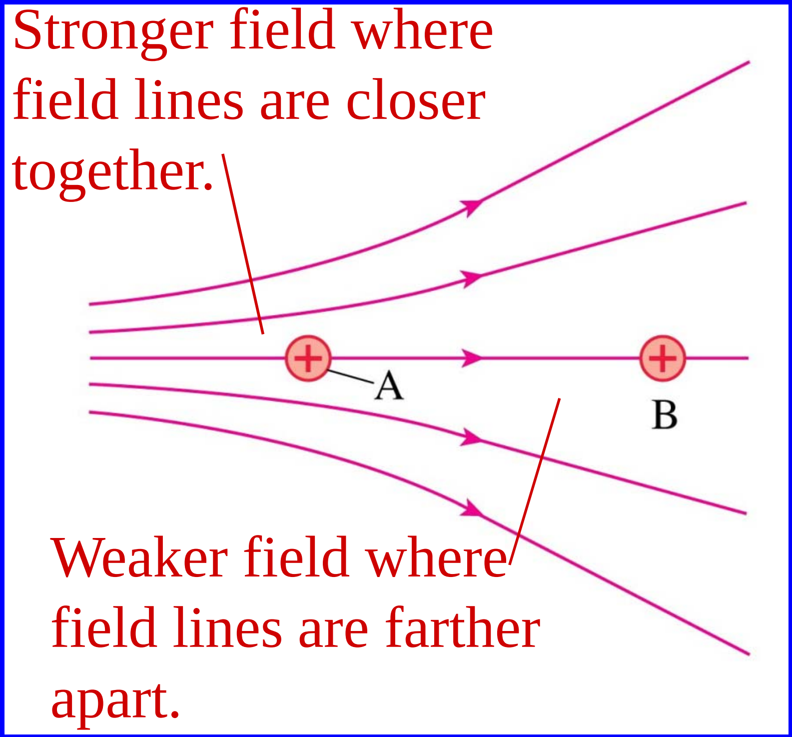
ConcepTest Electric Field Lines

Two protons, A and B, are in an electric field. Which proton has the larger acceleration?

A) Proton A.

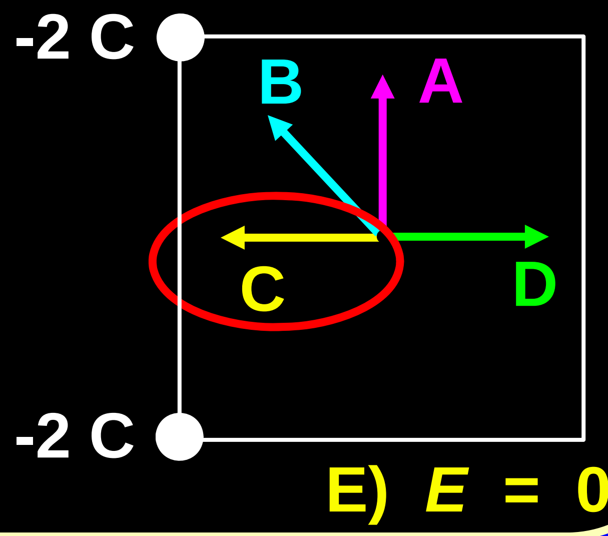
B) Proton B.

C) Both have the same acceleration.

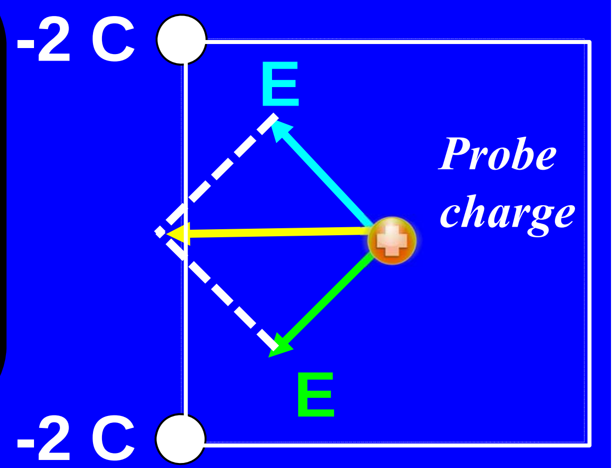


Channel 61 ***ConcepTest*** **Electric Field**

What is the electric field at the center of the square?



For the upper charge, the *E* field vector at the center of the square points toward that charge. For the lower charge, the same thing is true. Then the vector sum of these two *E* field vectors points to the left.



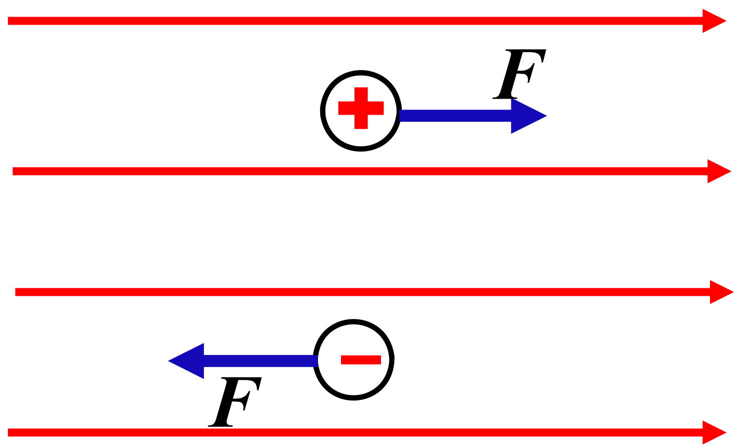
Follow-up: What if the lower charge were +2 C?
What if both charges were +2 C?

Once *E* is known, it is easy to get *F*

A charged particle with charge q at a point in space where the electric field is E experiences an electric force:

$$\vec{E} = \frac{\vec{F}_{on\ q}}{q} \quad \Rightarrow \quad \vec{F}_{\text{on } q} = q\vec{E}$$

E *(Uniform Electric Field)*



If q is positive, the force on the particle is in the direction of E.

The force on a negative charge is opposite the direction of E.

Channel 61

ConcepTest Uniform Electric Field

In a uniform electric field in empty space, a 4 C charge is placed and it feels an electric force of 12 N. If this charge is removed and a 6 C charge is placed at that point instead, what force will it feel?

A) 12 N

B) 8 N

C) 24 N

D) no force

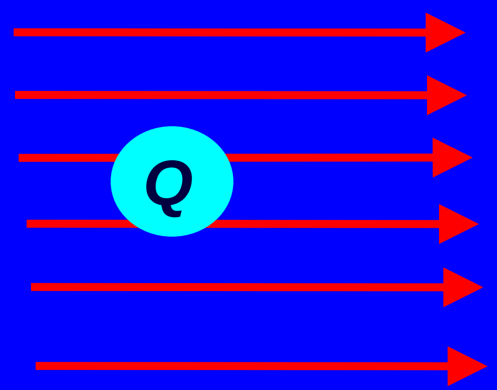
E) 18 N

Since the 4 C charge feels a force, there must be an electric field present, with magnitude:

$$E = F / q = 12\text{ N} / 4\text{ C} = 3\text{ N/C}$$

Once the 4 C charge is replaced with a 6 C charge, this new charge will feel a force of:

$$F = q E = (6\text{ C})(3\text{ N/C}) = 18\text{ N}$$



Follow-up: What if the charge is placed at a *different position* in the field?

Channel 61 **ConcepTest** **Electric Field Lines**

What are the signs of the charges whose electric fields are shown at right?

A) + −

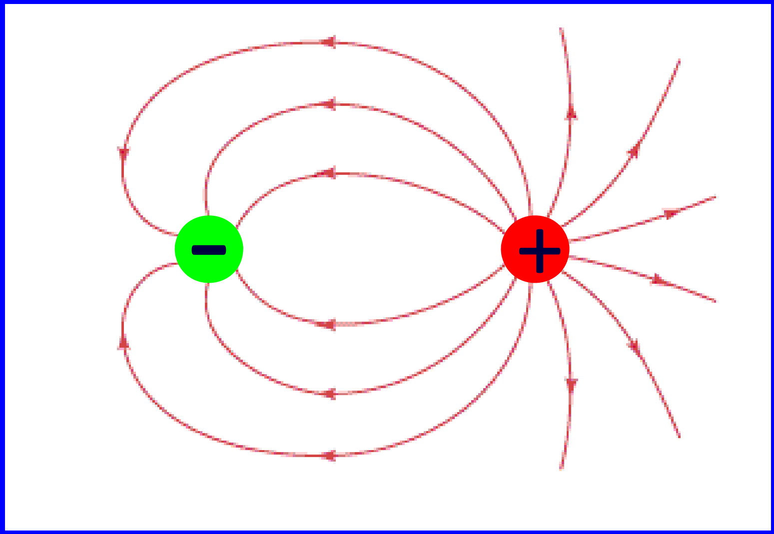
B) − +

C) − −

D) + +

E) no way to tell

Electric field lines originate on positive charges and terminate on negative charges.



We know how to treat point charges

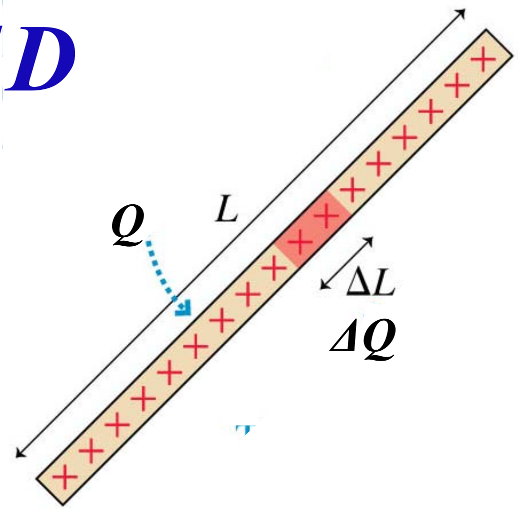
Now we need to learn how to deal with charges

continuously distributed

Charge density of a uniformly charged objects

Charge Q on a rod of length L

1D



The linear charge density is:

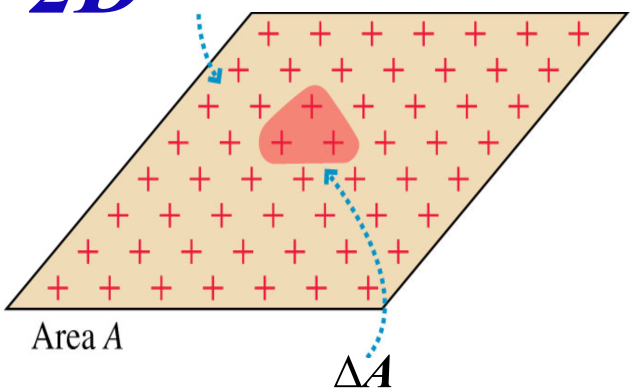
$\lambda = Q/L$

The charge ΔQ in ΔL is:

$\Delta Q = \lambda \Delta L$

Charge Q on a surface of area A

2D



The surface charge density:

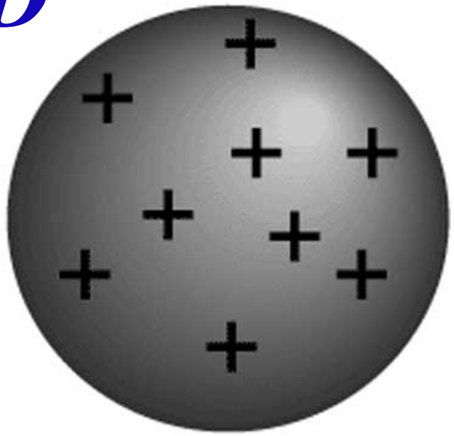
$\eta = Q/A$

The charge ΔQ in ΔA is:

$\Delta Q = \eta \Delta A$

Charge Q in volume V

3D



The volume charge density:

$\rho = Q/V$

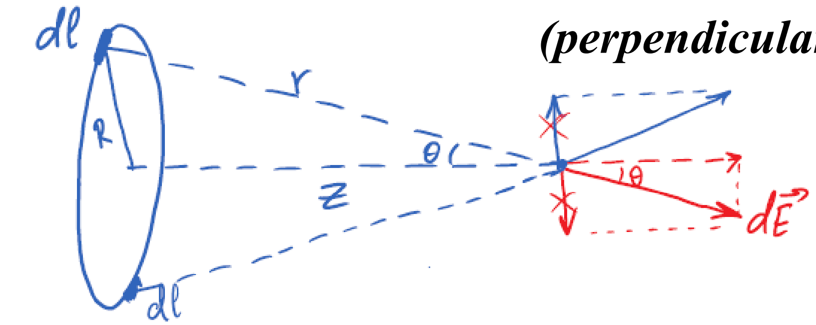
The charge ΔQ in ΔV is:

$\Delta Q = \rho \Delta V$

Electric field of a charged ring (Example 26.4)

Example 26.4

A thin ring of radius R is uniformly charged with total charge Q. Find the electric field at a point on the axis of the ring (perpendicular to the ring)



Take an infinitesimal length dl of the ring. We can treat dQ of dl as a point charge:

$dE = k\frac{dQ}{r^2}$; dQ – amount of charge on dl. r – distance from dl to P

$dQ = \lambda \cdot dl = \left\| \lambda = \frac{Q}{2\pi R} \text{ - Charge per unit length} \right\| = \frac{Q}{2\pi R} \cdot dl$, so

$dE = k\left(\frac{Q\,dl}{2\pi R}\right)\cdot\frac{1}{r^2} = \left\| r^2 = R^2 + z^2 \right\| = \frac{kQ}{2\pi R}\cdot\frac{dl}{R^2+z^2}$

Before integration, let's use symmetry of the problem. Note that there is a similar segment dl (diametric. opposite) which creates a similar dE. Perpendicular components of them cancel each other. Thus, by symmetry $\vec{E}$ at P will have zero y component. So, we need to deal with only x component

$$dE_x = dE\cdot\cos\theta = \left\| \cos\theta = \frac{z}{r} = \frac{z}{\sqrt{R^2+z^2}} \right\| = \frac{dE\cdot z}{\sqrt{R^2+z^2}}$$

$$E = \int dE_x = \int\left(\frac{kQ}{2\pi R}\cdot\frac{dl}{R^2+z^2}\right)\cdot\frac{z}{\sqrt{R^2+z^2}} = \int\left(\frac{kQz}{2\pi R\,(R^2+z^2)^{3/2}}\right)dl =$$

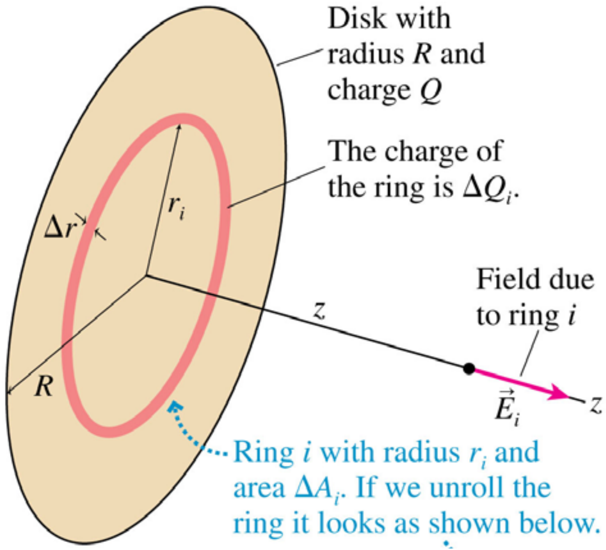
(= const)

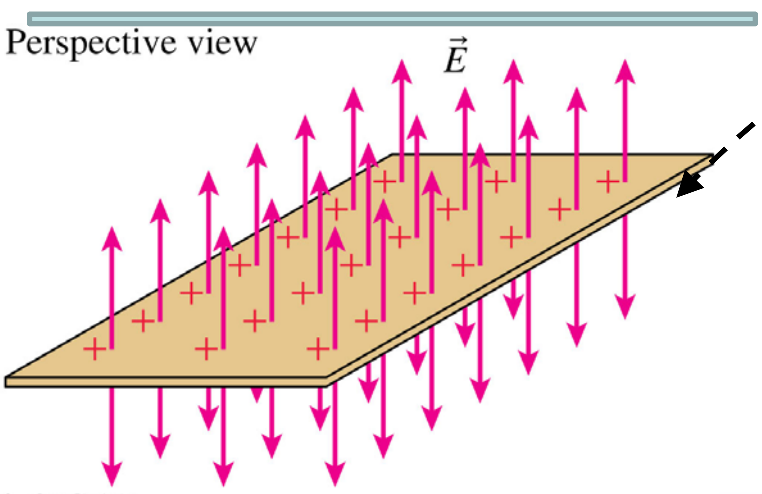
$$= \frac{kQz}{2\pi R\cdot(R^2+z^2)^{3/2}}\int dl \; (= 2\pi R) \; = \boxed{\frac{k\cdot Q\cdot z}{(R^2+z^2)^{3/2}} = E(z)}$$

limits: $z \gg R$ (far away from the ring)

- $E \simeq \frac{kQz}{z^3} = k\frac{Q}{z^2}$ – it looks like a point charge.
- at $z = 0$ (ring center) $\Rightarrow E = 0$

Electric field of a disc of charge





The ring results can be extended to calculate the electric field of a uniformly charged disc.

Without derivation:

$$E = \frac{\eta}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right]$$

In the limit $z \ll R$, it becomes

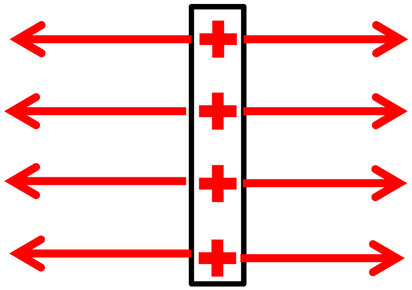
$$E = \frac{\eta}{2\varepsilon_0}$$

El. field created by an infinite charged plate
η - surface charge density

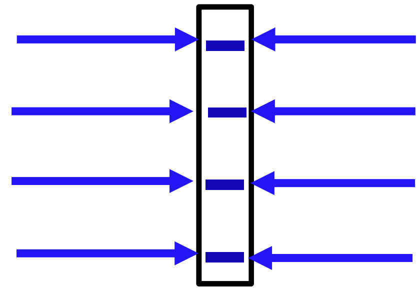
Interesting!!!
The field strength is independent of the distance z

E field of a parallel-plate capacitor

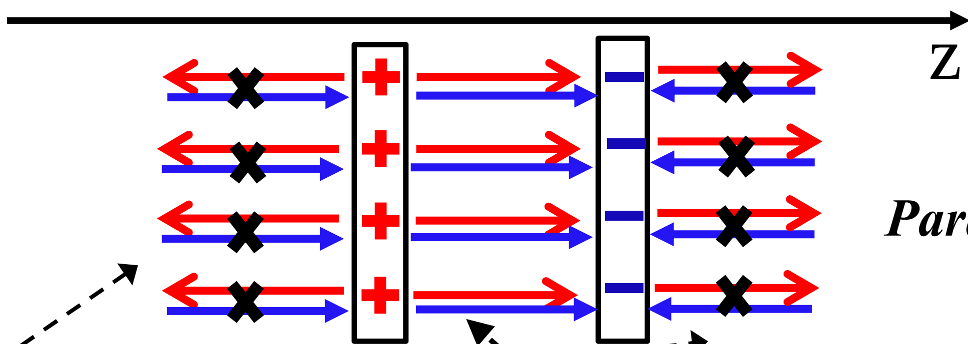
Single plates



$$E = \frac{\eta}{2\varepsilon_0}$$



Let's bring them very close to each other



It is called a Parallel-plate capacitor

Outside the plates $E = \frac{\eta}{2\varepsilon_0} - \frac{\eta}{2\varepsilon_0} = 0$

Inside the plates $E = \frac{\eta}{2\varepsilon_0} + \frac{\eta}{2\varepsilon_0} = \frac{\eta}{\varepsilon_0}$

*The field inside a parallel-plate capacitor is **uniform***

$$E = \frac{\eta}{\varepsilon_0}$$

What you should read

Chapter 26 (Knight)

Sections:

- ***26.1***
- ***26.2 (skip part about "The electric field of a dipole)***
- ***26.3***
- ***26.4 (skip part about "The Disc of charge")***
- ***26.5***

Thank you

See you on Tuesday